A Magnetic Curve Tracer

By F. E. HAWORTH

An apparatus for photographically recording hysteresis loops and initial magnetization curves is described. It employs a rotating drum and a fluxmeter, the restoring torque of the latter being completely counter-balanced by a photoelectric cell arrangement. With this apparatus curves may be taken so slowly that eddy currents are negligible. The accuracy of the instrument is intrinsically as great as that of a ballistic galvanometer. An analysis of sources of error is included.

FOR accurate determinations of hysteresis loops and initial magnetization curves of magnetic specimens, a laborious routine involving the use of a ballistic galvanometer is usually necessary. This article describes an apparatus by means of which these curves may be obtained photographically with quantitative accuracy. Attempts to devise such a scheme have previously been made. Ewing [1] describes one which was used with short, thick specimens in a magnetic yolk. Fleming [2] invented a device, the Campograph, which made use of a magnetometer and had the advantage of making possible the use of long, thin, specimens, thus reducing eddy current and demagnetization effects. J. B. Johnson [3] describes the most recently published design, embodying a vacuum tube amplifier and a Braun tube oscillograph. This hysteresigraph is used with frequencies of the order of five cycles per second, or higher, and consequently introduces an eddy current loss, a disadvantage in a great many measurements.

The greatest difficulty has always been to devise an instrument which would accurately record the total change in magnetic flux in the specimen. The ideal instrument would be a fluxmeter with no restoring force and no friction. Fluxmeters are on the market in which the restoring force is negligible only over short periods of time or in which there is no restoring force but where the friction is appreciable; but if it is required that the magnetic cycle have a period of more than a few seconds, such fluxmeters are out of the question. In addition they require that the search coil be of such low resistance that it must have too few turns for use with long thin specimens, in which the flux is small. These difficulties have been overcome in the apparatus described below, in which the principal feature is the use of a

[1] J. A. Ewing, "Magnetic Induction in Iron and Other Metals," 3d ed., p. 118.

[2] J. A. Fleming, *Proc. Phys. Soc. Lon.*, 27, 316–27 (1915).

[3] J. B. Johnson, *Bell System Tech. Jour.*, 8, 286–308 (1929).

fluxmeter in which the suspended coil has its restoring torque counterbalanced for all deflections within a range sufficient for accurate delineation of magnetic curves.

Description of the Apparatus

The operation of the apparatus is as follows: a long, sensitive, photoelectric cell is fitted with a V-shaped slit, as shown in Fig. 1; a beam of

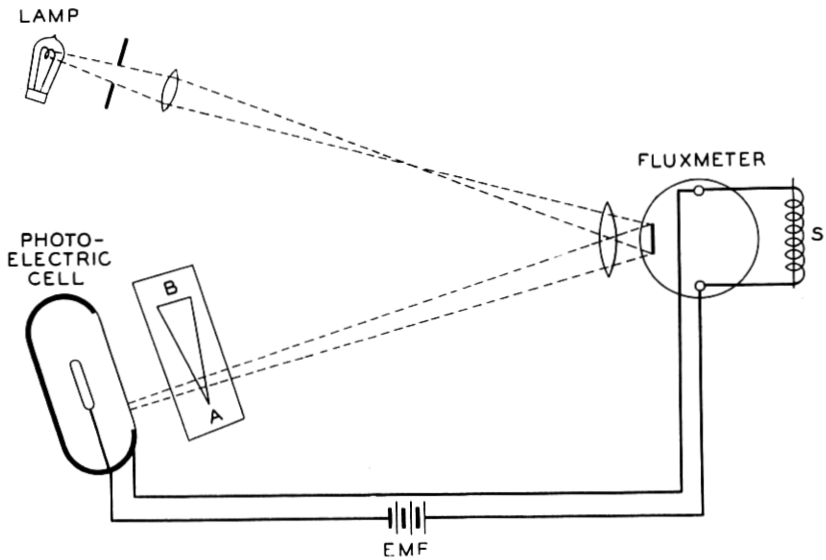

Fig. 1—The photoelectric cell circuit.

light is reflected from the mirror of the fluxmeter and focused on the slit of the photo-electric cell, which is connected, in series with a source of e.m.f., across the terminals of the fluxmeter; the e.m.f. is adjusted once for all to such a value that, if the beam is at rest when at the narrow end of the slit, at any other position the current controlled by the cell will develop a torque in the fluxmeter coil which just balances the restoring torque of the suspension. The fluxmeter deflection will then be proportional to the change of flux which has occurred within the search coil *S*. It may be found necessary to shape the slit empirically to correspond to the unequal sensitivities of the photo-electric cell at different spots. The fluxmeter used is a Leeds and Northrup type 2290 HS galvanometer. It has a critical damping resistance of about 100,000 ohms, and when used with about one hundred ohms in the external circuit it is much over damped.

The apparatus for registering the deflections photographically, and for changing the magnetic field in the specimen, is shown in Fig. 2. A drum D, carrying photographic paper, is placed in a light-tight box provided with a long, narrow slit parallel to the axis of rotation of the drum. A beam of light from a second lamp is reflected by the fluxmeter mirror and focused on the slit. This beam is reflected by the same mirror which reflects the beam onto the photo-electric cell, the two

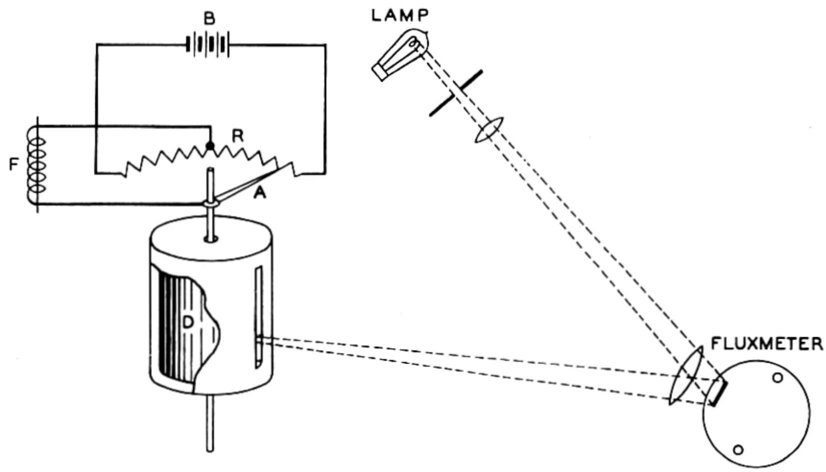

Fig. 2—The field current circuit and the photographic drum.

beams being incident at different angles. Attached to the shaft of the drum is an arm A, which slides along the rheostat R. A battery B is connected across R, and a center tap soldered to it. Between the arm A and the center tap a varying e.m.f. is produced which is applied to the field coil F. This e.m.f. reverses its sign every time the arm A slides past the center of the rheostat, and the latter is curved in a manner calculated so that the field current will be proportional to the angle of rotation of the drum from the position for zero current. The search coil S of Fig. 1 is placed within F, and consequently when D is rotated it moves the photographic paper past the slit so that the distance moved is proportional to the change in field current, while at the same time the fluxmeter deflects the beam of light along the slit so that the deflection is proportional to the time integral of the changes of flux within S. As the drum is turned from one position to another, a curve with rectangular axes is thus registered, the scales of which may be calibrated in terms of B and H. Figs. 4 to 7 are some examples of curves taken with the apparatus.

In Fig. 3 the electrical circuits are shown in detail. R_7 is the rheostat controlling the field current, and A is the arm which rotates with the drum. The battery B_2 supplies the field current, and B_3 furnishes the e.m.f. for the photo-electric cell, the value of the potential applied to the latter being regulated by R_1. The potential divider R_3, and dry cell B_1, in series with the 10 megohm resistance R_2, are used to balance out thermo-electric potentials and current from the photo-electric cell

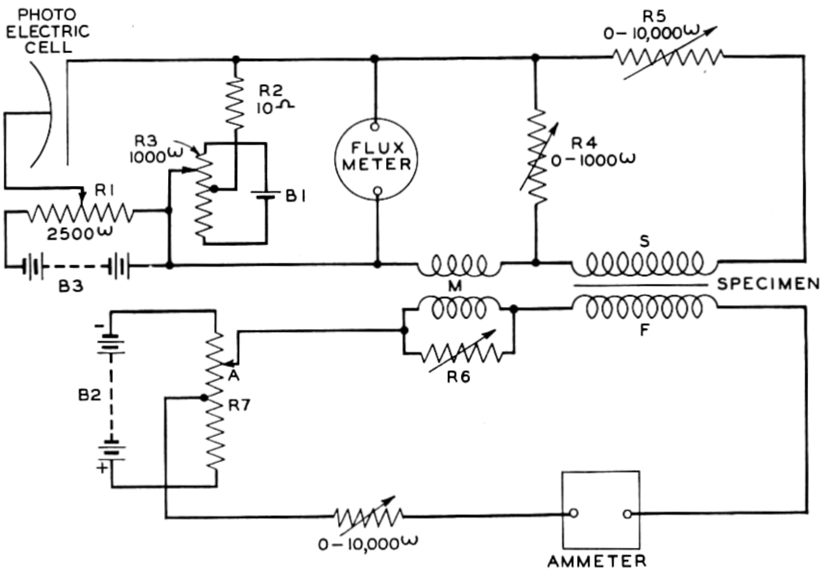

Fig. 3—Detailed diagram of the electrical connections.

due to stray light. R_4 and R_5 are adjusted according to the amount of flux in the specimen, in order to keep the maximum deflection within the desired limits. The mutual inductance M is used to balance out the potentials produced in S when no specimen is within it, so that the

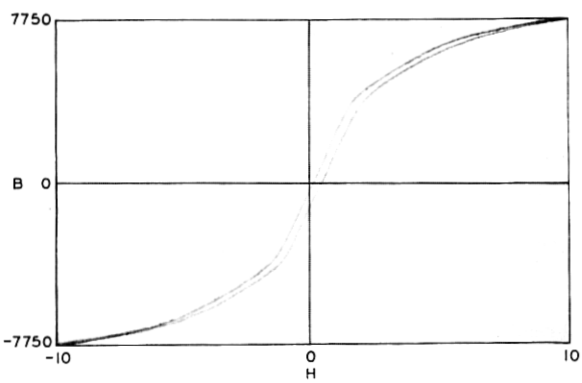

Fig. 4—Hysteresis loop of annealed iron.

fluxmeter deflection is proportional to the change in $B - H$. The drum is conveniently rotated by an electric motor, connected by gears so that the drum makes about one revolution in two minutes, and it is desirable to have this rate variable. The motor may be reversed, so that complete hysteresis loops may be recorded.

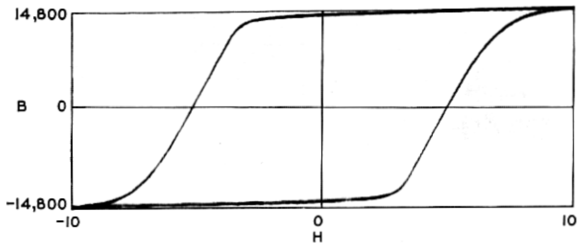

Fig. 5—Hysteresis loop of hard iron.

In setting up the apparatus the photo-electric cell may be conveniently placed above or below the drum, and one lamp above the other. The lamp used to illuminate the photo-electric cell should furnish a brilliant beam, and it was found that a 250 watt Mazda projection lamp was quite satisfactory.

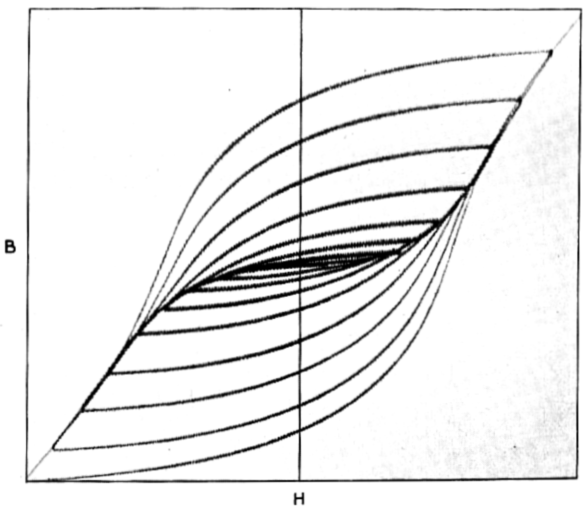

Fig. 6—Hysteresis loops of hard iron, with increasing maximum fields.

Calibration of the Circuit

The circuit is calibrated by passing a known current through the primary of a known mutual inductance, the secondary of which is connected in series with the search coil S. By measurement of the deflection produced the relation between the quantity of electricity passing through the fluxmeter and its deflection can be determined. From this relation and other known constants the change in induction of a magnetic specimen producing a given deflection may be calculated.

This calibration may be done in the following manner: Let the

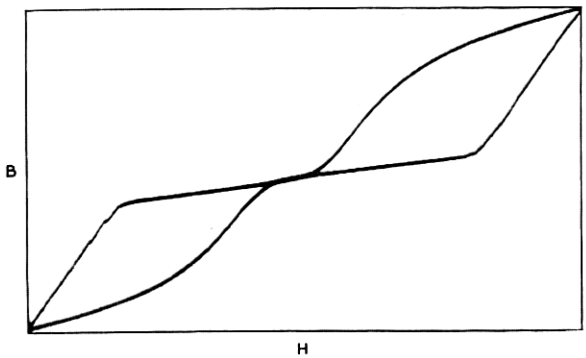

Fig. 7—Hysteresis loop of perminvar, showing the "wasp waisted" loop.

magnetic specimen be removed, R_4 and R_6 be set on infinite resistance, the magnetizing coil F be shorted, and a change in the field current made which will give a convenient deflection on the drum, as shown in Fig. 8.

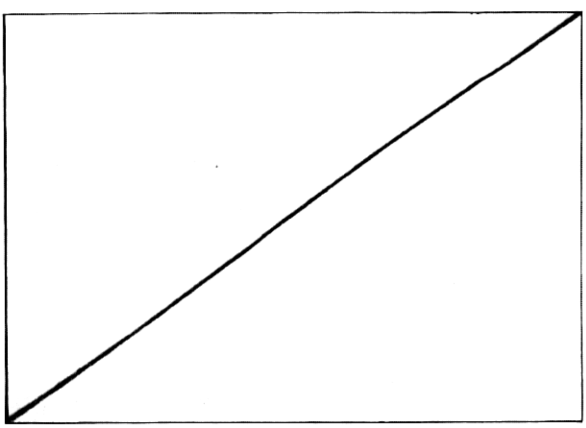

Fig. 8—Line taken for calibrating the apparatus.

Let i_M = instantaneous primary current,
i_2 = instantaneous secondary current,
r_2 = resistance of secondary circuit,
M = mutual inductance of M,
L_2 = self inductance of secondary circuit.

Then:

$$\frac{L_2}{r_2}\frac{di_2}{dt} + \frac{M}{r_2}\frac{di_M}{dt} + i_2 = 0.$$

Integrating from time $t = 0$ to $t = t_0$, the time at any later instant,

$$\frac{L_2}{r_2}\int_0^{t_0} di_2 + \frac{M}{r_2}\int_0^{t_0} di_M = -\int_0^{t_0} i_2 dt.$$

Now if i_M is changed slowly enough

$$\frac{L_2}{r_2}\int_0^{t_0} di_2$$

is negligible and we have:

$$\frac{M}{r_2}\int_0^{t_0} di_M = -\int_0^{t_0} i_2 dt,$$

or

$$\frac{M}{r_2} i_M = -Q_M,$$

where Q_M is the quantity of electricity that has passed through the fluxmeter in time t_0. Now let $Q_M = -K\delta_M$, where δ_M is the deflection produced when Q_M flows. Then:

$$\frac{M}{r_2} i_M = K\delta_M,$$

and

$$K = \frac{Mi_M}{r_2\delta_M},$$

and the quantity of electricity which has passed through the fluxmeter for any other deflection is

$$Q = -\frac{Mi_M}{r_2\delta_M}\delta. \tag{1}$$

This equation makes it possible to determine $B - H$, calculated from Q as described below, by observing the deflection δ. Relation (1) may be determined once for all as it is a constant of the fluxmeter only. The parts of Q passing through R_2 and the photo-electric cell will be negligible on account of their high resistances.

Now suppose a magnetic curve recorded with R_6 adjusted until the deflection is due solely to the magnetization of the specimen. Let the resistance of the fluxmeter plus that of the secondary of M be denoted by R_g, and that of S plus R_5 be denoted by R_s. Then if the field current i_H is varied slowly enough, the time lag in the secondary circuit will be negligible and we shall have for the instantaneous current in the fluxmeter:

$$i_g = \frac{e}{R_s + R_g + \dfrac{R_s R_g}{R_4}}.$$

Now the e.m.f. in the search coil is

$$e = -AN\frac{d(B-H)}{dt},$$

where A is the cross sectional area of the specimen and N is the number of turns in the search coil. Then

$$i_g = \frac{-AN\dfrac{d(B-H)}{dt}}{R_s + R_g\left(1 + \dfrac{R_s}{R_4}\right)}$$

and therefore

$$Q = \int_0^{t_0} i_g dt = \frac{-AN}{R_s + R_g\left(1 + \dfrac{R_s}{R_4}\right)} \int_0^{t_0} d(B-H).$$

But by Eq. (1)

$$Q = -K\delta$$

therefore

$$\Delta(B-H) = \frac{K\delta\left[R_s + R_g\left(1 + \dfrac{R_s}{R_4}\right)\right]}{AN}, \qquad (2)$$

where

$$K = \frac{M\Delta i_m}{r_2 d_M},$$

r_2 being the total secondary resistance when K was determined. This equation, then, gives $B - H$ for any given deflection δ, in terms of known constants. For any fluxmeter, K is determined once for all by passing the current i_M through a mutual inductance and measuring the deflection δ_M on a photographic record. The other constants are changed in a calculable way when the number of turns in the search coil, the resistance settings, and the cross-sectional area of the sample are changed.

Sources of Error

Since it is the voltage applied to the magnetizing coil F which is proportional to the angle through which the drum has rotated, there is a lag in the field current behind the field registered on the drum, due to the self inductance of the coils. Added to this there is a lag in the secondary due to its self inductance, and another lag due to the time required for the fluxmeter to act. The effect of these is to widen the loop. In Fig. 9 is shown a curve traced with no magnetic sample in the field coil, and with dH/dt so great that the lag is appreciable.

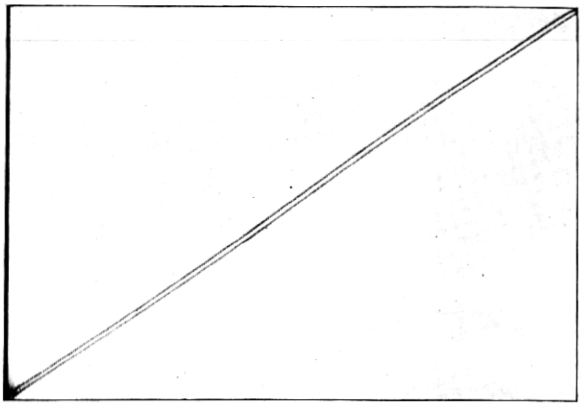

Fig. 9—Loop made with an air core mutual inductance at a very high dH/dt.

Fig. 10 shows two loops, the outer one representing a loop as taken on the apparatus, and the inner one the true loop corresponding thereto. Let B be some induction near zero, on the traced loop. B will be incorrect for the indicated value of H by an amount $B_0 - B$, such that if the field were held constant at that point while the drum continued to rotate the curve would approach B_0 as an asymptote, as indicated by the dotted curve. If dH/dt is not zero, B may be regarded as momentarily approaching B_0 as an asymptote. The equation for B at any instant is:

$$\lambda_1 \frac{dB}{dt} + B = B_0, \tag{3}$$

where λ_1 is the time constant of the circuit and B_0 is not a constant but a function of H and t. If we assume that dB/dH is constant for a small region in the neighborhood of $B = 0$, we have, putting ΔH_c equal to the error in coercive force H_c,

$$B_0 - B = \frac{dB}{dH} \Delta H_c.$$

Combining this with Eq. (3), we have

$$\Delta H_c = \lambda_1 \frac{dH}{dt} \cdot \tag{4}$$

Data taken with no magnetic specimen inserted show that this linear relation actually exists. Added to this there is an increase in H_c due to

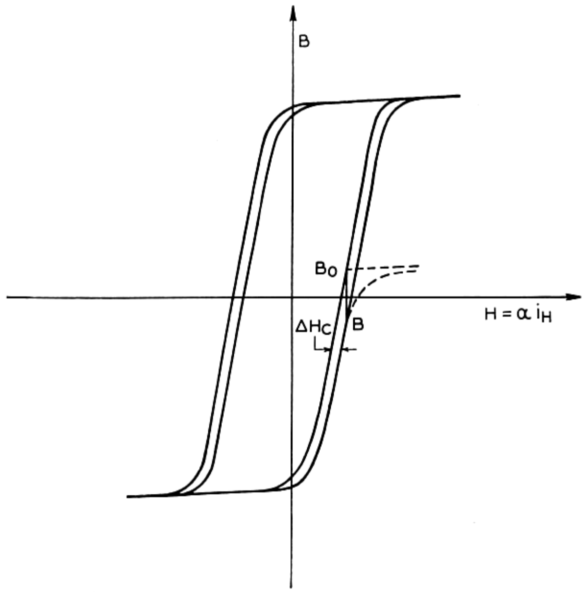

Fig. 10—A diagram to illustrate the widening of a loop due to inductance.

eddy current lag. Johnson [3] has derived an equation for this, and with a slight modification to make it applicable to cylindrical specimens, it is:

$$\Delta H_c = \frac{\Pi}{2} \frac{10^{-9}}{\rho} r^2 \frac{dB}{dH} \frac{dH}{dt}, \tag{5}$$

where ρ is the resistivity of the specimen, and r its radius. This gives us for the total error,

$$\Delta H_c = \left(\lambda_1 + \frac{\Pi}{2} \cdot \frac{10^{-9}}{s} r^2 \frac{dB}{dH} \right) \frac{dH}{dt}$$
$$= (\lambda_1 + \lambda_2) \frac{dH}{dt} \cdot$$

This equation was tested experimentally by taking a series of loops with varying dH/dt. The specimen used was a cylinder of 81 per cent Ni permalloy, 60 cm. long and 0.1 cm. in diameter, and was placed in a magnetic yolk. Its hysteresis loop, as shown in Fig. 11, has an

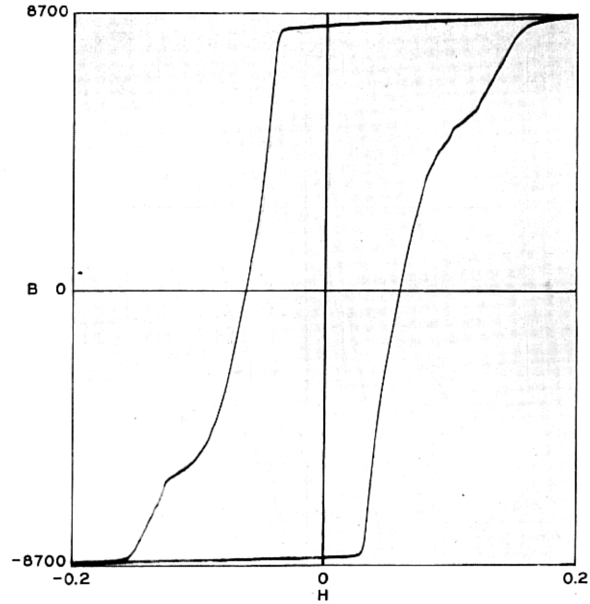

Fig. 11—A hysteresis loop of permalloy containing 81 per cent nickel.

unusual slope, 225,000 at $B = 0$. This gives $\lambda_2 = .055$ sec. From this series of curves the straight line shown in Fig. 12 was obtained, for which $\lambda_1 + \lambda_2 = 0.314$ sec. By another set of loops in which the

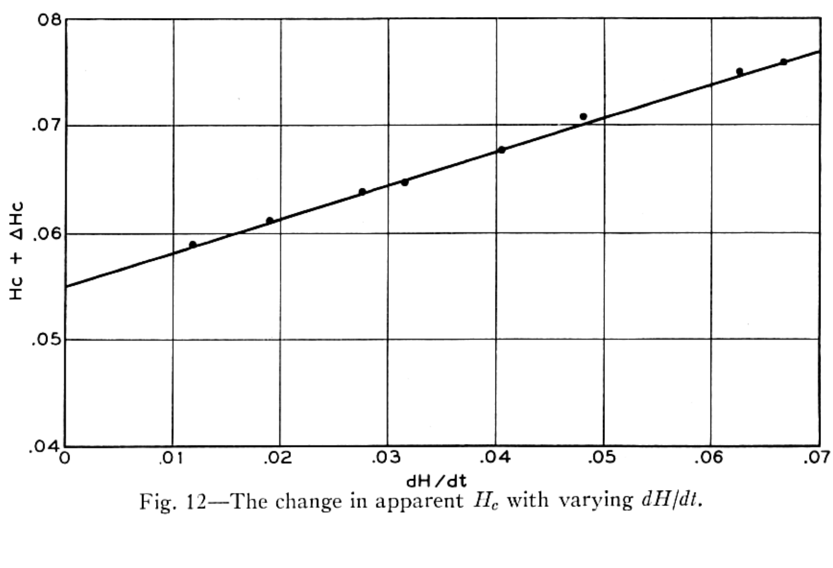

Fig. 12—The change in apparent H_c with varying dH/dt.

deflection is produced by an air core mutual inductance, λ_1 is found to be 0.134 sec. This determines λ_2 as 0.180 sec., in disagreement with the value 0.055 sec. calculated from Eq. 5. Johnson assumes in his derivation that dB/dH is constant and hence that the shape of the curve before H_c is reached has no effect on ΔH_c. It is probable that if the equation were changed to allow for dB/dH being a function H,

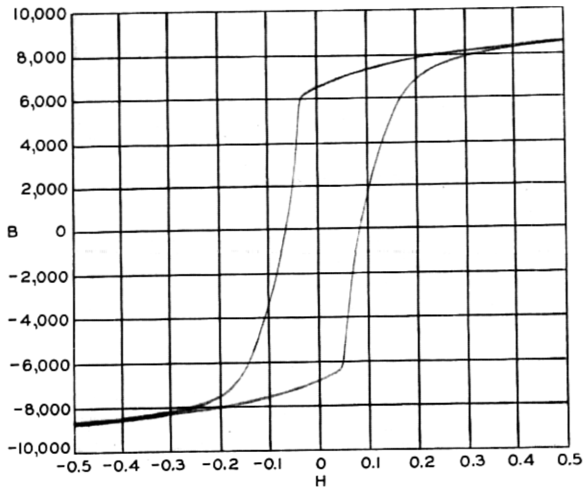

Fig. 13—A hysteresis loop of permalloy containing 78.1 per cent nickel.

that the difference could be accounted for. At any rate, this error is negligible for all but specimens with exceptionally high dB/dH or great thickness, and the true coercive force can always be found by taking two loops with different values of dH/dt and extrapolating to $dH/dt = 0$.

Another possible source of error is the passage of a large fraction of the photo-electric cell current through the search coil, the field being thereby altered. The maximum photo-electric cell current used is on the order of $5(10)^{-7}$ amperes. Since the search coil is unlikely to have more than about 400 turns per centimeter, this would make the maximum error in H about $2.5(10)^{-4}$ gauss, which is negligible for most measurements.

As a test of the accuracy of the instrument, a comparison was made with curves made by ballistic galvanometer measurements. Fig. 13 shows a loop taken of the specimen which Bozorth used in some previous measurements.[4] Both the coercive force and the maximum induction taken by the two methods agreed to within less than one

[4] R. M. Bozorth, *Phys. Rev.*, 32, 124–132 (1928).

per cent. Fig. 14 shows an initial magnetization curve which gives a value of the initial permeability agreeing accurately with the value determined ballistically.

A fluxmeter with no restoring torque is also useful in certain types of current measurements. If the average value of a current which fluctuates too much to be read on a slowly moving meter is desired, it

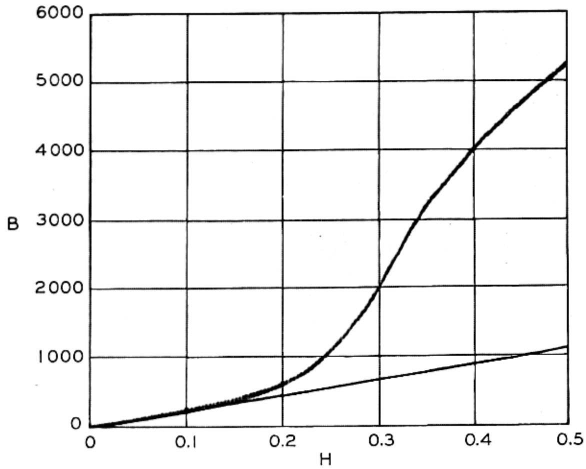

Fig. 14—An initial magnetization curve of the specimen of 78.1 per cent nickel permalloy.

can be integrated on the fluxmeter, and the average value obtained by dividing the total quantity of electricity which has passed through by the time during which the measurement was made. Also if a current is too small to be read directly on a galvanometer it may be possible to maintain it for a sufficient length of time to give a readable deflection on the fluxmeter, and again the current will be obtained by dividing by the time.

In conclusion I wish to thank Dr. R. M. Bozorth for suggestions given during the development of the apparatus, and Mr. A. W. Metz for his assistance in taking the curves.