The Bell System Technical Journal

April, 1924

High Frequency Amplifiers

By H. T. FRIIS and A. G. JENSEN

IN this paper, a simplified mathematical treatment of the theory of high frequency amplifiers is presented, and the theory is verified by experiment. This method of mathematical analysis provides a ready means of predicting the performance and action of an amplifier from a knowledge of the fundamental constants of its circuit and places the design of high frequency amplifiers on a precise and rational basis. The paper also includes a description of various methods for quantitatively determining the amount of amplification at high frequencies.

In order to make the discussion more easily followed, it is started with the simple close-coupled non-resonant transformer amplifier of the kind used at audio frequencies, and it is pointed out that good efficiency at higher frequencies requires that the transformer be used

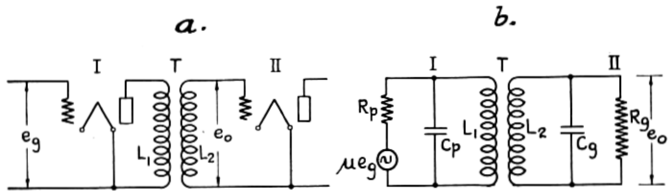

Fig. 1—Schematic of One Stage Transformer Coupled Amplifier

at its natural frequency, i.e., the transformer inductance and distributed capacity must be in resonance. We have therefore, next treated the simplest type of resonance circuit amplifier, namely, a single tuned circuit amplifier, and it is shown that exactly the same method can be used for a choke coil amplifier or a close coupled transformer amplifier. Finally, it is shown that a loosely coupled transformer amplifier can be treated like two coupled tuned circuits.

Considering a low frequency transformer-coupled amplifier, in Fig. 1 (a) there is shown an amplifier tube I with its output transformer T working into another tube II and in Fig. 1 (b) is given the corresponding equivalent circuit. The equivalent circuit is obtained by the theorem, that the plate circuit of a vacuum-tube may be treated as an ordinary a.c. circuit, consisting of the external impedance in series with a resistance R_p, and in which the impressed emf. is μe_g, R_p being the internal plate impedance of the tube, μ the amplification constant of the tube and e_g the voltage applied to the grid.

In Fig. 1 (b) C_p is the plate to filament capacity of tube I, and the input impedance of tube II is represented by a resistance R_g in parallel with a condenser C_g.

The maximum amplification which can be obtained by this amplifier is given by the well-known expression

$$K=\frac{e_o}{e_g}=\tfrac{1}{2}\mu\sqrt{\frac{R_g}{R_p}}, \tag{1}$$

but this maximum amplification can only be obtained when

$$\left.\begin{aligned} &\omega L_1 >> R_p, \\ &\omega L_2 >> R_g, \\ \text{and}\qquad &\frac{\omega L_1}{R_p}=\frac{\omega L_2}{R_g}. \end{aligned}\right\} \tag{2}$$

Large reactances ωL_1 and ωL_2 can only be obtained at low frequencies because at higher frequencies the effects of internal tube capacities and the distributed capacity of the coil become large. This may best be illustrated by means of the table given below:

TABLE I

Coil No.	Inductance L	Natural Tuning Frequency f	Reactance at Half Natural Tuning Frequency $\pi f L$
1	.0025 henries	10^6 cycles	8,000 ohms
2	.25 "	10^5 "	80,000 "
3	25 "	10^4 "	800,000 "

The tube capacity plus the distributed capacity of each of the three coils for which these data are given is assumed to be 10 $\mu\mu f$. Since transformers in order to give a flat band must work below their natural frequency a much higher impedance than given by $\pi f L$ in the Table can therefore not be obtained. It is thus seen that only at audio frequencies is it possible to build a transformer with an impedance which is high compared with the tube resistances, the plate resistance being of the order of 6,000–50,000 ohms for ordinary receiving tubes and the grid resistance R_g being as high as 4×10^6 ohms but often limited to 500,000 ohms by an added resistance.

At higher frequencies sufficiently high impedances can only be obtained by working at the natural frequency of the transformer, and to illustrate this we shall in the following give some results of experiments made with ordinary tuned circuit amplifiers, choke coil amplifiers and loosely coupled transformer amplifiers at high frequencies.

TUNED CIRCUIT AND CHOKE COIL AMPLIFIERS

In Fig. 2 there are shown to the left two different ways of connecting up a tuned circuit amplifier, and to the right are given the corresponding equivalent circuits. The input impedance to the next tube is assumed to be a pure resistance R_g, thus neglecting the grid-

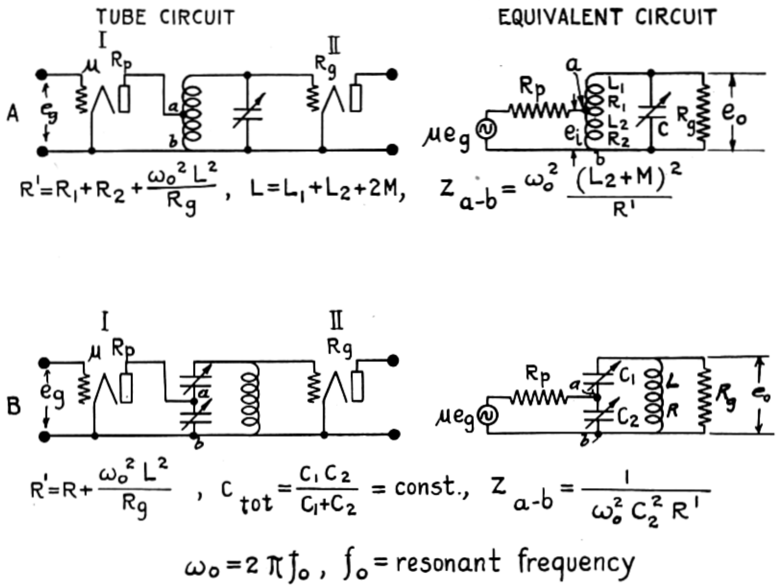

Fig. 2—Schematic of Tuned Amplifier Circuits

filament capacity and the grid-plate capacity of this tube. The effect of the grid-filament capacity, however, will only be to detune the circuit a little and can, therefore, be compensated for by retuning the condenser C (or C_1 and C_2) and the effects of the coupling through the grid-plate capacity of the second tube will be treated specially later.

Fig. 2 gives the well-known formulas for the equivalent series resistance R' of the circuit at resonance and for the impedance of the circuit Z_{a-b} measured between points a and b at resonance.

From this we then get in Case (A)

$$e_i = \mu e_g \frac{Z_{a-b}}{Z_{a-b}+R_p} = \mu e_g \frac{\omega_o^2(L_2+M)^2}{\omega_o^2(L_2+M)^2+R_pR'}, \tag{3}$$

and, assuming that $R_p >> \omega_o (L_2+M)$,

$$e_o = e_i \frac{L}{L_2+M}.$$

Hence, defining the voltage amplification K of the first stage as the voltage impressed upon the grid of tube II divided by the voltage impressed upon the grid of tube I, we have

$$K = \frac{e_o}{e_g} = \mu \frac{\omega_o^2 L(L_2+M)}{\omega_o^2(L_2+M)^2+R_pR'}. \tag{4}$$

In order to find the step-up ratio, which gives maximum amplification we have to solve for L_2+M in the equation $\delta K/\delta(L_2+M)=0$, which gives

$$R_p = \frac{\omega_o^2(L_2+M)^2}{R'} = Z_{a-b}, \tag{5}$$

and by inserting this in equation (4) we get

$$K_{max} = \frac{\mu}{2} \frac{1}{\sqrt{R_p}} \frac{\omega_o L}{\sqrt{R'}}. \tag{6}$$

From equation (5) it is seen that the condition for maximum voltage amplification is exactly the same as the well-known condition for maximum power amplification; namely, that the external impedance Z_{a-b} inserted in the plate circuit must be equal to the internal tube impedance R_p.

By repeating the calculations given above for Case (B) in Fig. 2, it will be found that this condition again holds good, and also it will be found that the expression for K_{max} is the same.

As already mentioned the resistance R' in the formulas above includes the equivalent series resistance introduced in the tuned circuit by the impedance of the input circuit of tube II, but in many cases this extra resistance will be negligible as compared to the resistance of the coil itself, and equation (6) thus gives us the very interesting information that the maximum amplification obtainable with a tuned circuit amplifier is proportional to the *ratio of the inductive reactance to the square root of the resistance.* In the case of an ordinary selective circuit such as a tuned loop antenna the output voltage developed is proportional to the ratio of the inductive reactance to the *first power of the resistance.* This does not mean that low resistance is less desirable in amplifier coils than in ordinary tuned circuits but it does mean that the penalty exacted by increasing the resistance is not as great.

In order to test the formulas given by equations (5) and (6) a series of experiments have been carried out.

For measurements of the maximum amplification of a tuned circuit amplifier a circuit as shown in Fig. 3 was used.[1] The grid of the am-

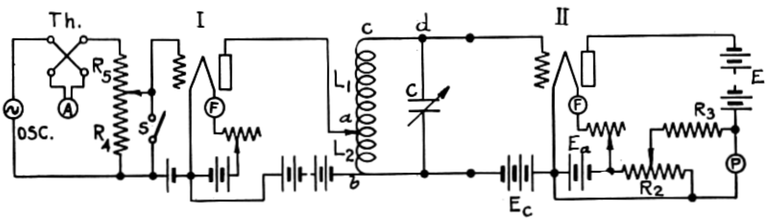

Fig. 3—Method of Measurement of Tuned Amplifier

plifier tube I is connected up to a known resistance, through which is passed a known current, and the voltage across the tuned circuit is measured by means of the tube-voltmeter II. The inductance L_1L_2 is made up of a single layer solenoid closely wound with 173 turns of solid wire and its value was 1.63×10^{-3} henries.

Keeping the frequency and the input from the oscillator constant the circuit is tuned to resonance by means of the variable condenser C and the lead from the plate to the coil is then moved along the coil until a point is reached which gives maximum deflection of the tube-voltmeter. During this process it is necessary to retune the circuit for each new point tried. Having thus obtained the right step-up for a certain frequency we then measure the amplification for different frequencies and get the amplification curves shown in the upper half of Fig. 4. On the lower half of Fig. 4 are given the number of turns (L_2) across the plate of the amplifier tube and also the capacity of the condenser C for each of the four cases shown.

[1] For a more detailed description of the method of measurement, see section entitled "Measurements" below.

In order to calculate the maximum amplification from formula (6) it is necessary to know the resistance R' of the circuit, the voltage amplification factor μ, and the internal plate impedance R_p of the amplifier tube. R' was obtained by running resonance curves for the circuit with the tubes connected up as usual, but with no filament current in the amplifier tube, in which case R_p may be regarded as being infinite.

These resonance curves are shown in the lower part of Fig. 4, and the resistance is then calculated from the well-known formula

$$R' = 2\pi(f_1 - f_2)L, \tag{7}$$

in which f_1 and f_2 are the frequencies, for which $E = E_{max}/\sqrt{2}$.

The resistance R' may also be obtained from the amplification curves as these can be regarded as resonance curves for the tuned circuit with the resistance R_p across part of the coil, and since this

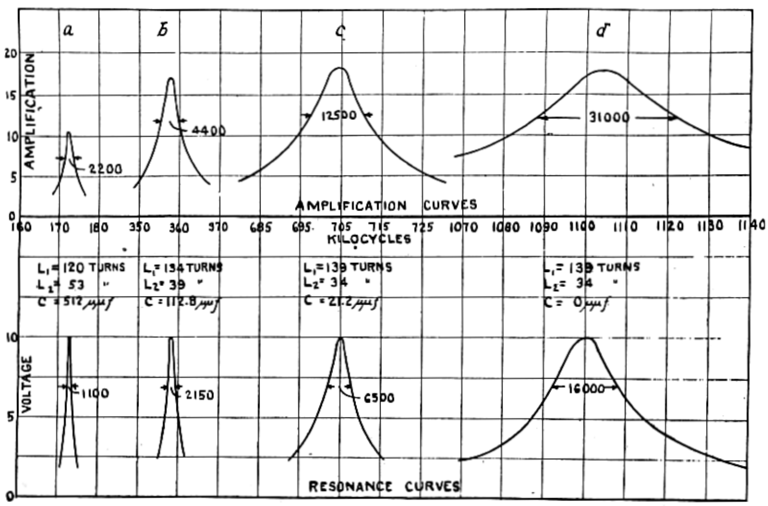

Fig. 4—Experimental Amplification and Resonance Curves of Tuned Circuit Amplifier

part of the coil is chosen so as to give $Z_{a-b} = R_p$, the equivalent series resistance should have increased to exactly twice the value found in formula (7). By comparing the widths of the amplification curves in Fig. 4 with the widths of the corresponding resonance curves it is seen that this actually was the case.

The internal plate impedance R_p and the amplification factor μ were obtained from the slope of the static characteristic of the amplifier tube used (a Western Electric 215-A or "peanut" tube).

The results of the calculations are given in Table II and the calculated values of K_{max} are seen to agree very well with the measured values given in the last column.

TABLE II

$L=1.63\times10^{-3}$ henries, $R_p=22{,}000$ ohms, $\mu=6{,}1$

Frequency	f_1-f_2	$R'=2\pi L(f_1-f_2)$	Calculated Maximum Amplification $K_{max}=\frac{\mu}{2}\frac{\omega L}{\sqrt{R_p\cdot R'}}$	Measured Maximum Amplification
172,000	1,110	11.4	10.7	10.3
357,200	2,150	22.1	15.9	16.6
704,000	6,500	66.6	18.1	18.1
1,100,000	16,000	164	18.1	17.8

The amplification of the amplifier was also measured with no step-up, i.e., with the plate of the amplifier tube connected across the whole

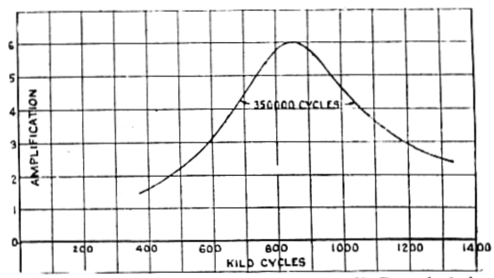

Fig. 5—Amplification Curve of Choke Coil Coupled Amplifier

coil and the tuning condenser C omitted (choke coil amplifier) the amplification curve shown in Fig. 5 being obtained.

From a resonance curve the following value is obtained:

$$R'=2\pi L(f_1-f_2)=2\pi\times1.63\times10^{-3}\times15{,}000=153 \text{ ohms,}$$

and, therefore,

$$R_{tot}=R'+\frac{\omega^2L^2}{R_p}=153+\frac{4\pi^2 853{,}000^2\times163^2\times10^{-6}}{22{,}000}$$

$$=153+3320=3523 \text{ ohms,}$$

which inserted in formula (7) gives

$$f_1-f_2=\frac{R_{tot}}{2\pi L}=\frac{3523}{2\pi\times1.63\times10^{-3}}=345{,}000 \text{ cycles}$$

while the amplification curve gives $f_1-f_2=350{,}000$ cycles.

As a final check of formula (6) by means of this tuned circuit, the maximum amplification was measured at 170,000 cycles with different values of extra resistance, R_{ext}, inserted in the circuit between c and d in Fig. 3. The results of these measurements agree very well with the formula as will be seen from Table III. For $R_{ext}=160$ it was found necessary to connect the plate across the entire coil in order to get maximum amplification and thus a further increase of R_{ext} beyond 160 ohms will make it impossible to obtain maximum amplification with this circuit.

TABLE III

$f=170{,}000$ cycles, $L=1.63\times10^{-3}$ henries, $R_p=22{,}000$ ohms, $\mu=6.1$.

R_{ext}	Total R'	Calculated Maximum Amplification $K_{max}=\frac{\mu}{2}\frac{\omega L}{\sqrt{R_p \cdot R'}}$	Measured Maximum Amplification
0	11.6	10.2	10.7
10	21.6	7.8	7.7
20	31.6	6.4	6.4
40	51.6	5.2	5
80	91.6	3.95	3.7
160	171.6	2.85	2.7

The variation with frequency of the resistance of the coil is shown on Fig. 6. These resistance values are obtained from the resonance curves in Fig. 4, and hence indicate also the losses in the variable condenser and the loss due to the input impedance R_g.

The curve in Fig. 6 gives what may be called the "true" resistance of the circuit, which is to be distinguished from the "apparent" resistance of the circuit as measured for instance by the well-known resistance variation method. By this latter method, the resistance of the coil is assumed to be equal to such an amount of extra resistance, as inserted in the circuit will decrease the resonance current to half its former value, but this assumption is only true when the distributed capacity of the coil is negligible as compared with the capacity of the variable condenser C, or when the resistance is introduced in the center of the coil.

It follows from formula (6) that for a given coil the maximum amplification is proportional to $\frac{\omega L}{\sqrt{R'}}$, and the measurements mentioned above seem to indicate that the maximum of this ratio has already been passed in the last case (d, Fig. 4) when the coil is used simply as a choke coil or auto-transformer (without any extra condenser). This, however, will depend upon the kind of wire used in making the coil. The coil used in the measurements above was made of No. 28

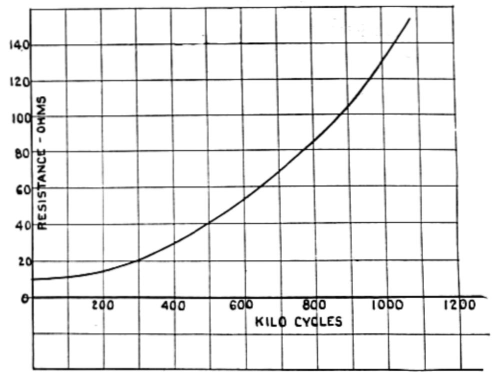

Fig. 6—Effective Resistance of Choke Coil

solid wire, but earlier results obtained by other investigators have shown that solid wire is superior to stranded wire at high frequencies, and thus it may be expected that the maximum of the ratio $\frac{\omega L}{\sqrt{R'}}$ for a given inductance will occur at a lower frequency when the coil is made of stranded wire.

For constant frequency the maximum amplification is proportioned to the ratio $\frac{L}{\sqrt{R'}}$ as already mentioned. It is thus desirable to adopt a construction for the coil, which will increase L without increasing $\sqrt{R'}$ proportionally. The highest amplification will in general be obtained when L is as large as possible for the frequency in question; in other words, it will be possible to obtain a higher amplification when the tuning condenser in the tuned circuit amplifier is reduced to zero, giving a simple choke coil amplifier.

For a tuned circuit amplifier with an ordinary good inductance coil made of stranded wire and of an inductance of, for instance, 200 microhenries and a high-frequency resistance of about 5 ohms, the amplification at 800 kilocycles will not be higher than about 9 times, according to formula (6) (using the same kind of tubes as in the experiments above), while with a choke coil an amplification as much as 18 times was obtained. This means that in order to get high amplification, small coils made of fine, solid wire and with large inductance and small distributed capacity should be used, rather than large coils made of stranded wire and with smaller inductance, but with larger distributed capacity.

In practice, it is not important to go to extremes in order to reduce the distributed capacity by one or two $\mu\mu f$. because the coil will always be shunted by the tube capacities, which are of the order of 10 $\mu\mu f$. It may be mentioned that the distributed capacity of the coil used in the above experiment is 3.5 $\mu\mu f$. This means that the constructional details of such a coil are not very important, and the coil may be made as a single layer coil or as a coil wound in one or several sections of rectangular or square cross-sections, but in all cases it will be found that coils of the same inductance will have very closely the same resonance frequency provided that the same tubes and leads are used in all cases.

Some experiments made with a choke coil (or auto transformer) at about 50,000 cycles show that the formulas given above may be also used here.

The coil used in these experiments was wound on a core of iron dust and made with square cross-section. The total inductance of the coil was .33 henries and provisions were made so that the plate of the amplifier tube could be tapped across any part of the coil.

The circuit diagram was the same as that given in Fig. 3 with the exception that the condenser C was omitted. The maximum amplification curve for this coil, used as a choke coil, is given by Fig. 7, curve A. The step-up ratio necessary to obtain maximum amplification was 1:16; i.e., the plate was connected across 1/16 of the total number of turns.

The resistance of the coil is obtained from a resonance curve as before:

$$R' = 2\pi L(f_1 - f_2) = 2\pi \times .33 \times 1300 = 2700 \text{ ohms},$$

and inserting this in formula (6) gives:

$$K_{max} = \frac{6.1}{2} \frac{2\pi \times 54800 \times .33}{\sqrt{22{,}000 \times 2700}} = 45$$

while the experiment gave 44.5.

On Fig. 7 are also given the amplification curves B, C and D for a step-up of 1:4, 1:1 and 1:48, respectively.

In the two cases B and C, the selectivity of the circuit is determined almost entirely by R_p, the resistance of the circuit itself being negligible, while in case D the selectivity is practically determined by the resistance of the coil itself.

It is seen that the amplification curve C for a step-up ratio of 1:1 is extremely flat as compared with the amplification curve shown in Fig. 5 for a choke coil working at 850 kilocycles.

In connection with these experiments with tuned circuits and choke coils it may be mentioned that in order to separate the DC plate voltage from the DC grid voltage, it will often be found of advantage to replace the coil by a transformer with very close coupling. In all our experiments, we have found that the amplification curves obtained in the two cases are identical when the coupling coefficient for the two windings of the transformer is nearly unity.

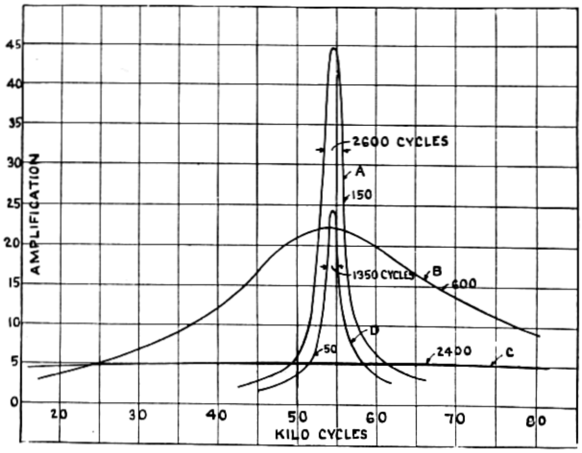

Fig. 7—Curve Showing the Effect of Ratio of Transformation in the Characteristic of a Choke Coil Coupled Amplifier

Loosely Coupled Transformer Amplifier

From the amplification curves obtained with choke coils, it will be seen that the frequency range obtainable with a choke coil amplifier is not as wide as might be desirable in some cases. This is especially true for higher frequencies between 300,000 and 1,000,000 cycles, and where a wide frequency band is desired these choke coils have, therefore, been replaced by transformers with a rather loose coupling, in which case the transformers will have the characteristics of two ordinary coupled circuits and give an amplification curve with two peaks.

It has been found by experiment that such transformers can actually be treated just as ordinary coupled circuits and the amplification curves can be computed by means of the well-known formulas for current and voltage conditions in two coupled circuits.

Before going into the details of these experiments, it is worth while to consider briefly the general relations involved as indicated by the curves obtained with two coupled circuits, each tuned to 52,000 cycles. These curves are shown in Fig. 8. The coils used

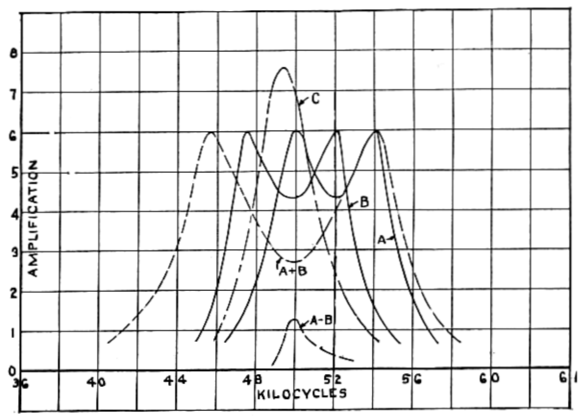

Fig. 8—Curves Showing the Effect of Coupling Inductive and Capacitive, on Amplification Characteristic of Coupled Tuned Circuits

had an inductance of 10 millihenries and were tuned by condensers. The circuit of the apparatus employed in obtaining the curves is given in Fig. 9.

Curve A gives the amplification for inductive coupling alone.

Curve B is for capacitive coupling alone.

Curve $A+B$ is for both capacitive and inductive coupling aiding each other, each coupling having the same value respectively as in curves A and B.

Curve $A-B$ is with the two couplings opposing each other and

Curve C is the same as $A-B$ but with different value of the inductive coupling.

The curves have the same shape as the well-known resonance curves for two coupled circuits with the oscillator input in series with the primary circuit, where the peak frequencies are given by the following approximate formulas:

$$\text{Inductive coupling: } f' = \frac{f_o}{\sqrt{1-k}}, \quad f'' = \frac{f_o}{\sqrt{1+k}},$$

Capacitive coupling: $f' = f_o$, $f'' = f_o\sqrt{\dfrac{C}{C+2C'}}$,

where $f_o = \dfrac{1}{\sqrt{LC}}$, $k = \dfrac{M}{L}$ = coefficient of coupling.

Having thus demonstrated the general shape of the amplification curves for a two coupled tuned circuit amplifier, the action of a loosely coupled transformer amplifier for high frequencies will be treated.

The transformer used in this experiment was made up of two similar pancake coils, 2″ diameter, wound with 210 turns of solid wire. Fig. 9 shows the circuit diagram. Curves A and B in Fig. 10a

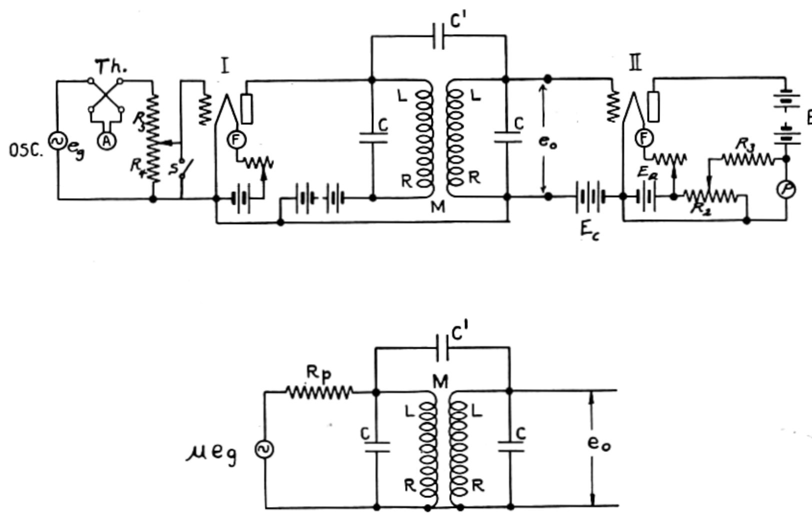

Fig. 9—Method of Measurement of Loosely Coupled Transformer Amplifier

show the measured amplification curves for a 3/8″ distance between the windings. The coupling condensers C' were omitted but even then there was some capacity coupling left due to the distributed capacity between the coils. The curves A and B correspond respectively, to an aiding and an opposing action of this capacitive coupling. Interchanging the leads to either coil changes the amplification curve from one type to the other.

The self inductance and mutual inductance of the coils were measured at low frequency and found to be:

$$L = 2.1\times10^{-3} \text{ henries}, \quad M = .95\times10^{-3} \text{ henries}.$$

The resistance of the coils was measured as described before by taking resonance curves at different frequencies. The distributed capacity of each coil was 14×10^{-12} farad (including tube capacity). By means of these values, the curve C was calculated.[2] The unknown capacity coupling makes it impossible to predict the exact shape of a transformer coupled amplifier from the constants of the circuits. However, the calculated curve C (calculated for inductive coupling only) will give a general idea of the shape of an experimental curve A.

Curves A and B, Fig. 10b, show the amplification curves for the case of capacity coupling alone. A is the experimental and B the calculated curve and they are seen to give fair agreement. The coupling capacity was 21×10^{-12} farad and the distributed capacity of the coils was 19.3×10^{-12} farad, the increase, as compared with the case of inductive coupling, being due to the ground capacities of the coupling condenser.

In connection with this type of amplifier it may be mentioned that a higher amplification naturally can be obtained if the plate of the amplifier tube is connected across a part of the primary circuit only, maximum amplification corresponding to the circuit impedance being equal to the plate impedance. However, the same effect will take place here as was shown for the tuned circuit amplifier, namely, that the band width will decrease with increase in amplification. Using transformers at their natural frequency instead of coupled tuned circuits with outside condensers will give broader bands or higher amplifications corresponding to the single tuned circuit amplifier.

[2] The following two formulas have been used for calculating the amplification of the circuit shown in Fig. 9.

Inductive Coupling:

$$\text{Amplification} = \frac{e_o}{e_g} = \frac{\omega M}{Z}\frac{1}{\omega C}\frac{1}{R_p(1-\omega^2 L^1 C)+R^1+j(\omega L^1+R_p R^1 \omega C)} \cdot \mu$$

where $Z = \sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$, $R^1 = R\left(1+\left(\frac{\omega M}{Z}\right)^2\right)$, $\omega L^1 = \omega L - \left(\frac{\omega M}{Z}\right)^2\left(\omega L - \frac{1}{\omega C}\right)$.

Capacitive Coupling:

$$\text{Amplification} = \mu\frac{R+j\omega L}{A+jB}$$

where $A = R_p\left(2-2\omega^2 LC+2\frac{C}{C'}-\omega^2 LC\frac{C}{C'} - \frac{1}{R^2+\omega^2 L^2}\frac{\omega L}{\omega C'}\right) + R\left(1+\frac{C}{C'}\right)$,

$$B = RR_p\omega C\left(2+\frac{C}{C'}\right) - \frac{RR_p}{\omega C'(R^2+\omega^2 L^2)} + \omega L\left(1+\frac{C}{C'}\right) - \frac{1}{\omega C'}.$$

AMPLIFIERS WITH SEVERAL STAGES. "FEED BACK" ACTION

The experiments so far have shown, that with one stage of amplification and with the amplifier working into a detector tube without grid condenser and leak, it is always possible to calculate the amplification curve from the constants of the tubes and of the coils, regardless

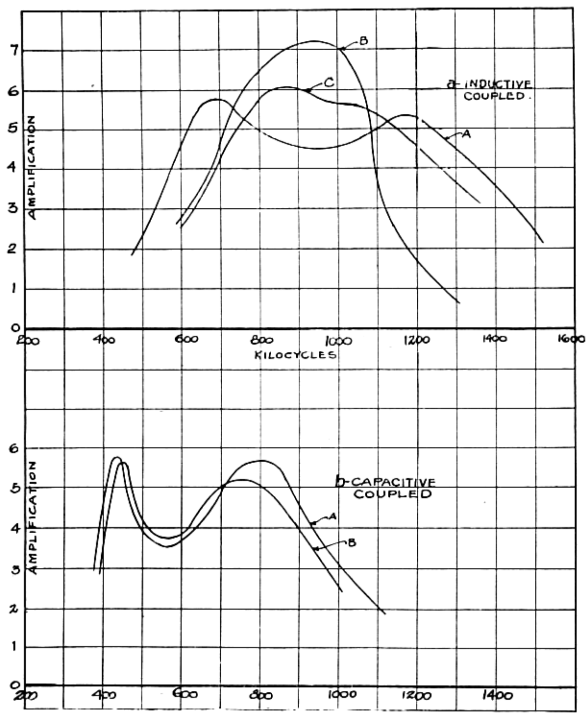

Fig. 10—Amplification Curves of a Loosely Coupled Transformer Amplifier Showing Effect of Coupling

of whether the connection between the amplifier and the detector consists of a simple tuned circuit, a choke coil, two coupled circuits or a loosely coupled transformer, the circuits being treated simply as ordinary tuned circuits.

Also the experiments have shown that a higher amplification can be obtained in the 50,000 cycles region than around 1,000,000 cycles, as might be expected from formula 6.

The next question is: What happens when more than one stage of amplification is used? If, for instance, the amplification for one stage is 10, will then the amplification for two stages be 100 or, in other words, in a multiple-stage amplifier is it possible to get the total amplification curve from the curve for the amplification per stage by multiplying them together?

The answer to this question is that the total amplification of a multi-stage amplifier will, in general, be lower than the value obtained by multiplying the amplification values per stage, and the reason for this is to be found in the input impedance of the tubes. So far, we have assumed the input impedance of the tube after the amplifier to be high as compared with the impedance of the tuned circuit (or transformer) and this is correct for a plate curvature detector, in which the impedance of the load in the plate circuit is negligible at the frequency of the amplified current but if the next tube is another amplifier it is only true at lower frequencies. It has been shown [3] that the input impedance of a vacuum tube can readily be calculated by means of the constants of the tube and the output impedance.

For the tubes used in the foregoing experiments we have the following approximate constants:

C_{g-p} = Grid to plate capacity $= 3\times 10^{-12}$ farad
C_g = Grid to filament capacity $= 5\times 10^{-12}$ farad
C_p = Plate filament capacity $= 5\times 10^{-12}$ farad
R_p = Plate impedance = 20,000 ohms
μ = Amplification constant = 6.

The output impedance including the plate-filament capacity will be assumed to be a resistance equal to the plate impedance.

If the input impedance is represented by an apparent resistance R'_g in parallel with an apparent capacity C'_g, we get for R'_g and C'_g the values given in Table IV.

TABLE IV

$$R'_g=\frac{\omega^2 R_p^2 C_{g-p}^2+4}{\omega^2 R_p C_{g-p}^2\,(\mu+2)} \qquad C'_g=C_g+2C_{g-p}\,\frac{\mu+2}{\omega^2 R_p^2 C_{g-p}^2+4}$$

Frequency	R'_g	C'_g
10^3 cycles	7×10^{10} ohms	17×10^{-12} farad
10^4 "	7×10^8 "	17×10^{-12} "
10^5 "	7×10^6 "	17×10^{-12} "
10^6 "	7.5×10^4 "	16×10^{-12} "

[3] H. W. Nichols, *Phys. Rev.*, Vol. 13, p. 405, 1919. John M. Miller, Bureau of Standards—Scien. Pap. No. 351, 1919.

From this table it is seen that the effect of the input impedance is negligible at frequencies up to about 100,000 cycles, but for frequencies in the broadcasting range, the input impedance will introduce an appreciable loss in the preceding circuit, which will result in a drop in amplification below the value obtained for a single stage amplifier. It is seen that the input impedance R'_g for broadcasting frequencies

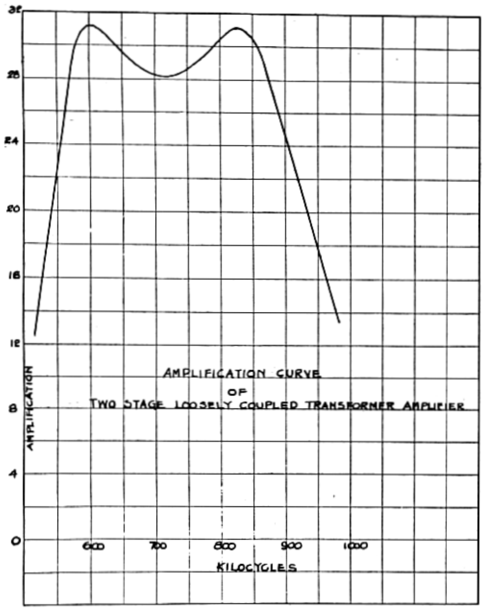

Fig. 11—Amplification Curve of Two Stage Loosely Coupled Transformer Amplifier

is of the same order of magnitude as the plate impedance R_p, which means that it will be of no advantage to use much step-up in choke coils or tuned circuits for an amplifier with more than one stage, since the amplification in no case will be much higher than μ per stage, except for the last stage, which is working into the detector.

The loosely coupled transformers of the type already discussed will, on the other hand, work very well in a two-stage amplifier, since there is no step-up used in these, and the amplification will be very nearly twice the amplification for a one-stage amplifier, as will be seen from the amplification curve shown in Fig. 11. The width of such an amplification curve can be increased by proper adjustment of the transformer inductances but the amplification will naturally drop correspondingly.

The values of R'_g and C'_g given in Table IV were calculated on the assumption of a pure resistance load R in the plate circuit. If the load in the plate circuit is an impedance $Z=R+j\,x$, it will be found that the sign of the apparent shunt resistance R'_g will depend upon the

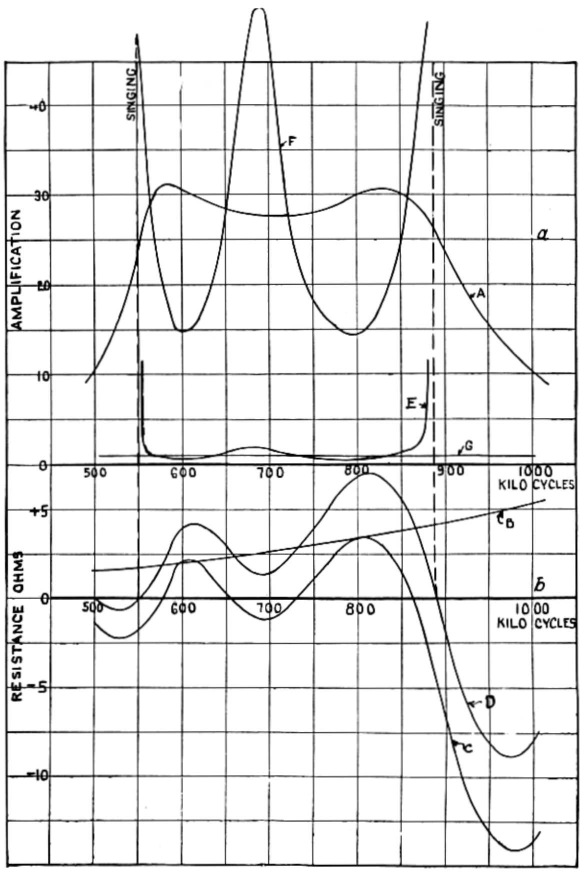

Fig. 12—Total Amplification of Transformer Coupled Receiver and Effect of "Feed Back" on Loop Resistance

sign of the reactance x. For a capacitive load, the resistance R'_g will always be positive, but for an inductive load, R'_g may in some cases become negative and we then have "feed back" or regeneration occurring through the tube. The negative resistance introduced in the circuit below the resonance frequency may in certain cases be so high that it more than neutralizes the positive resistance of this circuit which means that the set will start to oscillate or "sing."

As an illustration of the effect of this "feed back" action, there are given in Fig. 12 some curves obtained for a two-stage high frequency amplifier with loosely coupled transformer stages. The input circuit to the amplifier consisted of a loop antenna circuit tuned to the frequency of the induced signal.

Curve *A* shows the straight high frequency voltage amplification of the set, as measured with resistance input to the grid of the first high frequency amplifier. (Same as curve shown in Fig. 11.)

Curve *B* gives the actual resistance of the loop used with the set.

Curve *C* gives the resistance introduced in the loop due to "feed back" action from the first stage.

Curve *D* gives the resulting apparent resistance of the loop (Curve *B*+Curve *C*) and

Curve *E* shows the "feed back" amplification of the set. (Curve *B:* Curve *D*.)

Curve *F* shows the total amount of amplification obtained by the set which is the product of the ordinary voltage amplification (Curve

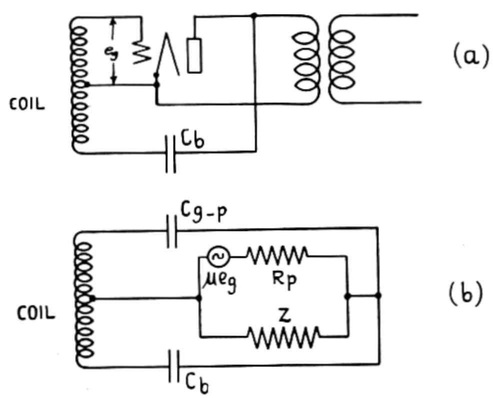

Fig. 13—Schematic of Balancing Condenser Action

A) and the "feed back" amplification (Curve *E*) and it is thus seen that the feed back action makes the total amplification vary irregularly in a very undesirable manner, and also makes the set "sing" at certain frequencies.

In order to avoid this, it is necessary to provide some means of balancing out the effect of the grid plate capacity of the tubes, and Fig. 13 (a) shows how this may be done.[4] The filament of the tube

[4] See Patent No. 1,183,875 issued to R. V. L. Hartley, and Patent No. 1,334,118 issued to C. W. Rice.

is connected to the middle of the coil, the grid to one end and the plate is connected through a small balancing condenser to the other end of the coil. In Fig. 13 (b) is given a schematic diagram of the circuit, which shows that the effect of C_b upon the coil circuit is just opposite the effect of C_{g-p}, so that the circuit can be regarded as an ordinary bridge circuit. It will, therefore, always be possible by proper adjustment of the condenser C_b to neutralize the effect of the feed-back action as shown by curve G in Fig. 12.

The same kind of an arrangement can be used between the different stages in a multi-stage high frequency amplifier, and it is thus seen that by proper use of such balancing condensers, it will be possible to obtain for a multi-stage amplifier a total amplification which is practically equal to the product of the amplifications per stage. This is true for a multi-stage tuned circuit coupled amplifier but for transformer coupled amplifiers, where it is more difficult to obtain a 180° phase difference of voltages, the advantage of the balancing condenser is not so great.

Of course, this favorable result presupposes that the wiring of the amplifier is properly done and the different stages shielded carefully from each other so that no external coupling exists between them.

Resumé

What has been said about amplifiers in the preceding sections can be summarized as follows:

With a given type of amplifier the same general shape of the amplification curve is obtained regardless of the frequency range at which the amplifier is designed to operate.

Thus, a low amplification over a wide frequency range will be obtained by using loosely coupled transformers or choke coils without any step-up, while a high amplification over a narrow range of frequencies can be obtained by using choke coils or tuned circuits with a proper step-up. In this last case, it will be necessary to use a small tuning condenser across the coils in order to make the frequency range of the amplifier wider, and the higher amount of amplification is, therefore, obtained only by a sacrifice of tuning facilities of the set. In a multi-stage amplifier it may, however, often be found of advantage to use a combination of low amplification stages and high amplification stages so that, for instance, one tuned circuit stage with high step-up and variable condenser is used in connection with one or several stages of choke coils or loosely coupled transformers with low amplification and a wide frequency range. The maximum amplification obtained with any kind of an amplifier will, in general, be higher at the lower frequencies, due to the lower loss and the higher ratio of L over C obtainable.

The width of the frequency band for a choke coil amplifier will be smaller, the higher the frequency due to the decrease in ωL with increasing frequency, and at broadcasting frequencies it will, therefore, in general be found advantageous to use loosely coupled transformers rather than choke coils, whenever a wide frequency band is desired. In addition to giving a wider frequency band, lower frequency amplifiers have the advantage of a smaller grid-plate feed back action.

Amplification Measurements at High Frequencies

In order to make a thorough study of radio frequency amplification, it is necessary to have a dependable method of measurement. Such a method developed in our laboratory and used very successfully will be described here.

In order to obtain an accurate comparison between different types of amplifiers, in which any type of resonant coupling is used, it is essential that these amplifiers be operated from a resistance input and not from an input containing a tuned circuit. With a tuned circuit it is not only very difficult to obtain an accurate measure of the voltage impressed upon the amplifier but considerable regeneration may occur between this input circuit and the output circuit of the first amplifier tube. There is, naturally, also a feed-back action in connection with a resistance input circuit, but its effect is negligible when the resistance is only a few hundred ohms. When the characteristic of a radio frequency amplifier with a resistance input has been accurately determined, its characteristic when used with a tuned circuit input may be determined as will be described later.

A schematic circuit diagram of the apparatus as used is shown in Fig. 3. To the left is shown the input apparatus which consists of an oscillator, a sensitive thermocouple and a potentiometer. The drop across the resistance R_4 of the potentiometer is used as the input to the amplifier stage I. The output of the amplifier stage is measured by the tube voltmeter II shown to the right in Fig. 3. The tube voltmeter II may be a low frequency detector in the case of amplification measurements of an actual receiver set.

It is necessary first to calibrate the tube voltmeter or detector II which is done by disconnecting it from the amplifier and connecting it directly across the potentiometer $R_4 - R_5$. R_4 is then adjusted to, say, 500 ohms and the current through it adjusted to some convenient value, such as 1 milliampere. This voltage of .5 volt will be sufficient with most tubes to give a change in the plate current of 30 to 40 microamperes.

The tube voltmeter is then reconnected to its normal place in the circuit and the resistance R_4 is connected to the input of the amplifier. Keeping the current constant at the value of 1 milliampere, the resistance R_4 is adjusted until the change in the detector plate current is the same as before. It is immediately apparent that the amplification will be the ratio of the known voltage on the grid of the detector, that is .5 volt, to the voltage on the input of the amplifier, as indicated by the product of the resistance R_4 and the current through it. The current having been kept constant, the amplification is the quotient of the 500 ohms used when calibrating the detector and the resistance value obtained with the amplifier included.

Considerable precaution must be observed to make sure that no energy is getting into the amplifier circuit except that which may be measured by the voltage drop across the resistance R_4. This necessitates the most careful shielding especially when the amplification is more than 50 times.

With the measuring apparatus described a dependable input voltage as small as 1 millivolt can be obtained. The maximum amplification which can be measured directly is, therefore, of the order of 500 times when the output voltage to the detector is of the order of one half of a volt.

For the measurement of higher amplification the following indirect method may be used.

The amplification is artificially decreased in some manner such as reducing the number of stages in the circuit and this reduced amplification is measured in the usual manner. The input current is then reduced and the input resistance increased keeping the plate current of the detector constant, the voltage impressed on its grid being determined by the previous calibration. The amplification is now increased to its normal value and the input resistance decreased until the detector plate current has its original value. The ratio of decrease in input resistance will thus give the increase in amplification and the total amplification will be the product of this and the smaller amplification as first measured.

The smaller current through the input resistance, which is obtained by this method and which will generally be less than can be determined by the most sensitive thermocouple, will reduce the pick-up to a sufficiently low value to give satisfactory results. In this connection it may be noted that an excellent test for the presence of undesirable pick-up is the closing of a switch (*S*) placed at the input of the amplifier. With this switch closed there should be no appreciable input to the detector.

Direct high frequency amplification measurements require input units made up of very carefully constructed attenuation boxes or potentiometers and well shielded oscillators. Such units have been developed in connection with field measurements and are described in a paper on "Radio Transmission Measurements," by Messrs. Bown, Englund and Friis and "Note on the Measurements of Radio Signals," [5] by Englund.

On the right in Fig. 3 is shown, as mentioned before, the circuit diagram of a "tube voltmeter" such as is used in many high frequency measurements. The tube voltmeter is essentially a plate current curvature detector. The grid is made negative by means of the grid battery E_c, so that the normal plate current of the tube is very small (of the order of 50 microamperes or so), and this plate current is further balanced out by means of the potentiometer arrangement R_2, R_3, so that the plate current meter reads zero when the input to the tube voltmeter is short-circuited. This arrangement has the advantage of making it possible to utilize the entire scale of the meter and to obtain the measured voltage from a single reading instead of the difference of two readings. Such a tube voltmeter built with an "N" tube will give a deflection of 1 microampere for an input voltage of about 1/5 of a volt, and the calibration will stay remarkably constant for several months and is *independent* of the *frequency* at which it is calibrated. The values of the resistances in the resistance boxes used at high frequencies may, therefore, be checked by using the boxes for calibrating a tube voltmeter first at 60 cycles and afterwards at, for instance, 1,200 kilocycles. If the two calibration curves obtained are exactly identical, then the resistance has not changed appreciably within this frequency range.

In measuring the amount of "feed-back" amplification in a receiving set, it is not possible to use a method as direct as described above. The "feed-back" or regeneration in a set is, as already mentioned, due to the coupling between the grid circuit and the plate circuit of the tubes through the grid-plate capacity, and will depend upon both the load in the plate circuit and the nature of the input circuits. If, for instance, it is desirable to measure the amount of "feed-back" amplification due to the coupling between the loop circuit and the

[5] Proc. Inst. R. E., Vol. 11, No. 1, February, 1923. Proc. Inst. R. E., Vol. 11, No. 2, April, 1923.

plate circuit of the first amplifier in a high frequency amplifier set, it will not be possible to measure this with a resistance input to the amplifier since in this case the "feed-back" has no appreciable effect. In order to get the correct value for the "feed-back" amplification, the set must be connected up to the same loop with which it is going to be used and the measurements can then be made in the following way.

A resistance box is inserted in the middle of the loop and a tube voltmeter is connected across half of the loop in addition to the receiving set as shown in Fig. 14. With the filament circuit of the set open, a strong high frequency emf. is induced in the loop and the loop circuit is tuned until the tube voltmeter reads a maximum.

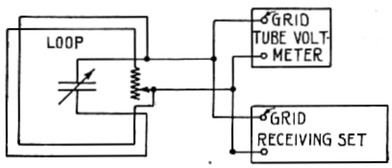

Fig. 14—Method of Measuring "Feed Back" Amplification

A "feed-back" action in the set will then produce a change in the tube voltmeter reading when the filament current is switched on. If the "feed-back" action is positive, i.e., if the resistance introduced in the loop is negative, then the tube voltmeter reading will increase, and in order to bring it back to its former value, the resistance of the loop is increased by an amount R' by means of the resistance box. If, on the other hand, the "feed-back" action is negative, the resistance of the loop must be decreased in order to obtain the former value of the tube voltmeter reading.

R' represents the equivalent series resistance introduced in the loop circuit by the "feed-back" action, and the apparent resistance of the loop is, therefore, $R-R'$, where R is the actual resistance of the loop. The voltage impressed upon the grid of the first tube is inversely proportional to the apparent resistance of the loop, and the amount of "feed-back" amplification is, therefore, defined as the ratio $K'=R/(R-R')$ where R' must be taken with the proper sign. This ratio is seen to be a direct measure of the increase (or decrease) in input voltage due to the "feed-back" action in the set and the total amount of amplification in a set at a certain frequency will then be given by the product of the ordinary voltage amplification factor K and the "feed-back" amplification factor K'.

In determining K', it is necessary to know the actual resistance R of the loop and this may be conveniently obtained by the reactance variation method using a tube voltmeter across half of the loop as the voltage indicating device. It has been found that the loss introduced by such a tube voltmeter is negligible, a fact which can be easily checked by connecting two similar tube voltmeters across the loop and determining the maximum reading of one of them. When the other one is then disconnected and the loop condenser slightly re-adjusted so as to again give maximum reading of the first tube voltmeter, it will be found, that the two readings obtained are exactly the same.

The discussion of the two types of amplification measurements of high frequency amplifiers may be summarized as follows:

The *ordinary voltage amplification* K is defined as the ratio of the amplified signal voltage impressed on the grid of the low frequency detector and the signal voltage impressed on the grid of the first amplifier tube. This amplification is measured by using a resistance input to the amplifier and includes the effect of "feed-back" action between the stages in the amplifier. This "feed-back" action between stages can naturally be analyzed by a method similar to the one used to determine the "feed-back" action between the amplifier and its tuned input circuit.

The *"feed-back" amplification* K' is defined as the increase (or decrease) of signal voltage due the "feed-back" action between amplifier and its tuned input circuit. The "feed-back" amplification depends upon the selectivity of the input circuit and will only vary slightly from unity when the resistance of this circuit is very large, while large variations, as shown in Fig. 12, may be found when a selective input circuit is used.

The total amplification is defined as the product of the ordinary amplification K and the "feed-back" amplification K'.