A Test for Polarization of Electron Waves by Reflection¹

By C. J. DAVISSON and L. H. GERMER

A homogeneous beam of electrons is directed at 45° incidence against a {111}-face of a nickel crystal. The beam regularly reflected from this face impinges upon a second similar face at the same incidence angle. A Faraday collector is set to receive electrons regularly reflected from the second crystal, but only such electrons are accepted into the collector as have survived the two reflections without appreciable loss of kinetic energy. The collector and second crystal are rigidly joined, and may be rotated about the axis of the beam proceeding from the first to the second crystal. Measurements of the intensity of the twice reflected beam have been made at bombarding potentials from 10 to 160 volts. Within this range selective reflections (intensity maxima) are observed at 20, 55, 77, 103 and 120 volts.

These five selectively reflected beams have been separately tested for polarization by measuring the current received by the collector as a function of the azimuth of the movable system. If electron waves are polarized by reflection the intensity of the twice reflected beam should be greatest when the planes of incidence of the two reflections coincide, and least when they stand normal to one another. No such variation of the current to the collector is observed within the limits of error of the measurements—about one half of one per cent of the total current. Our observation is that electron

waves are not polarized by reflection.

HE experiment described in this article was undertaken to determine whether or not a beam of electron waves is polarized by reflection from the surface of a nickel crystal. It is similar in certain respects to the experiment with double Nörrenberg mirrors by which one demonstrates the polarization of light by reflection from glass, and in others to the experiment by which Barkla established that X-rays may be polarized. It resembles most closely, however, the variation of the Barkla experiment performed by Mark and Szilard in which the first of the radiators was a crystal and a Bragg reflection beam proceeded to the second radiator. A homogeneous beam of electrons is directed at 45 degrees incidence against a {111}-face of a nickel crystal, and the beam proceeding in the direction of regular reflection from this crystal is then reflected at the same angle of incidence from a second similar crystal. A double Faraday box is placed to receive electrons which have been regularly reflected from the second crystal, but only such electrons are allowed to enter the collector as have retained all or nearly all of their kinetic energy through the two reflections; those which have lost more than a small fraction of their kinetic energy are excluded by a retarding potential of suitable strength.

The second crystal and the collector are joined rigidly together, and may be rotated about an axis which coincides with the axis of the beam proceeding from the first to the second crystal. It is possible, there-

¹ Phys. Rev., Vol. 33, May, 1929, pp. 760-772

fore, to vary the dihedral angle between the plane of incidence of the second reflection and that of the first. There are two positions of the movable system for which these planes coincide. For these "parallel" positions the current entering the collector should be at a maximum provided the electron beam is unpolarized initially and becomes asymmetric at reflection; for the intermediate "transverse" positions the current should be at a minimum. In the analogous experiment in optics the intensity *I* of the twice reflected beam satisfies the formula

$$I = I_0(1 + p \cos 2\theta),$$

where θ represents the azimuth angle of the movable system measured from either of its parallel positions, and p an amplitude coefficient which serves as a convenient measure of the polarization effect.

In the experiment with electrons our procedure has been to measure the intensity of the twice reflected beam for various values of θ —though chiefly for the values corresponding to the cardinal positions—to assume the same form of relation between intensity and angle as in optics, and to evaluate the coefficient p.

The reflection of electrons from a crystal surface is, like that of X-rays, "selective in wave-length"; the intensity of the reflected beam attains maximum values at various critical wave-lengths or speeds of bombardment. This effect is, of course, accentuated in a beam which has suffered two reflections. In the test for polarization we have made observations in the range of bombarding potentials from 10 to 200 volts, chiefly at five different electron speeds at which there are intensity maxima of the beam twice reflected.

Preliminary observations indicated that at each of these critical speeds the intensity of the reflected beam is, to a first approximation, independent of angle. The acutal values found for p were some of them positive and some negative, and none greater absolutely than 0.02, which was about the order of uncertainty involved in the determinations of the collector currents. These results were described in a letter to the Editor of "Nature." 2

In the present article the experiment is described more fully, and additional data are adduced from which it is concluded that the value of p, if different at all from zero, cannot be greater than 0.005.

The principal parts of the apparatus are the gun for supplying a homogeneous beam of electrons, the two crystal reflectors, and the collector. These are contained in two metal boxes or enclosures shown in longitudinal sections in Fig. 1. The right hand or gun enclosure contains the electron gun and the first reflector, and is attached rigidly

² C. J. Davisson and L. H. Germer, Nature, 122, 809 (1928).

to the framework of the apparatus. The left-hand or collector enclosure contains the second reflector and the collector, and is supported from the frame of the apparatus through bearings by which it can be rotated about a horizontal axis. Communication between the enclosures is through the right-hand bearing which is hollow. The sections of the enclosures at right angles to the plane of the drawing are square.

The electron gun is similar in construction to the one described in an earlier paper ³ to which we refer for the details. The apertures are circular and those which define the beam are 2 mm. in diameter.

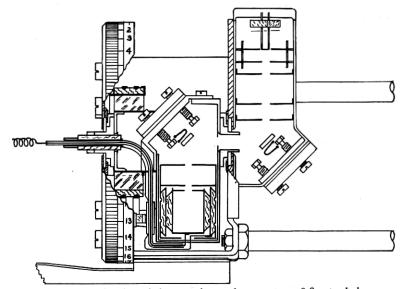


Fig. 1—Cross-section of the experimental apparatus—0.8 actual size.

The reflectors were cut from a single crystal of nickel formed by the slow freezing of pure nickel in vacuum. Their faces, which were polished to fairly good optical flats and then lightly etched by acid, are approximately 4×4 mm. in extent. The normals to these faces diverge from one of the $\{111\}$ -directions of the crystal structure by only about 10 minutes of arc.

The reflectors are so mounted that for each of them the incident beam lies in what we have designated as a {111}-azimuth of the crystal structure, as illustrated in the schematic diagram, Fig. 2. This adjustment may be unimportant, but was made because it has not yet been established that the selectivity of reflection is independent of the azimuth of the incident beam. The {111}-azimuth was chosen rather

³ C. J. Davisson and L. H. Germer, Phys. Rev., 30, 705 (1927).

than any other because our earlier observations on electron reflection were made with the incident beam in this azimuth, and several of the critical electron speeds for 45 degrees incidence were already known.

Each reflector is attached to a triangular frame which is supported from the diagonal wall of the enclosure through three adjusting screws. Two only of each set are shown in Fig. 1. The frames to which the crystals are attached and other accessory parts have been omitted from the drawing in the interest of clearness.

Small tungsten filaments, mounted one behind each of the reflectors, are supported by stiff wires from quartz plates which are clamped to the outer walls of the enclosures. Electrons emitted by these filaments

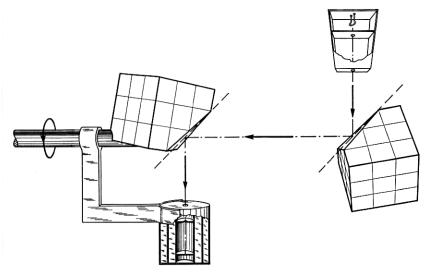


Fig. 2—Schematic diagram illustrating the principle of the experiment.

are used for heating the reflectors by bombardment. The reflectors are not insulated from the enclosures, which in fact contain no insulating material whatever except that incorporated in the gun and the collector.

The metal parts of the collector comprise an inner and an outer box of circular cross-sections and a cylindrical guard electrode of intermediate diameter. These parts are separated by cylinders of pyrex glass, and the assembly constitutes a unit which fits into the end of the collector enclosure. The aperture in the outer box is circular and 2 mm. in diameter; that in the inner box is of the same form but of slightly greater diameter. The guard cylinder is interposed to intercept the leakage current which would flow otherwise from the outer to the inner box. It was anticipated that the electron current entering

the collector would be excessively small and that this leakage current, unless guarded against, might prove an intolerable disturbance.

The lead wire from the inner box is guarded from the frame of the apparatus at all points of support within the tube by electrodes connected with the guard cylinder. This lead wire and the wire from the guard electrodes leave the tube through remote seals as indicated in Fig. 3. The isolation of the latter of these seals was a matter of convenience rather than of precaution.

Four electrical connections are required to parts of the movable system—two to the filament and one each to the collector and to the guard electrodes. These are maintained, with the exception of that to the collector, through platinum tipped molybdenum brushes which bear upon platinum rings. The connection to the collector is through a flexible spiral of tungsten wire lying in the axis of rotation.

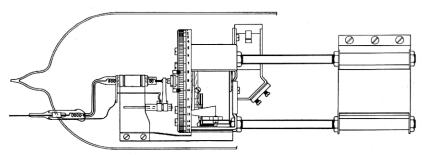


Fig. 3—Outside view of the experimental apparatus.

It may be well to add to this description of the tube a few words in regard to the adjustment of the reflectors to their proper positions and Each reflector is attached, as has been already mentioned, to a triangular frame which is supported through three adjusting screws from the diagonal wall of the enclosure. By turning these screws the reflector can be rotated through small angles about any axis parallel to the wall, and by the same means its distance from the wall can be varied. These adjustments were sufficient for locating the beam reflected from the first mirror in the axis of rotation, and that reflected from the second in the axis of the collector. They were not sufficient, however, for meeting the further requirement that the incident beam should lie in a {111}-azimuth of the crystal structure. For this adjustment we relied upon orienting the reflector correctly with respect to the triangular frame at the time of its attachment. mosaic of sharply defined triangular etch pits was visible under the microscope on the surface of the crystal reflector, and it was only necessary to relate these properly to the triangle formed by the frame to insure the desired orientation of the crystal with respect to the incident beam. Short wires attached to the reflector and protruding from it were rested upon the frame, and the reflector was then turned until the triangles on its surface stood in opposition to the triangle formed by the frame, this being the necessary relation. This adjustment was made with the frame and reflector mounted on the movable stage of a tool maker's microscope. One of the wires was then electrically welded to the frame. The adjustment was disturbed slightly by this operation, but the disturbance was corrected for by bending slightly the attached wire before proceeding with the second weld. This alternation of adjustment and welding was continued until all wires were attached. As finally adjusted the orientation of the reflector may have been wrong by one or two degrees, but hardly by more.

In adjusting the first reflector for position two conditions sought were, first that the intersection of the axis of the gun with the axis of the movable system should lie in the surface of the reflector, and second that the normal to the reflector should bisect the angle formed by these These were attained by removing the collector enclosure from the frame of the apparatus and the filament from the gun, and collimating the collector enclosure bearings with the images of the gun apertures formed by the reflector. For making the similar adjustment of the second reflector an aperture was formed in the center of the rear wall of the collector so that a view of the reflector might be had along the collector axis. The gun enclosure which had been detached from the frame of the apparatus during the adjustment of the second reflector was then replaced, and the adjustment of the two reflectors was checked by directing a beam of light along the axis of the gun and finding that the twice reflected beam proceeded accurately along the axis of the collector.

The preparation of the tube—the preheating of the metal parts, the baking, the exhausting, and the sealing off—was the same essentially as described in an earlier paper to which the reader is referred for particulars. (*Phys. Rev.*, loc. cit.)

In operation, the tube is mounted in a cradle with its axis inclined 30 degrees from the horizontal, so that an auxiliary tube lying in the axis and containing charcoal may be kept submerged in liquid air. The movable system swings to the lowest part of its arc, and its angular position with respect to the frame of the apparatus is read against the circular scale shown in Fig. 3. To alter this azimuth angle θ the tube is rotated about its axis; actually the "movable system" remains at rest relative to the earth, and all other parts are rotated.

No means were provided for measuring the current of electrons incident upon the first crystal. We had found, however, from a preliminary investigation of the characteristics of the gun, that currents of the order 2×10^{-4} amperes could be obtained from it. It was known also from these tests that the electrons ejected from the gun are very nearly homogeneous in speed. Given this value for the current in the primary beam, it was possible from our previous observations on the regular reflection of electrons at 45 degrees incidence to estimate the order of magnitude of the current of full speed electrons which might reach the collector after two such reflections. The estimated magnitudes were from 10^{-12} to 10^{-11} amp, and the currents of selectively reflected electrons actually observed have had values within this range.

In measuring these small currents we have had the use of a direct current vacuum tube amplifier designed and built by Dr. J. M. Eglin. It is the type of amplifier described recently by Wynn-Williams,⁴ but embodies certain improvements described by Dr. Eglin at a recent meeting of the American Physical Society.⁵ Conditions for observing were best when the amplification factor was about 2,000, so that the currents actually measured were of the order of 10^{-8} amp.

A few preliminary observations were made before heating the crystals to free their surfaces from adsorbed gas. The relation between the current entering the collector and the bombarding potential for a fixed angle θ was quite different in these first tests from that observed after the crystals had been heated. The principal feature of this initial current-voltage relation is a strong maximum at 20 volts. Tests were made for polarization with the crystals in this condition but no evidence of such a phenomenon was obtained.

The current-voltage curve characteristic of reflection from the crystals in a thoroughly cleaned condition is shown as Curve A in Fig. 4. The data from which this curve has been plotted were obtained with the faces of the reflectors parallel to one another as illustrated in Figs. 1 and 2. It will be convenient to designate this position of the movable system as the position $\theta = 0^{\circ}$. The "parallel" positions are then the positions $\theta = 0^{\circ}$ and $\theta = 180^{\circ}$ and the "transverse" positions are those for which $\theta = 90^{\circ}$ and $\theta = 270^{\circ}$. A curve similar to Curve A is obtained whatever value is chosen for θ . In this and in all other tests the inner box of the collector was maintained at a potential 2 volts above that of the midpoint of the filament.

Curve B of Fig. 4 exhibits, on a different scale of ordinates, the relation between current and voltage observed for angle of incidence 45

C. E. Wynn-Williams, Proc. Camb. Phil. Soc., 23, 811 (1927).
 J. M. Eglin, Phys. Rev., 33, 113 (1929).

degrees in our earlier experiments on the single reflection of electrons incident in the {111}-azimuth. The locations of the maxima of this curve are indicated in a diagram which forms a part of a report of these experiments.⁶

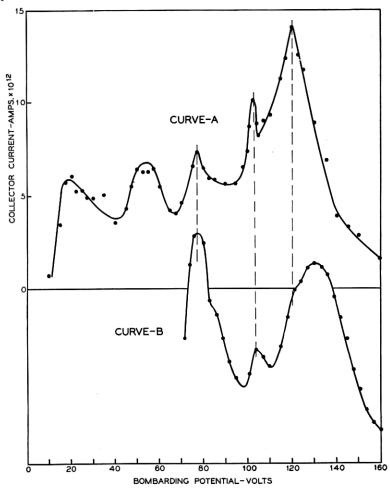


Fig. 4—Variation with bombarding potential of the intensity of beams reflected at 45°. Curve A is the doubly reflected beam of this experiment. Curve B is the singly reflected beam previously reported. (*Proc. Nat. Acad.*, loc. cit.)

The agreement between the curves of Fig. 4 is on the whole satisfactory; each displays three maxima in the voltage range common to both, two of which occur at the same voltages in Curve A as in Curve B. The voltages at which the third maxima occur—those on the ex-

6 C. J. Davisson and L. H. Germer, Proc. Nat. Acad. Sci., 14, 619 (1928), Fig. 3.

treme right—differ by about 10 volts. We believe that the position of this maximum is given correctly by Curve A, and that in Curve B it is shifted to the right owing to an eccentricity of the tube used in the earlier experiments. It will be noted that the position of the maximum in Curve A is marked in Curve B by a shoulder which protrudes from the side of the peak. It is with respect to the positions of the maxima only that the curves of Fig. 4 may be legitimately compared, the ordinates in the two cases being proportional to different quantities. Those of Curve A are proportional to the current of full speed electrons entering the collector, while those of Curve B are roughly proportional as has been explained (loc. cit.), to the ratio of full speed electrons, entering the collector to the corresponding current of electrons of all lower speeds.

There is some doubt in our minds as to whether the maximum in Curve A which occurs at 20 volts truly indicates a maximum in the reflecting power of the crystals for electrons of corresponding speed. The current to the collector is determined primarily by the product of the primary current by the square of the coefficient of reflection, so that a maximum in the collector current must correspond to a maximum in the reflecting power if the current in the primary beam is almost or quite independent of voltage, but not otherwise. This condition is known to be reasonably well satisfied in the range of bombarding potentials above 30 or 40 volts. Below this range, however, the current from the gun is limited partly by space charge, and its variation with voltage is rapid. A maximum in the current to the collector in this region must therefore be regarded with a certain suspicion; it may be due to a maximum in the reflecting power of the crystal with which, however, it will fail to coincide in voltage, or it may signify only that the reflecting power has a trend opposite to that of the primary current. We are not, however, greatly concerned in this investigation with the interpretation of this maximum, nor even of the other maxima of Curve A.

Measurements have been made of the intensity of the twice reflected beam as a function of the angle θ for bombarding potentials corresponding to the five maxima of Curve A. In some cases intensities have been measured at intervals of 5 or 10 degrees around the entire circle; but for the most part measurements have been made only at the cardinal positions $\theta = 0$, 90, 180 and 270 degrees. The total number of measurements of this kind is about 500. The complete data for bombarding potential 77 volts, corresponding to the third maximum of Curve A, and for $\theta = 270$ degrees are given in Table I.

TABLE I

Bombarding Potential 77 volts, Azimuth angle $\theta=270$ degrees. $R_0=$ zero reading of galvanometer, R= galvanometer reading. $(R-R_0)=D=$ deflection, $\overline{D}=$ arithmetic mean of deflections. $|D-\overline{D}|=S=$ deviation from mean, $\overline{S}=$ mean deviation.

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36.0 (36.2) 39.4 36.4 3.2 36.4 .10 26.5 (27.1) 28.9 26.5 (27.1) 2.9 30.1 30.1 30.1 30.1 30.1 30.1 30.1 30.1		40.0	3.5	.40		41.9	3.3	1.40
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37.2 (37.3) 40.3 3.0 .10 (28.2) 31.5 3.0 .20 37.4 33.6 (28.7) (28.7) 32.5 32 .10 33.6 (33.85) 37.2 3.35 .25 29.9 32.5 3.2 .10 34.1 (34.35) 37.1 2.75 .35 (29.8) 33.0 3.2 .10 34.6 (35.15) 37.9 2.75 .35 (31.0) 34.1 3.1 .00 35.7 (36.15) 39.5 3.35 .25 (31.85) 34.8 2.95 .15 36.6 (36.75) 40.0 3.25 .15 (32.45) 35.5 3.05 .05 36.9 37.4 33.8 33.0 37.6 3.2 .10 38.2 (38.5) 41.6 3.1 .00 (35.45) 38.5 3.05 .05		40.0	2.2	10		30.1	3.0	10
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33.6 (33.85) 37.2 34.1 (34.35) 3.35 37.1 37.1 37.9 37.9 37.7 (36.15) 3.35 37.9 37.9 37.9 37.9 37.9 37.9 37.9 37.9		40.3	3.0	.10	(28.2)	31.5	3.0	.20
(33.85) 37.2 3.35 .25 29.9 29.4 33.0 3.2 .10 34.1 (34.35) 37.1 2.75 .35 (29.8) 33.0 3.2 .10 34.6 (35.15) 37.9 2.75 .35 (31.0) 34.1 3.1 .00 35.7 (36.15) 39.5 3.35 .25 (31.85) 34.8 2.95 .15 36.6 (36.75) 40.0 3.25 .15 (32.45) 35.5 3.05 .05 36.9 37.4 33.0 33.8 37.6 3.2 .10 38.2 (38.5) 41.6 3.1 .00 (35.45) 38.5 3.05 .05						22.5	2.2	10
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35.7 (36.15) 39.5 36.6 (36.75) 3.35 40.0 .25 3.25 31.8 (31.85) 31.9 (32.45) 34.8 31.9 32.45 2.95 35.5 30.5 .15 30.5 36.9 37.4 (37.8) 38.2 (38.5) 40.9 40.9 40.9 40.9 40.9 40.9 40.9 40.9		27.0	2.75	35		34.1	3.1	00
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(36.75) 40.0 3.25 .15 (32.45) 35.5 3.05 .05 36.9 37.4 33.0 33.8 33.8 33.8 37.6 3.2 .10 38.2 38.5 41.6 3.1 .00 (35.45) 38.5 3.05 .05 (38.5) 41.6 3.1 .00 (35.45) 38.5 3.05 .05	(36.15)	39.5	3.35	.25		34.8	2.95	.15
36.9 37.4 (37.8) 38.2 (38.5) 41.6 3.1 3.2 3.0 33.0 33.0 33.8 (34.4) 35.0 (35.45) 38.5		40.0	2.25	1.5		25.5	2.05	05
37.4 (37.8) 40.9 3.1 .00 33.8 (34.4) 37.6 3.2 .10 38.2 (38.5) 41.6 3.1 .00 (35.45) 38.5 3.05 .05		40.0	3.25	.15		33.3	3.03	.03
38.2 (38.5) 41.6 3.1 .00 (35.45) 38.5 3.05 .05					33.8			
(38.5) 41.6 3.1 .00 (35.45) 38.5 3.05 .05		40.9	3.1	.00		37.6	3.2	.10
(66,5)		41.6	2 1	00		38.5	3.05	05
		41.0	3.1	.00		30.3	3.03	.03

Number of observations, N = 36; $\overline{D} = 3.104$; $\overline{S} = 0.154$

Probable error, $\Delta = 0.845 \frac{S}{N^{1/2}} = .022$ approx.

 \therefore Deflection, $D = 3.104 \pm .022$.

That the deviations from the mean value of the deflections are distributed in this, and in other cases, in close accordance with the normal error function is illustrated by diagrams displayed in Fig. 5.

The value obtained in Table I for the deflection at $\theta = 270^{\circ}$ is shown again in Table II, together with the values similarly obtained for the same bombarding potential at the other cardinal positions.

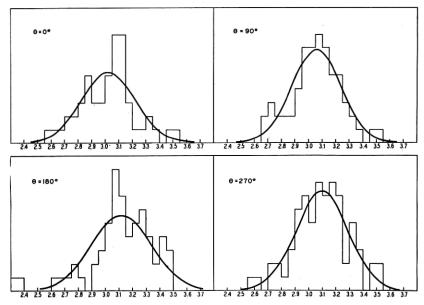


Fig. 5—Plots of all data taken at 77 volts for the four cardinal positions— $\theta=0^{\circ},90^{\circ},180^{\circ},270^{\circ}$. The solid curves represent calculated normal error function curves. The data plotted here are summarized in Table II.

TABLE II

	Bombarding Potent	ial 77 volts.	Wave-length	1.39 A.
Angle	e θ	No. of Obs.		Deflection
0 d	leg	26		$3.023 \pm .026$
90.		31		$3.057 \pm .022$
180.		31		$3.116 \pm .027$
270.		36		$3.104 \pm .022$

The values of these deflections and their probable errors have the characteristics of four measurements of one and the same quantity. There are certain values of deflection common to two of the error ranges but none common to three. This is the situation most likely to be met with if we are measuring the same quantity in each case; the maximum number of overlapping ranges should be one half the total number of ranges. It is of some interest, however, to pretend that the different

values found for the deflections correspond to actual differences in the current to the collector, and to attempt to evaluate the amplitude coefficient of the polarization effect. We shall find actually that observations at four positions only are insufficient to determine this constant precisely.

It will be appreciated that differences in the collector current at different angles may arise from mechanical defects in the apparatus—improper alignments, etc.—as well as from polarization, and that in general we shall require for the expression of D as a function of θ a complete Fourier series such as

$$D = D_0 \left[1 + \sum_{1}^{\infty} a_n \cos (n\theta + \alpha_n) \right].$$

The data available for evaluating the constants of this series consist of four values of D, corresponding to the four values of $\theta - 0^{\circ}$, 90° , 180° and 270° . We designate these four values respectively by D_1 , D_2 , D_3 and D_4 . From the four simultaneous equations formed by writing these pairs of values of θ and D into the series we obtain the following relations:

$$D_1 + D_2 + D_3 + D_4 = 4D_0 \left[1 + \sum_{1}^{\infty} a_{4n} \cos \alpha_{4n} \right],$$

$$D_1 - D_3 = 2D_0 \sum_{0}^{\infty} a_{2n+1} \cos \alpha_{2n+1},$$

$$D_4 - D_2 = 2D_0 \sum_{0}^{\infty} (-1)^n a_{2n+1} \sin \alpha_{2n+1},$$

$$D_1 - D_2 + D_3 - D_4 = 4D_0 \sum_{0}^{\infty} a_{4n+2} \cos \alpha_{4n+2}.$$

If we make the definite assumption that all periodic terms of orders greater than the second may be neglected, these reduce to the relations

$$D_1 + D_2 + D_3 + D_4 = 4D_0,$$

 $D_1 - D_3 = 2D_0a_1\cos\alpha_1,$
 $D_4 - D_2 = 2D_0a_1\sin\alpha_1,$
 $D_1 - D_2 + D_3 - D_4 = 4D_0a_2\cos\alpha_2,$

from which we may obtain expressions for a_1 , α_1 and $a_2 \cos \alpha_2$, but not, unfortunately, for a_2 and α_2 separately; the fourth observation is used up in fixing D_0 in which we have no interest. Observations at one additional angle would have been sufficient to resolve a_2 and α_2 , but this was not appreciated at the time the measurements were made.

If we write Δ_1 , Δ_2 , etc. for the probable errors involved in the measurements of D_1 , D_2 , etc. we find on solving for amplitudes and phases, and compounding errors ⁷ that

$$a_1 = \frac{\left[(D_1 - D_3)^2 + (D_4 - D_2)^2 \right]^{1/2}}{2D_0} \pm \delta(2 + a_1^2)^{1/2},$$

$$\tan \alpha_1 = \frac{D_4 - D_2}{D_1 - D_3} \pm \frac{(\Delta_1^2 + \Delta_3^2)^{1/2}}{D_1 - D_2} (1 + \tan^2 \alpha_1)^{1/2},$$

$$a_2 \cos \alpha_2 = \frac{D_1 - D_2 + D_3 - D_4}{4D_0} \pm \delta(1 + a_2^2 \cos^2 \alpha_2)^{1/2},$$
re

where

$$\delta = (\Delta_1^2 + \Delta_2^2 + \Delta_3^2 + \Delta_4^2)^{1/2}/4D_0.$$

Substituting into these formulas the values of D and Δ contained in Table II, we find

$$a_1 = 0.0169 \pm .0080,$$

 $\tan \alpha_1 = -0.50 \pm .45$
 $(136^{\circ} < \alpha_1 < 177^{\circ}),$
 $a_2 \cos \alpha_2 = -0.0018 \pm .0040.$

The last of these quantities includes the amplitude of the polarization effect as one of its components. To make this explicit we may restrict a_2 and α_2 to represent the amplitude and phase angle of variations of twice the fundamental frequency due to mechanical imperfections only, and use p to represent the amplitude of the polarization effect. We may then write, since the phase angle associated with p is zero,

$$p + a_2 \cos \alpha_2 = -0.0018 \pm .0040$$

and from this we wish to infer that p is itself a small quantity, the same in order of magnitude as $(p + a_2 \cos \alpha_2)$.

It may be urged, of course, that nothing in regard to the value of p is to be inferred from the value of $(p + a_2 \cos a_2)$, and this in a strictly mathematical sense is true enough; the individual terms may both be large, and the small value of their sum may be entirely fortuitous. While one must recognize this as a possibility, he must recognize also that the likelihood of the occurrence of chance compensations of such perfection in the case not only of this beam, but in the cases of the others as well, is extremely small. The values found for $(p + a_2 \cos a_2)$ for all five beams have been set down in Table III. It will be seen that,

 $^{^7}$ In calculating the probable errors of these functions we have disregarded the small differences in precision involved in the measurements of the various deflections.

with the possible exception of the value for the 103 volt beam, all are equal sensibly to zero.

TABLE	Ш

Beam No.	Bombarding Potential	No. of Obs.	a_1	$lpha_1$	$p + a_2 \cos \alpha_2$
1 2 3 4 5	20 volts 55 77 103 120	111 108 124 30 146	$\begin{array}{c} 0.013 \pm.012 \\ 0.015 \pm.013 \\ 0.017 \pm.008 \\ 0.065 \pm.011 \\ 0.021 \pm.004 \end{array}$	133° to 194° 107° to 264° 136° to 177° 113° to 127° 102° to 121°	$\begin{array}{c} 0.0089 \pm.0058 \\ -0.0025 \pm.0065 \\ -0.0018 \pm.0040 \\ 0.0230 \pm.0057 \\ 0.0053 \pm.0020 \end{array}$

The large values found for both $(p + a_2 \cos \alpha_2)$ and a_1 in the case of the 103 volt beam are due, we believe, to some departure from the usual conditions of the experiment which occurred while observations on this beam were being made. Fewer measurements were made on this beam than on any of the others, and the discordant values found for its constants are traceable to an exceptionally low value—based on eight readings only—which was obtained for $D_2(\theta = 90 \text{ degrees})$. discordance of this value with those obtained for the other deflections is evident from the figures set down in the last column of Table IV. These are the differences between the various deflections and the mean of the deflections D_1 , D_3 and D_4 . It will be noted that the departure of the value obtained for D_2 from this mean is three times as great as that for any of the others. The fact that this single unusual departure is responsible for the exceptionally large value not only of $(p + a_2 \cos \alpha_2)$ but of a_1 as well, is reason, we think, for regarding it as accidental. We believe, therefore, that we are justified in disregarding the result obtained in this case, and in concluding from the values found for $(p + a_2 \cos \alpha_2)$ for the other beams that the amplitude of the polarization effect is zero within the limits of uncertainty of our measurements—that is, within about one half of one per cent.

TABLE VI 103 Volt Beam

· · · · · · · · · · · · · · · · · · ·	No. of Readings	Deflections	$D - \frac{D_1 + D_3 + D_4}{3}$
0 deg 90 180 270	7 8 9 6	$D_1 = 4.829 \pm .070$ $D_2 = 4.481 \pm .052$ $D_3 = 5.133 \pm .031$ $D_4 = 5.033 \pm .061$	$ \begin{array}{r} -0.17 \\ -0.52 \\ +0.13 \\ +0.03 \end{array} $

Experiments designed to test for the polarization of electrons by

reflection have been made also by Cox, McIlwraith and Kurrelmeyer, by Joffé, and by Wolf. The experiment by the first-named three 8 is similar in principle and arrangement to our own; the intensity of a beam of electrons which has been twice reflected through 90 degrees is measured while the second reflector and collector are revolved about the direction of incidence of the second reflection. But in other respects the experiments differ. The electrons constituting the primary beam are β -rays from a sample of radium, the reflectors are plates of polycrystalline gold, and the collector is a point-discharge electron counter. The authors report that the shielding between the electron source and the counter was inadequate to suppress entirely an effect due to the gamma radiation, and further that rapid changes in the characteristics of the discharge point made it difficult to obtain consistent data. The results which they publish are ratios of the current received by the collector in one of the "parallel" positions to that received in one or the other of the "transverse" positions, and the ratios of the currents received in the two "transverse" positions. The values found for the first of these ratios depart from unity by much more than the probable error, and show a bias in favor of polarization. The authors do not point this out, however, but lay emphasis instead upon a rather slight departure from unity of the values obtained for the second ratio—that of the currents in the two transverse directions.

The experiment by Joffé is mentioned by Darwin ⁹ in a short article on the Sixth Congress of Russian Physicists which was held last summer. Darwin remarks that at one of the meetings Joffé reported that he had looked for a polarization of electrons by reflection, but had failed to detect such an effect. So far as we are aware no report of this work has been published.¹⁰

In the experiment by Wolf ¹¹ a beam of low speed electrons (accelerating potentials of about 10 volts) is deflected in a magnetic field and caused, while still in the field, to impinge at 45 degrees incidence upon a target which in various tests was a plate of brass, a cleft crystal of galena and a crystal of copper. The currents to the target and to an enclosing electrode are measured as the target is revolved about the direction of incidence, and are found to be independent of azimuth. This result is susceptible of two interpretations at least; it may mean that the incident beam is not polarized by the magnetic field, or it may mean that none of the targets serves as an analyser. The latter interpretation, which leaves unanswered the question of polarization in a magnetic field, is consistent with the result which we have obtained.

⁸ Cox, McIlwraith & Kurrelmeyer, Proc. Nat. Acad. Sci., 14, 544 (1928).

⁹ Darwin, Nature, 122, 630 (1928).

¹⁰ A brief account of these experiments has appeared recently in the Comptes Rendus; Joffé and Arsenieva, C. R. 188, 152 (1929).

¹¹ Wolf, Zeit. f. Phys., 52, 314 (1928).

The question of the result to be expected from the wave theory of the electron in experiments of the kind here described has recently been considered by Darwin.¹² The conclusion which Darwin reaches is that a beam of electrons initially unpolarized will remain unpolarized after diffraction by a grating provided the forces in the grating responsible for the scattering are electric rather than magnetic, and that therefore any experiment designed to detect polarization by successive reflections from crystals can lead only to a negative result. The result of our experiment is in accord with this prediction.

It is a pleasure to express our best thanks to Mr. G. E. Reitter for the great care with which he constructed the special apparatus used in this experiment, and to Mr. C. J. Calbick for valuable assistance in collecting and reducing the data.

¹² Darwin, Proc. Roy. Soc., 120, 631 (1928).