A High Precision Standard of Frequency 1

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Synopsis: A new standard of frequency is described in which three 100,000 cycle quartz crystal-controlled oscillators of very high constancy are employed. These are interchecked automatically and continuously with a precision of about one part in one hundred million. They are checked daily in terms of radio time signals by the usual method employing a clock controlled by current maintained at a submultiple of the crystal frequency. Specially shaped crystals are used which have been adjusted to have temperature coefficients less than 0.0001 per cent per degree C.

O meet the demands for increased precision in measurement and greater reliability of operation a new reference standard frequency system has been developed in the Bell Telephone Laboratories having an absolute accuracy that may be relied upon at all times to better than one part in a million. This reference standard is similar in many respects to one described by J. W. Horton and W. A. Marrison a little over a year ago,² but a number of important changes have been made which have contributed to increased accuracy and reliability.

The standard is based on the quartz crystal-controlled oscillator, with a synchronous motor-driven clock, used to determine its rate. It differs from others of the same general type in having a number of similar crystal-controlled oscillators which may be interchanged at will and which are intercompared continuously and automatically with a precision of one part in one hundred million. A number of improvements have been made in the crystal and mounting, and in the circuit, which justify this precision of measurement.

By far the most important element in a crystal-controlled oscillator is the crystal itself and great care was taken in selecting the type to be used in the new standard. A crystal was required as nearly independent as possible of ordinary variations in temperature and pressure and which could be mounted so as to vibrate freely. effect of temperature appeared to be especially serious as the changes in frequency thus obtained with an ordinary crystal, even with the best commercial thermal regulators available, are greater than are caused by any other single factor in the new standard.

It has been known for some time that plates of quartz cut in the plane of the optic and electric axes usually have positive temperature coefficients and that plates cut in the plane of the optic axis but perpendicular to an electric axis have negative coefficients. It has

¹ Presented before Institute of Radio Engineers, April 3, 1929. ² "Precision Determination of Frequency," by J. W. Horton and W. A. Marrison, Proceedings of Institute of Radio Engineers, Vol. 16, pp. 137–154, Feb. 1928.

also been known that oscillations may be produced in a crystal either parallel to the impressed electric field or perpendicular to it, the so-called longitudinal and transverse effects. There is a certain amount of mechanical coupling between such different modes of vibration within the crystal, more or less close, depending upon the shape, and in particular depending upon the ratio of dimensions in the principal directions of vibration. In view of these facts it was thought probable that crystals could be produced with such coupling between the modes which have inherently positive and negative coefficients that the resultant temperature coefficient would be nil.

Series of crystals of rectangular and circular shape were made to test this fundamental assumption. It was found that the temperature coefficient does vary with the shape of a resonator and, in particular, that crystals may be proportioned so as to have a coefficient that is practically nil. The relations between the temperature coefficient



Fig. 1—Crystals used in preliminary temperature coefficient tests.

and the dimensions in the case of rectangular plates have been further studied in detail by F. R. Lack ³ of the Bell Telephone Laboratories.

In the first experiment performed with circular discs for this study a large disc was first cut and smaller ones cut from it, after measurement, to insure constancy of material, thickness and orientation with respect to the crystal axes. The parts remaining after three sizes of discs had been cut in this way, with the remainder of the slab from which they were obtained, are shown in Fig. 1. The slab is shown in the partly assembled original crystal in Fig. 2 to show the manner of cutting. With such circular discs it was found that at least one diameter could be found for which the temperature coefficient is very small throughout the entire room temperature range.

Low temperature coefficient crystals obtained in this way are subject to the usual mounting difficulties, namely that the friction on the mounting considerably increases the decrement, and by an amount

³ "Observations on Modes of Vibrations and Temperature Coefficients of Quartz Plates," by F. R. Lack, presented before the Institute of Radio Engineers, April 3, 1929.

which may vary with time. A form of crystal is desired which can be mounted so that the parts vibrating at relatively large amplitude do not bear heavily on any portion of the mounting.

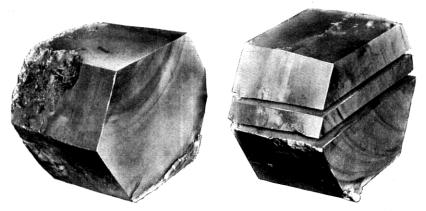


Fig. 2—Partly assembled crystal showing the relation of the slab to the crystal axes.

Further study of temperature coefficients showed that the rings remaining after the small discs had been cut from the larger one,

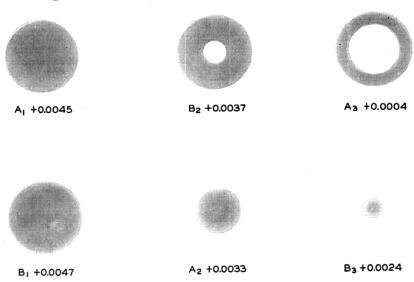


Fig. 3-Temperature coefficients of some discs and rings.

shown in Fig. 1, have a temperature coefficient lower than discs of the same diameter and thickness. This is further illustrated in Fig. 3 which gives the temperature coefficient of two discs and the four

parts remaining after holes of different diameters had been trepanned in them.

It is possible to make ring-shaped crystals having negligible temperature coefficients in a considerable range of frequencies, and, since the ring shape permits of an improved method of mounting in which there is very little friction on the holder, they have been adopted for use in the present standard. Such a crystal having a frequency of 100,000

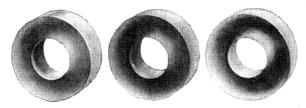


Fig. 4—Three 100,000-cycle low temperature coefficient rings used in frequency standard.

cycles is of substantial size and is reasonably easy to make and adjust. Three of the crystals used in the present standard, adjusted to 100,000 cycles, and having temperature coefficients less than one part in a million per degree C., are shown in Fig. 4.

The variation of frequency with temperature for one of the ringshaped crystals is given in Fig. 5, showing that it is very small over

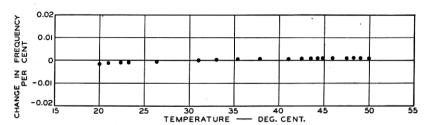


Fig. 5—Variation of frequency with temperature for a 100,000-cycle ring crystal adjusted for low temperature coefficient.

the usual room temperature range. All of the ring-shaped 100,000-cycle crystals made thus far are alike in having a coefficient which is small throughout this range.⁴ The temperature coefficient of a disc of the same frequency having the same outside dimensions as the 100,000-cycle rings, is approximately 30 parts in a million per degree C., more than thirty times that of the adjusted crystal.

⁴ Where an accuracy of the order of only one part in 100,000 is desired, as in some portable standards, such a crystal could be employed without any form of temperature control.

The manner in which the ring-shaped crystals are mounted in their operating position is shown in Fig. 6. The hole is shaped so that when the crystal hangs on a horizontal cylinder the point of contact is at a theoretical node for mechanical vibration. There is evidence of slight vibration where the central plane intersects the double conical hole but it is small in comparison with that obtained at the outer surface where a crystal is usually supported. The decrement of the crystal when so mounted is considerably less than when it is supported on one of its plane surfaces.

In the mounting the crystal is spaced from the electrodes and is kept approximately central by means of paper spacers on each side. The crystal is free to move laterally in a narrow region but, since it is

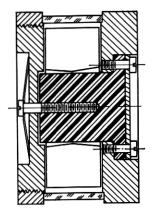


Fig. 6—Section of crystal mounting showing point support.

centrally located, the frequency is at a maximum value and hence a slight motion of the crystal to either side has only a second order effect on the frequency.

The variation of frequency with total electrode spacing is appreciable, but variations due to this factor are avoided by keeping the electrodes accurately spaced by means of a ring of pyrex glass. The temperature coefficient of pyrex is about one quarter of that of crystal quartz perpendicular to the optic axis, so the variation in spacing that is obtained is due almost entirely to the expansion of the crystal. The effect on the frequency due to the differential thermal expansion of the crystal and crystal holder is, however, less than one part in 107 per degree C. and so it may be neglected. If it is desired to eliminate this effect entirely a spacer should be used having the same temperature coefficient of expansion as quartz perpendicular to the

optic axis, but for practical purposes a material such as pyrex glass or fused quartz is entirely satisfactory.

The crystal holder is constructed so that a slight variation can be made in the total spacing between electrodes. Since the frequency varies with electrode spacing this can be used for making a slight adjustment of frequency. The thread on the adjustable plate is kept tight by spring tension to prevent the spacing from varying irregularly. A crystal holder and a 100,000-cycle crystal are shown in Fig. 7.

Even though the crystal and its mounting have very low temperature coefficients, it is desirable to control their temperature, for which

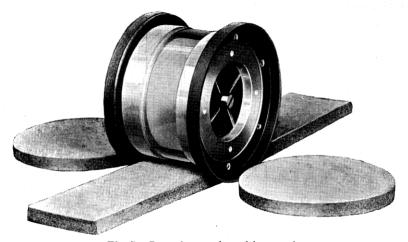


Fig. 7—Crystal mounting with crystal.

purpose the temperature controlling device shown in Fig. 8 has been constructed. It consists of a cylindrical aluminum shell with a wall about one inch thick, with a heater (not shown), and with a temperature responsive element in the wall to control the rate of heating. The aluminum shell has a metal plug that screws into the open end forming a chamber for the crystal which is then completely closed except for a small hole for electrical connections.

Since aluminum is a good thermal conductor the shell equalizes the temperature throughout the chamber and thus avoids the use of a fluid bath. The main heating coil is wound in a single layer over the whole curved surface of the aluminum cylinder, being separated from it only by the necessary electrical insulation. Auxiliary heating coils are wound also on the ends so as to distribute the heating as uniformly as possible. This, in effect, makes the short cylinder behave like a

section from an infinite cylinder. To protect the thermostat from the effect of ambient temperature gradients the heating coil has an outside covering consisting of four layers each of thin felt and sheet

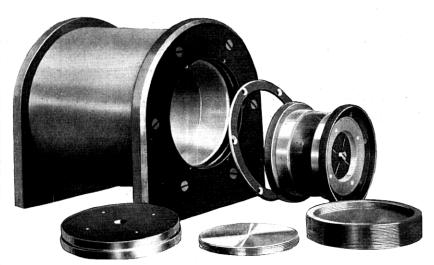


Fig. 8—Temperature control chamber with crystal mounting.

copper spirally wound so that alternate layers are of copper and felt, the innermost layer being of felt and the outer one of copper. This is the covering that appears on the complete device shown in Fig. 9.

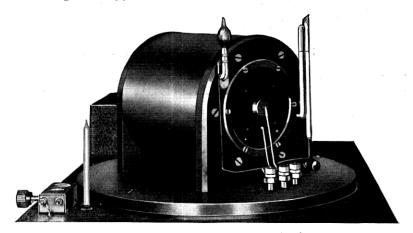


Fig. 9—Complete temperature control unit.

This covering is very effective in reducing surface gradients since the conductivity in directions parallel to, and perpendicular to, the surface differ by a large ratio.

The temperature of the shell rises and falls periodically by about 0.02° C. but even this variation is prevented from reaching the crystal in its mounting by a layer of felt about half a centimeter thick surrounding the crystal holder. At the period of thermostat operation obtained the temperature variations actually reaching the crystal are reduced more than a thousand-fold. The complete temperature controlling device is shown mounted in its operating position in Fig. 9. One of the mounted crystals wrapped in its felt protecting layer is shown in Fig. 10.

To protect the resonator from humidity and pressure variations it is kept under a bell jar at a pressure slightly below atmospheric.

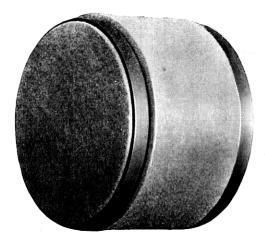


Fig. 10—Crystal mounting with felt insulation.

With the crystals used the frequency varies approximately one part in a million for 10 cm. of mercury change in pressure. It is aimed, therefore, to maintain the pressure constant to about \pm 1 mm. A small mercury gauge within the bell jar indicates the pressure, which may be adjusted by a vacuum pump through a valve in the surface plate. The pressure within the bell jar is affected somewhat by the temperature, and in order to keep it within the required limits it is necessary to maintain a rough control of the temperature within the jar. The pressure gauge does not indicate a change in pressure due to a change in temperature but will indicate any slow leak into the jar that may develop. A thermometer within the bell jar indicates the temperature, from which the change of pressure, and the correction of frequency due to it, may be computed if desired.

Since the frequency varies with the pressure surrounding the crystal an approximate adjustment of the frequency may be made conveniently by an adjustment of pressure.

The circuit of the crystal-controlled oscillator and the first amplifier stages is shown in Fig. 11. The oscillator is of the familiar type in which the crystal electrodes are connected to grid and ground and in which a tuned plate circuit is used. The great advantage in being able to ground one electrode was the major consideration in choosing this circuit. With this circuit, as in the one described a year ago,² it has been found possible to choose plate tuning elements such that slight variations in either the inductance or the capacity have little effect upon the frequency. For certain values of inductance and

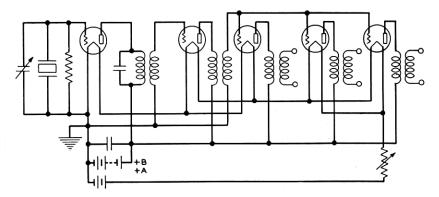


Fig. 11—Circuit of standard frequency oscillator.

capacity the frequency, as a function of their product, takes on a maximum value. The adjustment that gives the maximum value of frequency is used, therefore, so that slight variations, such as those due to temperature coefficient and aging of the tuning elements, will have a negligible effect on the frequency.

The output circuit of the oscillator is very loosely coupled to three independent output amplifiers. This arrangement provides three independent output circuits free from mutual interference and unable to react to an appreciable extent on the crystal oscillator.

The final adjustment of frequency is made with a small cylindrical condenser, having a capacity of about 5 mmf., connected in parallel with the crystal electrodes. The size of this condenser is chosen such that an adjustment of one division on the dial corresponds to a change of frequency of about one part in a hundred million. There are 100 divisions on the dial and a total of 10 turns may be made

corresponding to a total possible adjustment of about one part in 100,000. This condenser is shown at C in the circuit drawing.

The oscillator circuit, showing the tubes and transformers, the plate tuning elements, the filament and plate current meters, and the frequency adjusting condenser is shown in Fig. 12. The adjusting condenser is mounted between the meters and is controlled by the large knob and dial.

One complete oscillator unit, consisting of a 100,000-cycle crystal controlled oscillator with three independent 100,000-cycle outputs,

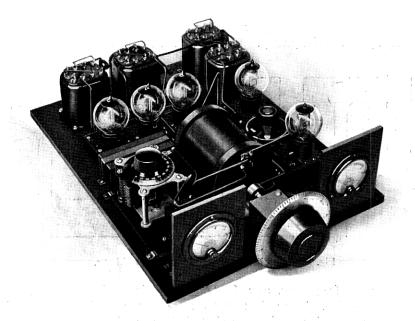


Fig. 12—Standard frequency oscillator without shield.

having a self-contained temperature and pressure controlled crystal, and having a temperature controlled, electrically shielded circuit, is shown in Fig. 13.

The submultiple generator circuit that is used to obtain outputs at 10,000 cycles and 1,000 cycles is shown in Fig. 14. It consists of an inherently unstable vacuum tube oscillator with the tube operating on the curved part of its characteristic. The frequency of this oscillator may be controlled readily by any frequency which is a small multiple or submultiple of it. In this instance the oscillator is controlled by an input having the frequency of its tenth harmonic, the controlling high-frequency input being resistance coupled into the

plate circuit of the lower frequency oscillator. The frequency of the controlled oscillator remains indefinitely at an exact submultiple of the controlling frequency.

Two such circuits are used, one to obtain current at 10,000 cycles and one for 1,000 cycles. Of course, additional amplifier circuits are required in order to supply outputs of considerable magnitude at



Fig. 13—One complete 100,000-cycle standard frequency unit.

these frequencies, and in such a way that there can be no reaction on the controlling circuits due to load variations or to stray currents at other frequencies fed backward through the output circuits.

A 1,000-cycle motor, operated by current controlled at the 100th submultiple of the standard, drives generators for producing current at 100 cycles and 10 cycles. There are available, therefore, frequencies

in decade steps from 100,000 cycles to 10, all controlled by the 100,000-cycle primary oscillator.

The 1,000-cycle motor is geared to a clock in such a way that, when the controlling frequency has its nominal value exactly, the clock keeps accurate time. In order to check the frequency of the system, therefore, it is only necessary to observe changes in rate of the clock so controlled.

An error of 0.864 second per day in the rate of the clock corresponds to an error in the frequency controlling it of one part in 100,000. It is possible to check the rate of the clock visually with an accuracy of about 0.2 second from audible time signals but obviously this is not sufficiently accurate for our purpose, giving an accuracy of only about one part in 400,000 in a day's observation. In order to facilitate the

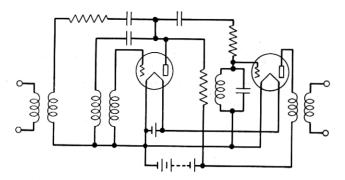


Fig. 14—Circuit of submultiple generator.

comparison with time signals, a contact operated by a cam driven by the 1,000-cycle synchronous motor makes a contact once each second, or to be exact, once for each 100,000 cycles of the primary oscillator. This contact operates one element of a two element recorder while time signals operate the other. Comparisons may thus be made by actual measurements on tape and can be made with greater accuracy than can be judged by eye.

The 1,000-cycle synchronous motor, with its two generators and induction starting motor geared to the clock, is shown in Fig. 15. In this figure the seconds contact mechanism may be seen on the vertical shaft intermediate between the shaft of the motor and the second-hand shaft of the clock.

The assembled rotor of this motor is shown in Fig. 16. The large disc is the 1,000-cycle motor rotor. The disc below it is a hollow steel flywheel filled with mercury used to reduce hunting. The small

rotor below the flywheel is the rotor of the unipolar 10-cycle generator. The disc above the motor rotor is the armature of the 100-cycle generator. The squirrel cage armature of an induction motor for starting is immediately above the 100-cycle generator rotor.

A single constant frequency generator is no longer sufficiently reliable as a standard of frequency of high precision and, as has been the practice where accurate time standards are maintained, three



Fig. 15-1,000-cycle synchronous motor, generators, and clock.

similar units have been installed. Means are provided for interchecking them continuously and automatically with the highest precision justified. The use of three such generators with means for interchecking them makes it possible to determine very quickly if and when one generator fails to operate properly.

Only one submultiple generator, clock, and multiple output amplifier is provided, but a special 3-way switch is used by means of which any one of the three primary oscillators may be selected and used as the controlling unit. The oscillators may be interchanged in any order without interrupting the circuits controlled by them.

In the method used for automatic interchecking a fourth oscillator unit is used, identical with the other three except that the frequency is maintained at a slightly different value. The difference between the frequency of this oscillator and that of the other three is kept at about 1 cycle in 10 seconds. The number of beats between the fourth oscillator and each of the other three oscillators is recorded automatically during each 1,000 second interval. The number of beats thus recorded is approximately 100 during each interval.

In 1,000 seconds each oscillator generates approximately one hundred million waves. The numbers that are recorded are, therefore, the number of parts in one hundred million by which oscillator No. 4

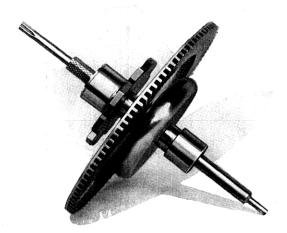


Fig. 16—Rotor of motor-generator.

differs from each of the three primary oscillators during the interval. If the numbers recorded during successive intervals remain the same, the oscillators either did not vary, or they all varied in the same direction by the same number of cycles. If the numbers recorded in successive intervals vary by say 1, 2 or 5, it means that the oscillators have drifted, relative to each other, by so many parts in one hundred million.

Designating the frequencies of the four oscillators by

$$f_1, f_2, f_3, f_4,$$
 (1)

the three numbers recorded are

$$1,000 (f_4 - f_1), 1,000 (f_4 - f_2), 1,000 (f_4 - f_3).$$
 (2)

The mean of these numbers is

$$1,000\left(f_4 - \frac{f_1 + f_2 + f_3}{3}\right) \tag{3}$$

If we subtract each of the original three recorded numbers from the mean we obtain

$$1,000\left(f_1 - \frac{f_1 + f_2 + f_3}{3}\right) = \delta_1,\tag{4}$$

$$1,000\left(f_2 - \frac{f_1 + f_2 + f_3}{3}\right) = \delta_2,\tag{5}$$

$$1,000\left(f_3 - \frac{f_1 + f_2 + f_3}{3}\right) = \delta_3. \tag{6}$$

Thus we may compute readily the performance of each of the four oscillators referred to the mean of the three similar primary oscillators. It is obvious that the accuracy of intercomparison of the three similar oscillators does not in any way depend upon the constancy of oscillator No. 4. For convenience in reducing the results, however, it is controlled as carefully as the others.

The records and computed results for approximately ten hours are given in Table 1. During this time the largest relative variation between any two of the four oscillators taken in pairs was 5 parts in 108. The random variations between 1,000 second periods appear to be in the order of one or two parts in a hundred million. These random variations are superposed on slow drifts of a quasi-periodic nature probably caused by temperature changes in the circuit and amounting to less than one part in ten million. In addition to these effects a slow, steady drift is expected due to a settling-down of the oscillator circuit and the crystal in its mounting as well as due to aging of the vacuum tubes and even of the crystal itself. The effects of aging can, of course, only be determined after long continued operation.

It is preferable in some cases to refer the performance of each of the four oscillators to the mean performance of all four. This is in the event that all four oscillators are equally reliable in which case the mean of all four makes a better reference standard than the mean of any three. If we designate the numbers

1,000
$$(f_4 - f_1)$$
, 1,000 $(f_4 - f_2)$, and 1,000 $(f_4 - f_3)$

TABLE 1

A Ten-Hour Record Obtained by Means of the Beat Recorder.

The columns δ_1 , δ_2 and δ_3 indicate the difference between each oscillator and the mean of the three during each 1,000 second interval expressed in parts in one hundred million

million. Serial	Mean	(f_4-f_3) δ_3	(f_4-f_2)	δ_2	(f_4-f_1) δ_1
45	86	92 +		+12	69 –17
44	86	92 +		+12	$\frac{68}{68}$ -18
43	87	92 +		$+12 \\ +12$	6018
42	87	93 +	5 99	$^{+12}_{+12}$	70 -17
41	86	93 +		$^{+12}_{+13}$	68 -18
40	87	92 + 94 +	7 98	$^{+13}_{+11}$	68 -19
39	86	93 +	7 99 7 99	$^{+11}_{+13}$	$\begin{array}{ccc} & -19 \\ & 67 & -19 \end{array}$
38	87	93 + 94 +	7 99	$^{+13}_{+12}$	$\begin{array}{ccc} 67 & -19 \\ -20 & -20 \end{array}$
37	86	93 +	7 99	+13	79 —17 68 —18 68 —19 67 —19 67 —20 67 —19
36	87	95 +	8 99	$^{+13}_{+12}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
35	86	93 +	7 98	$^{-12}_{+12}$	67 -19
34	87	95 +	3 100	$^{+12}_{+13}$	66 -21
33	87	95 + 94 +	7 100	$^{+13}_{+13}$	66 -21
32	87	95 +	3 101	$^{+13}_{+13}$	66 -21 66 -21 65 -22 66 -21 65 -22 66 -21 65 -22 66 -21 65 -22 66 -21 66 -22 68 -20 68 -21 68 -21
31	87	95 +	3 101	$^{+13}_{+14}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
30	87	94 +	7 101	+14	$\frac{66}{6}$ -21
29	87	94 +	7 101	+14	$\frac{-21}{65}$
28	87	95 +	3 101	+14	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
27	87	95 + 94 +	7 102	+15	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
26	87	95 +	101	+14	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
25	88	94 +	101	+15	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
24	88	94 +		+15	68 -20
23	89	96 +	103	+14	$\frac{68}{68}$ -21
22	89	95 +	103	+14	$\frac{68}{68}$ -21
$\overline{21}$	89	95 +	103	+14	68 - 21
20	90	95 +	105	+15	69 - 21
19	89	95 +	103	+14	69 - 20
18	89	95 +	104	+15	68 -21
17	88	94 +		+14	68 -21 69 -21 69 -20 68 -21 68 -20 69 -20 69 -20 69 -20 69 -20
16	89	95 +	104	+15	69 -20
15	89	95 +	5 102	+13	69 -20
14	89	95 +	103 102	+14	69 -20
13	89	95 +	102	+13	69 -20
12	89	94 +	103	+14	$ \begin{array}{ccc} 70 & -19 \\ 69 & -20 \end{array} $
11	89	95 +	5 103	+14	69 -20
10	90	95 +	103	+13	71 -19 70 -19
	89	95 +	5 103	+14	70 -19
9 8 7	89	95 +	103	+14	69 -20
7	89	94 +	103	+14	70 -19

by a, b, and c respectively, it can be shown readily that the numbers

$$a - \frac{a+b+c}{4}$$
, $b - \frac{a+b+c}{4}$, $c - \frac{a+b+c}{4}$ and $-\frac{a+b+c}{4}$ (7)

represent the difference between each of the oscillators Nos. 1, 2, 3 and 4 respectively and the mean of all four, expressed in parts, in one hundred million. This method treats all four oscillators symmetrically. The symmetry is evident if we substitute in (7) the values (2) assigned to a, b, and c, whence we get:

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$$a - \frac{a+b+c}{4} = 1,000 \left(\frac{f_1 + f_2 + f_3 + f_4}{4} - f_1 \right),$$
 (8)

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$$b - \frac{a+b+c}{4} = 1,000 \left(\frac{f_1 + f_2 + f_3 + f_4}{4} - f_2 \right), \tag{9}$$

$$c - \frac{a+b+c}{4} = 1,000 \left(\frac{f_1 + f_2 + f_3 + f_4}{4} - f_3 \right),$$
 (10)

$$-\frac{a+b+c}{4} = 1,000 \left(\frac{f_1 + f_2 + f_3 + f_4}{4} - f_4 \right). \tag{11}$$



Fig. 17—Four 100,000-cycle oscillators with auxiliary equipment.

The four oscillators used in the equipment described are shown in Fig. 17. The panels on which the controlling and measuring circuits are mounted are at the right of the picture. The 1,000-cycle motor-driven clock is at the top of the nearest panel. A schematic of the apparatus showing the general arrangement of parts is given in Fig. 18.

The circuit of one element of the modulator for producing low-frequency beats is shown in Fig. 19. The input circuits A and B are supplied from oscillator No. 4 and one of the other three, respectively. The plate circuit includes the windings of a balanced relay

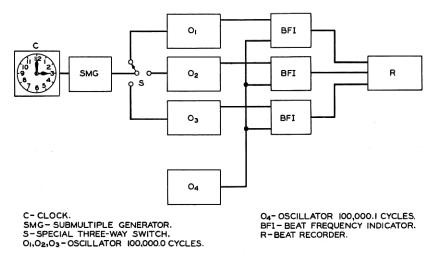


Fig. 18-Schematic of complete frequency standard system.

which makes a contact once for each cycle difference between the input frequencies at A and B and which operates the recording mechanism accordingly.

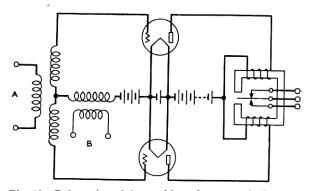


Fig. 19—Balanced modulator of beat frequency indicator.

The beat recorder is a counter arranged to count these relay operations for a definite time interval, in this case for 1,000 seconds, and then to print the total and reset to zero. Five such units are provided which print on a wide strip of paper similar to that used in

an adding machine. Three of the counters are used as outlined above. The fourth counter is to be used for recording the mean of these three numbers, computed automatically by an auxiliary device. The remaining one is a serial counter which is used to record the time, either directly, or by numbering the 1,000 second intervals consecutively.

The five element counter is shown in Fig. 20 with the cover removed to show part of the mechanism. The energy for actuating the counting and resetting mechanism is obtained from a small motor running continuously. These elements are operated at the proper times by clutches controlled by electro-magnets which are selected by the relays in the modulator circuits described above. The counting, printing,

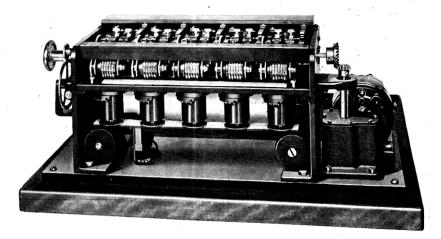


Fig. 20-Automatic beat counter.

and resetting operations are interlocked by means of cams and relays so that no counts may be missed through superposition of operations. The electrical circuit of one unit of the recorder is shown in Fig. 21.

The 1,000 second intervals are determined by a cam operated by the 1,000-cycle synchronous motor. It might be questioned whether one of the crystals being checked should be used to determine the 1,000 second intervals. No serious error arises from this, however, since the percentage variation in the interval due to a change in rate of the crystal is only one millionth of the percentage variation in the recorded beat number. Thus, using one crystal to determine the intervals, used in comparing its rate with other crystals, makes the final measurement subject to an error from this cause of only about 0.0001 per cent.

In order to determine extremely small relative variations in frequency between two oscillators a method of measurement is used in which the duration of individual beats between two oscillators may be measured with an accuracy of about one part in 10,000. This is equivalent to saying that the average beat frequency during each 10 second interval is measured with an accuracy of one part in ten thousand. Since the percentage error in the beat frequency is one million times greater than the percentage error in the primary frequency, to measure the beat frequency with an accuracy of one part

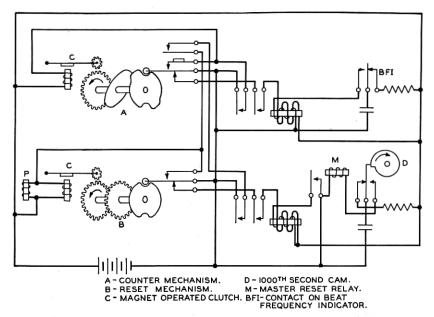


Fig. 21—Electrical circuit of one element of beat counter.

in ten thousand is equivalent to intercomparing the primary frequencies with a precision of one part in ten thousand million. By such a precise method of measurement a great deal can be learned about the nature of the variations that do occur.

The circuit of this device is shown in Fig. 22. Fig. 23 is a photograph of the actual apparatus used in obtaining the data given in Fig. 24. A circular transparent scale S, having 100 numbered divisions, is driven at 10 revolutions per second by a 1,000-cycle synchronous motor controlled by one of the crystals. A modulator operates the balanced relay at the difference frequency between the crystal controlled oscillators. This relay discharges condenser C_1

through the primary of an induction coil, which discharge produces a spark across the gap below the circular scale. The portion of the scale illuminated by the spark is photographed on slowly moving

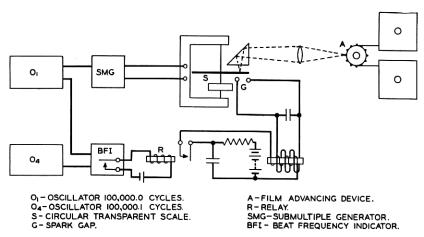


Fig. 22—Circuit of device for determining beat periods accurately.

film at A. In this manner, assuming a beat frequency of 0.1 cycle per second, 360 checks per hour may be obtained with practically no supervision.

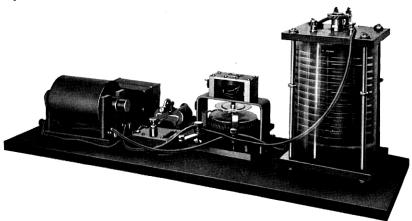


Fig. 23—Apparatus used for measuring beat periods.

If the sparks occur at exactly an even number of revolutions of the motor apart, the same portion of the scale will be photographed by each spark. If the intervals differ from such a value by 0.001 second, the successive photographic images will differ by one scale division.

Thus, the length of the beat periods may be read directly from the photographic records with an accuracy of 0.001 second, which determines the length of the 10 second periods with an accuracy of one part in ten thousand. Thus a variation of one division on the photographic record corresponds to a relative variation of one part in ten billion between the two frequencies, compared during an interval only ten seconds long. The whole number of revolutions of the scale may be determined readily by auxiliary means.

The two graphs in Fig. 24 show typical variations between two crystal oscillators operating under rather unfavorable conditions. One crystal was not in its sealed bell jar and one circuit was only

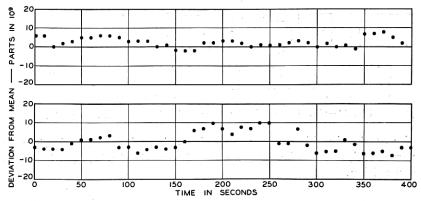


Fig. 24—Relative rates of two pairs of crystal oscillators showing small random variations in frequency.

partially shielded and was exposed to draughts of varying temperature. Even under these conditions, however, the variations from the mean did not exceed one part in 10⁸ during the test.

The checking methods just described are intended primarily to indicate more or less rapid changes in frequency. Slow changes, and, of course, the absolute rate, can best be determined in terms of standard time. This is done, as previously indicated, by checking the rate of a clock controlled by one of the crystals against radio time signals. Unfortunately no long checks have been obtained as yet in this way, but several tests made over periods of a few days indicate a constancy of rate in the order of 0.01 second a day.

The measurements made so far indicate that the frequency of a crystal controlled oscillator such as described when suitably controlled, may be expected to be constant to at least one part in 10⁷ over periods of seconds or over periods of days. It is hoped that it will be possible in the near future to present accumulated data on the performance of the frequency standard system described.