Vacuum Tube Oscillators—A Graphical Method of Analysis

By J. W. HORTON

Introduction

THE vacuum tube oscillator is fast becoming one of our most versatile circuits and the requirements which are being imposed upon it are constantly increasing in severity. In some cases it is asked to efficiently convert several kilowatts of direct current power to alternating current power. At other times, it may be called upon to deliver an alternating current having a frequency which shall remain constant within extremely narrow limits. It may be required to operate at a few cycles per second or at several million.

The question of frequency stability has recently taken on considerable importance. The need for currents of accurately known frequency is being felt in all branches of the electrical communication art, particularly in the field of multiplex transmission over wires by means of carrier currents and in radio broadcasting. The factors affecting the frequency of an oscillator will for this reason be given attention in the following discussion.

The operation of a vacuum tube oscillator or, in fact, of any system maintained in continuous oscillation, has certain unique features. In order for such a system to be in stable equilibrium its several elements must adjust themselves until certain necessary conditions are established. It is important, in an analytical study of oscillators, to know the manner in which this adjustment takes place.

If any operating condition may be defined by an equation made up of independent variables, it is a relatively simple matter to predict the result of changes in a single one. When, however, a change in one quantity is accompanied by a general readjustment of all the others, it is quite difficult to obtain a clear picture of what occurs from an equation. Graphical methods are better suited to a study of the manner in which a number of inter-dependent variables arrive at an equilibrium condition. Such a graphical treatment will be described in the following paragraphs and its application to the design of a circuit to perform certain specified duties will be discussed.

Graphical Method for Determining Conditions of Stable Operation

It is sometimes convenient to think of an electrical transmission system as being made up of a number of units, each delivering energy to the next succeeding unit, and thereby controlling the energy which that unit, in turn, delivers to the next. In case such a unit is made up of a vacuum tube amplifier circuit with its associated power supply batteries, it will be capable of passing on to succeeding units a greater amount of energy in a given time than it receives from preceding units. If a transmission unit does not contain some source of energy, it will, in general, deliver less power than it receives. In many cases these units may be arranged so as to form a complicated network. Whenever in such a network, a group of units forms a closed loop, that particular group is said to constitute a regenerative system. If a regenerative system is capable of maintaining a continuous flow of energy around the loop without receiving energy from any unit

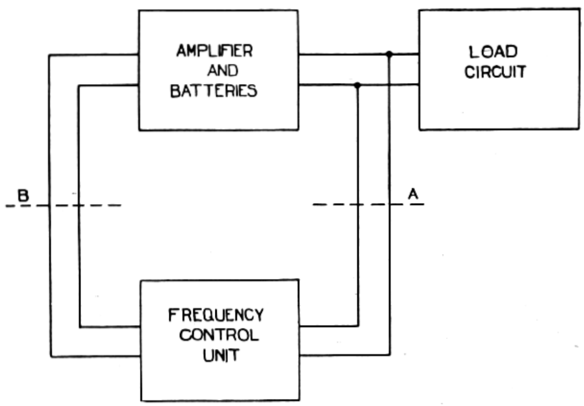

Fig. 1—Elements of an oscillating system

of the transmission network external to the loop, the system is said to be oscillatory.

For the purpose of this discussion let us think of an oscillatory system as made up of three units, the amplifier with its associated power supply source, a frequency control unit and an energy absorbing load unit. The arrangement of these units is as shown in Fig. 1. Now it is quite possible to determine the individual characteristics of the amplifier and of the frequency control units considered separately. The problem is to find the relation between these individual characteristics and the characteristics of the system.

In order that the regenerative circuit shall be in stable equilibrium, there are two conditions which must be met. The first of these is that the increase in power from the point B to the point A, through the amplifier unit, must be exactly equal to the decrease in power from the point A to the point B, through the frequency control unit. Due account must be taken of any energy delivered to the load circuit. In other words, when a given amount of energy flows into the amplifier across the junction B, it must be transmitted around the regenerative loop and returned to this junction unchanged in amount. The second condition is that the phase displacement of the wave transmitted from A to B through the frequency control unit must be equal in amount and opposite in sign to the phase displacement of the wave transmitted from B to A through the amplifier. That is, a wave which enters the amplifier at the junction B must be transmitted through the regenerative system and returned to this junction with no resultant phase displacement.[1] The individual characteristics of the amplifier and of the frequency control unit which permit these two conditions to be satisfied fix the operating point of the system.

Although the reasoning to be used in the succeeding paragraphs may, in general, be applied to any oscillatory system, it will be easier to follow if described in terms of familiar electrical circuits. In Fig. 2

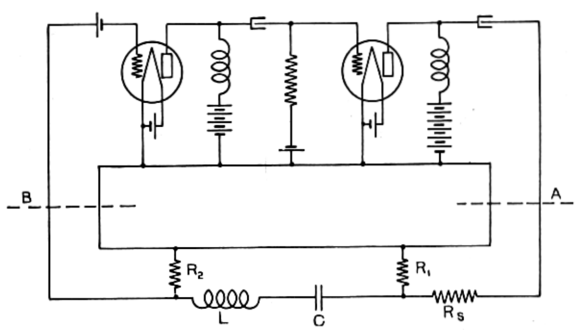

Fig. 2—Circuit of an elementary oscillator

the amplifier and associated batteries are shown in the form of an elementary vacuum tube circuit. It is necessary to use a two-stage circuit if the voltages at A and at B are to have the same sign. This is because the grid voltage, which is obtained as a potential drop due to current from an external source flowing through a resistance connected between the grid and the filament of the vacuum tube, reduces the current flowing from the filament of the tube to the plate, through a second external resistance, as the current flowing from the grid to the filament is reduced. That is, a change in the voltage drop across

[1] This condition for stable equilibrium will also be satisfied if the total phase shift around the loop is equal to $2\pi n$ where n may be any whole number.

the grid circuit resistance causes a change of opposite sign in the voltage drop across the plate circuit resistance, the two voltage drops being referred to the potential of the filament. The frequency control unit is in the form of a series circuit containing inductance, capacity, and resistance. Two resistance elements are used for coupling to the input and to the output of the amplifier.

Let us first consider the properties of the vacuum tube amplifier. In Fig. 3 the voltage developed across the junction A is plotted as a function of the voltage across the junction B. Let us assume, for the present at least, that this curve holds for all frequencies. Obviously

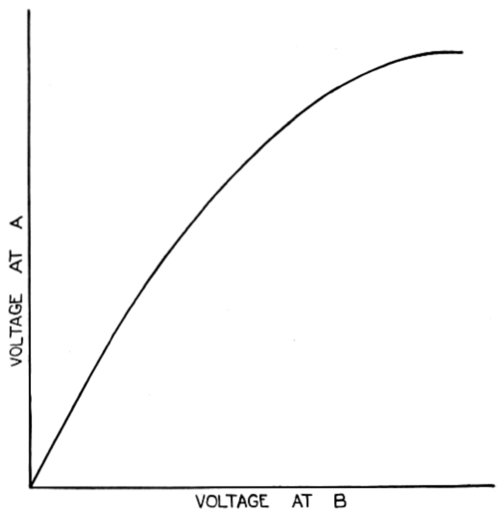

Fig. 3—Amplifier output characteristic

the voltage across the junction A depends upon the impedance looking into the frequency control unit, but if the resistance R_1 is small in comparison with the resistance R_s the voltage will be practically independent of the frequency control unit. This curve represents a familiar characteristic of the vacuum tube amplifier. It shows that as the voltage upon the grid of the first tube is increased, a point is reached where the amplitude of the output is no longer proportional to the amplitude of the input. If this is carried far enough a point is ultimately reached where a continued increase in the voltage on the grid fails to produce any further increase in the voltage across the output. For our present purpose the data contained in this curve will be more useful if plotted in a less familiar form.

In Fig. 4 the ratio of the voltage across the junction B to the voltage across the junction A is plotted as a function of the voltage across the junction A. This curve is obtained from the same data as the curve of Fig. 3 and tells the same story. Assuming that this curve holds for all frequencies a family of curves may be plotted for the amplifier unit, as shown by the horizontal lines in Fig. 5. In these

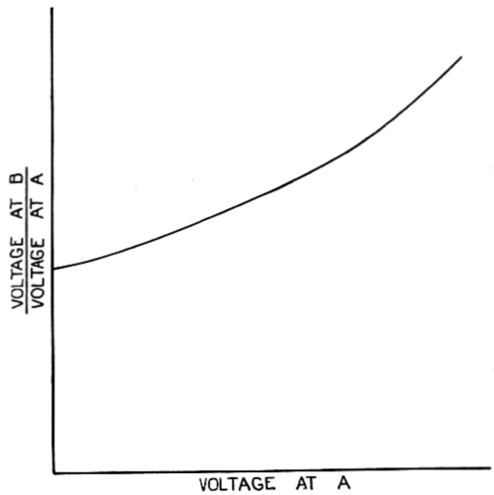

Fig. 4—Amplifier gain characteristic

curves the ratio of the voltage received by the unit to the voltage delivered by it is plotted against frequency. The numbers associated with each curve indicate the voltage at A, in arbitrary units, for which the curve holds.

A similar family of curves may be plotted for the frequency control unit. Since the impedance of the series resonance circuit varies with frequency from relatively high values above and below the resonance frequency to a minimum value at the resonance frequency, it follows that, for a fixed voltage across the junction A, the current through the inductance, the capacity and the resistance R_2 will vary with frequency. Consequently the voltage drop across the resistance, which is impressed across the junction B, will vary with frequency. The relation between this voltage and frequency, for a fixed voltage across the junction A, is given by the familiar resonance curve. As the voltage across the junction A is increased, currents of considerable magnitude may be caused to flow through the inductance, particularly in the neighborhood of the resonance point. If this inductance has an iron core an increase in the current will result in increased damping which, at a fixed frequency, acts to reduce the ratio between the voltage set up across the resistance R_2 and the voltage impressed on the junction A. The resonance curves given

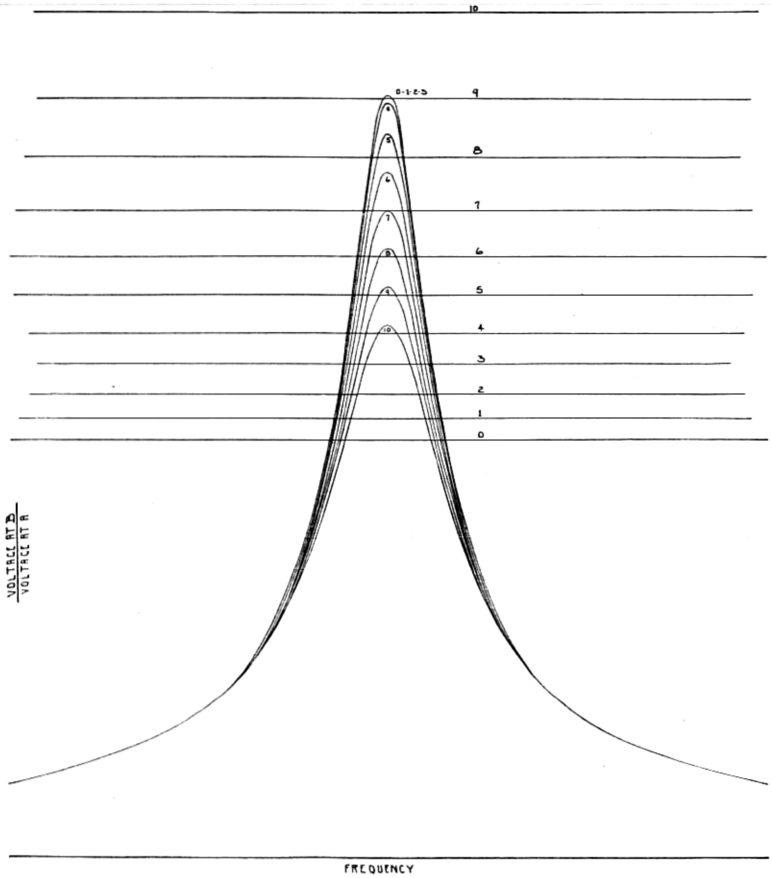

Fig. 5—Families showing relation between power gain, or loss and frequency for various power levels

in Fig. 5 show the relation between the ratio of the voltage across the junction B to the voltage impressed upon the junction A and frequency. The numbers again indicate the voltage at A, in the same arbitrary units as were used for the amplifier family.

Selecting any of the values given for the voltage across the junction A, it will be found that there are two curves showing the relation between the ratio of the voltage at the junction B to that at the junction A and the frequency. One of these is a characteristic of the amplifier, the other of the frequency control unit. It is, of course, apparent that the voltage across any junction in a transmission system may be taken as a measure of the rate at which energy crosses this junction. Therefore, points of intersection of these lines satisfy the first condition which was imposed upon the oscillating system in order that it should be in stable equilibrium, namely, that the increase of power through one portion should be equal to the decrease in power through the remaining portion. Such points of intersection define values for the amplitude of the voltage at the junction A and of the frequency for which this condition is met. Similar pairs of lines, plotted for other values of the amplitude of the voltage at the junction A, have intersections indicating the corresponding frequency for which the energy relations are again satisfied. For each of these points, then, the amplitude of the voltage at A, the frequency and the ratio of the amplitude at B to the amplitude at A have the same values for the amplifier unit that they have for the frequency control unit. In the curve A, of Fig. 6, the first of these

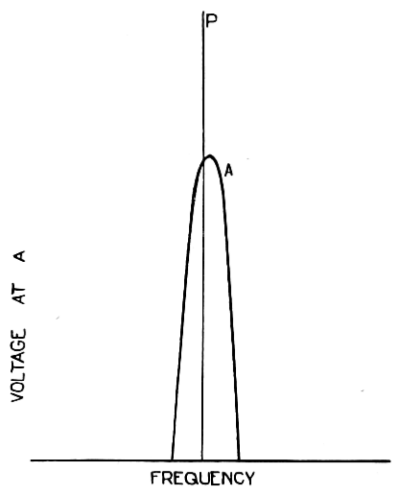

Fig. 6—Amplitude and phase equilibrium curves

variables is plotted against the second. The curve, therefore, shows the magnitude of the voltage delivered by the amplifier unit which, for a given frequency of the wave transmitted by the system, permits energy equilibrium to be maintained.

If energy considerations alone determined the stability of the oscillating system, it would appear that the operating point might be anywhere along this line. It is necessary, however, to consider the phase displacements occurring in the two units as well. The phase difference between the voltage across the output resistance of the frequency control unit and the voltage impressed across the junction *A* varies with frequency as indicated by the family of curves shown in Fig. 7. The assumption is made in drawing these curves

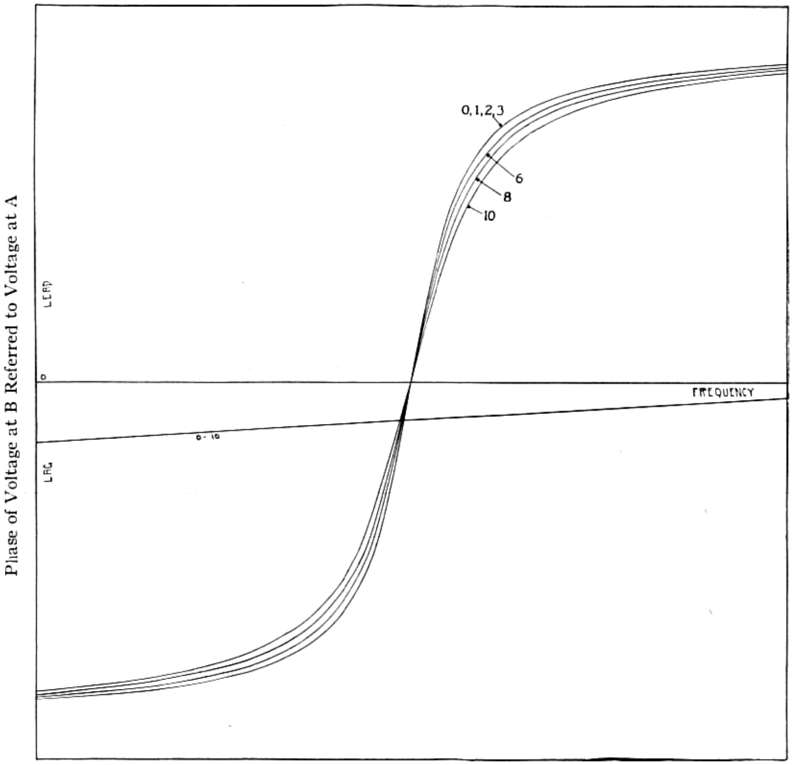

Fig. 7—Families showing relation between phase displacement and frequency for various power levels

that the change in damping is due entirely to a change in the resistance of the coil and that the inductance and the natural frequency of the circuit are unaltered as the load is increased. For low damping in the resonant circuit the phase shift changes rapidly with frequency in the neighborhood of the resonance point. As the amplitude of the voltage across the junction *A* is increased, thereby increasing the damping, the rate of change of this phase shift is reduced. The several curves correspond to different values of the voltage amplitude at the point A.

The phase relation between the voltage wave impressed upon the input to the amplifier and the wave delivered by it is indicated in Fig. 7 by the single straight line. That is, we are assuming that the phase displacement of the wave transmitted through the amplifier varies but little over the frequency range covered by the diagram and that it is independent of the power which is being delivered by the amplifier. The numbers associated with the several curves have the same significance as those used with the power ratio families.

From these two sets of lines it is possible to determine a series of values of the voltage at the junction A and of the frequency for which the resultant phase displacement around the loop is zero; exactly as we determined a series of values for which the resultant power change was zero. These values are plotted in Fig. 6 as shown by the line P. If the condition for zero phase shift were the only one which the system had to satisfy, it is obvious that it would be in equilibrium at any point on this curve. Since, however, the system is called upon to satisfy two conditions, one defined by the curve A and the other by the curve P, any intersections which they may have are the only points at which the system can be in equilibrium.

This method of analyzing the relations between the characteristics of the several members of an oscillating system, and their mutual adjustment to an equilibrium condition may be summarized in general terms. The system is considered as a regenerative transmission circuit divided into two portions. For each of these portions a family of curves is plotted showing the relation between the rate at which energy crosses one of the junctions, which will be used as a reference point, the ratio between the rates at which energy crosses the two junctions and the frequency. Any two of these variables may be chosen as the coordinates for these families of curves, the remaining variable being the parameter. The intersections of a curve in one family with the curve of the same parameter in the other family define pairs of values of the frequency and of the power at the reference junction for which the system is in energy equilibrium.

For each portion of the regenerative system, a second family of curves is plotted showing the relation between the power at the reference junction, the phase displacement of the transmitted wave between the two junctions and frequency. Intersections of a curve in one of these families with the curve having the same parameter in the other family define pairs of values of the frequency and of the power at the reference junction for which the system is in phase equilibrium.

The relations between these two quantities—frequency and power—may be expressed by two curves, one indicating the values necessary for energy equilibrium, the other indicating the values necessary for phase equilibrium. The intersections of these curves correspond to the only values meeting both conditions. The several elements must, therefore, adjust themselves to operate at the frequency and at the power indicated by such an intersection.

Effect of Variations in Circuit Elements

In addition to determining the frequency and power at which a given system is in stable equilibrium, it is important to be able to predict the effect upon these quantities of such changes as may be expected to occur in the elements composing the system. It is then a relatively simple matter to so redesign these elements that some particular effect shall be reduced, or increased, as desired. The circuit which has already been described may be used for illustrating the application of the graphical method in answering some of the questions occurring most frequently in connection with vacuum tube oscillators.

One of the more important problems concerns the reaction on the oscillating circuit of the load absorbing system. Let us imagine that an impedance, to which energy is to be supplied, is connected across the junction A. If this impedance is a complex quantity it will alter both the amplitude and the phase of the voltage across the junction. This will affect both families of curves—Figs. 5 and 7—which define the operation of the amplifier. If, for simplicity in the present discussion, we assume the load impedance to be a pure resistance, the major change will be a reduction in the voltage across A for a given voltage across B. The ratio of the voltage at B to that at A will be increased and the family of curves defining the power ratio relations between frequency and power ratio in the amplifier will thus be moved upward. The reaction upon the energy equilibrium curve—curve A, Fig. 6—will be to decrease both its height and its breadth. Assuming that the phase equilibrium curve remains unchanged, it is apparent that the frequency at which the system is in stable equilibrium will be increased and that the power delivered to the junction A will be decreased. Any change affecting the amplification of the vacuum tube circuit would react in much the same way.

Another question concerns the effect upon the amplitude at which the system operates as its frequency is altered by readjustment of the frequency control elements. If, for example, the capacity in the series resonant circuit is increased, the resonance curves of Fig. 5 will move to the left. Their shape also will be altered very slightly. Since the power ratio curves defining the operation of the amplifier are horizontal straight lines there will be a correspondingly slight change in the shape of the curve indicating the possible conditions for energy equilibrium. It will, of course, be displaced to the left by the same amount as are the resonance curves. If, however, the resistance of the resonant circuit varies directly with frequency, as it might through changes in hysteresis and eddy-current losses, the current through the series resonant circuit and through the resistance, R_2 will be increased. This increases the ratio of the voltage across B to the voltage across A and consequently lengthens the ordinates of the resonance curves shown in Fig. 5. The shapes of the curves will also be changed due to the change in the ratio of the reactance to resistance. Under these conditions the energy equilibrium curve, in addition to being moved to the left, will be increased both in height and in breadth.

The phase curves of the frequency control unit—Fig. 7—will be moved to the left by the same amount as the resonance curves. Due to the slope of the phase family of the amplifier which we have assumed to be coincident straight lines, the intersections of the two phase families must move away from the point of zero phase displacement. The separation between the members of the phase family of the frequency control unit is greater here and consequently the phase equilibrium curve is less nearly vertical than before. The slope of the phase family of the amplifier also causes the phase equilibrium curve to move to the left by a slightly greater amount than the displacement of the resonance point of the tuned circuit. It is apparent, therefore, since the phase equilibrium curve moves farther than the amplitude equilibrium curve, that their intersection will move to a position corresponding to a lower value of the voltage at the junction A. This is true, of course, only if the change in the shape of the amplitude equilibrium curve due to the change in resistance of the inductance coil is small. It is also evident that the change in the frequency of the current delivered by the oscillator is greater than the change in the resonant frequency of the inductance and capacity.

These two examples are undoubtedly sufficient to demonstrate how a change in the constants of a single element of an oscillating system necessitates a general readjustment of the other elements and how this readjustment reacts upon the operating point.

During the last few years the need for oscillating circuits of exceptionally high frequency stability has become more and more pressing. The requirements of multiplex telephony and telegraphy by means of carrier currents set particularly severe limits on the constancy of frequency of the alternating currents used. The efficient use of the ether in radio communication also places a very narrow tolerance upon any frequency variation in the carrier generators. It may be of interest, therefore, to consider some of the fundamental factors affecting the frequency stability of the vacuum tube oscillator.

Two lines of attack are open; we can design the several elements so as to reduce the possibility of a change in the value of their constants, or we can adjust the system so that unavoidable changes produce the least effect. It is in this latter connection that the graphical method of analysis is particularly helpful.

A change in the constants of any element of the oscillating system is going to result in a displacement or in a change in shape in at least one of the two equilibrium curves shown in Fig. 6. For a given change in either curve the horizontal displacement of their intersection will depend upon the slope of the other curve. The steeper one curve is, the less will be the frequency change resulting from any variation in the other. It, therefore, follows that we should make both curves as nearly vertical as possible.

Referring to the gain and loss families, Fig. 5, it will be seen that the slope of the amplitude equilibrium curve, and consequently the magnitude of the frequency change corresponding to a given change in the voltage at the reference junction, is determined by three things:

1. The separation between the lines defining the power gain in the amplifier; the less this separation, the less will be the frequency change accompanying a given change in the voltage.

2. The separation between the resonance curves defining the power loss in the frequency control unit; the less this separation, the less will be the frequency change accompanying a given change in the voltage.

3. The slope of the resonance curves; the steeper these curves, the less will be the frequency change accompanying a given change in voltage.

It appears then, that the change in frequency resulting from a given change in phase displacement, that is, accompanying any change in the phase equilibrium curve, may be reduced by operating the vacuum tubes considerably below their overloading point, where the gain changes but little as the output is increased; by operating the tuned circuit at low power levels, where the damping, and consequently the loss, varies but little with changes in the input; and by keeping the damping as low as possible.

The slope of the amplifier gain family is, of course, a factor, but in practice it is found undesirable to permit the gain of the amplifier to vary with frequency. The slope of the phase equilibrium curve, which determines the change in frequency corresponding to a given change in transmission gain or loss, depends upon three things, as may be seen from Fig. 7. These are:

1. The distance from the point of zero phase displacement at which the phase family of the amplifier intersects the phase family of the frequency control unit; the less this distance, the less will be the frequency change accompanying a given voltage change.

2. The slope of the phase family of the frequency control unit; the more nearly vertical these curves are made, the less will be the frequency change accompanying a given voltage change.

3. The separation between the members of the phase family of the frequency control unit; the less this separation, the less will be the frequency change accompanying a given amplitude change.

If the phase family of the amplifier is not a single line, the separation between its members would be a factor. The slope of the curve also has a slight effect. The distance from the point of zero phase displacement, at which the two families intersect, may be reduced by reducing such reactive impedances as appear in the amplifier circuit. The slope of the frequency control unit family may be increased by reducing the damping of the tuned circuit. It may also be increased by reducing the phase displacement in the amplifier, thereby operating nearer the point of zero phase displacement where the rate of change of phase shift with frequency is greatest. The separation between the members of the phase family of the frequency control unit may be reduced by reducing the magnitude of such changes as occur in the damping. Moreover, since for various amounts of damping the several members of the phase family approach coincidence at the resonance point, it is again desirable to reduce any phase displacement of the amplifier in order to work as near this point as possible.

It has just been suggested that any reduction in the phase shift through the amplifier will make the phase equilibrium curve more nearly vertical. It will be noticed, however, that this causes the intersection between the phase equilibrium curve and the amplitude equilibrium curve to occur in a portion of the latter where it approaches the horizontal. It would appear desirable, therefore, if frequency stability is to be pushed to the limit, to permit a slight phase displacement to occur in the amplifier in order that the intersection might be located at a place where the amplitude equilibrium curve is steeper. Any decrease in the slope of the phase equilibrium curve will be more than compensated for by the increase in slope of the amplitude curve. It will be apparent from the curves that such an adjustment reduces the amount of frequency change accompanying any change in phase displacement at the expense of amplitude stability. In practice, phase changes can be made smaller than transmission gain changes and we are consequently justified in placing most of the burden of holding the frequency to narrow limits on the phase equilibrium characteristic.

As a result of the foregoing analysis it appears that the amplifier should be designed so that, at the normal operating point, its gain varies but little with load and so that it introduces as small a phase shift as possible. The tuned circuit should have little damping and the variation in damping with load should be reduced to a minimum. Although these conclusions have been based upon the characteristics of a specific circuit, they apply equally well to other circuits of the same general form.

Design of Circuit for High Frequency Stability

The arrangement of an oscillating circuit embodying the features which the preceding section has shown to be essential, if the generated current is to be maintained within narrow frequency limits, is given in Fig. 8. The frequency control unit is a shunt resonant

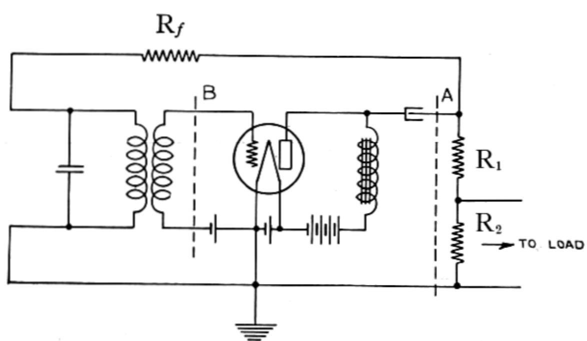

Fig. 8—Circuit of constant frequency oscillator

circuit coupled to the output of the amplifier, at the junction A, by a high series resistance, R_f, and to the input of the amplifier, at the junction B, through a winding coupled directly to the inductance.

The amplifier is designed to have ample load carrying capacity so that its gain varies but little with changes in load. This, as we have seen, is necessary in order to make the amplitude equilibrium curve steep and the frequency less subject to variation through unavoidable changes in phase displacement. Moreover, the voltage which appears at the junction A, as a result of a given voltage impressed upon the junction B, is stabilized by making the sum of the resistances R_1 and R_2 low as compared with the load impedance and with the impedance of the frequency control unit. In particular, R_2, across which the load is connected, is so small in comparison with the impedance of the load that changes in the latter are entirely negligible. Such an arrangement does not, of course, lead to high efficiency, but we must be prepared to make concessions in one direction in order to secure benefits in another. By making the effective load applied to the tube largely resistance the phase displacement occurring in the amplifier is made very small.

In this circuit it is not necessary to use two tubes to obtain the proper phase relations. At resonance the apparent impedance of the shunt resonant circuit approaches a pure resistance and the voltage drop across it is consequently in phase with the e.m.f. acting in the plate circuit of the tube. The current through the inductance, however, lags 90° behind this voltage. The e.m.f. set up in the oscillator input winding is 90° out of phase with the current in the primary, thus making it possible to secure in the frequency control unit the necessary 180° phase reversal between the plate and grid circuits of the tube. Care must, of course, be exercised to connect the windings of the oscillating coil in the proper direction.

The damping of the resonant circuit may be made small by giving the primary winding of the oscillating coil a high time constant. The coupling impedances introduce additional damping which may be made small by making both the feed-back resistance, R_f, and the input impedance of the tube high. The tube impedance may be made very high by using sufficient negative grid bias to prevent the filament-grid circuit from becoming conductive. Of the two coupling impedances, the feed-back resistance, together with the other impedances associated with the tube output, introduces the greater damping. Now it can be shown that for the frequency control unit to have a given transmission efficiency, the total added damping due to coupling is a minimum when the damping due to the input coupling is equal to that due to the output coupling. It is, therefore, desirable to make the coupling to the input of the tube as efficient as possible in order to permit the coupling to the output to be reduced. For this reason the mutual impedance of the oscillating coil has been kept as high as is practicable.

By increasing the feed-back resistance the ratio of the voltage at B to the voltage at A may be reduced, thereby decreasing the ordinates of the power loss family of the frequency control unit—Fig. 5. This affords a control by means of which the system may be adjusted so that both the amplifier and the frequency control unit are operated in regions where their power outputs are nearly porportional to the power inputs or, in other words, where the separation between the members in the gain and loss families is practically negligible.

There is another advantage in keeping the feed-back resistance high. In making it the major element in the network shunted across the resonant circuit, the effect of any variations in the output impedance of the tube or in the load impedance is reduced.

It is evident from an examination of the power ratio families which define the operation of the two elements of the regenerative system that before the system can come into equilibrium, at least one of these elements must enter a region where the relation between the power which it receives and the power which it delivers is non-linear. This means that the wave delivered by this element does not have the same form as the wave received by it. Distortion of this kind is manifest in the presence of harmonics of the fundamental frequencies in the current delivered by the oscillator. In most cases the amplifier is the distorting element and we find in the output all multiples of the fundamental. It has, however, been found advantageous in some instances to so adjust the system that the iron core of the inductance element in the frequency control unit overloads before the amplifier. In this case, the resulting distortion is such that only the odd multiples of the fundamental frequency are present.

By the proper choice of circuit elements, it has been found possible to design commercial oscillating circuits, covering the range of frequencies between 100 and 100,000 cycles per second, in which the frequency is but little affected by changes in elements external to the frequency control unit. In one such commercial oscillator it has been found, for example, that the average deviation in the frequency observed with any one tube from the mean frequency obtained with a number of tubes is approximately 0.02%. In this same circuit, as the plate potential changes from 100 to 150 volts, the frequency change does not exceed 0.04% at any portion of the frequency range. Similarly, if the filament current is changed from 1.1 to 1.4 amperes, the average frequency change is 0.03%. Changes in the frequency resulting from changes in the load impedance are practically negligible.

Such frequency changes as occur in the oscillator referred to above, are due, to a large extent, to variations in the inductance consequent upon the variations in power level which accompany the particular circuit changes referred to. The stability of the system may, therefore, be increased considerably beyond the limits indicated if the electrical constants of the elements used in the frequency control unit are independent of the power level. The use of an air core coil in place of an iron core coil improves the stability to a very marked extent provided, of course, that the same time constant is obtained in both cases.

In oscillators which have been designed primarily for frequency stability, it is found that the largest frequency variation is due to the variation in the electrical constants of the frequency control unit with temperature. When iron core coils are used, the temperature coefficient of frequency of the oscillating system is approximately 0.01% for 1° *C*. Using suitably designed air core coils, the temperature coefficient of the oscillator becomes approximately 0.003% for 1° *C*. The change in frequency in this case is due almost entirely to the change in capacity of the mica condensers used in the frequency control unit.

Although the method of analysis which we have just considered has been discussed largely in terms of the relation of the frequency of an oscillating electrical system to the constants of the several members of the system, it is by no means limited to such consideration. It is, in fact, applicable to practically all types of oscillating systems, including those containing mechanically resonant devices. It should, however, be remembered that while an analytical study of this type may assist materially in furnishing a qualitative picture of the conditions existing in some piece of apparatus, it is by no means a substitute for a rigorous quantitative treatment.