Optimum Reverberation Time for Auditoriums

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The suggestion is made that the sound damping material in an auditorium should be such that the loudness of tones will decay at the same rate for all frequencies. To attain this the reverberation time at 80 cycles must be twice what it is at 1000 cycles.

The change of optimum reverberation time with volume is shown to be derivable from a single hypothesis.

I. Reverberation Time vs. Frequency

THERE is very little published data in regard to the change in reverberation time with frequency in auditoriums which are considered near ideal. It is often mentioned by engineers and physicists that to secure the best acoustical results, the reverberation time should be the same for all frequencies in any one room. This specifies that the sensation level shall decay at the same rate for all frequencies of interest.

It seems more reasonable, however, to specify that the loudness of all pure tones shall decay at the same rate for all frequencies since it is the loudness of a tone which takes into consideration not only the energy level but also its ultimate effect upon one's brain. In Fig. 1[2] are plotted data which show the relation between the loudness as judged by a considerable number of observers and the sensation level. It will be seen that for frequencies between 700 and 4000 cycles per second these two quantities are equal to each other so that the two points of view mentioned above demand identical conditions throughout this frequency band. Outside of this band, however, any change in the sensation level gives a greater change in the loudness, as may be seen.

The maximum loudness in which we are interested at present is about 73.[3] In the figure the curves may be replaced by straight lines which represent fair approximations to the observed data up to this loudness. This family of straight lines may be represented by the expression

$$L_t = A_f S_t, \tag{1}$$

where A_f is the slope of the line adopted to fit the data for the fre-

[1] Presented before Acoustical Soc. of Amer., Dec., 1929. Jour. Acou. Soc. Amer., Jan., 1930.

[2] This is Fig. 108 from "Speech and Hearing" by H. Fletcher.

[3] This is the loudness that the source chosen in Part II of this paper will produce in a room of 1000 cubic feet having a reverberation time of 0.8 seconds.

quency f. The values of A_f chosen from this figure are given by the next, Fig. 2. This approximation simplifies our calculations very much and introduces errors which are not intolerable.

Referring back to Fig. 1, if we wish to adjust the absorption of the room so that the loudness of all pure tones will decay at the same rate, say for the moment 60 units per second, it is seen that the sensation level must drop 60 db per second for frequencies between 700 and

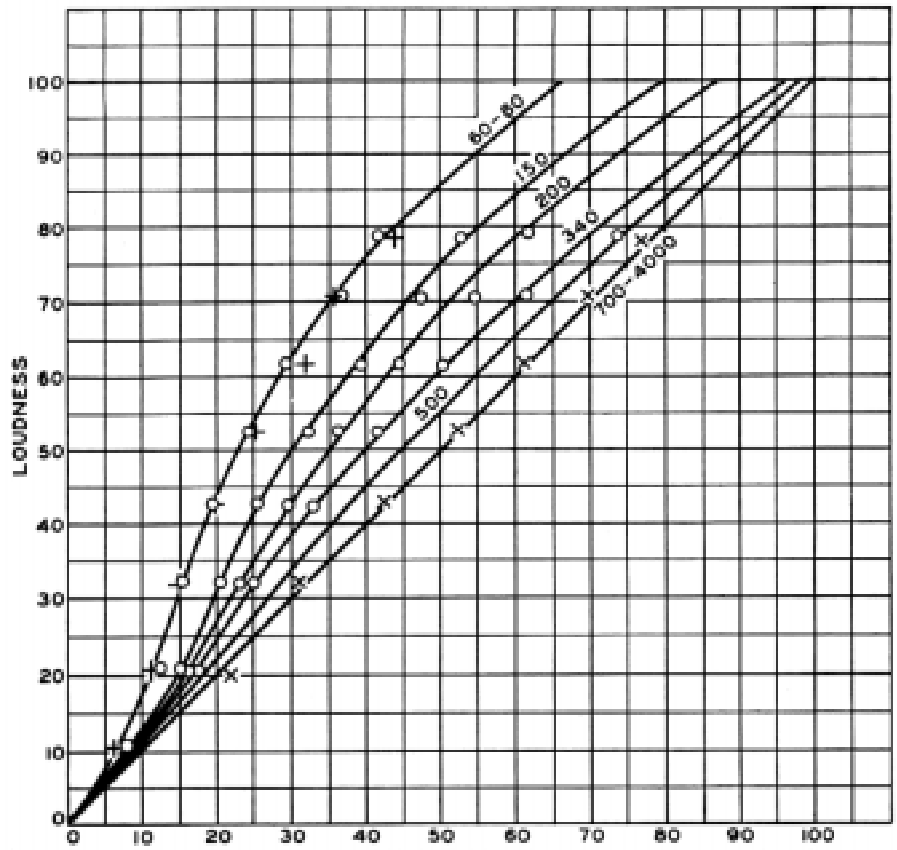

Fig. 1.—Loudness of pure tones.

4000 cycles and for other frequencies the sensation level must drop $60/A_f$ db per second; or in other words, the reverberation time for frequencies between 700 and 4000 cycles should be one second and outside of this band it should be A_f seconds. Fig. 2, then, which is a plot of A_f vs. frequency now becomes also an illustration of the shape of the reverberation time vs. frequency curve which a room should have in order that the loudness of pure tones of all frequencies shall decay at the same rate.

According to Sabine's well known formula the reverberation time is inversely proportional to the number of absorption units in the room so that, if we assume this, we may immediately infer the shape of the curve which represents the number of absorption units necessary at any frequency, referred to the amount required at 1000 cycles, to obtain our required condition. These values are plotted in Fig. 3. If it should happen that the greater part of the sound absorption in a room is caused by one particular kind of surface, then the curve in Fig. 3 is the shape of the absorption curve that this material should have.

A pertinent observation on which every one seems to agree is that if

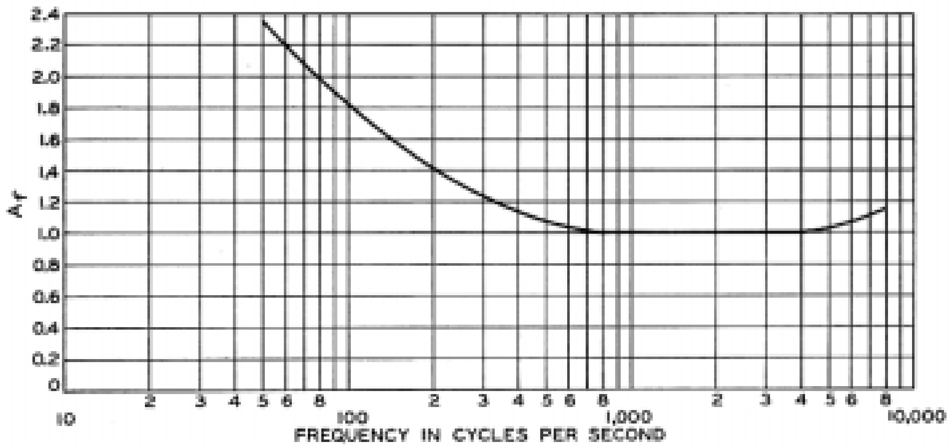

Fig. 2.—Values of A_f vs. frequency.

an auditorium has an unusually long reverberation time and consequently is of little use, when empty, it attains excellent acoustic conditions when filled with a large audience. In these cases a very large part of the absorption is caused by the audience. The absorption of an average audience has been measured by W. C. Sabine [3] and his results are also plotted in Fig. 3. The close agreement between this curve and the one we have obtained from our hypothesis gives considerable confidence in our general viewpoint.

II. Reverberation Time vs. Volume

It is generally accepted that the best acoustical conditions in a room are obtained when the reverberation time is adjusted to a definite value known as the optimum reverberation time. Observations reported in literature agree that the value of the optimum reverberation time in-

[3] "Collected Papers on Acoustics," page 86.

creases with the size of the room in the way shown in Fig. 4 where the curves are the choices reported by Watson,[4] Lifschitz,[5] and Sabine.[6] These experimental results have served as the basis of successful adjustment and design of many auditoriums. One naturally seeks the factor which determines a choice of reverberation time of two seconds for a million cubic feet theatre and on the other hand a choice of near one second for a 10,000 cubic foot music room. It is our purpose now to point out the factor which apparently does this.

We will set down a condition which we believe to be this factor and then will show that the requirements demanded by it agree quite

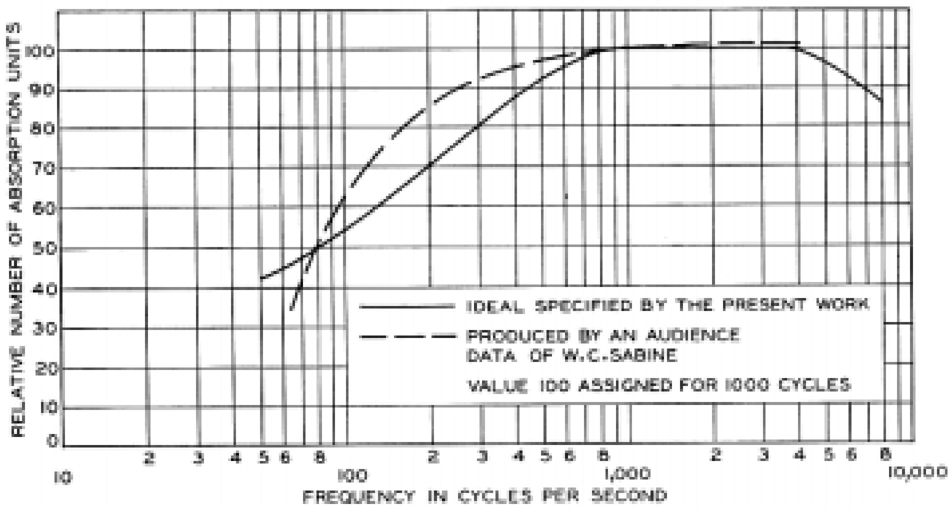

Fig. 3.—Relative number of absorption units vs. frequency.

closely with the empirical results illustrated in Fig. 4 and mentioned above. The condition is

$$\int_{t_0}^{t_1} L_t dt = -K, \tag{2}$$

in which t_0 is the time a sustained source of sound $\bar{E}$ is cut off, t_1 the time the sound becomes inaudible, L_t the loudness of the sound at any instant t, and K a constant. As shown in Fig. 1, the loudness of a one thousand cycle note is equal to the sensation level, that is,

$$L_t = S_t \text{ for 1000 cycles.}$$

[4] Watson, *Architecture*, May, 1927.

[5] Lifschitz, *Phys. Rev.*, 27, 618, 1926.

[6] Sabine, *Trans. of S.M.P.E.*, XII, 35, 1928.

Since, during the time of decay, S_t decreases uniformly with time, and therefore L_t also, then for a thousand cycle note, evaluating our integral we have

$$L_{t_0}T_1 = 2K \tag{3}$$

or

$$S_{t_0}T_1 = 2K,$$

where

$$T_1 = t_1 - t_0.$$

This last expression is practically in the form in which this condition was first stated by Lifschitz.[5] In (3) there are three unknowns and a fourth is implied, namely, the power of the source, $\bar{E}$.

We now turn our attention to finding the relation between the volume of a room and the reverberation time dictated by the stated condition. Following P. E. Sabine let us take the rate of emission of the source, $\bar{E}$ to be 10^{10} cubic meters (35.3×10^{10} cubic feet) of sound of threshold density per second. Now[7]

$$T_1 = \frac{4V}{ca}\log_e \frac{4 \times 35.3 \times 10^{10}}{c \cdot a},$$

where V is the volume of the room in cubic feet.

c is the velocity of sound, 1120 feet per second.

a is the number of absorption units in sq. feet and[8]

$$L_{t_0} = S_{t_0} = 10 \log_{10} \frac{4 \times 35.3 \times 10^{10}}{c \cdot a}.$$

If we should substitute these values in (3) we would obtain a relation between V, a, and K which must be satisfied when condition (2) is satisfied. In other words, this relation would specify the amount of absorption, for a one thousand cycle note, a room should have if it complies with (2).

If we assume Sabine's well known formula, namely,

$$T = \frac{.05V}{a},$$

where T is the reverberation time in seconds we may express this relation in terms of V, T, and K with the result

$$10.40 + \log T_{op} - \log V = \frac{(2K)^{1/2}}{1.283T_{op}^{1/2}}, \tag{4}$$

[7] See Crandall "Theory of Vibrating Systems and Sounds," page 211.

[8] See Crandall "Theory of Vibrating Systems and Sounds," page 210, and the definition of sensation level.

where T_{op} is the value of T imposed by our condition (2) for a thousand cycle tone.

Referring to Fig. 4 it will be seen that all three observers agree rather closely that the reverberation time for an auditorium of 1,000,000 cubic feet should be 2.0 seconds. This value refers to a tone of 512 cycles, the customary frequency used for experimental observation. It has been shown above that the reverberation time for

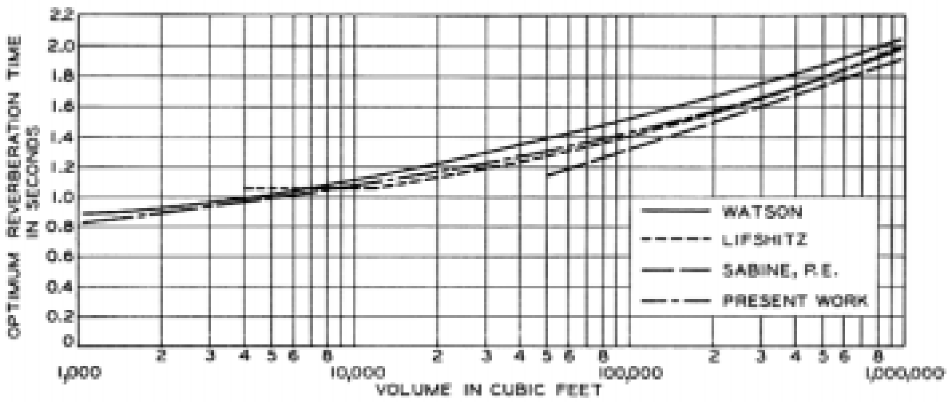

Fig. 4.—Optimum reverberation time vs. volume in cubic feet for 512 cycles.

1000 cycles should be 92.5 per cent of the reverberation time for a 512 cycle tone, so that the 2.0 seconds above corresponds to 1.85 seconds for 1000 cycles. We can evaluate K in (4) by adapting this latter value of T_{op} for a volume of 1,000,000 cubic feet. This gives $K = 32.6$. Substituting this value in (4) we obtain

$$\log V = 10.40 + \log T_{op} - \frac{6.35}{T_{op}^{1/2}}. \tag{5}$$

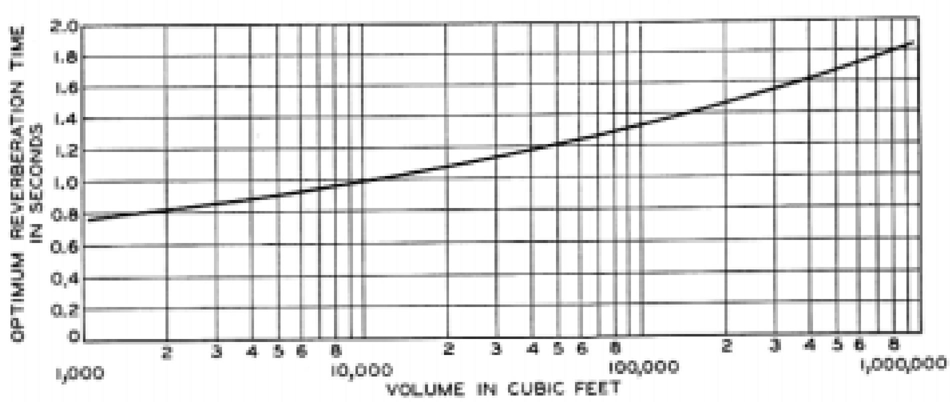

Fig. 5.—Optimum reverberation time vs. volume in cubic feet for 1000 cycles.

From (5) we may obtain T_{op} for 1000 cycles for any volume. See Fig. 5. As mentioned above these values of T_{op} are 92.5% of T_{op}

for 512 cycles so that these latter may be easily deduced for comparative purposes. These values are plotted to give curve number 4 in Fig. 4. It is seen that this curve agrees very well with those showing the choices of competent judges.

III. The More General Hypothesis

Equation (2) may be written as follows, since we have assigned a value to K:

$$\int_{t_0}^{t_1} L_t dt = -32.6 \tag{6}$$

and it will be remembered that we have considered L_{t_0} to be the loudness set up by a certain standard source. Allowing V to vary with f constant (1000 cycles) we have obtained a relation between the optimum reverberation time and volume of rooms for 1000 cycles. We wish to point out now that exactly this same condition (6) with V constant and f variable, will give the same results that we have obtained in Part I of this paper with the only further requirement that for other frequencies than 1000 cycles the strength of the source $\overline{E}$ shall be such that the loudness L_{t_0} set up in the room at the frequency considered shall be exactly the same as the loudness which our standard source would set up at 1000 cycles.

In Part I of this paper our stated condition was that the loudness of all pure tones shall decay at the same rate for all frequencies. Since we have specified that the loudness at the time t_0 shall be the same for all test frequencies and also that the loudness at the time t_1 shall be zero for all frequencies, it is quite evident that the above integral can have the same value at all these frequencies only when the loudness decays at the same rate for all frequencies concerned. In other words, this condition stated as an integral specifies exactly the same requirement on the decay of loudness that we expressed in our statement early in Part I of this paper.

IV. Conclusions

To recapitulate, we have set down an equation, together with a specification of the strength of the virtual source in each case, from which we obtain the value the reverberation time for any frequency tone should have in any sized room according to the condition which apparently controls the choice of observers.

One naturally turns to see what meaning may be attached to this significant expression, namely, the integral of the loudness taken throughout the time of decay to inaudibility. Since this integral has the same value for all auditoriums which are considered ideal, it implies that one's brain is a ballistic instrument which is concerned with not only the maximum value of loudness but also with the effect of loudness integrated throughout a considerable interval of time.