Effects of Phase Distortionon Telephone Quality *

By JOHN C. STEINBERG

This paper discusses the effects of the type of phase distortion found in low pass filters and the loaded line on telephone quality. The effects are ascribed to three factors; the first involves the slopes of the phase characteristic at various frequencies in the range of interest, the second involves the intercept values on the phase shift axis of the tangents to the phase curve, the third involves the interference caused by portions of one sound overlapping portions of a succeeding sound. The first factor appears to be the one of most importance.

IN the engineering of telephone systems it is convenient to define their transmission properties in terms of the changes that occur in transmission in the amplitude and the phase of steady state sinusoidal waves of different frequency. The terms attenuation characteristic and phase characteristic refer, respectively, to the amplitude change, usually expressed in decibels, and to the phase shift, expressed in radians or degrees, as functions of frequency. That distortion which is attributable to the attenuation characteristic is spoken of as attenuation distortion, and that attributable to the phase characteristic, as phase distortion.

To be of greatest use in evaluating a system the steady state properties must be experimentally correlated with the satisfactoriness, or quality in its broad sense, of the system from the viewpoint of the individual receiving the signals. If the signals are speech, quality involves the recognizability of the speech sounds and their naturalness. If the signals are music, the second factor is the one of chief concern. A reasonably quantitative measure of the recognizability of the received speech sounds may be obtained by means of the articulation test which is described in a later paragraph. Naturalness is considerably less definite, and the procedure in this case has been to compare the distorted signals, speech or music, with the original or undistorted signals and obtain the amounts of distortion that cause just noticeable differences.

The purpose of this paper is to discuss the effects of phase distortion on the quality of speech.[1] Brief reference will be made also to a small amount of data that have been obtained for music.

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[1] A companion paper by C. E. Lane on "Phase Distortion in Telephone Apparatus" shows the types of phase characteristics found in various networks and discusses their relation to the transmission properties. For a discussion of methods of measuring phase characteristics the reader is referred to a companion paper on "Measurement of Phase Distortion" by H. Nyquist and S. Brand.

Nature of Speech and Hearing

Since the manifestation of phase distortion depends upon the type of signal and the method of observation, it is of interest to first consider the nature of the waves of speech sounds and certain facts of audition.

Speech waves may be regarded as non-periodic in that they start at some time, take on some finite values, and then approximate zero again as may be seen from the wave form [2] of the word "seems" in Fig. 1. In this particular word the wave form of each sound and the transition periods are readily distinguishable. Although in other cases of connected speech this may not be done so easily it is usually possible to

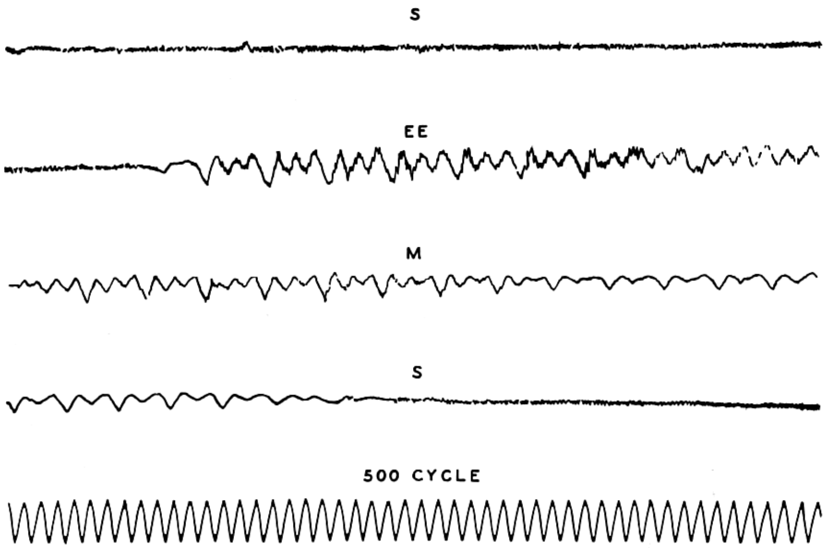

Fig. 1—Wave form of the word "seems."

approximately distinguish between sounds and to ascribe to each an initial period of growth, an intermediate period which in some cases approximates a steady state and then a final period of decay. The duration intervals of different sounds vary from about .03 to as much as .3 or .35 seconds.

Hearing appears to be concerned more with the spectra of sound waves, i.e., something corresponding to the amplitudes of the Fourier components, than with the actual wave form of the disturbance. For the type of steady state complex waves that the speech sound waves approximate for a considerable portion of their duration intervals, it has been observed that phase changes in the component waves cause

[2] Speech and Hearing, H. Fletcher, D. Van Nostrand Co., 1929.

little if any change in the character of the sound. A possible exception may arise for complex waves of large amplitude because of non-linearity in the hearing mechanism. It would be expected then that the observable effects of phase shift would arise from the intervals preceding and following the steady state intervals of the sound waves. For this reason the wave of a speech sound is regarded as non-periodic and when an amplitude frequency distribution is spoken of a Fourier Integral is implied.

Types of Phase Characteristics

The determination of the effects of phase distortion on quality involved the characteristics of the experimental system as a whole although, for convenience, the distortion usually originated in a specific network in the system. The procedure that was followed was to make the system, except for the network, as distortionless as possible. In most cases the characteristic of the system from the viewpoint of distortion, was the insertion characteristic of the network.

Before taking up experimental results on phase distortion it will be helpful to briefly review certain conclusions bearing on the relation between phase shift and wave distortion that have been obtained by analytical methods.[3] A phase characteristic of interest is one of form $B = B_0 + B_1\omega$, where $B =$ phase shift in radians and $\omega = 2\pi f$. If the original wave be represented by a Fourier Integral, the expression for the received wave may be obtained by shifting the phases of all of the sinusoidal components in the original in accordance with the above expression, assuming negligible attenuation distortion. For convenience the two terms in the above expression may be considered separately. If this is done, it can be shown by inspection that a constant shift of B_0 in the phases of all of the sinusoidal components gives a wave which is the sum of two waves, one the original wave multiplied by an amplitude factor $\cos B_0$, the other a wave obtained from the original by shifting all of its components by $\pi/2$ radians and multiplying by an amplitude factor $\sin B_0$.

If the phases of the sinusoidal components in the original are shifted by the amounts $B_1\omega$, where $\omega = 2\pi$ times the frequency of the component, it can be shown that the resulting wave differs from the original only in that the origin of time is displaced by an amount B_1 or the slope $dB/d\omega$ of the above expression.

[3] Transient Oscillations in Electrical Wave Filters, Carson and Zobel, *Bell Sys. Tech. Jour.*, July 1923. Building Up of Currents in Long Periodically Loaded Lines, Carson, *Bell Sys. Tech. Jour.*, Oct. 1924. Phase Distortion and Phase Distortion Correction, Mead, *Bell Sys. Tech. Jour.*, Apr. 1928. Phase Compensation, Sandeman, *Electrical Communication*, 1929. Transient Solutions of Electrical Networks, Mason, *Bell Sys. Tech. Jour.*, Jan. 1929. Phase Distortion in Telephone Apparatus, Loc. cit.

As will be seen later, it is convenient to regard these two operations as occurring in sequence. The second term of the expression introduces a definite time delay of B_1, sometimes called the envelope delay, and no distortion in the form of the original wave. Following this operation, the phases of all of the sinusoidal components of the delayed original are shifted by the constant amount B_0 the resulting wave being the received wave.

If B_0 equals zero or even multiples of π, the amplitude factor sin B_0 equals zero, so that, the received and original waves are identical in form. If B_0 is an odd multiple of π the received wave is reversed in sign only. In both cases the received wave is delayed by an amount $dB/d\omega$, and the wave cannot appear until this time has elapsed.

For all other values of B_0 the form of the received wave differs from that of the original to a greater or less degree depending upon the original wave form and the value of B_0. In this case the delay in the received wave as a whole cannot be spoken of precisely for no point on the received wave can be said to correspond to a point on the original wave. Theoretically the received wave may begin to appear at some earlier time than $dB/d\omega$, as has been shown by Mr. T. C. Fry in some unpublished work for the case of a wave having the form of a telegraph dot.

When the phase characteristics are curved over appreciable portions of the frequency range, as is usually the case in actual systems, exact statements of the above nature are difficult to make. It seems best, therefore, to confine the discussion to particular characteristics and to the case of speech waves.

A qualitative picture of what happens for the type of characteristic shown in Fig. 2 may be seen by regarding it as the limiting case of a characteristic made up of a number of straight lines of different slopes, each line approximating the curved characteristic for a frequency range Δf. As discussed above, the wave of a speech sound may be regarded as made up of steady state component waves of different frequency. The resultants of the component waves in various frequency ranges Δf are subject to the phase distortion discussed in the preceding paragraphs, that is, the original forms of the resultants are delayed by times $dB/d\omega$, and then undergo a distortion that depends upon the terms B_0 or the intercepts of the straight lines with the vertical axis.

As mentioned above it is convenient to regard these operations as taking place in sequence, the first introducing definite delays and no distortion in portions of the original signal corresponding to frequency ranges Δf, the second introducing a constant amount of phase shift in the component waves of each delayed portion. Thus the first operation spreads the wave out on a time scale, the part of the original wave corresponding to the frequency range having the minimum slope arriving first and followed successively by portions corresponding to the remaining frequency ranges. The relative delay for a range Δf is given by the difference between the slope of the phase characteristic

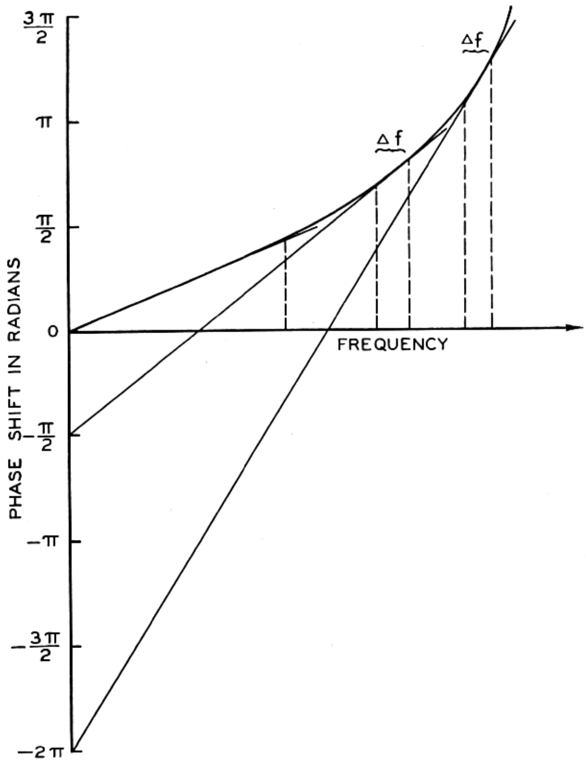

Fig. 2—A curved phase characteristic.

in the range Δf and the minimum slope. This difference or the expression $[(dB/d\omega)_f - (dB/d\omega)_{min}]$ is spoken of as the delay distortion. The delay distortion characteristic is simply a graph of the above expression plotted against frequency. The second operation distorts the wave forms of the delayed portions corresponding to the frequency ranges Δf, thus making it impossible to speak of the delay of the final or received portions or of the received wave as a whole.

This may be described in other terms by saying that a network having a characteristic of the type shown in Fig. 2 may be thought of as if it were made up of two sets of networks, the two sets being connected in series. The first set consists of a number of networks in parallel, each network passing a frequency range Δf and having a phase characteristic of the form $B_1\omega$, where B_1 is the slope of the straight line approximating the curved phase characteristic in the range Δf. The second set consists of a number of corresponding networks having phase characteristics of the form B_0, where B_0 for a network passing the range Δf, is the constant term in the equation of the straight line approximating the curved phase characteristic in this range. The phase distortion thus consists of two operations in sequence, the first introducing definite delays in various portions of the original wave, the second introducing constant phase shifts in the component waves of each delayed portion.

A definite contribution to the recognizability of a speech sound may be associated with each frequency range Δf in the undistorted state. At the output terminals of the first set of networks the various portions corresponding to the ranges Δf do not combine to form an exact copy of the original wave, because of the different delays that have been introduced. It is supposed that their normal contributions to recognizability are decreased by a factor depending upon the delay distortion and the duration time of the speech sound. This factor is referred to here as the "time factor" and it would be expected to operate even though the second set of networks were non-existent.

Since the constant phase shifts of the second set of networks are not all multiples of 2π, additional distortion will be introduced by this set of networks. To take account of this another factor called the "intercept factor" is introduced. As will be seen later this factor seems to be negligible for the case of speech waves due in part, no doubt, to the sustained character of the waves and to the mechanism of hearing as previously described.

In addition to the above, when we deal with a succession of speech sounds, as in connected speech, a third factor might be expected to operate because the delayed frequency ranges of one sound may overlap the least delayed ranges of a succeeding sound and interfere with its perception in the manner of an extraneous noise. As will also be seen later on this factor appears to be negligible for the type of characteristic shown in Fig. 2 because the so-called noise and the signal with which it interferes do not have components in a common frequency range. When this is true, noise in general interferes much less than when the signal and the noise have components in a common range.

Phase Distortion and Quality

Quite aside from the recognizability viewpoint, when speech from a system having phase distortion is compared with that from a system having negligible distortion, it is noticed that the distorted speech is accompanied by certain audible effects which appear to be extraneous to the speech and transient in character. As discussed above phase changes in the component frequencies of steady state waves cause little if any change in the character of the sound. This would indicate that the so-called audible effects of phase distortion arise in the transition periods, i.e., in the period between the approximate steady state of one speech sound and that of the succeeding sound, and are due to the spreading out effect of phase distortion. Data on the amount of distortion that will cause just noticeable effects will be discussed in a later paragraph.

Before discussing the various factors affecting the recognizability, it is of interest to consider the importance as obtained from articulation tests,[4] of different portions of the duration intervals of speech sounds, and also the importance of different portions of the speech frequency range.

Fig. 3 shows the effects upon sound articulation of limiting the transmitted frequency range by means of high or low pass filters in a system having neglible distortion in other respects. Although the curves do not intersect at 50 per cent nor do the articulation values of complementary filters add up to 100 per cent, they may be used with qualifications, to measure the contribution or importance to articulation of a portion of the speech frequency range. Thus the slope vs. cut-off frequency for the low pass filter curve gives a measure of the contribution to articulation of a frequency range Δf when contiguously added to the range 275 to f.

Articulation tests that have been made with voice operated relays give an indication of the importance to articulation of portions of the duration intervals of speech sounds. In the tests, syllables of the consonant-vowel-consonant type were spoken at intervals of about 3 seconds. A circuit having a relay adjusted so as to break contact almost simultaneously with the beginning of a syllable, was used. The contacts of a second relay formed a short circuit across the receiver. The operation of the first relay caused the second relay to break contact

[4] Articulation Testing Methods, H. Fletcher and J. C. Steinberg, *Bell Sys. Tech. Jour.*, Oct. 1929. In an articulation test lists of syllables are spoken into the transmitter of a system having phase distortion and observers at the receiving end write down the sounds which they hear. The observed lists are compared with the spoken lists and the errors determined. The percentage of the total number of spoken syllables that are correctly observed is called the syllable articulation. A syllable is considered to be incorrectly observed if one or more of the fundamental speech sounds which it contains are mistaken. The percentage of the total number of spoken speech sounds which are correctly observed is called the sound articulation. When attention is directed toward a specific sound such as "e," the term individual sound articulation is used. It is the percentage of the number of times that "e" was spoken that it was observed correctly.

after an interval of time depending on the time constants of the relay circuit alone. The time taken for the second relay to operate represents the time clipped from the initial consonants of the syllables. Fig. 4 shows the initial consonant articulation plotted against the operating time of the second relay. If equal elements of time in the

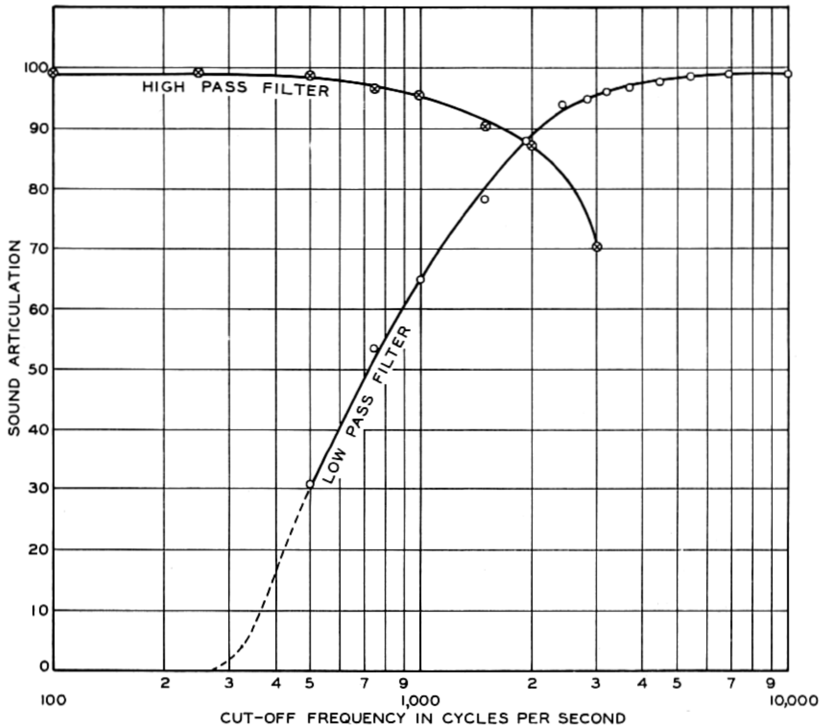

Fig. 3—Importance of frequency range to articulation.

duration intervals of the consonants are of equal importance, then the clipping by an amount ΔT should decrease the articulation by a factor,

$$K = (1 - \Delta T/T), \tag{1}$$

where T is the average duration time of a consonant. In the above tests, the syllables were spoken separately. Oscillograms taken for syllables spoken in a similar manner show an average duration time of .16 seconds for the consonant sounds.[5] The straight line in Fig. 4 was calculated by multipling the articulation obtained for zero operating time by a factor K as determined from the above equation with

[5] Speech and Hearing, H. Fletcher, D. Van Nostrand Co., Inc., 1929.

$T = .16$ seconds. The data indicate that equal elements of time in the duration intervals of the sounds are of approximately equal importance to the average articulation of a group of sounds. It should be pointed out that this might not be the case for individual speech sounds and also for certain types of speech distortion. In the above tests a carbon transmitter was used of a type which introduced attenuation distortion. Tests have also been made with speech having negligible attenuation distortion. Although they indicated a relation of the

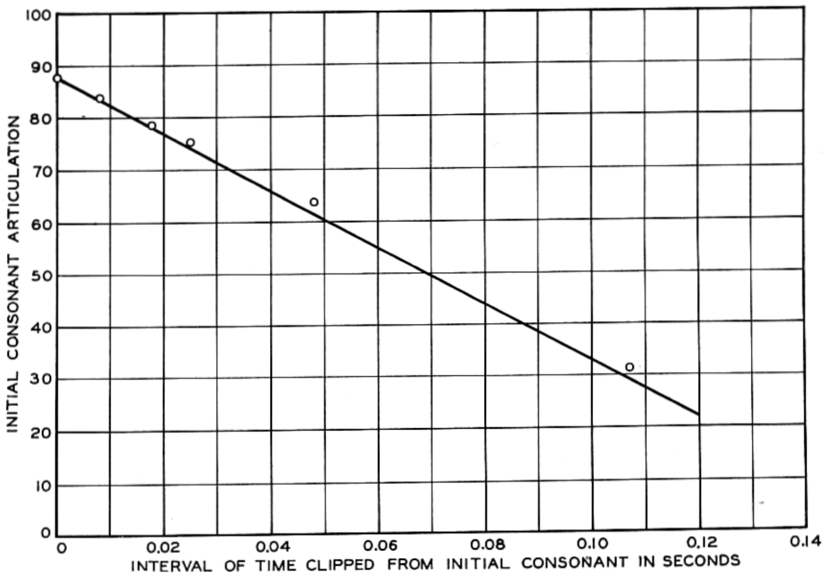

Fig. 4—Importance of an element of time to articulation.

above type the data are somewhat questionable because of uncertainties in the operating time of the relay.

In the next series of articulation tests a nominal undistorted speech frequency range of 0 – 4500 cycles was divided into two parts by means of filters and each part transmitted through a different channel. After transmission the two parts were recombined. The phase characteristic of each channel approximated a straight line over the greater part of the frequency range. The slope of the characteristic of one channel could be increased by various amounts over that of the other channel. One channel thus introduced a definite time delay, in the sense used here, with respect to the other channel, i.e., a delay given by the difference in the slopes of their phase characteristics. The observed sound articulations plotted against time of delay are shown in Fig. 5. The articulation values decrease with increasing delay and approach the articulation of the more intelligible band which was also the least delayed band. For convenience it is spoken of as the non-delayed band.

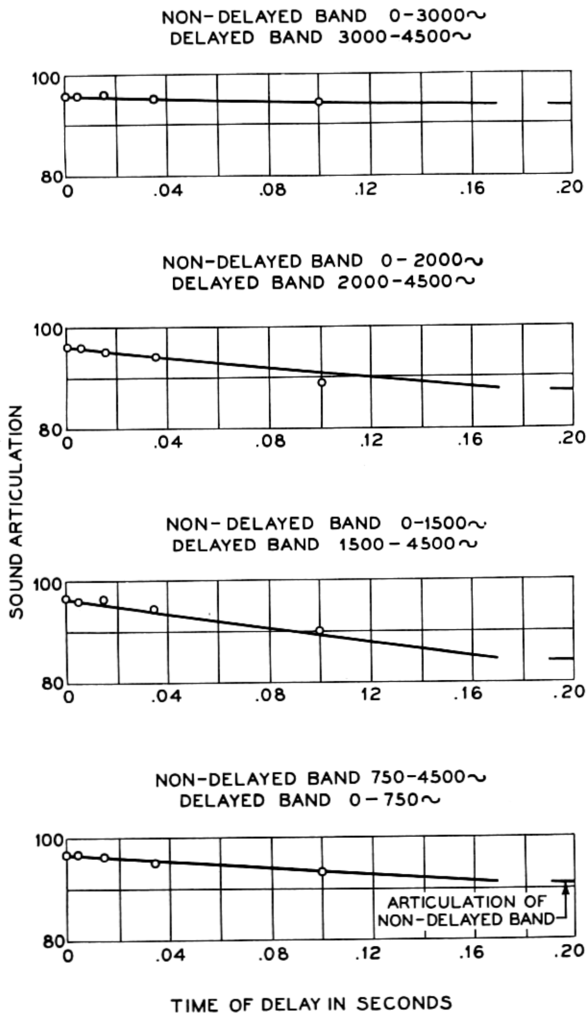

Fig. 5—Articulation vs. time of delay.

If the decrease is due primarily to the inability of the delayed range to contribute to articulation for a time equal to the delay interval, then if,

A_1 = sound articulation for zero delay
A_2 = sound articulation of the non-delayed band

the sound articulation for a delay ΔT should be

$$A = A_2 + K(A_1 - A_2), \tag{2}$$

where K is a factor obtained from Eq. (1) for $T = .17$ seconds.

This value of T is used because in these tests the articulation syllables were spoken as parts of introductory sentences, such as, "The first syllable is *nif*," etc. Oscillograph records for this manner of speaking indicate an average duration time for vowel and consonant sounds of an order of .16 to .18 seconds. This is somewhat less than the duration time for sounds spoken in detached syllables, and somewhat greater than the duration time for sounds spoken in connected speech containing words of one or more syllables. For the latter case, the average duration time is probably more of an order of .08 to .12 seconds.

The solid lines shown in Fig. 5 were calculated from Eq. 2 which involved only the time factor. The agreement between observed and calculated results indicates that the two other factors were comparatively small in these cases. As regards the noise factor it should be noted that the frequency range of the so-called noise and that of the sound wave with which it interferes have no part in common. When this is true, as previously pointed out, the interference from noise is small.

The following tests were made with networks having curved phase characteristics of the type shown in Fig. 2. One of these networks was an all pass structure made up of two types of sections, a "B" section having a critical frequency of 2000 cycles and an "A" section having a critical frequency of 2500 cycles. By using different numbers of sections different amounts of delay distortion could be obtained. Fig. 6 shows the delay distortion for the conditions that were tested. The attenuation characteristic was equalized up to 2500 cycles, and a 2400 cycle low pass filter having negligible phase distortion was associated with the network. Fig. 7 shows the sound articulation values versus the number of sections. A time factor K_f for an element Δf in the frequency range 0 to 2400 cycles, may be obtained from Eq. 1 by setting $T = 0.17$ seconds and ΔT equal to the delay distortion given in Fig. 6, for the element Δf. Delays in some frequency ranges will impair the articulation much more than similar delays in other ranges because some frequency ranges are of greater importance to articulation. The importance of various frequency ranges is closely related to the slopes of the curves in Fig. 3. To obtain an effective factor $\overline{K}$, the values of K_f must be weighted in accordance with the importance of the frequency ranges considered. This may be done approximately for the 2400 cycle range by averaging the values of K_f

corresponding to successive elements Δf, the elements being chosen so as to represent equal increments of increase, say 5 per cent, on the sound articulation versus cut-off frequency curve for low pass filters (Fig. 3).

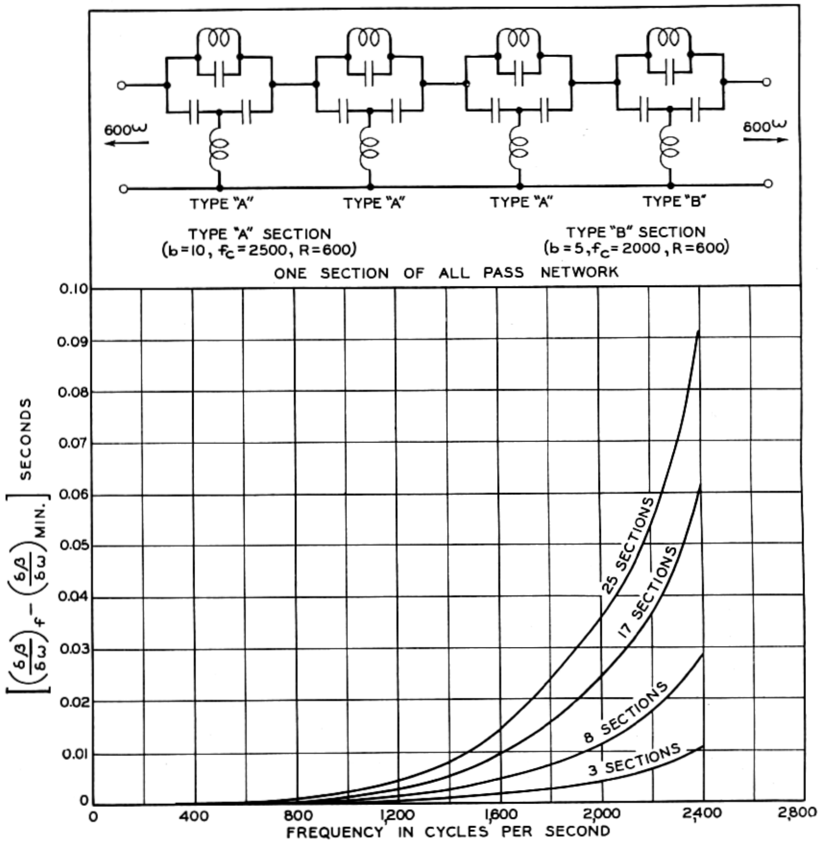

Fig. 6—Delay distortion for an all pass network.

The solid curve shown in Fig. 7 was calculated by multiplying the articulation obtained for zero phase distortion by the effective values of $\overline{K}$ obtained as described above.

Articulation tests were also made upon a system containing first one, and then twenty-five 5000 cycle low pass filters in series. In both cases the attenuation was equalized to 5000 cycles. Fig. 8 shows the delay distortion and the articulation results that were obtained. The calculated results were obtained in the manner described above.

The foregoing calculations have not been made for the purpose of setting up methods of determining the loss in articulation due to phase distortion, but rather to show the relative importance of the various factors for particular cases. In these cases the time factor appeared to be the one of most importance.

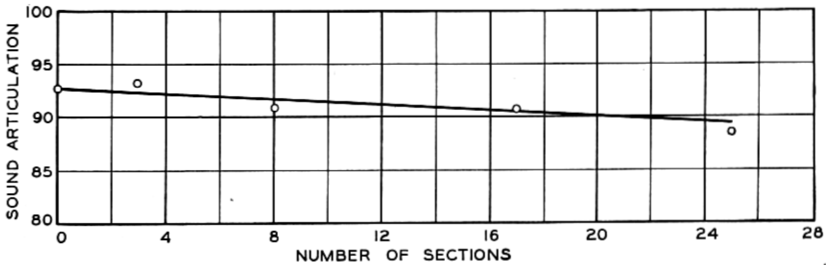

Fig. 7—Articulation vs. delay distortion.

It is evident that the primary effect of phase distortion was to effectively narrow the transmitted frequency range. The phase distortion

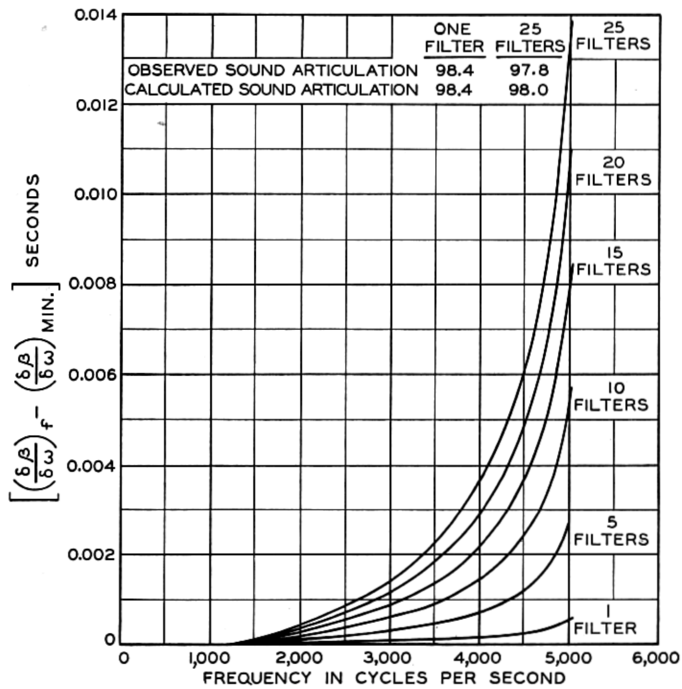

Fig. 8—Articulation and delay distortion for a 5000 cycle low pass filter.

caused by the twenty-five sections of the all pass network associated with a 2400 cycle low pass filter reduced the articulation from 92.7 per cent to approximately 89 per cent. Reference to Fig. 3 shows that lowering the cut-off of a low pass filter having negligible phase distortion from 2400 to 2000 cycles causes an equal change in articulation. Similar considerations show that the effective cut-off for twenty-five 5000 cycle low pass filters in tandem is of an order of 4700 cycles, although the attenuation distortion of the twenty-five filters was essentially the same as that of one filter. The effective transmitting range of a network is a function therefore of both the phase and the attenuation characteristics.

In addition to decreasing the understandability of speech, phase distortion introduces certain audible effects, as previously discussed, which may be a source of considerable annoyance to the listener. Their noticeableness depends upon the amount of delay distortion and the frequency range in which it occurs. For speech, it was found that one section of the all pass network when associated with a 2400 cycle low pass filter, had sufficiently small delay distortion so as to be just noticeable. This determination was made by alternately listening to speech from the system under two conditions, one, the filter alone, two, the filter with the all pass network. Judgments of which condition contained the network were correct about 50 per cent of the time and wrong about 50 per cent of the time for one section of the network. The total delay distortion at the cut-off frequency in this case, i.e. that due to the filter plus that due to one section of the network was about .006 seconds. When three sections were used the distortion was easily noticed.

Similar tests with the 5000 cycle low pass filters indicated that some number between five and ten filters in tandem would cause just noticeable distortion and that the distortion was clearly noticeable for 20 filters in tandem. The amount of delay distortion at the cut-off frequency for five filters is about .003 seconds, and for ten filters about .006 seconds.

The above figures depend somewhat upon the attenuation characteristic as the cut-off frequency is approached, small amounts of attenuation reducing the noticeability of the effects. The figures also vary somewhat with individuals depending upon their experience and hearing characteristics.

Tests on piano reproduction when single notes were struck or when a passage of music was played indicated that the distortion caused by twenty-five of the 5000 low-pass filters in tandem was not noticeable. As in the case of speech, it would be expected that the noticeableness of the distortion would depend upon the frequency range in which it occurs. In general, the effects of delay distortion on music are very much less noticeable than on speech which is probably due in part to the more sustained character of music.

Phase and Attenuation Distortion

To illustrate how phase and attenuation distortion operate to reduce the effective transmitted frequency range, the filter characteristics shown in Figs. 9 and 10 will be considered.[6] One of the uses of filters in telephone systems is to provide a number of channels for one transmission line by using filters in parallel, each filter transmitting a different frequency range. In a long line as many as twenty or twenty-five filters may be used in series for each channel. In this use it is

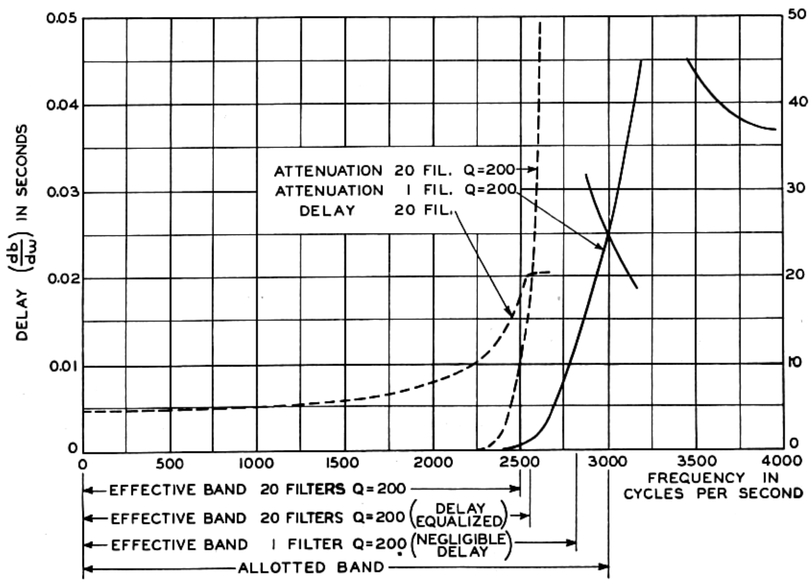

Fig. 9—Transmission for a filter having a slow rate of attenuation increase.

desirable that the attenuation at the edge of the transmitted band should increase at a very rapid rate, and that the delay distortion at the edge should be small.

The solid curve in Fig. 9 shows the attenuation distortion, i.e., the difference between the minimum attenuation and the attenuation for any frequency f, for a filter having a *slow* rate of attenuation increase. The delay for one filter is not shown, but it is about 1/20 of the delay shown by the dotted curve. The allotted frequency band is determined by the frequency value at which the attenuation curve of the filter crosses that of the filter transmitting the adjacent band. The attenuation at the crossover for the purposes of this discussion may be taken as

[6] For a discussion of the relation between these characteristics and the type of filter section, the reader is referred to the previously cited paper by C. E. Lane.

25 db. The effective band is the frequency range that will give the same articulation as that given by the filter. The delay distortion is so small for one filter of this type that its effects can be neglected and the effective band may be determined from the attenuation curve alone. The effective band is given approximately when the area bounded by the attenuation curve and a line parallel to the frequency axis through the 25 db point equals the area under the 25 db line between the frequency limits zero and f, where f is the upper limit of the effective band.

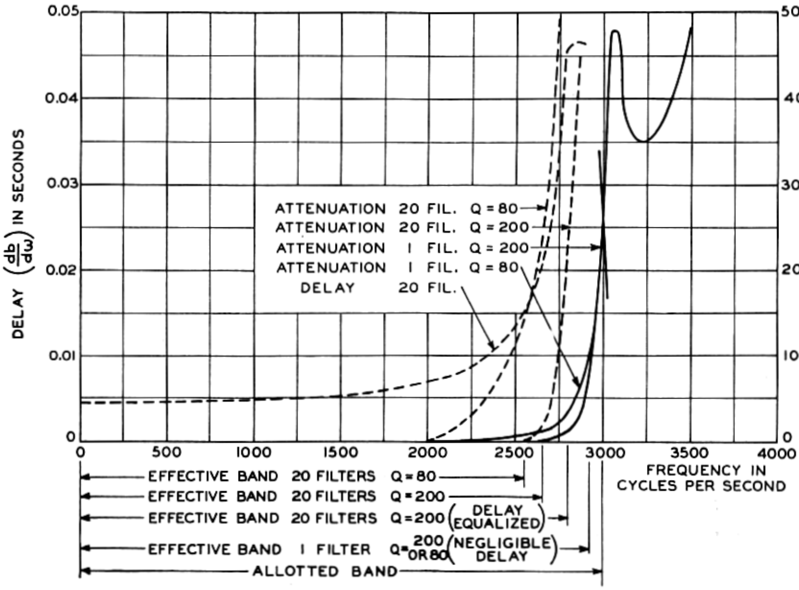

Fig. 10—Transmission for a filter having a rapid rate of attenuation increase.

This is true because the effect of attenuation near the cutoff frequency is to reduce the contributions of the various frequency ranges by a factor that is proportional to their attenuation, for the attenuation limits considered. In the above case this factor varies from zero at 3000 cycles to unity at 2400 cycles. The effective band width is 2825 cycles.

The dotted curves on Fig. 9 show the delay and attenuation values for 20 filters in tandem. The effective band width that is shown for "delay equalized" is that due to the attenuation distortion alone. It was obtained as described in the preceding paragraph except that the attenuation for zero articulation was taken as 40 rather than 25 db since there is no interference from the signals in the adjacent band. In this case the effective band due to attenuation distortion alone is 2550 cycles. The effect of delay distortion when taken into account in the manner previously described further reduces the effective range to 2500 cycles. Thus the additional reduction due to delay distortion is small.

Fig. 10 shows similar data for two filters having *rapid* rates of attenuation increase, one having coils with a Q of 80, the other[7] having coils with a Q of 200. For 20 filters ($Q = 200$) in tandem the effective band due to attenuation distortion alone (delay equalized) is 2800 cycles. The effect of delay distortion further reduces the effective range to 2650 cycles. Comparing these ranges with the corresponding ranges for the filters of Fig. 9 shows that the filters of Fig. 10 use the allotted band, which is the same for both figures, more efficiently from an articulation standpoint, i.e., the effective band is a larger fraction of the allotted band. The delay distortion, however, in the filters ($Q = 200$) of Fig. 10 is more noticeable. This will be seen by noting that the amount of delay near the cutoff for the filters of Fig. 9 is very much smaller than that of Fig. 10.

The noticeableness of the delay distortion may be decreased by equalizing for the delay distortion, or by using filters with coils of smaller Q. Any gain made by the former method is made at the expense of the minimum delay, i.e., the constant delay in the major part of the transmitted range. Any gain made by the latter method is made at the expense of the effective frequency range as shown by the effective band for 20 filters ($Q = 80$) of Fig. 10. Twenty filters of the type shown in Fig. 9 have about the same overall performance as 20 filters of the type shown in Fig. 10 having coils with a Q of 80.

It is evident that in the design of filters, compromises must be made between the rate of attenuation increase at the edge of the transmitted band, the minimum delay and the delay distortion. The compromises that are made in an actual system depend upon many factors and their discussion is beyond the scopeof this paper.

Although the time factor appeared to be the one of most importance for the phase characteristics that have been discussed here, it should be pointed out that this may not be true for all types of phase characteristics. Much work remains to be done for other types of phase characteristics, for example, characteristics which show irregular changes with frequency rather than the smooth changes of the type discussed here. It seems, however, that the main thing in securing transmission free from phase distortion is to provide a phase characteristic that is linear with frequency over the frequency range of interest.

[7] Q refers to the ratio of the reactance of a coil to its effective resistance.