Mathematics in Industrial Research[1]

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"Selling" Mathematics to the Industries

THE necessity for mathematics in industry was recognized at least three centuries ago when Bacon said: "For many parts of nature can neither be invented [discovered] with sufficient subtility nor demonstrated with sufficient perspicuity nor accommodated unto use with sufficient dexterity without the aid and intervening of mathematics." Since Bacon's time only a very small part of nature has been "accommodated unto use," yet even this has given us such widely useful devices as the heat engine, the telegraph, the telephone, the radio, the airplane and electric power transmission. It is impossible to conceive that any of these devices could have been developed without "the aid and intervening of mathematics." Present day industry is indeed compelled, in its persistent endeavors to meet recognized commercial needs, to make use of mathematics in all of the three ways pointed out by Bacon. The record of industrial research abundantly confirms his assertion that sufficient subtility in discovery, sufficient perspicuity in demonstration, and sufficient dexterity in use can be achieved only with the aid of mathematics.

There is throughout industry one vitally important common characteristic,—uncertainty. In one industry the uncertainty may be due to the supply of raw material, the supply of labor, the supply of brains or the supply of capital. In another industry the uncertainty may be due to the activity of competitors, to fluctuating public demand or to the passage and subsequent interpretations of statutory laws. Still other industries are the playthings of the weather. Whatever the sources of uncertainty it is of vital importance to the industry to reduce to a minimum the hazards due to each of the uncertainties to which it is subjected. To a limited extent hazards may be transferred by means of insurance; but most uncertainties cannot be disposed of in this manner—they must be met by the industry individually.

The practice of probabilities, therefore, has a place in every industry. In fact, it occupies the first place in industrial mathematics, barring only the elementary arithmetical operations. It is remarkable how subtle are the mathematical difficulties presented by ap-

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parently innocent problems in the theory of probability. For this reason, mathematicians who are entrusted with the application of probability to industry must have great insight and acumen. Even so, in applying probability to any industry, a beginning should be made with the simpler problems, going on by gradual steps to more and more complicated ones.

Each industry has its own special mathematical problems, which must be considered individually in order to determine where mathematics should be applied. No industrial problem can seem much more hopeless, as a field for exact mathematics, than the subject of electricity as understood in the time of Bacon. It was then a mere collection of curious observations, such as the evanescent attraction of rubbed amber. Persistent observation and careful correlation have, however, brought a large domain of present day electricity under quantitative relations. Electricity is now preeminently a field for mathematics, and all advances in it are primarily through mathematics.

Industrial mathematics will achieve but little unless it is undertaken by persons with suitable aptitudes working under favorable conditions, on problems which have reached the mathematical stage. Industrial mathematical research involves much more than the mechanical application of established mathematical formulas. It involves cooperation in determining the problems to be attacked, in deciding what experimental data are necessary, in obtaining these data, in formulating the mathematical problem, in carrying through the analytical and numerical work, in applying the results to the physical actuality and in practically testing the commercial results achieved. In this cooperation many individuals may be involved and many tentative trials may be necessary in order to determine the solution which best meets all of the commercial conditions.

The cooperation must be effective; it must produce results, and these promptly. Mathematical deductions must be made intelligible and convincing, so that they will eventuate in action even when the indications of theory are apparently contrary to practical experience. This is important because the most valuable theoretical results are often revolutionary.

On the part of the industrial mathematician, powers of observation, clear physical concepts, quick resourcefulness, creative imagination and constant persistency are required. These are rare human qualities.. Unless industrial mathematical work is made attractive to men possessing these high talents, the full measure of success cannot be expected. Industrial mathematics must offer a career in itself, since specialization is required—specialization of a type which eventually disqualifies most men from undertaking other lines of work most effectively.

Mathematics in Electrical Communication

In order to make the foregoing observations somewhat more specific, I will refer to a few applications of mathematics in the industrial research of the Bell Telephone System. This field is selected because I am more familiar with it than with other industrial activities.

Certainty of prediction is the basic requirement in the development and operation of the telephone system; no vital need of the system can be left to chance or to fortuitous development. For this reason, the Bell System is highly organized under research control. The telephone situation is studied as a whole; all departments cooperate; each problem is considered from every point of view. Every attempt is made to master a situation in advance of the necessity of action, so that the most effective and economical means for electrical communication may be adopted with each expansion of the system. Much more than the immediate requirements of the hour must be known; preparation for all eventualities must be made. Fortunately, the executives have carried out this program with a prophetic appreciation of the value and necessity of mathematics.

The importance of the theory and practice of probabilities was recognized as soon as the telephone reached a thoroughly commercial basis. It has proved invaluable during the great expansion which has already carried the number of telephones in the city of New York to over a million. Meeting the peak load demand of the million-odd telephones in New York City, on a practically no-delay basis, with the minimum amount of equipment, is a highly complex and important problem. Without probability studies of the situation, the equipment installed at one point would be inadequate, while at other points it would be superabundant. The superfluous equipment would involve a waste of capital, while the inadequate equipment would mean inconvenience to the public and a loss of possible revenue. Equipment engineering involves a large number of probability problems which are novel, difficult, and financially most important. The aggregate cost of all such studies is large, but the resulting saving to the telephone-using public is much greater. Satisfactory telephone service in metropolitan areas is as dependent upon applied probability as is the success of life insurance.

The telephonic ideal, which is the perfect reproduction of speech, with articulation which is indistinguishable from face-to-face conversation, involves extensive and exhaustive investigations in many fields, in particular in mechanics, acoustics and electromagnetism, since each telephonic conversation involves oscillations in the air, in solids and in the ether. Fortunately, the foundations of the mathematical theory in these three fields had been securely laid by the time Alexander Graham Bell effected their harmonious cooperation in his first telephone. It is impossible for us to be too well informed concerning the consequences of the mathematical laws in these three fields.

It is characteristic of many problems encountered in industry that a great number of independent variables are involved, far too great a number for the best solution to be reached simply by trained judgment. Consider the transposition problem of the telephone system, which is this: on pole lines, long lines between cities, for example, several wires—sometimes a great number of wires—are strung along in close proximity. Each pair of wires receives inductive effects from the electric waves carried by every other pair, producing so-called crosstalk. To reduce such effects, the pairs of wires are transposed according to a set plan; that is, the positions of the two wires are interchanged, an expedient analogous to the twisting of a pair of wires. It is necessary to consider not only the ideal location of the transpositions in each pair of wires, but also the practical irregularities which occur in the actual placing of the transpositions. One of the practical problems, in fact, is to determine the allowable tolerances limiting the irregularities in the location of loading coils and transpositions, since these irregularities modify the crosstalk and also the transmission efficiency by an amount which must be determined by the laws of probability.

Transpositions were originally introduced with complete success about thirty years ago, and yet at the present time this subject is being more actively studied than ever; this is due to the extended use of phantom circuits and the new uses of carrier frequencies, that is, high-frequency speech-carrying currents which are superposed on ordinary telephony.

To illustrate the way in which problems in industrial mathematics become, step by step, more complex by the progressive inclusion of one factor after another, brief reference may be made to the loaded cable circuit. The first successful telephone cable circuits could be treated mathematically on the basis of Kelvin's simple cable diffusion theory. To allow for the ignored inductance and to determine the effect of added inductance, Heaviside's much more complete transmission formulas were employed somewhat later. The next stage was to allow for the effect of inductance which was not uniformly distributed, but lumped at regular intervals. Here the steady state solution for sinusoidal vibrations of a loaded string was employed, and the cutoff frequency due to internal reflections at the loading coils determined. But with loaded cables of great length, extending from New York to Chicago and beyond, the transient state may be of such duration as to require consideration. The loaded line does not transmit the impulse as a whole, but breaks it up by reflection and transmission at each loading coil. Therefore some of the impulses arrive after a few short backward reflections, while other impulses may travel many times the length of the line, due to reflections back and forth at many of the thousand loading coils in the circuit. The calculation of the transient state at the receiving end, due to the arrival of these impulses in groups, one after another, involved the calculation of Bessel functions up to order 2000 and subsequent integration by an application of the principle of stationary phase to Fourier's integral.

Industrial Mathematics as a Career

It is true that the mathematician who takes up industrial work is not entirely free to set his own problems; the industry which he has chosen provides these and it demands concentration upon them. Such problems are often less inviting than the clear-cut, tractable problem which the pure mathematician is at liberty to set himself. Industrial problems may be most complicated to frame and they may admit only of approximate solution by laborious numerical methods. In addition to delimiting the nature of his problems, the imperative needs of industry set time limits for their solution, and the nature of industry demands a financial profit from industrial mathematics. But these restrictions of industry should not make the work less attractive. On the contrary, restrictions disclose the master. There is an inspiration in overcoming even the humblest difficulty standing in the path of progress. Restrictions, even in the case of the most gifted, may be beneficial in concentrating activities, thereby making up in depth what may seem lacking in breadth.

The industrial mathematician may have a chance to attack many large-scale investigations which would be impossible, except under the patronage of industry, because of the exceptional material equipment and widely sustained cooperation required. Some of the opportunities offered by cheap electrical power from Niagara, by high-voltage electric power lines, and by large steam turbines may be mentioned. It is often left to the industrial mathematician to reap the harvest from seed sown under adverse circumstances by pure mathematicians.

The industrial mathematician may hope to make some return for the debt which he owes the pure mathematician. He may introduce new mathematical problems, of which industry is an inexhaustible source. He may point out the application of pure mathematical results, stimulating further investigations along the same lines. He may assist mathematicians generally by promoting the preparation of needed tables and by creating a commercial demand for calculating machines and other brain-saving devices.

The opportunities presented by industrial mathematics are boundless, because mathematics is the key to extrapolation in time, and industry is absolutely dependent upon prediction. The position of mathematicians in industry must eventually correspond with the importance of the function which they may perform.

Training for Industrial Mathematics

In industry we are concerned with mathematics not as an objective, but only as a tool. It follows that the required training in mathematics should develop a wide acquaintance with the available mathematical tools and practical skill in their use. It is important to note the distinction between the using of tools and the making of tools. Under primitive conditions the workman makes his own tools, but in a highly organized society the tools are made by specialists, who provide the workman with an endless variety of implements superior to anything which he himself could make. By long experience the tool designer has discovered how best to adapt the tool to its intended use in order to economize the workman's time and energy as much as possible. Furthermore, the substitution of one tool for another with the minimum number of motions is made possible by the use of interchangeable parts and systematically arranged cabinets.

But no complete line of mathematical tools is for sale across the counter; only a limited number of numerical and algebraic tables and a few types of calculating machines are supplied as ready-made tools. By far the larger part of known mathematical tools must be sought for in the literature of the subject, but there they may be difficult to find and isolate in the form best adapted for the purpose in hand. What is very greatly needed at the present time is a compendium or unabridged dictionary of mathematical results concisely and uniformly stated, and systematically classified for convenient reference. What I have in mind is not a mere handbook of applied mathematics, but a statement of theorems and formulas and tabulated results expressed in the language of pure mathematics, and comparable in scope and size with the "Encyklopädie der Mathematischen Wissenschaften." Preparation of such a compendium would be a tremendous undertaking, but it would also be of the greatest value. To such a collection of tools the industrial mathematician would turn for the appropriate tool as each new problem arises.

I would have the university training of the industrial mathematician based upon such a compendium by means of judicious sampling, at many points, under competent leadership. He would thus become familiar with his source book as a whole and thereafter turn to it instinctively and use it with confidence. At the present time, when the average text-book is held in low esteem and nothing has been substituted which adequately fills the gap, the student of mathematics leaves the university with a five-foot shelf of notebooks, together with what he carries in his head. Neither the memory nor the notebook is likely to be a reliable source of information when a particular result is needed for the first time, ten years later. It then becomes necessary for him to take the time to deduce the result from first principles, or to hunt up lecture notes, a text-book or original paper and waste much valuable time picking up the thread of the argument. The sampling to which I have referred should not be that of a dilletante; it should be an intensive grounding in the fundamental concepts and methods of mathematics, and the development *ab initio* of several well distributed branches of mathematics.

The combination of mathematical ability with an observant mind is as desirable as it is rare. The university training should include non-mathematical courses adapted for developing the powers of observation, or at least an appreciation of the necessity of cooperating with others who are observant. A study of the natural sciences, accompanied by experimental work, should be of great value. It is of course, difficult to be reasonable and not ask the impossible of the university in the training of any specialist. We recognize that, at best, only a beginning can be made at the university, but this beginning should include the fundamentals and should not attempt to impart details of current industrial practice. These details are best acquired in the industrial environment itself. Self-training in fundamentals, on the other hand, is much more difficult, and is not likely to go far, unless a start has been made under the favorable conditions afforded by the university.

What I have tried to emphasize is that industry can realize its greatest possibilities only with the aid of mathematicians, and that mathematicians can find opportunities in industry worthy of their powers, however great those powers may be. To ensure the success of industrial mathematics the industry must inaugurate mathematical research as early as possible, so that ample time may be afforded for the gradual accumulation of information upon which mathematics may be securely based, and for deriving quantitative results before the necessity for commercial action arrives. The industrialist must also be ready to give the mathematician's conclusions a sympathetic trial even though they run contrary to established precedent. Above all, industry needs mathematicians of an especially broad type—men whose interests naturally extend beyond their special field, and who are flexible enough to cooperate with non-mathematicians. These industrial mathematicians must inspire confidence by their firm grasp of physical realities, by the relevance of their mathematics, and by the ability to present their results clearly and convincingly.