Mutual Impedances of Ground-Return Circuits

Some Experimental Studies *

By A. E. BOWEN and C. L. GILKESON

This paper describes some of the results of the work of the Joint Development and Research Subcommittee of the National Electric Light Association and Bell Telephone System on the mutual impedances of ground-return circuits.

The first part of the paper deals with some experiments which were performed to establish an experimental background for the testing of theoretical ideas. Different theories, one involving an "equivalent ground-plane," a second a d. c. distribution in the earth, and a third an a. c. distribution in the earth, are discussed in the light of the experimental results. While none of these is adequate to explain all the observed phenomena, each has a field in which it can be made useful.

The second part of the paper is devoted to a description of practical means for predetermining the mutual impedances of power and telephone lines. This involves an experimental determination of a curve of mutual impedance as a function of separation in the region of the proposed exposure and the calculation of the overall mutual impedance between the proposed lines from this curve and the dimensions of the exposure. The results of trials of this method in two locations are given which indicate that itshould be of sufficient accuracy for engineering purposes.

Introduction

THE magnitude of the inductive coupling between power and telephone lines is a factor of fundamental importance in problems of coordination to prevent interference between these two classes of lines. Accordingly, this is one of the subjects under investigation by project committees of the Joint Committee on Development and Research of the National Electric Light Association and the Bell Telephone System. It is the purpose of this paper to present the results of some work which has been done under the auspices of the Committee on one phase of this problem, namely, the mutual impedance of ground-return circuits.

The mutual impedance of two ground-return circuits is determined by measuring the ground-return current in one circuit (the "disturbing" circuit) and the open-circuit voltage at the terminals of the second circuit (the "disturbed" circuit). The vector ratio of the open-circuit voltage to the ground-return current is then defined as the mutual impedance of the two circuits.

For any normal or abnormal operating condition of a power system,

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the currents, either at fundamental frequency or at any harmonic frequency, in any of the lines can be resolved into components, some of which are entirely confined to the wires while another component flows in a circuit composed of all the wires as one side with the ground as a return path. The work which is described in this paper deals with the magnitude of the induced voltages on exposed telephone lines caused by the latter component. It has been directed to two ends, first, the establishment of an experimental basis for the study of the physical factors involved in the inductive coupling of ground-return circuits, and second, the development of practical methods to enable the advance calculation of the mutual impedances of power and telephone lines. The work is accordingly presented in two parts; first are given the results of tests made at a field laboratory in which testing conditions could be controlled, and second, tests in which the practical side of the problem was investigated are described.

Cross Keys Tests and Theoretical Background

Cross Keys Tests. An extended series of measurements was made at a field laboratory operated by the subcommittee near Cross Keys, New Jersey, about 20 miles southeast of Camden. A single conductor, located about 34 ft. above the ground and 8500 ft. in length, was available for the disturbing circuit. For disturbed circuits, 500-ft. lengths of insulated wire were laid on the earth parallel to the disturbing conductor, at several separations, as shown on Fig. 1. Grounds were pro-

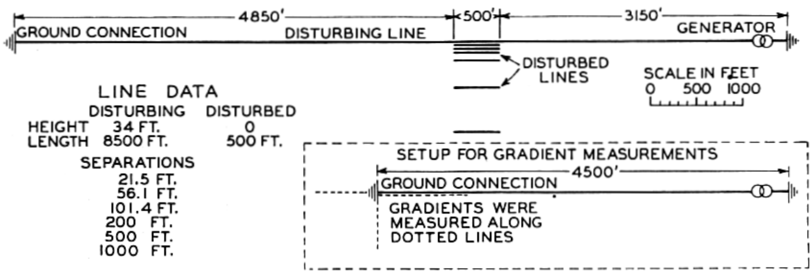

Fig. 1—Cross Keys tests—experimental setup.

vided at the ends of each line. Ground-return current was transmitted over the disturbing line at 60 cycles from a commerical source, or at frequencies between 100 and 1000 cycles from a vacuum-tube oscillator with power amplifier. The measuring instrument was an a.-c. potentiometer, equipped with suitable filters so that the observations were unaffected by the presence of harmonics in the disturbing current.

At several frequencies within the range from 60 to 1000 cycles the

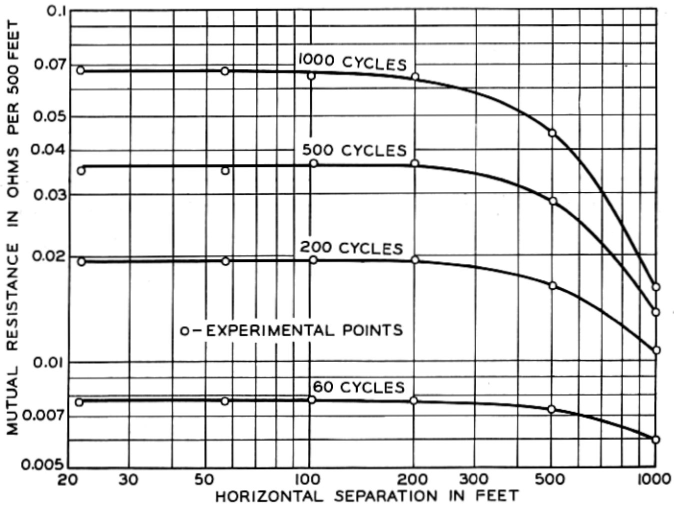

Fig. 2—Cross Keys tests— mutual resistance.

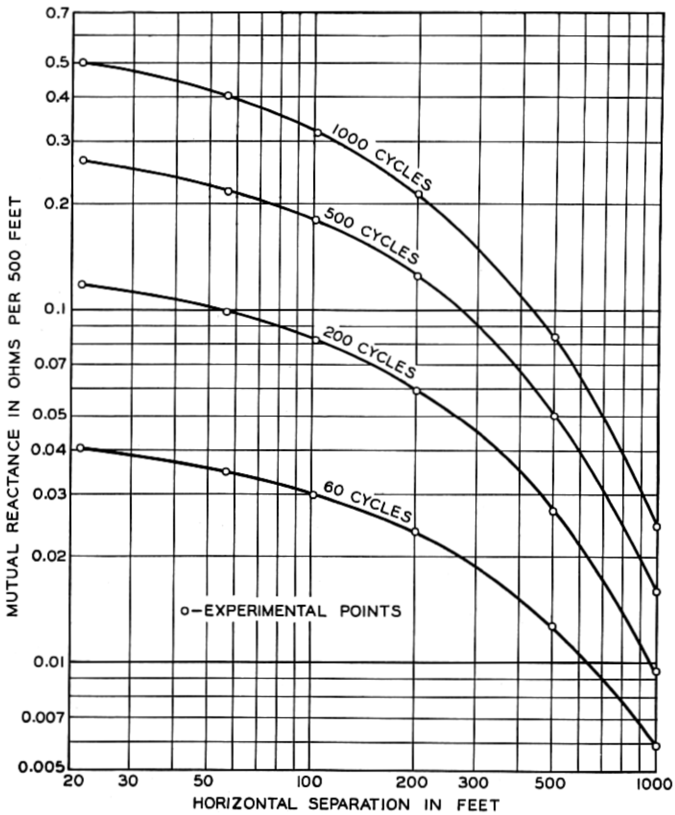

Fig. 3—Cross Keys tests—mutual reactance.

current in the disturbing line, the open circuit induced voltage in each of the short ground-return circuits, and the phase angle between these two quantities were measured. The mutual impedances were derived from the ratio of the induced voltage to the inducing current, in accordance with the definition. The results of these tests are given on Figs. 2, 3, and 4. Fig. 2 shows the resistance components, Fig. 3 the reactance components, and Fig. 4 the magnitudes of the mutual impedances.

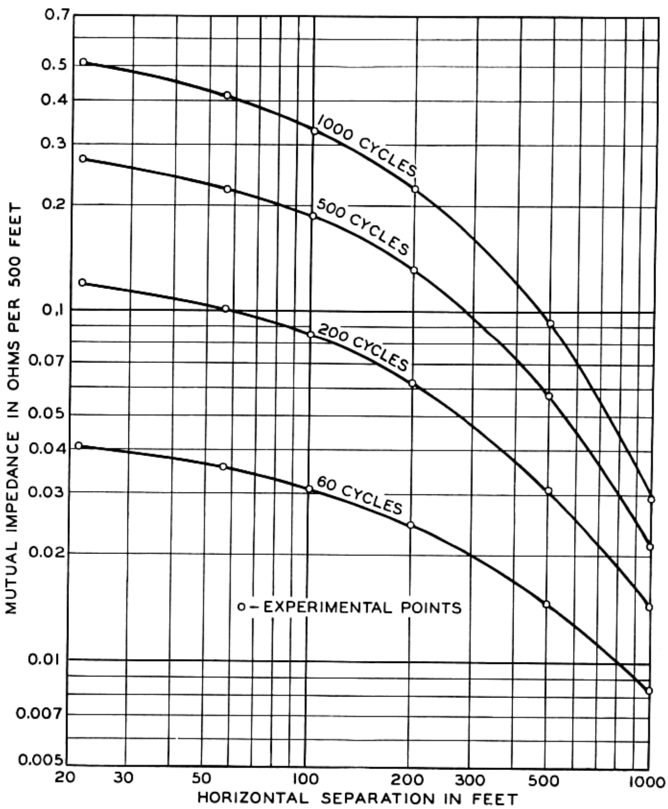

Fig. 4—Cross Keys tests—mutual impedance.

The measurements described above were made with the object of investigating the mutual impedances of ground-return circuits in which the ground connections on the disturbing line are sufficiently removed from those on the disturbed circuit so that effects due to proximity of the grounds may be ignored. The results presented above were supplemented by observations demonstrating that the induced voltage in a

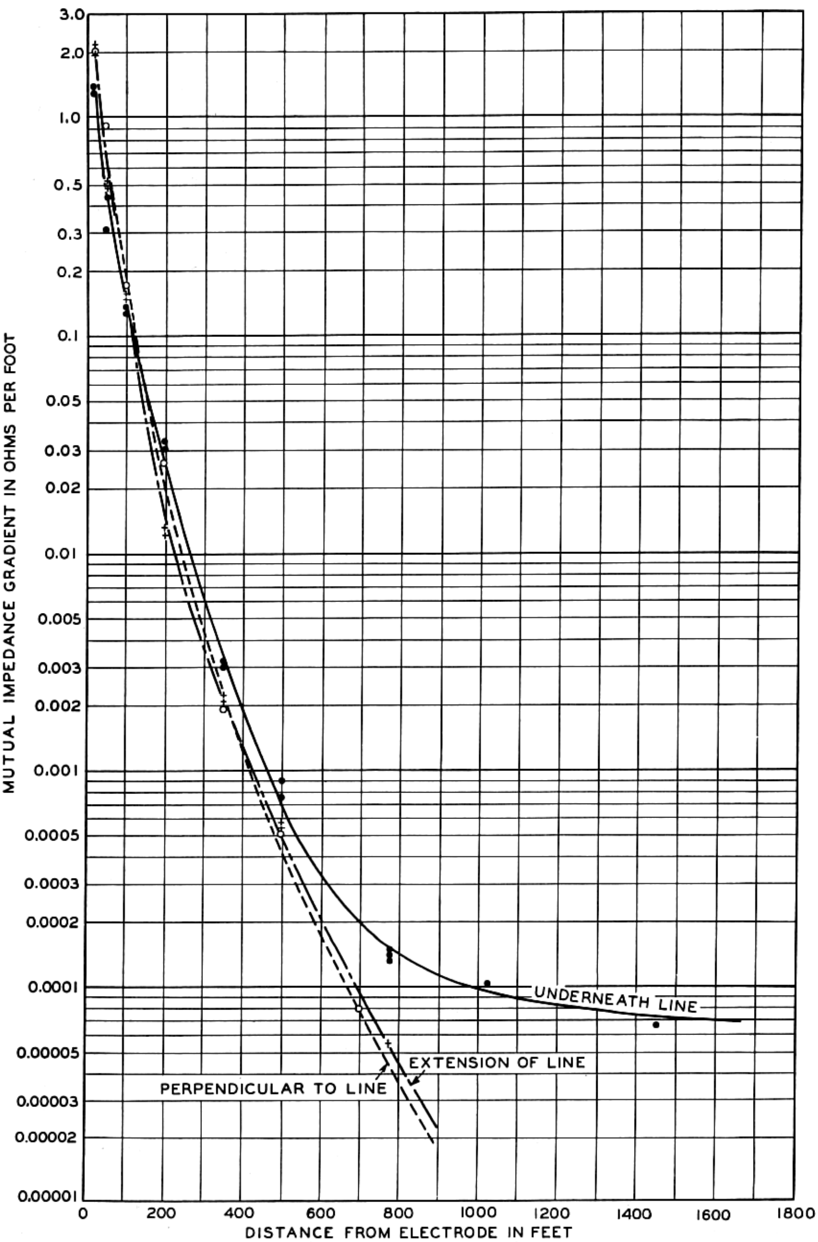

Fig. 5—Cross Keys tests—mutual impedance gradient in vicinity of grounding electrode. Experimental curves.

parallel circuit was closely proportional to the length of the circuit and that the voltage induced in a ground-return circuit extending perpendicular to the disturbing line was exceedingly small.

As the points of grounding on the disturbed circuits approached those of the disturbing circuit this proportionality no longer existed nor was the voltage in a perpendicular circuit of negligible magnitude. A second series of tests was therefore conducted to determine the nature of this effect and the area in which it was of importance.

In these tests voltages were measured in very short disturbed circuits extended along radii converging on the ground electrodes on the disturbing line. At each location the circuit was made progressively shorter until the quantity measured per unit length was practically independent of the length. Thus the gradient of the mutual impedance, in the direction of the radius at the point of measurement was determined. These measurements were made only at a frequency of 60 cycles.

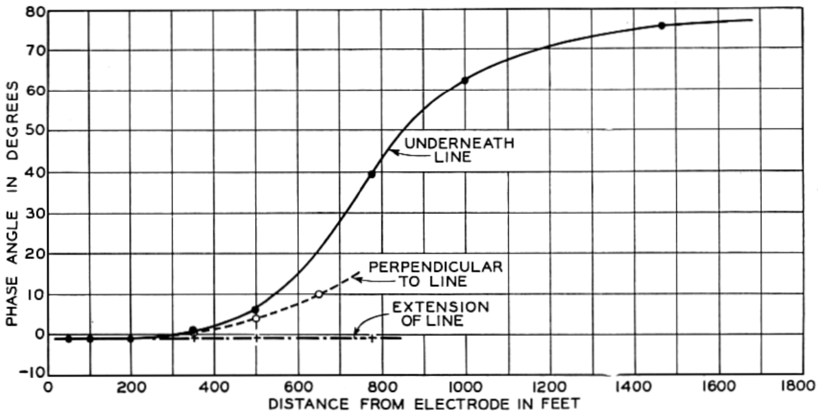

Fig. 6—Cross Keys tests. Phase angle of mutual impedance gradient in vicinity of grounding electrode. Experimental curves.

The resulting observations are shown in Figs. 5 and 6. Fig. 5 shows the magnitude of the mutual impedance gradient along three radii, one radius being directly under the disturbing line, the second perpendicular to it, and the third along the extension of the line. Fig. 6 shows the corresponding phase angles. Under the disturbing line, as the distance from the grounding point is increased, the gradient approaches a constant value and the phase angle changes rapidly from a very small value to an angle approaching 80 degrees. Along the latter two radii, however, the magnitude of the gradient appears to decrease indefinitely and the phase angles are smaller.

A more complete analysis of the results of both groups of tests is given in connection with the discussion of theory which follows:

Equivalent Ground-Plane Theory. The equivalent ground-plane method of computing the mutual impedances of ground-return circuits utilizes a very simple formula and has been in common use for a number of years. A derivation and discussion of the formula together with some experimental results are given in the report published by the California Railroad Commission in 1919.[1]

This method assumes that the returning earth current may be considered as flowing in a hypothetical plane surface of perfect conductivity located some distance below the actual surface of the earth. This surface is usually termed the "equivalent ground-plane." The depth of the equivalent ground-plane below the actual surface of the earth varies in different locations from about 50 ft. to 5000 ft. or more, depending upon the character and resistivity of the earth and the frequency.

This method is subject to the objection that it fails to represent completely the observed phenomena. For instance, the method represents the mutual impedance only with a reactive term, while the experimental results indicated a substantial resistance component, particularly at the wider separations and higher frequencies. Furthermore, no attempt is made to explain the phenomena observed in the neighborhood of the ground electrodes. However, in one respect the theory leads to results comparable to those observed; the magnitudes of the mutual impedances as observed under conditions in which end effects are negligible can be checked reasonably well with a suitable choice of ground-plane. Comparisons demonstrating this point are made on Fig. 7, where it will be seen that the curve of experimental mutual impedance for a frequency of 60 cycles can be fitted very well by a calculated curve with a ground-plane depth of 835 ft. That the depth of the equivalent ground-plane depends on the frequency is seen from the fact that to fit the experimental curve at 500 cycles requires the use of a ground-plane depth of 385 ft.

Method Assuming D.-C. Distribution in the Earth. For an earth of uniform conductivity, the distribution of the current in the earth for a ground-return circuit energized from a d.-c. source has been employed by G. A. Campbell[2] to derive formulas for the mutual resistance and inductance of ground-return circuits. The mutual resistance is expressed by a very simple formula which involves only the earth resistivity and the distances between the points of ground connection on the

[1] See Bibliography.

[2] See bibliography.

disturbing and disturbed circuits. For the calculation of the mutual inductance formulas and graphs requiring only a knowledge of the mutual arrangement of the wire parts of the disturbing and disturbed circuits with respect to each other and the earth are given. The mutual inductance is independent of earth resistivity. While these formulas are, of course, strictly applicable only for direct currents, it is to

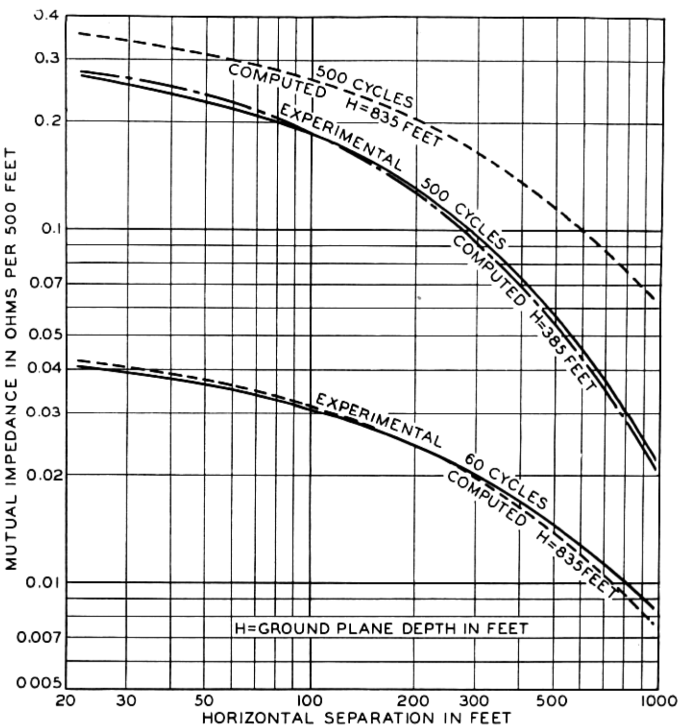

Fig. 7—Cross Keys tests. Ground plane theory. Comparison of measured and calculated values of mutual impedance.

be expected that at sufficiently low frequencies the ground-current distribution would not differ appreciably from that for direct current, and hence for these frequencies, these calculated d.-c. mutual resistance and inductance should approximate the actual values. In the paper referred to, some experimental results at frequencies of 25 and 60 cycles supporting this point of view are presented.

The experimental curves of Fig. 2, which were obtained from measurements at Cross Keys on the 500-ft. disturbed lines near the middle of the 8500-ft. disturbing line, indicate a pronounced increase in mutual resistance with increase in frequency in the range from 60 to 1000 cycles. These results have been replotted on Fig. 8, and it is apparent that for separations within the range of 20 to 500 ft. the mutual resistance increases rapidly in almost linear relation to the frequency. For the frequency range and circuit lengths involved in this series of tests, it would appear that a formula for the mutual resistance, based on a d.-c. distribution in the earth is inadequate.

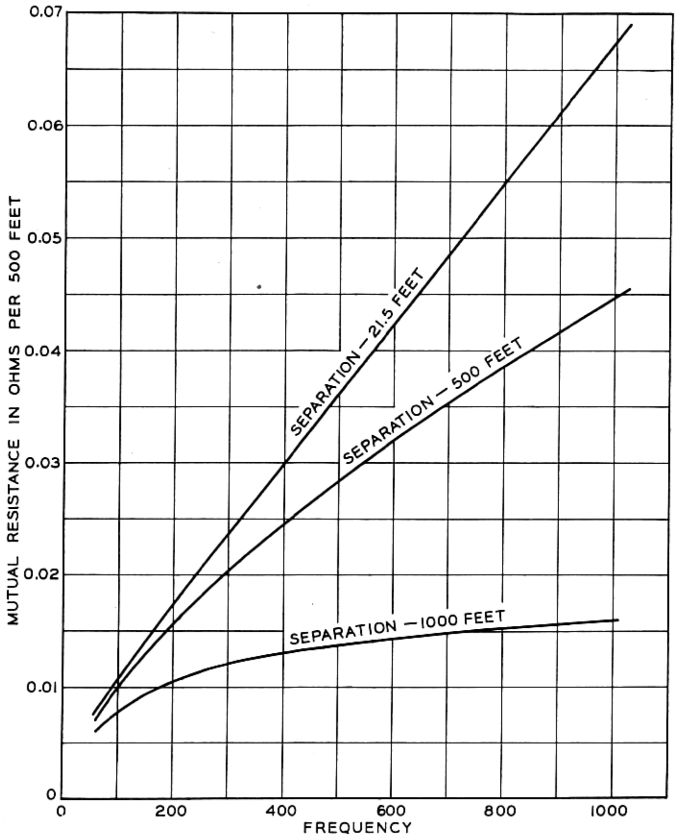

Fig. 8—Cross Keys tests. Variation in mutual resistance with frequency and separation.

The mutual-inductance curve of Fig. 9 has been computed according to the formulas given by Campbell, and for comparison purposes the mutual inductances derived from the mutual reactances shown on Fig. 3 are also plotted. It will be seen that the observed mutual inductances decrease as the frequency is increased, and that while the trend of the observed values is towards agreement with the calculated values as the frequency is decreased, the agreement is far from good at 60 cycles, the lowest frequency used in these tests.

In the immediate vicinity of the grounding electrode on the disturbing circuit, however, the experimental observations of mutual impedance gradient can be explained fairly well in terms of a d.-c. distribution. The curves of Figs. 5 and 6 show that in the immediate neighborhood of the electrode, the gradient along any radius diverging from the electrode decreases very rapidly with increase in distance from the

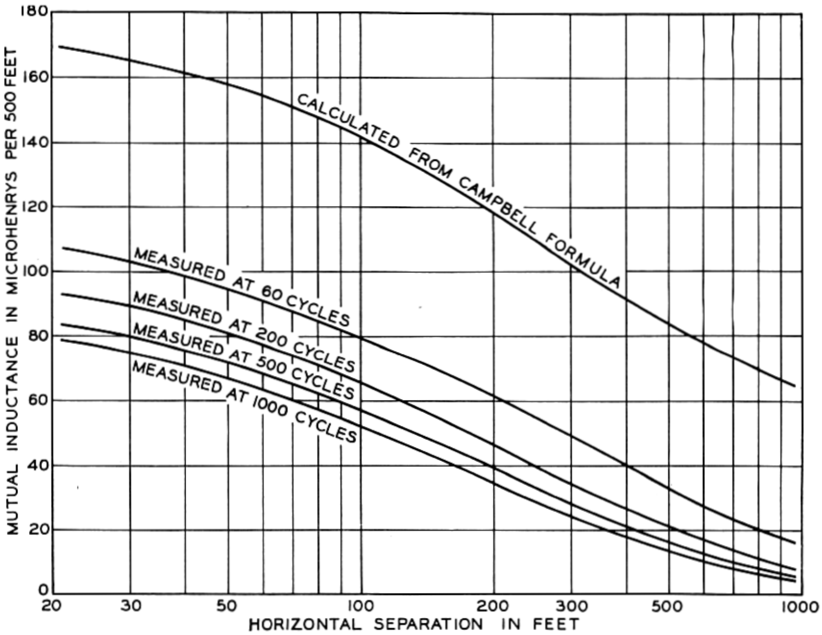

Fig. 9—Cross Keys tests. Campbell theory comparison of measured and calculated mutual inductances.

electrode, and is approximately in phase with the current. The gradient along the radius under the disturbing line approaches asymptotically a constant value, and beyond 300 ft. from the electrode the phase angle changes rapidly from a very small value to a value approximating 80 degrees. The gradient along the other radii, however, appears to decrease indefinitely and the phase angles are smaller. Such effects are in qualitative accord with predictions based on a d.-c. distribution, as will be seen by reference to Figs. 10 and 11.

On Fig. 10 are plotted the resistance and reactance components of the observed gradient under the disturbing line, with values computed using Campbell's formulas. Two calculated curves for the resistance component are plotted, for conductivities of 2.5×10^{-15} and 2.5×10^{-13} (abmhos per cm. cube). It will be seen that the experimental values lie between these two curves, tending towards the former for

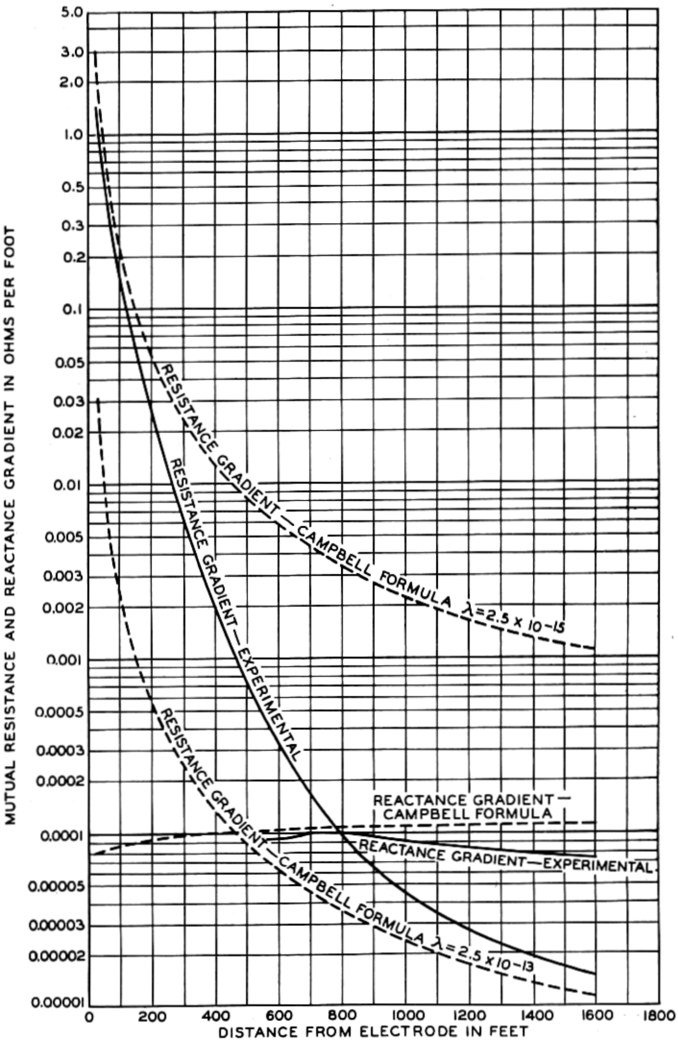

Fig. 10—Cross Keys tests. Mutual impedance gradient in vicinity of grounding electrode. Comparison with calculated values.

short distances from the electrode and towards the latter for long distances. As in the measurements previously described, the calculated mutual reactance component is greater than the measured value, although in this case the discrepancy is substantially smaller. On Fig. 11 are shown the phase angles of the gradient as computed from the calculated values of resistance and reactance components given on Fig. 10. Here also the measured curve falls between the two calculated curves.

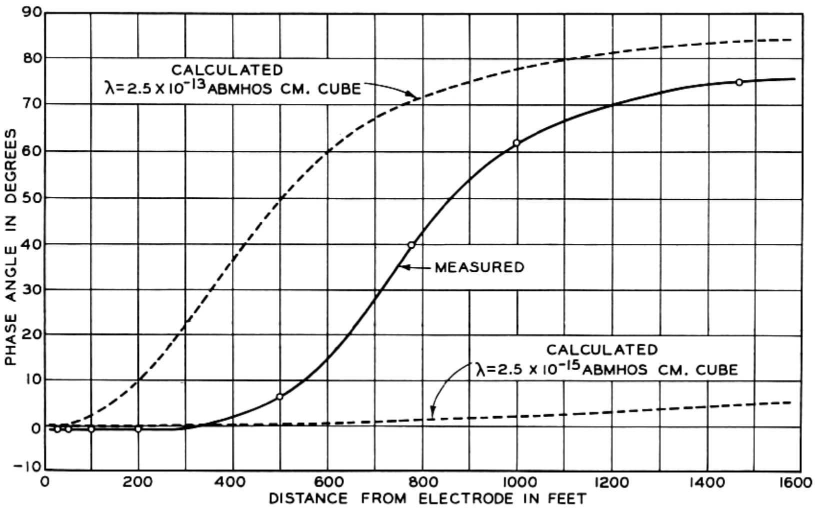

Fig. 11—Cross Keys tests. Comparison of measured and calculated (from Campbell's formula) angles of mutual impedance gradient.

Since the gradient near the electrode is obviously affected mainly by the conductivity of the earth in the immediate neighborhood, and that at remote points is influenced more by the conductivity of the earth at substantial depths, the possibility that the earth in this region is not homogeneous, but stratified, is suggested. These curves seem to support the conclusion that the earth in this neighborhood has at least two strata, the upper one having a very low conductivity and the lower one a conductivity approximately a hundred times greater. Further experimental evidence tending to the same conclusion has been obtained, and will be described presently. For the present it may be pointed out that this conclusion is supported by the geological data pertaining to this region, for which an upper layer of sand and gravel from 130 to 170 ft. in depth is indicated, superimposed on a mixed structure of sand, clay, and shale, with a substantial amount of ground water.

Methods Considering A.-C. Distribution of Earth Current. The problem of computing the mutual impedance of ground-return circuits, considering an a.-c. distribution in the earth has been attacked by several writers.[3] In the interpretation of the experimental results, the papers of J. R. Carson [7] and F. Pollaczek [6] have been used, since a minimum of assumptions was made in the solutions advanced by these writers. The assumptions made are that the disturbing circuit is straight and of great length,[9] that the propagation constant, in absolute units, of the circuit is very small compared to unity, and also that the earth is a homogeneous body of fairly good conductivity. With these assumptions it is found possible to solve the fundamental field equations for the magnetic and electric fields in the vicinity of the disturbing conductor at points remote from the ends of the circuit and thence to get the mutual impedance. Physically, this method recognizes and takes into account the fact that in a conductor of large extent, such as the earth, the distribution of alternating current will be influenced by the changing magnetic field. Qualitatively, the effects are similar to those involved in the well-known skin effect, and may be thought of in terms of a distribution of eddy currents in the earth. It is obvious that the distribution of the eddy currents will depend on the earth conductivity and also on the frequency. The resultant fields, and hence the mutual impedances, will then be functions of earth conductivity and of frequency.

Presentation of the formulas and graphs giving the results of the analysis is outside the scope of the present paper, and reference should be made to the original papers for these. As an illustration of the results, however, the curves of Fig. 12 have been prepared, showing the calculated mutual resistance and reactance of ground-return circuits at a frequency of 60 cycles for several values of earth conductivity, within the range of experimental values. Both the resistance and reactance components are seen to be pronouncedly affected by earth conductivity, particularly for the larger separations.

In applying this theory to the tests made at Cross Keys, the procedure adopted, in the absence of direct data on the earth conductivity at this location, was to choose an earth conductivity which would result in the best fit between the calculated and observed values, and to see whether a single value for earth conductivity would suffice to explain all the results. On Fig. 13 comparisons have been made between the experimentally determined mutual impedances for the 60- and 500-cycle frequencies; the curves were computed by use of the formulas given by Carson. It will be seen that in so far as the magnitude of the mutual impedance is concerned an excellent agreement can be made between

[3] See bibliography references 3 to 9, inc.
[7] See bibliography.
[6] See bibliography.
[9] See bibliography.

the calculated and observed values. However, for the best agreement it is found necessary to assume a different earth conductivity at 500 cycles than at 60 cycles. Thus, while at 60 cycles the indicated earth conductivity is 4.2×10^{-13} abmhos per cm. cube, at 500 cycles it is

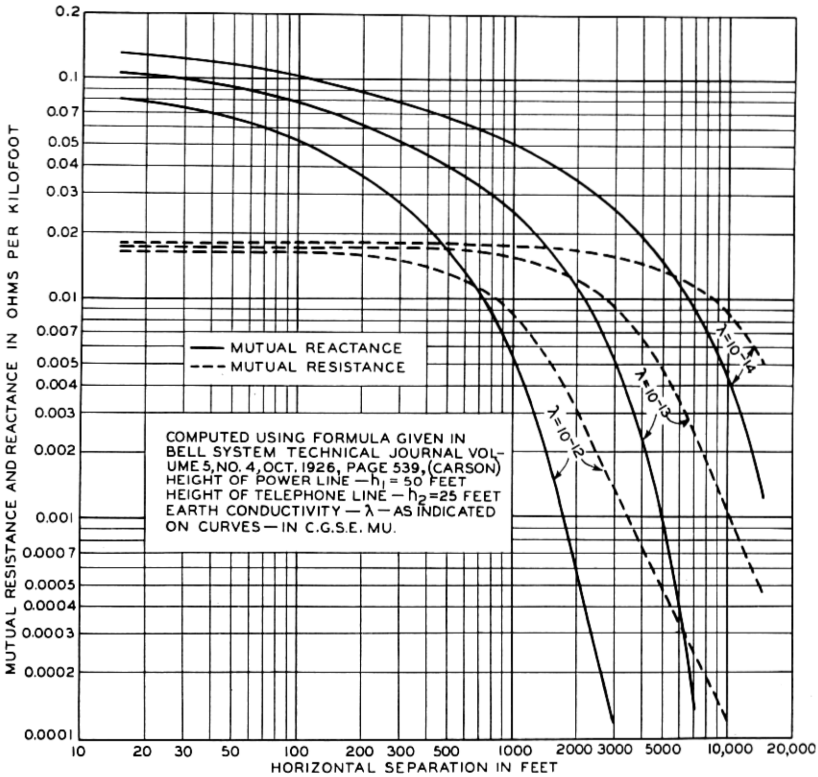

Fig. 12—Resistance and reactance of ground-return circuits. Frequency 60 cycles.

2.76×10^{-13}. However, a computed curve for 500 cycles, using a conductivity of 4.2×10^{-13} falls below the experimental curve by only 30 per cent. Table I gives the values of earth conductivity required to

TABLE I.

CROSS KEYS TESTS.

Earth Conductivity Giving Best Agreement Between Calculated and Measured Values of Mutual Impedance.

Frequency cycles	Indicated earth conductivity from Carson's formulas abmhos per cm. cube
60	4.2×10^{-13}
200	3.75×10^{-13}
500	2.76×10^{-13}
1000	2.0×10^{-13}

give the best fit to the curve of mutual impedance at each frequency. The range is not large, extending only from 4.2×10^{-13} at 60 cycles to 2.0×10^{-13} at 1000 cycles.

Turning to the components of the mutual impedance, however, the agreement is found to be not as good. Fig. 14 shows the measured values of mutual resistance and reactance at 60 cycles and at 500 cycles, also the computed values, the calculations at each frequency being made with the earth conductivity indicated in Table I. At 60 cycles

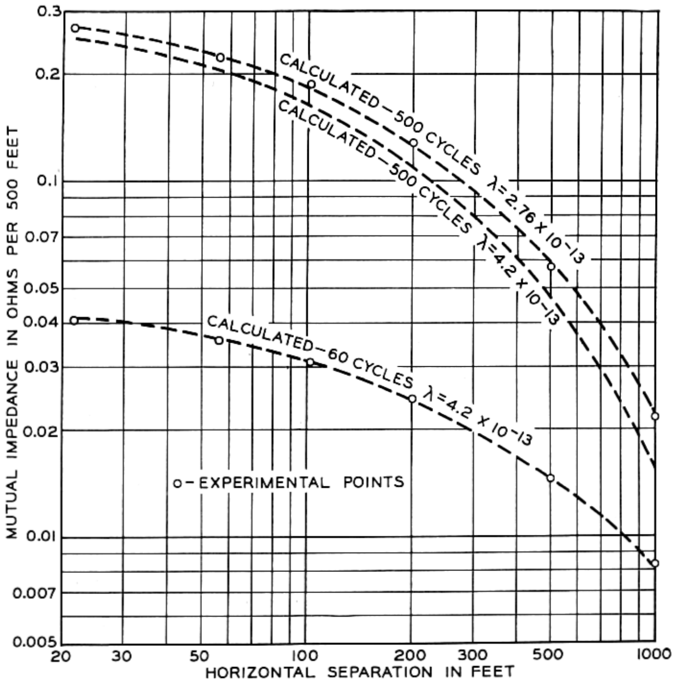

Fig. 13—Cross Keys tests—Carson theory. Comparison between calculated and measured values of mutual impedance.

the agreement is quite good, but at 500 cycles the departure between calculated and measured values is large. The measured mutual resistances are consistently lower than those calculated, while the measured mutual reactances are higher.

As indicated by the above comparisons, a theory of the kind under discussion leads to results which are in quite good quantitative agreement with the experimental results; it is of some interest to discover whether an extension of the theoretical ideas would lead to still closer agreement. It was stated previously that the measurements around the grounding electrode could be accounted for on the hypothesis that the earth in this neighborhood is stratified, with a conductivity of around 2.5×10^{-15} near the surface and 2.5×10^{-13} in the lower depths. Qualitatively, it is to be expected that with such an earth structure the mutual resistance would be less, and the mutual reactance greater, than the corresponding values for an earth of uniform conductivity, since the eddy currents near the surface of the earth will be less, due to the lower earth conductivity.

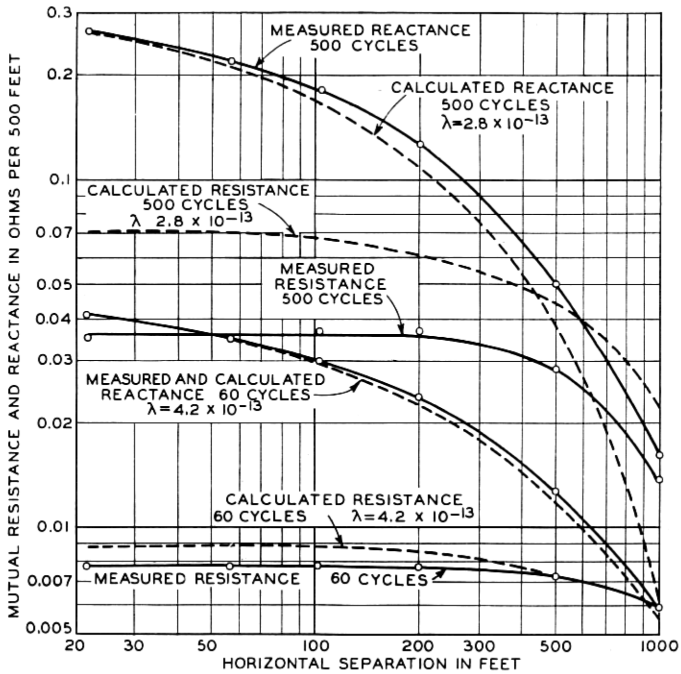

Fig. 14—Cross Keys tests—Carson theory. Comparison between calculated and measured values of mutual resistance and reactance.

Quantitatively, it would seem that a first approximation to the effect of a stratified earth in which the upper stratum has a much lower conductivity than that of the lower region could be obtained by assuming that the currents in the upper layer are negligible and hence that this layer can be abolished. The mutual impedances can then be worked out by the formulas applicable to a homogeneous earth, using the earth conductivity of the lower region and fictitious conductor heights, formed by adding the thickness of the upper stratum to the heights of the conductors above the actual earth's surfaces. Preliminary calculations have been made using this scheme, and it was found that using a conductivity of 5.0×10^{-13} for the lower stratum and a thickness of 130 feet for the upper stratum an excellent agreement could be found between calculated and observed values for all frequencies. The agreement extended not only to the magnitudes of the mutual impedances, but to the components as well.

Because of the simplifying assumption that the disturbing circuit is so long that the effects due to the ground connections at its ends can be neglected, the theory which we have been discussing is obviously inadequate to explain the phenomena in the neighborhood of the grounding electrode.

Practical Methods for Predetermining Coupling between Power and Telephone Lines

The ultimate purpose of the work of the committee is to develop simple methods to enable the calculation of the mutual impedance between power and telephone circuits before they are built. It is evident from the foregoing discussion that the use of any formula for the mutual impedance of ground-return circuits requires a knowledge of the conductivity of the earth or of the depth of the equivalent ground-plane. In the relatively few places in which tests have been made, a range of earth conductivity from 10^{-12} to 10^{-14} abmhos per cm. cube has been observed, and reference to Fig. 12 indicates that within this range of earth conductivity a variation in mutual impedances of 20 to 1 or more may exist. Therefore, other experimental work has been done with the object of developing relatively simple testing schemes, the results

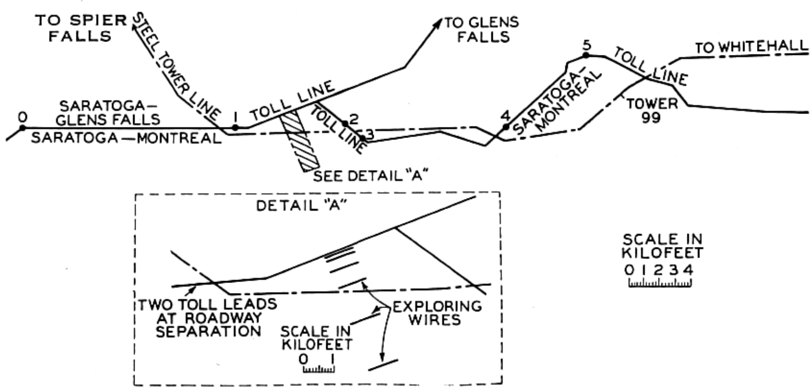

Fig. 15—Glens Falls tests.

	Circuit arrangements	
Tower 99	3-Phase conductors	Grounded
Circuit	Power	Telephone
Height	50 ft.	20 ft.
Frequency	60 cycles	

of which could be used to predict the coupling coefficients in advance of the construction of the power or telephone line. An obvious method is to determine an experimental coupling curve by performing tests similar to those made at Cross Keys, using short-length disturbed circuits and either an existing power or telephone line, or a specially laid out conductor, as the disturbing line.

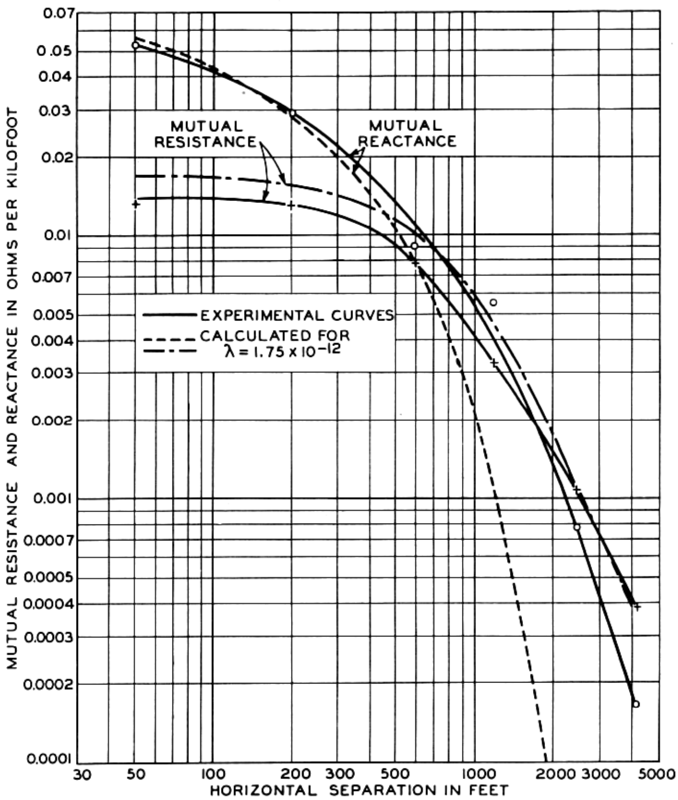

Fig. 16—Glens Falls tests. Experimental values of mutual resistance and reactance.

This experimental curve would then be used to compute the coupling between power and telephone lines. One advantage of using an experimentally determined coupling curve is that it obviates the necessity of knowing or assuming a structure and conductivity of the earth; the coupling curve can be used directly without reference to any theoretical formulas. To determine the practicability of such a scheme, 60-cycle tests have been made in two locations where existing exposures were present, for the purpose of determining the accuracy with which experimental observations could be predicted.

Tests at Glens Falls, N. Y. Fig. 15 shows the arrangement of circuits involved in tests made at Glens Falls, N. Y. A section of the Saratoga-Glens Falls telephone line about six miles in length was energized with ground-return current. Measurements were made of the voltages induced in short ground-return circuits laid on the ground parallel to the straight section of the telephone line. The resistance and reactance components of the mutual impedance derived from these measurements are given on Fig. 16. As a matter of interest the mutual resistance and reactance computed by the use of the Carson formulas for an earth conductivity of 1.75×10^{-12} are also given. This earth conductivity gives the best agreement between the calculated and observed magnitudes of the mutual impedances. The general agreement between the computed and observed quantities is much like that found from the Cross Keys tests.

Earth return current was then sent over the power line from Spier Falls to Tower 99, and induced voltages measured in the entire exposed section of the Saratoga-Montreal telephone line, and in several parts of the exposure as indicated on the sketch. In Table II, the observed

TABLE II.

GLENS FALLS TESTS

Measured Mutual Impedances of Power and Telephone Circuits and Comparison with Values Calculated from Coupling Curves of Fig. 16.

Section of telephone line	Measured mutual impedance—ohms	Calculated mutual impedance—ohms
0–1	.0586 /68.5°	.0614 /53.3°
1–2	.0294 /52.4°	.0564 /49.8°
2–3	.0476 /73.4°	.0382 /69.6°
3–4	.107 /56.4°	.113 /49.2°
4–5	.100 /44.4°	.0117 /35.8°
0–5	.347 /5[illegible].8°	.267 /55.3°

mutual impedances determined from this latter test are compared with values calculated by using the experimental coupling curve given on Fig. 16. The agreement between computed and observed values is, in general, only fair, although for two of the parts, the agreement is excellent. It is thought that the rather poor check for Sections 1-2 and 2-3 is due to the inductive effect of currents set up in the ground wire on another power line which extended through these sections. With regard to the extreme departure of the measured mutual impedance for Section 4-5 from that calculated it is impossible to decide the cause from the experimental data available. A possible explanation is that it is due to a large difference in earth conductivity in this region from that in the region in which the coupling curve was determined. The large difference in this section is reflected in the rather poor check in the overall coupling (Section 0-5).

Tests at Massillon, Ohio. Tests made at Massillon, Ohio, were similar to those at Glens Falls, except that the arrangements were somewhat more elaborate. The layout of the circuits involved is shown in Fig. 17. The exposure is about 16 mi. long with separation between the power and telephone line ranging from a crossing to about 4200 ft., a large part of the exposure being at a separation of about 3000 ft. A set of "exploring wires," each 200 ft. in length, was laid on the ground parallel to the telephone line as shown in the detail of Fig. 17. These were arranged in four groups and were distributed over an area approx-

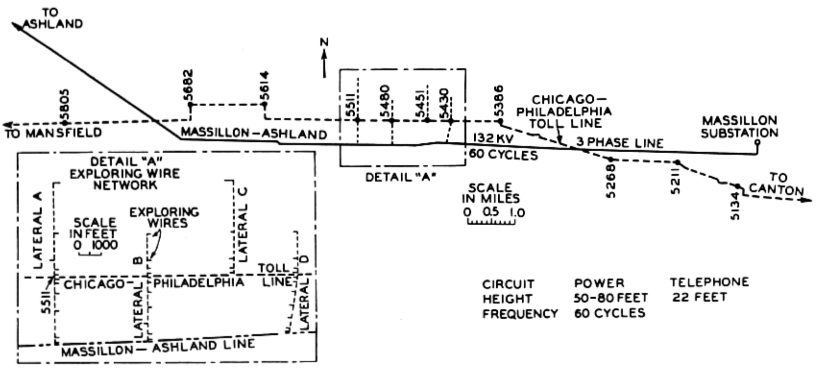

Fig. 17—Massillon tests—circuit arrangements.

imately 1½ by 2 mi. Coupling curves were determined from measurements of the voltage induced in the exploring wires for the condition of the telephone line energized with 6 amperes ground-return current, and also for the condition of the power line energized with 40 amperes ground-return current. The mutual impedances derived from the two sets of measurements are practically identical. Fig. 18 shows the resistive and reactive components of the coupling curve using the average of all measurements made on the exploring wires. A comparison of the measured curves with curves calculated by Carson's formulas for a value of earth conductivity of 3.6×10^{-13} abmhos per cm. cube show the same type of agreement as that observed at Cross Keys and Glens Falls.

The principal reason for using such a large number of exploring wires on this particular test was to investigate the effect of local irregularities of the earth upon an experimental coupling curve and to determine the minimum number and length of exploring wires which it is necessary to use in order to be reasonably confident of the accuracy of the results.

The data indicate that if only one of the seven groups of 200-ft. exploring wires had been used, the maximum deviation of any one point from the corresponding point on the average curve would have been less than 25 per cent and that the probable deviation would have been less than 10 per cent. This deviation could probably be reduced by using a somewhat longer exploring wire.

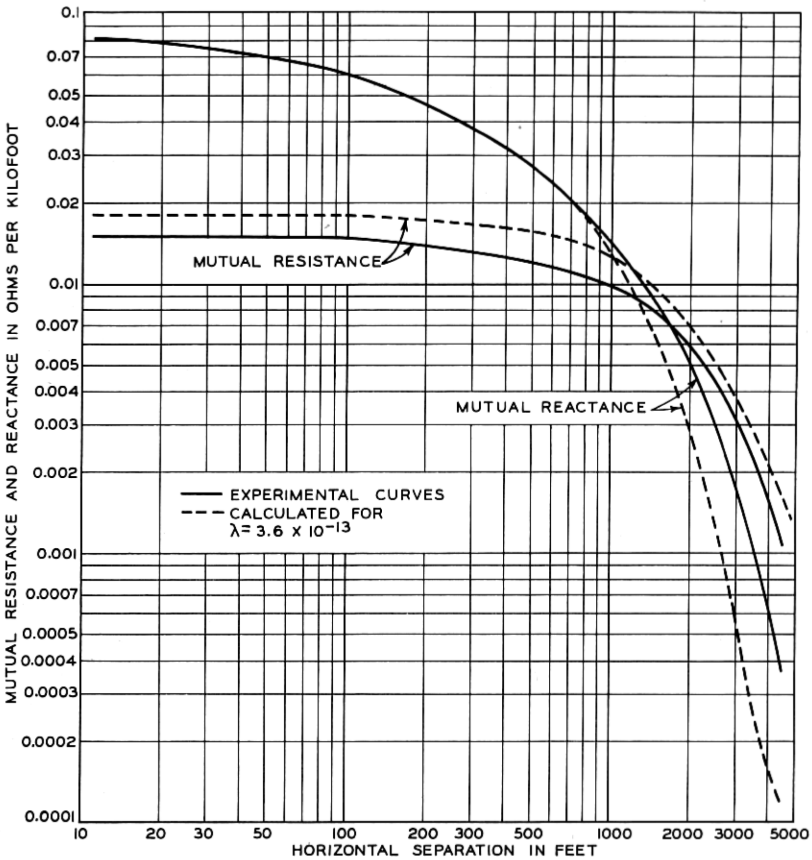

Fig. 18—Massillon tests. Experimental values of mutual resistance and reactance.

Measurements were also made of the voltage induced in sections of the telephone line when the power line was energized with ground-return current. A comparison of the measured values of the mutual impedance and those given from calculations using the coupling curves of Fig. 18 are given in Table III. With the exception of the section from pole 5134 to 5211, for which the calculations may be subject to some error due to proximity to the end of the disturbing circuit, and the section from pole 5614 to 5682, the agreement between measured and calculated values is statisfactory.

TABLE III.

MASSILLON TESTS.

Measured Mutual Impedances of Power and Telephone Circuits and Comparison with Values Calculated from Coupling Curves of Fig. 18.

Exposure		Measured mutual impedance-ohms	Calculated mutual impedance—ohms
From pole	To pole		
5134	5211	0.0308 /45.9°	0.0415 /35.5°
5211	5268	0.0532 /44.7°	0.0442 /34.2°
5268	5386	0.282 /63.7°	0.280 /64°
5386	5511	0.0707 /30.3°	0.0630 /28.2°
5511	5614	0.0386 /29.7°	0.0427 /28.5°
5614	5682	0.0164 /18.4°	0.0117 /14.8°
5682	5805	0.141 /62.4°	0.149 /59.2°
5134	5805	0.609 /53.7°	0.612 /52.5°

D.-C. Determination of Earth Conductivity. In considering the experimental results described above, particularly the reasonably good agreement between the experimental coupling curves and those calculated by means of the Carson formulas with suitably chosen earth conductivity, it seemed desirable to investigate whether an experimental value of earth conductivity alone would be sufficient information for the determination of coupling curves with enough accuracy for many purposes. With this in mind, an investigation has been undertaken of more direct methods for determining the conductivity of the earth or more generally of methods for determining the structure of the earth in a particular location (whether homogeneous or stratified and, if the latter, the thickness of the strata) and the earth conductivity. The work is at present only in an early stage, but a brief statement of the method followed and of the results so far obtained may be of interest.

The procedure followed amounts to an investigation of the mutual resistance of a number of suitably located ground-return circuits, with direct current. It will be recalled that in an earlier part of the paper it was stated that for a homogeneous earth the mutual resistance of two ground-return circuits, for direct current, can be easily derived, and expressed in a formula involving only the earth conductivity and the distances between the grounding electrodes. For a stratified earth, similar formulas have been worked out involving the distances between the grounding electrodes and the conductivities, and the thicknesses of the several strata. By means of measurements of the mutual resistance for direct currents of circuits with suitably located ground electrodes, the conductivities and thicknesses of the strata can then be determined.

Practically, the experimental work presents many problems, among them being the elimination of the effects of stray earth currents and evaluation of the effects local irregularities in earth conductivity. A preliminary trial of the method was made in connection with the tests at Massillon, and while local irregularities were found to be quite marked, yet the average earth conductivity in the region covered by the tests was about 1.5×10^{-13} abmhos per cm. cube, which is not greatly different from that indicated by the coupling tests. A quite extended series of tests at Cross Keys, using an improved technique, yielded results in excellent agreement with the hypothesis that at this location the earth is stratified, having an upper layer about 150 ft. thick with a conductivity about 3.4×10^{-15} and a conductivity in the lower stratum about 2.6×10^{-13}.

Conclusions

In conclusion, it is well to recall the end towards which the work described in this paper has been directed. It was desired, first, to obtain a sufficiently detailed experimental study of the mutual impedances of ground-return circuits to enable the formation of an adequate picture of the physical phenomena involved; also to test out the theoretical formulas available. Second, the aim was to investigate practical means for enabling the calculation of the ground-return mutual impedances of power and telephone lines.

With regard to the first item, it was found that an analysis in terms of an "equivalent ground-plane" was inadequate to represent completely the observed phenomena. However, when information is available as to the proper value of ground-plane depth, this method can be used to advantage in many cases where approximate results only are desired.

A theory based on the assumption of a d.-c. distribution in the earth gave a somewhat better explanation, particularly in connection with the mutual impedances of circuits in which the points of ground connection were in close proximity, but left much to be desired in the way of quantitative agreement with the experimental results. The results of a theory which considers the effect of eddy currents in the earth are shown to be in fair qualitative agreement with some of the test values, and by a slight extension of the theory, good quantitative agreement can be found. This theory, however, does not explain end effects.

In the investigation of practical means for enabling the calculation of the mutual impedances of power and telephone lines a scheme involving the experimental determination of a coupling curve has been found to give quite satisfactory results. Further work is to be done on this problem, and similar tests must be made in several other locations before the method can be considered completely satisfactory. Other methods are also being investigated.

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