Radio Propagation Over Plane Earth—Field Strength Curves

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Curves are presented to facilitate the calculation of radio propagation over plane earth. The magnitude and phase of the reflection coefficient for all conductivities of interest and for four values of the dielectric constant are presented in the form of curves from which the significant quantities may be read with the same degree of accuracy for all conditions. Simple equations, from which the effect of raising the antennas above the earth's surface may be readily calculated, are presented.

Introduction

THIS paper is intended to facilitate the calculation of radio propagation over plane earth. In Part I curves are presented that show the decrease of field strength with distance for antennas on the surface of the earth. In these curves the results obtained by Sommerfeld [1] and Rolf [2] are corrected and certain approximations [3] introduced by Rolf to reduce the number of variables to a workable number are eliminated. For a discussion of the Sommerfeld-Rolf curves, the reader is referred to a companion paper.[4]

Part II is concerned with the more general case of antennas above the surface of the earth. The complete equation that gives the field strength for antennas at any height above the earth is reduced to a simple equation which allows the calculation of the field under conditions of practical interest.

To facilitate field strength calculations, the values of the reflection coefficient are presented in the form of curves from which the significant quantities may be read with the required degree of accuracy for all angles of incidence.

Preliminary Remarks

A rectilinear antenna in free space generates an electric field whose effective value in the equatorial plane of the antenna at a distance large compared with the wave-length and the antenna length is

$$E_0 = \frac{60\pi HI}{\lambda d}, \tag{1}$$

where HI is equal to the line integral of the current taken over the

[1] Numbers refer to bibliography at end.

antenna.* If the antenna is placed above and perpendicular to a perfectly conducting plane and the antenna current is maintained the same, the electric field will be twice as great † or

$$E = 2E_0 = \frac{120\pi HI}{\lambda d}. \tag{2}$$

To maintain the current constant, however, it is now necessary to deliver more power to the antenna.

For a short doublet antenna in free space the radiation resistance is $R_0 = 80\pi^2 H^2/\lambda^2$ and hence the effective value of the received field strength is given as a function of the radiated power by ‡

$$E = \frac{3\sqrt{5}\sqrt{P}}{d}. \tag{3}$$

If this antenna is placed perpendicular to and very near a perfectly conducting plane the field strength pattern will be unchanged in the upper hemisphere but there will be no field below the perfectly conducting plane. The power that was required to produce the field in the lower hemisphere, which because of symmetry is half the total, is no longer radiated so that the same field strength will be produced by half the power,†† or

$$E = \frac{3\sqrt{10}\sqrt{P}}{d}. \tag{4}$$

If the transmitting antenna is removed so far from the ground that the reaction of the currents in the ground on the antenna current is negligible its radiation resistance is the same as if the ground were not present. The receiving antenna, however, still "sees" the image of the transmitting antenna in the ground. At a distance large compared with the height above ground, the transmitting antenna and

* The units are volts, amperes, meters and watts. H is the effective height of the antenna as defined in the most recent "Report of the Standards Committee" of the I.R.E. (1933).

† Under the hypothetical conditions taken by Sommerfeld, namely the antenna half in the ground and half in the air, the field is the same above a perfectly conducting plane as in free space. When the antenna is entirely above a perfectly conducting plane the field is the same as it would be if the plane were replaced by the image of the antenna in it. That is, the field is the sum of two equal components, one due to the antenna itself and the other due to its image. At distances large compared with the height of the antenna above the plane these two components are in phase and their sum is equal to twice either of them.

‡ For half-wave antennas the numerical factors in equations (3), (4) and (5) are respectively 7.0, 9.9 and 14.0.

†† Let E_1 be the received field strength in free space produced by a power P_1 and let E_2 be the field strength for an antenna perpendicular to and very near a perfectly conducting plane produced by a power P_2. Then $E_2 = E_1$ when $P_2 = P_1/2$, and by equation (3), $E_2 = E_1 = 3\sqrt{5}\sqrt{P_1}/d = 3\sqrt{5}\sqrt{2P_2}/d$, which is equivalent to equation (4).

its image are substantially the same distance from the receiver so that

$$E = \frac{6\sqrt{5}\sqrt{P}}{d}. \tag{5}$$

The way in which the ground currents affect the antenna resistance is given by the following equations which follow directly from more general cases considered by Sterba.[5]

$$\frac{R_V}{R_0} = \left[1 - 3\left(\frac{\cos v}{v^2} - \frac{\sin v}{v^3}\right)\right] = 2 - \frac{v^2}{10} + \frac{v^4}{280} - \cdots, \tag{6}$$

$$\frac{R_H}{R_0} = \left[1 - \frac{3}{2}\left(\frac{\sin v}{v} + \frac{\cos v}{v^2} - \frac{\sin v}{v^3}\right)\right] = \frac{v^2}{5} - \frac{3v^4}{280} + \cdots, \tag{7}$$

where v is equal to 4π times the height of the antenna above the ground in wave-lengths, and R_V and R_H are the radiation resistances of

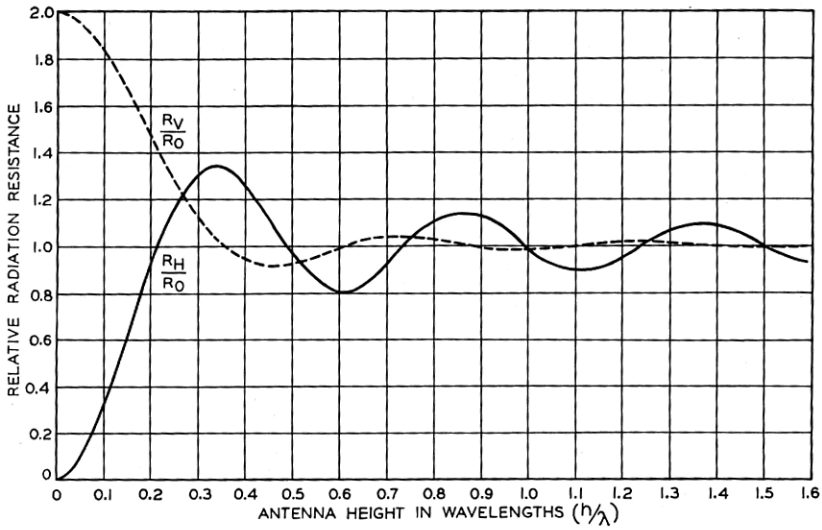


Fig. 1—Ratio of the radiation resistance of a short doublet antenna above perfectly conducting ground to that of the same antenna in free space.

short vertical and horizontal doublets above perfectly conducting earth respectively, and R_0 is the radiation resistance of the same antenna in free space. For the same input power the received field is inversely proportional to the square root of these ratios which are plotted in Fig. 1.

It is sometimes convenient to express the results in terms of the ratio of transmitted power to useful received power. The useful received power is the maximum power that can be transferred from the receiving antenna to the first circuit of the receiver. This is

$$P_r = \left(\frac{E\lambda}{8\pi\sqrt{5}}\right)^2 \tag{8}$$

for a short doublet. From equation (3) it follows that the ratio of transmitted to useful received powers for antennas in free space is *

$$\frac{P_t}{P_r} = \left(\frac{8\pi d}{3\lambda}\right)^2. \tag{9}$$

For short vertical doublets above the surface of a perfectly conducting plane this becomes,

$$\frac{P_t}{P_r} = \left(\frac{4\pi d}{3\lambda}\right)^2 \left(\frac{R_{V_1}}{R_0}\right)\left(\frac{R_{V_2}}{R_0}\right) \tag{10}$$

at distances that are large compared with the antenna heights. Here R_{V_1}/R_0 and R_{V_2}/R_0 are the ratios given by equation (6) and Fig. 1 for the transmitting and the receiving antenna respectively. When the antennas are more than a wave-length above the ground these ratios are substantially unity, and only one-fourth as much transmitted power is required as would be if the antennas were in free space. When both antennas are very near the surface of the earth, $R_V/R_0 = 2$ and the same transmitted power is required as in free space.

Part I—Vertical Antennas on the Surface of the Earth

In this section transmission between two short vertical antennas above and very near to the surface of plane earth will be considered. The attenuation factor will be taken as the ratio of the received field strength to that which would result if this plane surface had perfect conductivity.

In evaluating the electromagnetic field generated by a short vertical antenna on the surface of an imperfectly conducting plane it is convenient to first determine the auxiliary function $\mathbf{\Pi}$, called the Hertzian potential, from which the vertical component of the electric field may be obtained by means of the relationship,†

$$\mathbf{E} = -\frac{240 i\pi^2}{\lambda}\left(1 + \frac{\lambda^2}{4\pi^2}\frac{\partial^2}{\partial z^2}\right)\mathbf{\Pi} \quad \text{volts per meter.} \tag{11}$$

* For half-wave antennas the right-hand side of equation (9) must be multiplied by $(73.2/80)^2 = 0.837$.

† Bold face type is used to indicate a complex quantity. The same character in light face type represents its magnitude with which the radio engineer is concerned. The imaginary unit, $\sqrt{-1}$, is represented by i.

For an antenna on the surface of a perfectly conducting plane this function may be written *

$$\mathbf{\Pi} = 2\mathbf{\Pi}_0 = 2\frac{HIe^{-2\pi i R_1/\lambda}}{4\pi R_1} \quad \text{amperes,} \tag{12}$$

where $R_1 = \sqrt{d^2 + z^2}$ is the distance. For an antenna on an imperfectly conducting plane

$$\mathbf{\Pi} = 2W\mathbf{\Pi}_0, \tag{13}$$

where W is the ratio of the Hertzian potential due to an antenna on an imperfectly conducting plane to that on a perfectly conducting plane. W may be expressed as the sum of two infinite convergent series, A and D, which are defined in Appendix I (page 70).

$$W = A + D. \tag{14}$$

The series D becomes unwieldy for distances greater than the order of a wave-length. In order to facilitate computation, D may be transformed into an asymptotic expansion to which it is equivalent at large distances, so that

$$W = A - B/2 + F. \tag{15}$$

At still greater distances A also becomes unwieldy and it may in turn be replaced by its asymptotic expansion, which contains the term $B/2$, so that

$$W = C + F. \tag{16}$$

When the impedance of the ground is very different from that of the air,† F is small compared with $A - B/2 \approx C$. If the conductivity of the ground is not zero, F is exponentially attenuated so that it may be neglected in comparison with C in equation (16). Even if the conductivity is zero and the relative dielectric constant is as small as 4, the only effect of F in equation (16) is to produce oscillations in W of approximately 3 per cent from the magnitude of C. Even under these extreme conditions the received field strength may be calculated from equation (15) neglecting F without introducing an error greater than 3 per cent. As the transmitting antenna is approached, the approximations involved in equation (15) become poorer and poorer but at the same time the field strength becomes independent of W so that at no distance is there an appreciable error introduced

* The factor 2 occurs in equation (12) since Π_0 and E_0 refer to transmission in free space.

† This is true when the so-called "complex dielectric constant," $\epsilon - 2i\sigma/f$, differs sufficiently from unity.

by using equation (15) to calculate the field strength. This is fortunate since series $\boldsymbol{D}$ requires laborious calculations.

The attenuation factor may be obtained from $\mathbf{W}$ by means of the relation,*

$$\frac{E}{2E_0} = \frac{W}{1+\tau^2} + \frac{1}{1-\tau^4}\left[\frac{1}{2\pi i d/\lambda} + \frac{1}{(2\pi i d/\lambda)^2}\right], \tag{17}$$

where

$$\frac{1}{\tau^2} = \epsilon - 2i\sigma/f = \epsilon(1 - i/Q). \tag{18}$$

In this equation $2E_0$ is the inverse distance, or radiation, component of the field that would result from transmission over a perfectly conducting plane, and Q is the ratio of the imaginary component to the real component of the admittance of the ground. In other words, Q is the ratio of the dielectric current to the conduction current.† The parameter ϵ occurring in equation (18) is the relative dielectric constant (with respect to vacuum), a pure numeric that is numerically equal to the dielectric constant measured in electrostatic units.

If the value of W from equation (16) is substituted in equation (17) and terms which involve $(1/d)$ to powers higher than the first are neglected as may be done at the greater distances, we have

$$E \rightarrow \left[\frac{1}{1-\tau^4}\frac{1+\tau^2}{2\pi\tau^2 i d/\lambda}\right]\left[1 - \frac{\tau^5}{(1+\tau^2)}e^{\frac{2\pi i d}{\lambda}\left(1-\frac{1}{\tau}\right)}\right] 2E_0. \tag{19}$$

The magnitude of the second factor on the right differs from unity

* This expression may be obtained as follows. $\mathbf{\Pi}$ satisfies the wave equation which in cylindrical coordinates (z, θ, d) is

$$\left(\frac{1}{d}\frac{\partial}{\partial d}d\frac{\partial}{\partial d} + \frac{1}{d^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2} + \frac{4\pi^2}{\lambda^2}\right)\mathbf{\Pi} = 0.$$

Because of symmetry the second term is zero. Solving for the value of the last two terms and substituting it in equation (11) yields

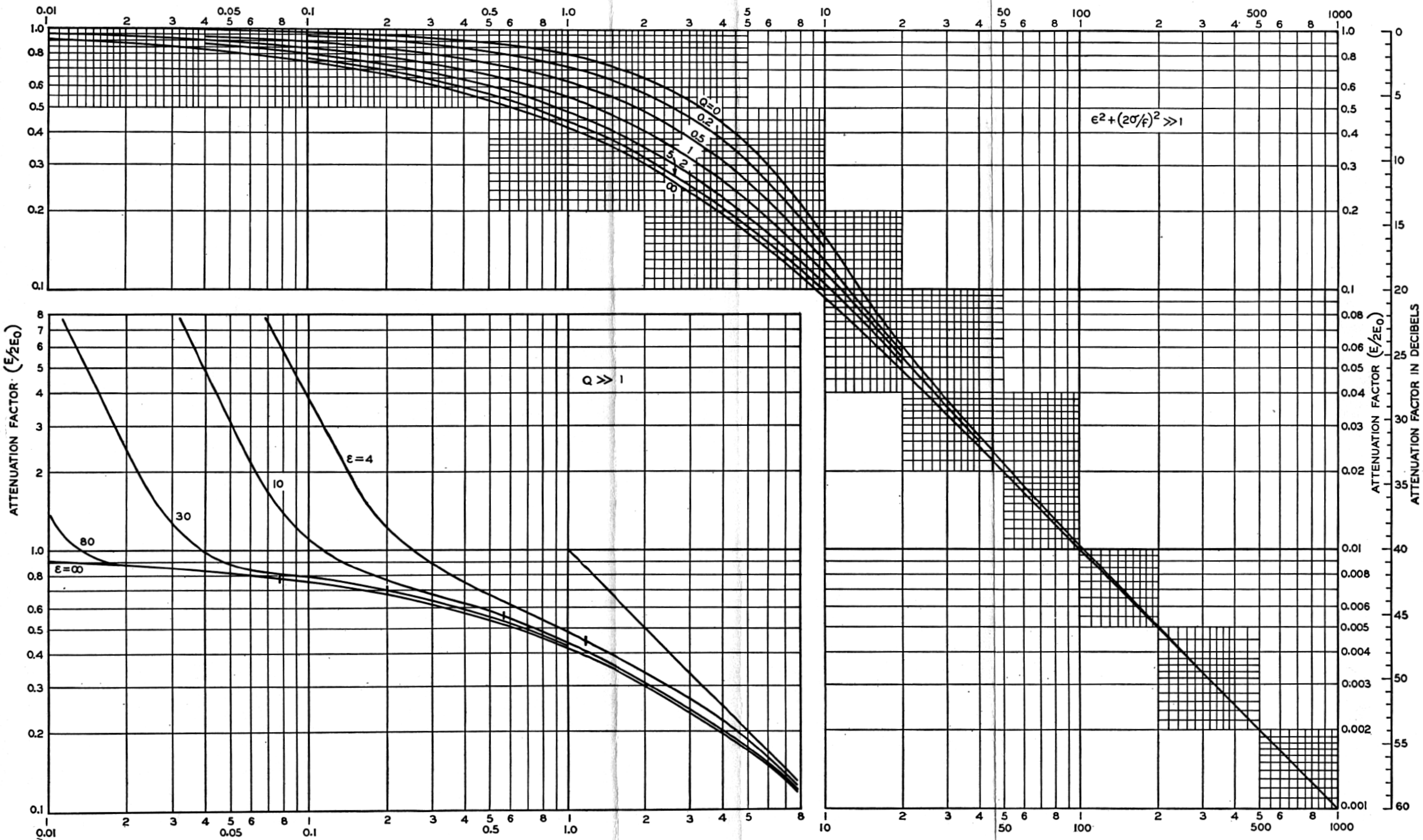
$$E = 60i\lambda\left[\frac{\partial^2}{\partial d^2} + \frac{1}{d}\frac{\partial}{\partial d}\right]\mathbf{\Pi}.$$

The differential equation given by Wise [11] for $\mathbf{\Pi}$ becomes

$$-\frac{\lambda^2}{4\pi^2}\left(\frac{\partial^2\mathbf{\Pi}}{\partial d^2} + \frac{1}{d}\frac{\partial\mathbf{\Pi}}{\partial d}\right) = \frac{\mathbf{\Pi}}{1+\tau^2} + \frac{2}{1-\tau^4}\left[\frac{1}{2\pi i d/\lambda} + \frac{1}{(2\pi i d/\lambda)^2}\right]\mathbf{\Pi}_0$$

when the value of $y = (1+\tau^2)\mathbf{\Pi}/2$ is substituted in his equation (7), and the result multiplied by $2/(1+\tau^2)$. Substitution of this relation in the preceding equation and division by $E_0 = -240i\pi^2\mathbf{\Pi}_0/\lambda$ gives equation (17) of the text. Since E_0 is the inverse distance component of the free space field, this relation follows from equation (11).

† In practical units $Q = 2\pi f\epsilon'/g$, where ϵ' is the dielectric constant in farads per meter and g is the conductivity in mhos per meter. On frequent occasions, the constants of the dielectric are expressed in electrostatic units; then $Q = f\epsilon/2\sigma$.



RELATIVE DISTANCE $$X = \frac{2\pi d/\lambda}{\epsilon}\,\frac{\sqrt{(1-1/\epsilon)^2+1/Q^2}}{1+1/Q^2} = \frac{2\pi d/\lambda}{\sqrt{\epsilon^2+(2\sigma/f)^2}}\sqrt{\frac{(\epsilon-1)^2+(2\sigma/f)^2}{\epsilon^2+(2\sigma/f)^2}} = \frac{2\pi d/\lambda}{\epsilon/Q}\,\frac{\sqrt{1+(1-1/\epsilon)^2Q^2}}{1+Q^2}$$

Fig. 3.

Fig. 2.

Fig. 2—Attenuation factor for radio propagation over plane earth. The number on each curve gives the value of the Q ($= \epsilon f/2\sigma$) to which it applies; σ is the conductivity and ϵ the dielectric constant in electrostatic units; f is the frequency in cycles per second; d/λ is the ratio of the distance to the wave-length.

Fig. 3—Attenuation factor for radio propagation over a dielectric plane. The number on each curve gives the value of the dielectric constant to which it applies.

by less than $2\frac{1}{2}$ per cent for values of ϵ greater than 4. Accordingly the magnitude of the first factor (which is equal to the first term of $\boldsymbol{C}$) gives the attenuation factor at great distances with a degree of accuracy sufficient for all practical purposes. If we choose our unit of distance such that

$$x = |2\pi\tau^2(1 - \tau^2)d/\lambda|$$

$$= \frac{2\pi d/\lambda}{\epsilon/Q} \frac{\sqrt{1 + (1 - 1/\epsilon)^2 Q^2}}{1 + Q^2} = \frac{2\pi d/\lambda}{\epsilon} \frac{\sqrt{(1 - 1/\epsilon)^2 + 1/Q^2}}{1 + 1/Q^2} \quad (20)$$

all of the attenuation curves will tend to coincide at the greater distances. This is done in Figs. 2 and 3. Figure 2 shows the variation of received field strength with distance for seven values of Q for the case where the impedance of the ground is very different from that of the air.* These curves give the correct attenuation factor for arbitrary ground constants at the greater distances. At any distance the above assumption introduces a significant error only when Q is large. Accordingly the curves of Fig. 3 have been calculated for various values of the relative dielectric constant when Q is large.† The short vertical line on each curve indicates the abscissa corresponding to a distance of one wave-length. The curves do not depart appreciably from that for an infinite dielectric constant except for distances less than this.‡ Since the error introduced in applying the curves of Fig. 2 to the general case is greatest for the conditions represented in Fig. 3, the curves of Fig. 2 may be used with confidence.

It should be emphasized that the curves of Fig. 2 give the ratio of the received field strength to that which would result from the same current in the same antenna on the surface of a plane earth of perfect conductivity. The antenna is assumed at the earth's surface so that the curves are strictly true only for short antennas. The error for half-wave doublets whose mid-points are not more than a half wave-length above the surface of the earth is negligible except in the immediate vicinity of the transmitter. The effect of height above the surface of the earth is taken up more fully in the next section.

* Since the writing of this paper, Part I of a paper by K. A. Norton on "The Propagation of Radio Waves Over the Surface of the Earth and in the Upper Atmosphere" has appeared in *Proc. I.R.E.*, **24**, 1367–1387, October, 1936. The curves of Fig. 2 in this paper are similar to those of Norton's Fig. 1, but by presenting the curves as a function of x their validity is extended to include a wider range of ground constants.

† The writer is indebted to Miss Clara L. Froelich for making these calculations.

‡ The ratio $E/2E_0$ is greater than unity at the shorter distances because E_0 is the inverse distance or radiation component of the free space field while E is the total field. At distances that are small compared with a wave-length, $E/2E_0$ is given by the second and third terms on the right of equation (17) and the effect of the ground is to increase the field by the factor $2/(1 - \tau^4)$.

The calculation of the field strength as a function of the radiated power requires a knowledge of the effect of imperfect conductivity on the resistance of the antenna. The reader is referred to papers by Barrow [6] and Niessen [7] on this subject. In the wave-length range where these curves are of greatest applicability, the practice is to minimize the ground losses by a ground system consisting of a counterpoise or a network of buried wires. When this is done the ground losses are properly part of the antenna losses and the radiated power may rightfully be taken as the rate of flow of energy past a hemisphere large enough to include the antenna and ground system. If this is done, the field strength is given by

$$E = \frac{3\sqrt{10}\sqrt{P}}{d} F(x), \tag{20a}$$

where $F(x)$ is the ratio plotted in Fig. 2.*

Part II—Antennas Above the Surface of the Earth

It is well known [8, 9] that calculations based on the physical optics of plane waves give the first approximation to the received field for radio propagation over plane earth. This approximation is accurate enough for all practical purposes if the antennas are sufficiently removed from the surface of the earth.† Under these conditions, the ratio of the received field strength to that which would be received in free space is given by ‡

$$E/E_0 = \sqrt{(1 - K)^2 + 4K \sin^2 (\gamma/2)}, \tag{21}$$

* In estimating the fraction of the total power input that is radiated the following papers may be helpful: George H. Brown, "The phase and magnitude of earth currents near radio transmitting antennas," *Proc. I.R.E.*, **23**, 168–182, February, 1935; and H. E. Gihring and G. H. Brown, "General considerations of tower antennas for broadcast use," *Proc. I.R.E.*, **23**, 311–356, April, 1935.

† This height depends upon the distance, wave-length and ground constants. The range of validity of this approximation is discussed more fully in connection with equation (27).

‡ Equation (21) gives the received field strength for either polarization for transmission along the ground. In this case the direct and reflected components are oriented in the same direction in space. It may also be used to calculate the effect of the ground for signals arriving at large angles by taking into consideration the space orientation of the components.

For horizontal antennas the orientation of the electric vector is horizontal for all angles of incidence so that equation (21) applies directly. For vertical antennas the electric vector makes the angle ξ_2 with the vertical, both in the direct and reflected wave. Hence if the ratio given in equation (21) is taken as the ratio of the vertical component of the received field to the total incident field it must be multiplied by $\cos \xi_2$. Even if the ground were not present, however, the vertical component would be reduced by this factor so that the effect of the presence of the ground on the field received by a vertical antenna is given by equation (21) as written without the $\cos \xi_2$ factor.

where K is the ratio of the amplitude of the reflected wave to that of the direct wave and $\gamma + \pi$ is their phase difference.

$$\gamma = \psi - \Delta, \tag{22}$$

where Δ is 2π times the path difference in wave-lengths and

$$\varphi = \psi \pm \pi \tag{23}$$

is the phase advance at reflection. The geometry is shown in Fig. 4. Δ may be calculated from the geometry by means of equation (47) of Appendix II (page 72).

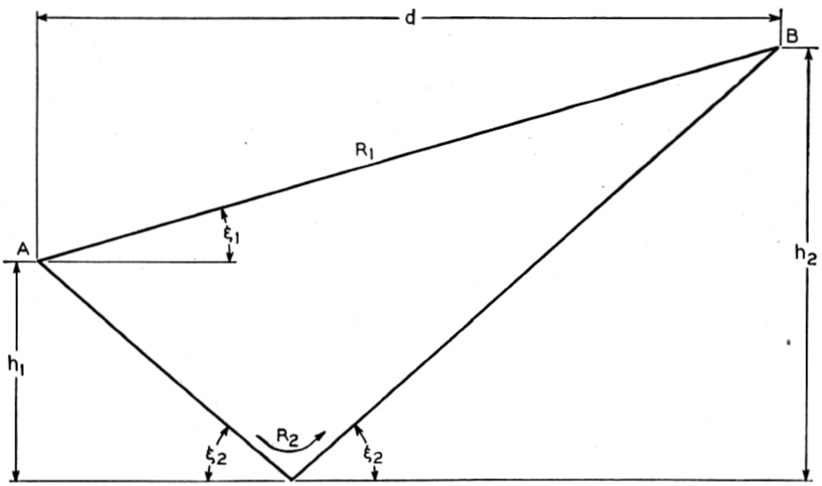


Fig. 4

The magnitude and phase of the reflection coefficient, $R = -Ke^{i\psi}$, for both polarizations are plotted in Figs. 5–12 for $\epsilon = 4, 10, 30$ and 80 and a series of values of $\epsilon/Q = 2\sigma/f$ differing by factors that are multiples of 10. The coordinate system has been chosen so that the quantities $1 - K$ and ψ that enter into the equation for the resultant field strength may be read with the same degree of accuracy for the entire range of the curves. To obtain values of $1 - K$ and ψ for smaller values of ξ_2 than shown on Figs. 5–12 use is made of the fact that both of these quantities are proportional to ξ_2 for small values of ξ_2. This linear relationship holds for the lowest cycle * of the curves so that the parts

* An exception to this occurs for large Q in the ψ-curves for vertical polarization. Under these conditions the difference between ψ and zero for values not shown on the chart is relatively unimportant. When Q is large and K is different from zero ψ is substantially 0° or 180°. For horizontal polarization ψ may be taken equal to zero for most practical purposes.

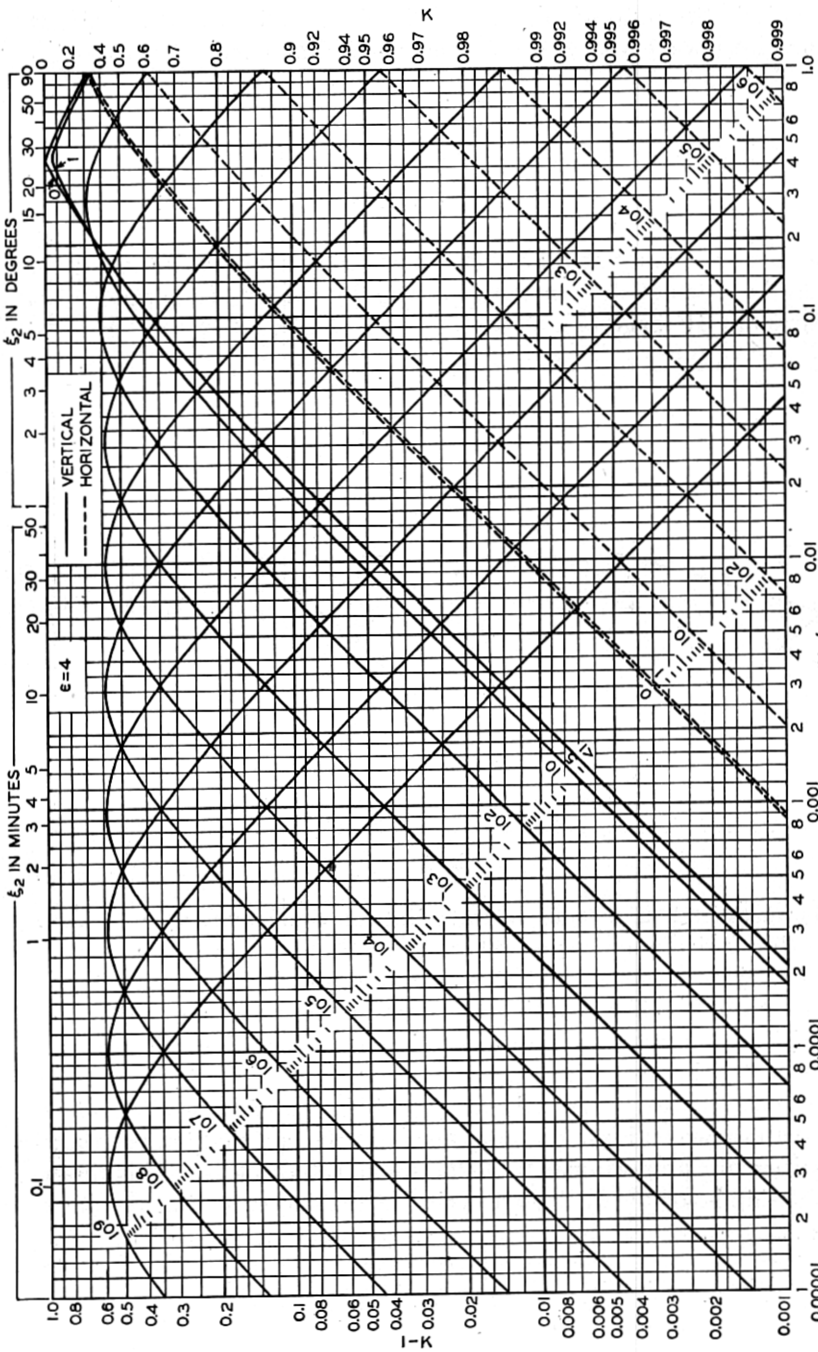


Fig. 5—Magnitude of reflection coefficient for $\epsilon = 4$. The number on each curve gives the value of q $(= 2\sigma/f)$ to which it applies. Vertical polarization is shown by solid lines, horizontal polarization by broken lines.

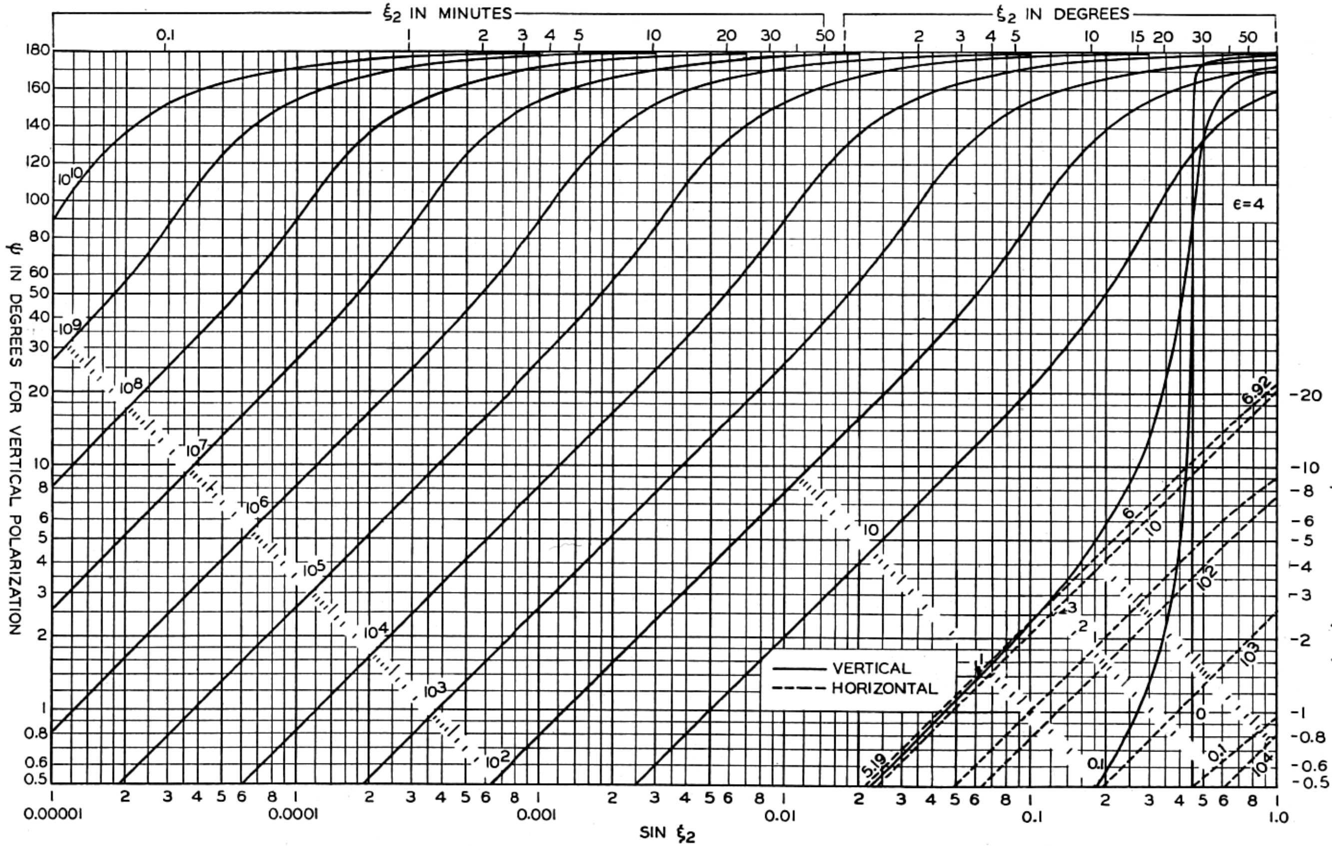


Fig. 6—Phase shift at reflection for $\epsilon = 4$. The number on each curve gives the value of q $(= 2\sigma/f)$ to which it applies. Vertical polarization is shown by solid lines, horizontal polarization by broken lines.

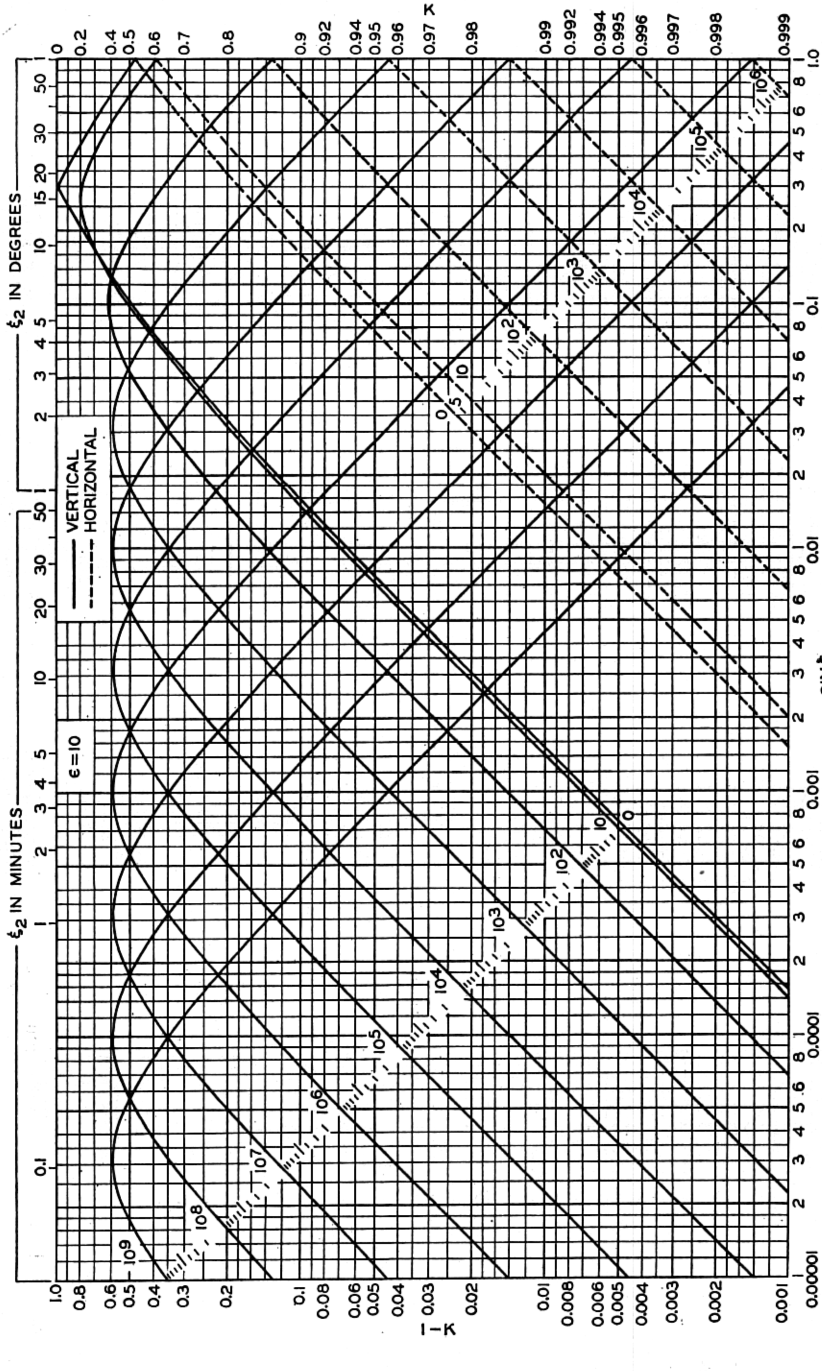


Fig. 7—Magnitude of reflection coefficient for $\epsilon = 10$. The number on each curve gives the value of q ($= 2\sigma/f$) to which it applies. Vertical polarization is shown by solid lines, horizontal polarization by broken lines.

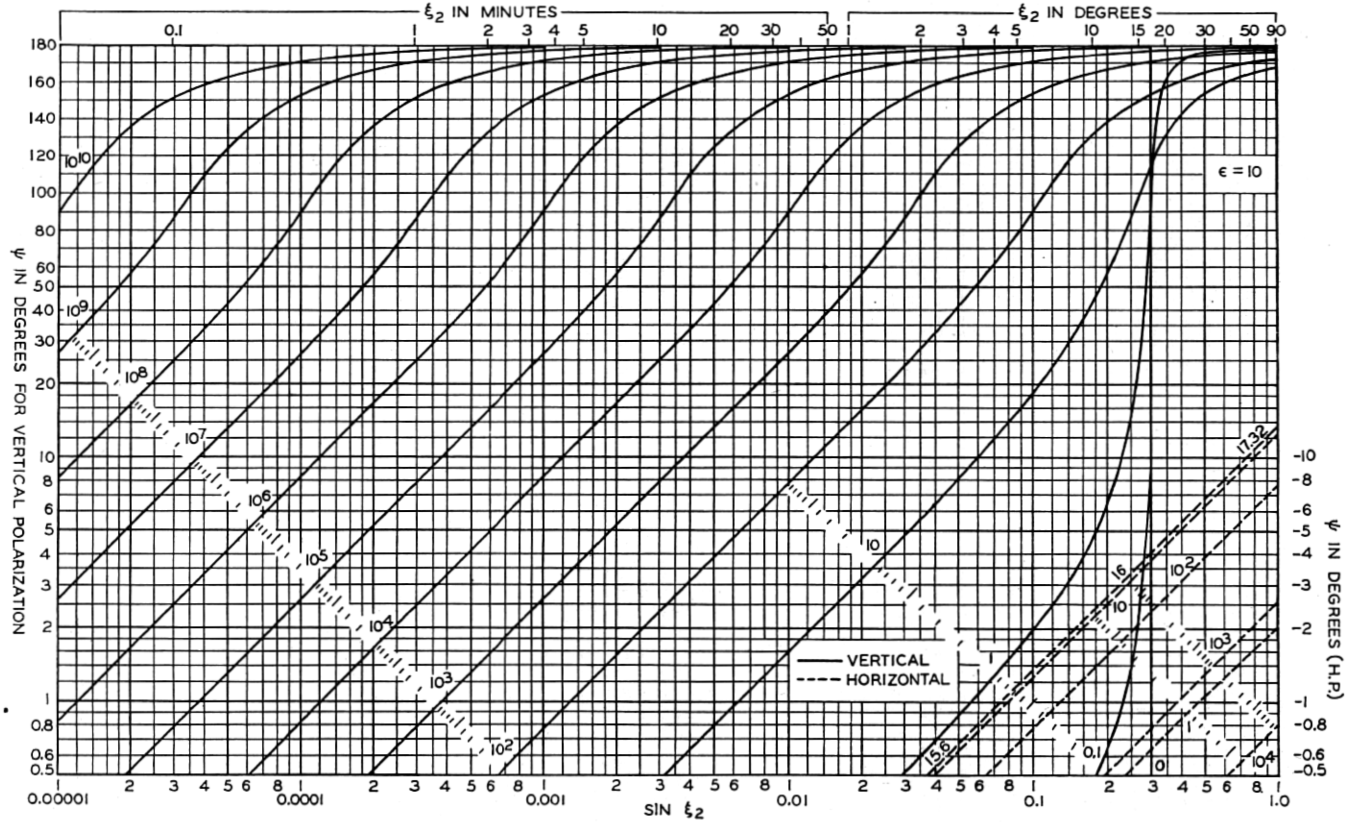


Fig. 8—Phase shift at reflection for $\epsilon = 10$. The number on each curve gives the value of q $(= 2\sigma/f)$ to which it applies. Vertical polarization is shown by solid lines, horizontal polarization by broken lines.

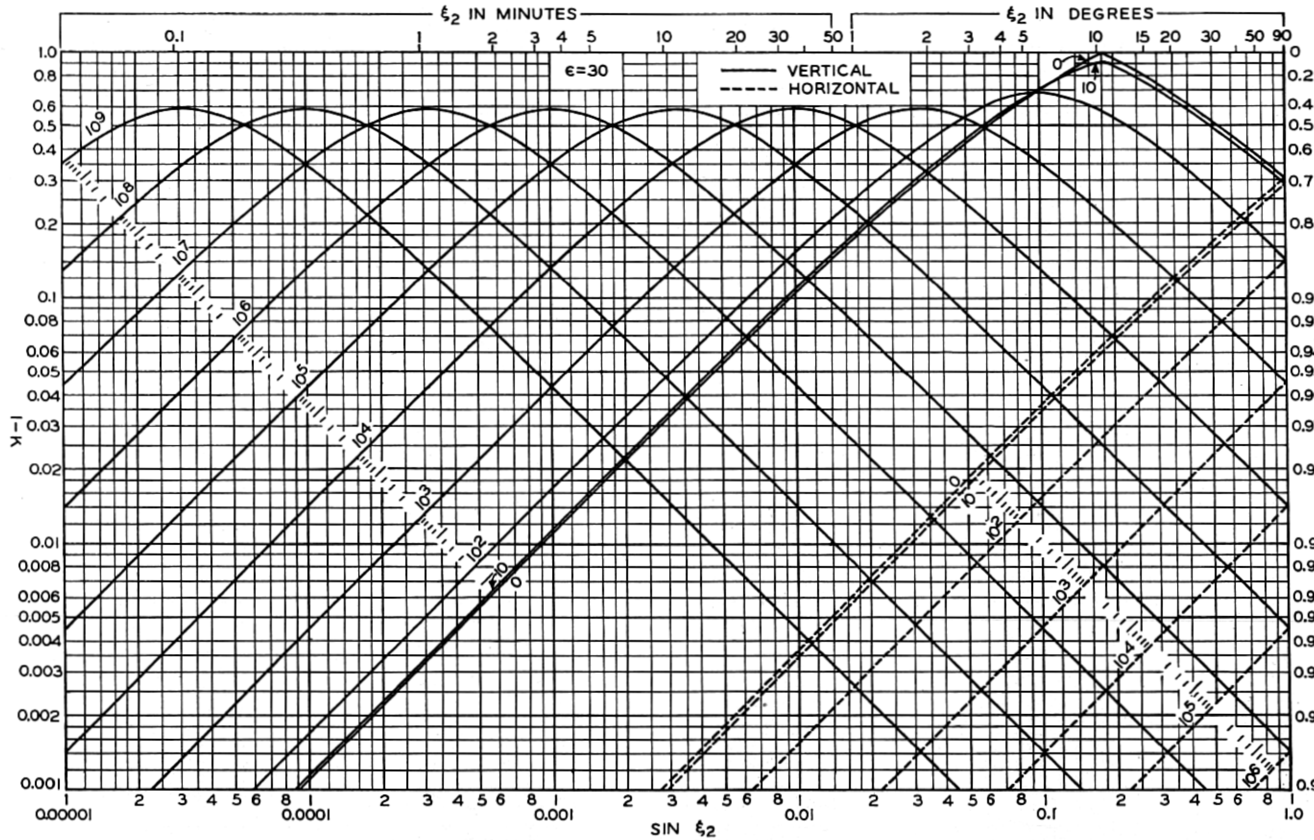


Fig. 9—Magnitude of reflection coefficient for $\epsilon = 30$. The number on each curve gives the value of q $(= 2\sigma/f)$ to which it applies. Vertical polarization is shown by solid lines, horizontal polarization by broken lines.

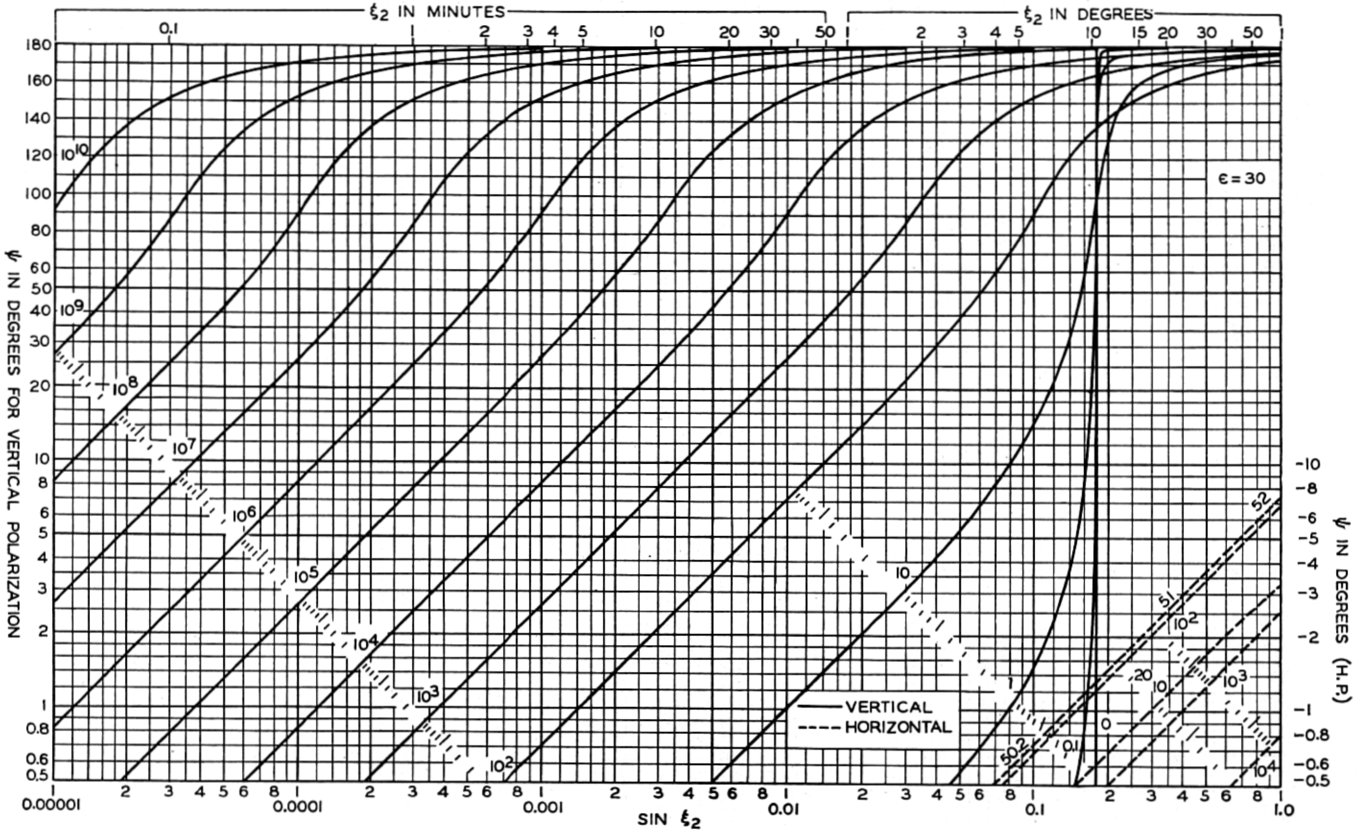


Fig. 10—Phase shift at reflection for $\epsilon = 30$. The number on each curve gives the value of q $(= 2\sigma/f)$ to which it applies. Vertical polarization is shown by solid lines, horizontal polarization by broken lines.

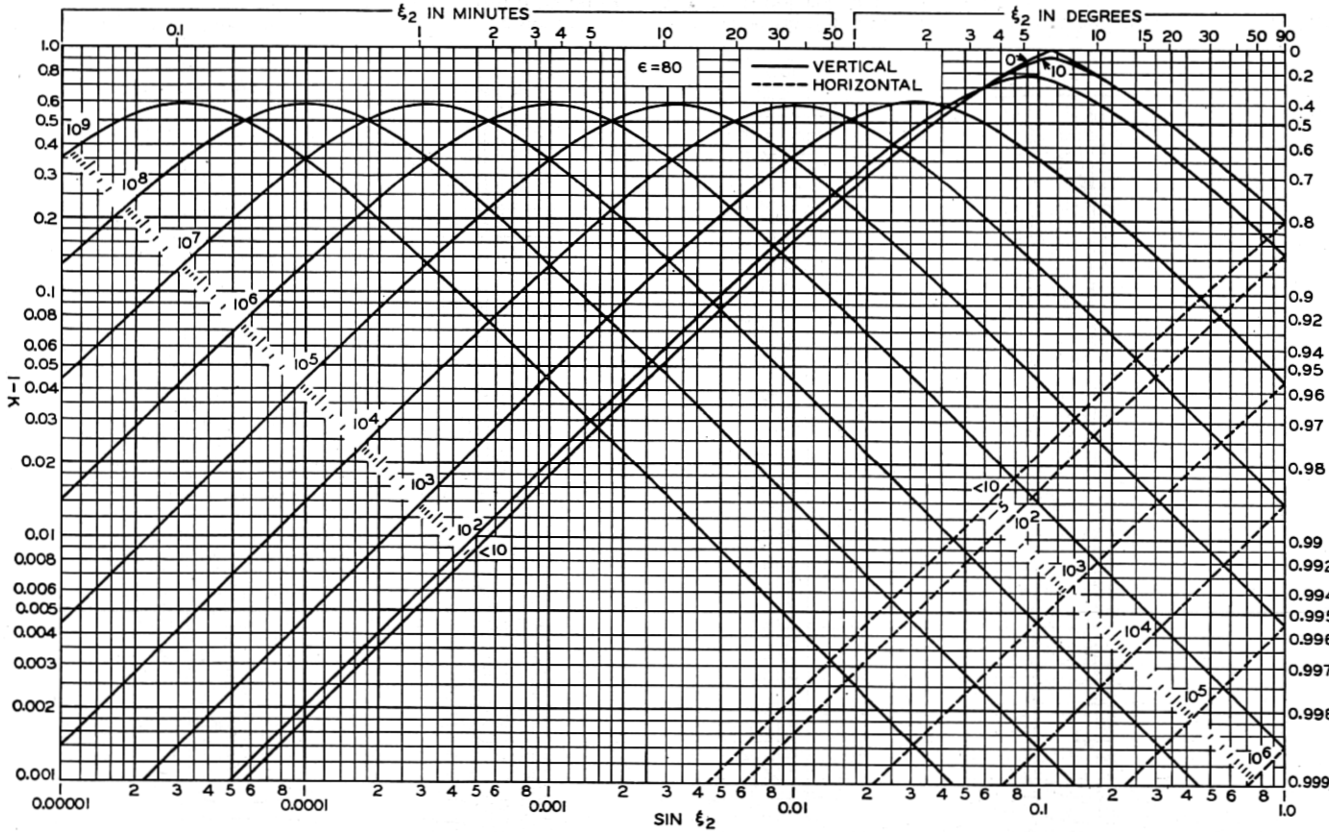


Fig. 11—Magnitude of reflection coefficient for $\epsilon = 80$. The number on each curve gives the value of q $(= 2\sigma/f)$ to which it applies. Vertical polarization is shown by solid lines, horizontal polarization by broken lines.

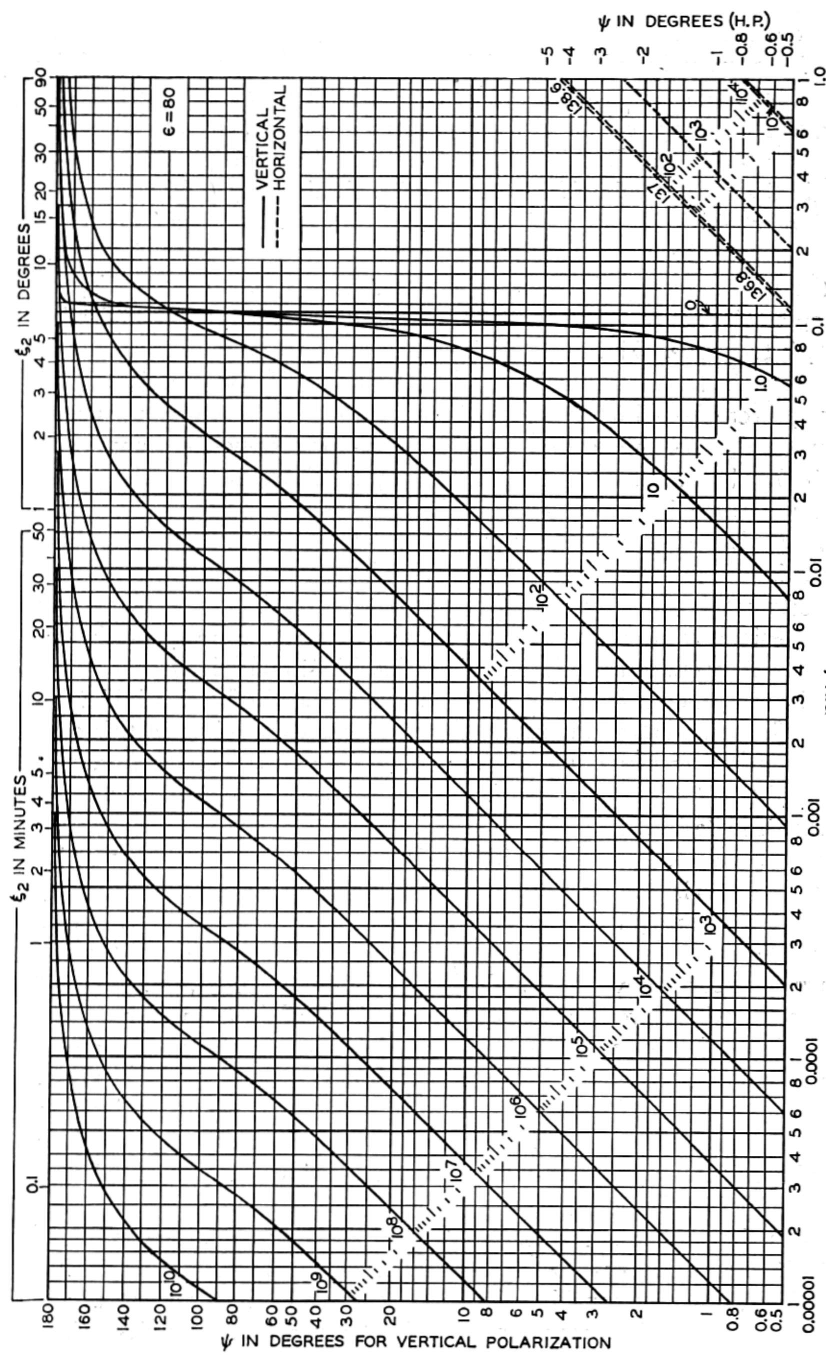


Fig. 12—Phase shift at reflection for ε = 80. The number on each curve gives the value of q $(= 2\sigma/f)$ to which it applies. Vertical polarization is shown by solid lines, horizontal polarization by broken lines.

of the curves in this cycle may be used to obtain the values of $1 - K$ and ψ for any value of ξ_2 for which the curves go below the edge of the charts, as follows. Multiply ξ_2 by the smallest power of ten that will give a value on the chart, read the value of $1 - K$ or ψ corresponding to this new ξ_2 and divide this value of $1 - K$ or ψ by this power of ten to obtain the desired value of $1 - K$ or ψ. To obtain values of $1 - K$ and ψ for larger values of $q = 2\sigma/f$ than shown on Figs. 5–12 use is made of the fact that both of these quantities may be expressed as functions of the parameter $\sqrt{q}\sin\xi_2$ for large values of q, $(Q \ll 1)$. That is, the shape of all the curves for values of Q that are small compared with unity is the same. Hence to obtain values of $1 - K$ and ψ for values of q greater than those for which curves are shown, divide the given q by some power of one hundred that gives a value of q for which a curve is drawn, and read the desired value of $1 - K$ or ψ opposite the value of $\sin\xi_2$ that is the same power of ten times the given $\sin\xi_2$ as the power of one hundred by which q was divided.

If the following characteristics of the curves are taken into consideration, interpolation is simplified. The similarity of the K-curves suggests relabelling the abscissa so that the value of K for some intermediate value of q may be read from one of the curves that is drawn. Any curve for a large value of q, $(Q \ll 1)$, such as for $q = 10^n$ may be relabelled $q = x \times 10^n$ if the value of the abscissa is divided by $\sqrt{x}$. The same method is useful for small values of q but in this case the quantity by which the abscissa must be divided depends on the value of Q. Also in this case the shape of the curves changes with Q so that it is desirable to read the values from the curves drawn for the nearest values of q on either side of the desired value. The factor by which the abscissa must be multiplied to obtain the desired result may be inferred from the interpolation scale on the curves. When q is large the same method of interpolation may be employed for the ψ-curves as for the K-curves. When q is small, $(Q \gg 1)$, the fact that ψ is proportional to q suggests the method of interpolation. (On vertical polarization when ξ_2 is greater than the Brewster angle, $\pi - \psi$ is proportional to q.) If the value of ψ for $q = x \times 10^{-n}$ is required, read the value of ψ from the curve for $q = 10^{-n}$ and multiply by x to obtain the desired value of ψ. If greater accuracy is required the values may be calculated from the equations and Table III of Appendix II without prohibitive labor.

When the antennas approach the ground, the ratio given in equation (21) approaches zero so that more terms of the complete solution must be taken into consideration. Wise [10] has derived an expression for the effect of the ground on the Hertzian potential which when added to the primary disturbance gives the following expression. Since it is now known[4] that no exponential term must be added to this expression, it may be used to calculate the received field.

$$\mathbf{\Pi} = \frac{HI}{4\pi}\left[\frac{e^{-2\pi iR_1/\lambda}}{R_1} + \frac{e^{-2\pi iR_2/\lambda}}{R_2}\sum_{n=1}\frac{g_n}{(-\,2\pi iR_2/\lambda)^{n-1}}\right], \tag{24}$$

where the geometry and nomenclature are given in Fig. 4.

$g_1 = \boldsymbol{R} = -\,Ke^{i\psi}$ is the reflection coefficient and

$$g_{n+1} = \frac{n-1}{2}g_n - \frac{\sin\xi_2}{n}g_n' + \frac{\cos^2\xi_2}{2n}g_n'', \tag{25}$$

where $\pi/2 - \xi_2$ is the angle of incidence and the primes denote differentiation with respect to $\sin\xi_2$. Performing the operation indicated in (11) on (24) the complete expression for the received field strength on vertical polarization is found to be

$$\begin{aligned} \boldsymbol{E} = -\frac{60\pi iHI}{\lambda}\Bigg\{ & \frac{e^{-2\pi iR_1/\lambda}}{R_1}\cos^2\xi_1 + \frac{e^{-2\pi iR_2/\lambda}}{R_2}g_1\cos^2\xi_2 + \frac{e^{-2\pi iR_1/\lambda}}{R_1}\frac{1-3\sin^2\xi_1}{2\pi iR_1/\lambda} \\ & + \frac{e^{-2\pi iR_2/\lambda}}{R_2}\frac{g_1(1-3\sin^2\xi_2)+2g_1'\sin\xi_2\cos^2\xi_2 - g_2\cos^2\xi_2}{2\pi iR_2/\lambda} \\ & + \frac{e^{-2\pi iR_1/\lambda}}{R_1}\frac{1-3\sin^2\xi_1}{(2\pi iR_1/\lambda)^2} + \frac{e^{-2\pi iR_2/\lambda}}{R_2}\Bigg[\frac{g_1(1-3\sin^2\xi_2)+5g_1'\sin\xi_2\cos^2\xi_2}{(2\pi iR_2/\lambda)^2} \\ & + \frac{-g_1''\cos^4\xi_2 - g_2(1-5\sin^2\xi_2) - 2g_2'\sin\xi_2\cos^2\xi_2 + g_3\cos^2\xi_2}{(2\pi iR_2/\lambda)^2}\Bigg] \\ & + \sum_{n=3}\frac{e^{-2\pi iR_2/\lambda}}{R_2}\Bigg[\frac{g_{n-1}(n-1)(1-[n+1]\sin^2\xi_2)}{(-2\pi iR_2/\lambda)^n} \\ & + \frac{(2n+1)g_{n-1}'\sin\xi_2\cos^2\xi_2 - g_{n-1}''\cos^4\xi_2}{(-2\pi iR_2/\lambda)^n} \\ & + \frac{-g_n(1-[2n+1]\sin\xi_2) - 2g_n'\sin\xi_2\cos^2\xi_2 + g_{n+1}\cos^2\xi_2}{(-2\pi iR_2/\lambda)^n}\Bigg]\Bigg\}. \end{aligned} \tag{26}$$

The first term on the right of equation (26) is the vertical component of the electric field radiated by a vertical electric doublet in free space. The second term is the corresponding component reflected from the earth. The third and fifth terms are sometimes referred to as the induction and electrostatic components respectively. The remaining terms complete the effect of the ground. When the antennas approach the ground $R_1 \to R_2$, $\cos\xi_1 \to \cos\xi_2 \to 1$ and $g_1 \to -1$ so that the first two terms tend to cancel. Under these conditions the sum of the first four terms of equation (26) may be written *

$$\frac{E}{E_0} = 1 + \left[R + \frac{(R+1)^2\lambda d}{4\pi i(h_1 + h_2)^2}\right]e^{-4\pi i h_1 h_2/\lambda d}. \tag{27}$$

At the greater antenna heights this expression also gives the correct result provided the distance, d, is large compared with the sum of the antenna heights, $h_1 + h_2$. For smaller antenna heights this expression is limited to distances for which the magnitude of the second term within the bracket is small (say less than 0.1). If this term is not small more terms of equation (26) must be taken into consideration. While equation (26) applies to vertical polarization only, equation (27) applies to both polarizations within the region for which it is valid provided the appropriate reflection coefficient is employed.

For antenna heights sufficiently small that the exponential factor of equation (27) may be replaced by the first two terms of its series expansion,

$$\frac{E}{E_0} = \frac{4\pi i h_1 h_2}{\lambda d}\left[1 + \frac{(a - ib)\lambda}{4\pi i h_1}\right]\left[1 + \frac{(a - ib)\lambda}{4\pi i h_2}\right], \tag{28}$$

where a and b are given in Table III of Appendix II (page 72). The first factor gives the well known expression for ultra-short-wave propagation over level land. The second two factors are important for antennas near the ground. When $h_1 \rightarrow h_2 \rightarrow 0$ this becomes

$$\frac{E}{E_0} = \frac{(a - ib)^2\lambda}{4\pi i d}, \tag{29}$$

which is equivalent to the first term of the asymptotic expansion of the attenuation factor given in Part I.

A more useful form of equation (28) is

$$\frac{E}{E_0} = \frac{4\pi h_1 h_2}{\lambda d}\left[1 + a_1\left(\frac{1}{h_1} + \frac{1}{h_2}\right) + a_2\left(\frac{1}{h_1} + \frac{1}{h_2}\right)^2 + a_3\frac{1}{h_1 h_2} + a_4\frac{1}{h_1 h_2}\left(\frac{1}{h_1} + \frac{1}{h_2}\right) + a_5\frac{1}{h_1^2 h_2^2}\right]^{1/2}, \tag{30}$$

* Equation (27) differs from equation (26) only in the dropping of terms in $1/d^3$. By leaving the exponential factor and the coefficient of reflection unexpanded the useful range of this formula is increased. The term

$$\frac{1}{2\pi i d/\lambda}\left[1 + Re^{-4\pi i h_1 h_2/\lambda d}\right]$$

has been omitted from the right side of equation (27) since it can be shown that this term is always small compared with the remaining terms when $2\pi d/\lambda \gg 1$. In order to facilitate calculations by means of the reflection coefficient curves, $-g_2$ is replaced by $(R + 1)^2 d^2/2(h_1+h_2)^2$ to which it is equal to the required order of approximation. Another form of equation (27) that may be preferred in some cases is given as equation (35) in the conclusions. This form results from substituting for $-g_2$ its value $(a - ib)^2/2$.

TABLE I

	a_1	a_2	a_3	$a_4 = a_1 a_2$	$a_5 = a_2{}^2$
1. Both V.P. and H.P.	$-\dfrac{b}{2\pi/\lambda}$	$\dfrac{a^2+b^2}{4(2\pi/\lambda)^2}$	$-\dfrac{a^2-b^2}{2(2\pi/\lambda)^2}$	$-\dfrac{a^2b+b^3}{4(2\pi/\lambda)^3}$	$\dfrac{(a^2+b^2)^2}{(4\pi/\lambda)^4}$
2. V.P. in general	$-\dfrac{\sqrt{2}[q\sqrt{s+r}-\epsilon\sqrt{s-r}]}{s(2\pi/\lambda)}$	$\dfrac{\epsilon^2+q^2}{s(2\pi/\lambda)^2}$	$-\dfrac{2[r\epsilon^2+(2\epsilon-r)q^2]}{s^2(2\pi/\lambda)^2}$	———	$\dfrac{(\epsilon^2+q^2)^2}{s^2(2\pi/\lambda)^4}$
3. V.P. $Q \ll 1$	$-\dfrac{\sqrt{2q}}{2\pi/\lambda}$	$\dfrac{q}{(2\pi/\lambda)^2}$	$-\dfrac{2(\epsilon+1)}{(2\pi/\lambda)^2} \to 0$	$-\dfrac{q\sqrt{2q}}{(2\pi/\lambda)^3}$	$\dfrac{q^2}{(2\pi/\lambda)^4}$
4. V.P. $Q \gg 1$	$-\dfrac{2q/\sqrt{\epsilon-1}}{2\pi/\lambda} \to 0$	$\dfrac{\epsilon^2/(\epsilon-1)}{(2\pi/\lambda)^2}$	$-\dfrac{2\epsilon^2/(\epsilon-1)}{(2\pi/\lambda)^2}$	$-\dfrac{2q\epsilon^2/(\epsilon-1)^{3/2}}{(2\pi/\lambda)^3} \to 0$	$\dfrac{\epsilon^4/(\epsilon-1)^2}{(2\pi/\lambda)^4}$
5. H.P. in general	$\dfrac{\sqrt{2}\sqrt{s-r}}{s(2\pi/\lambda)}$	$\dfrac{1/s}{(2\pi/\lambda)^2}$	$-\dfrac{2r/s^2}{(2\pi/\lambda)^2}$	$\dfrac{\sqrt{2}\sqrt{s-r}/s^2}{(2\pi/\lambda)^3}$	$\dfrac{1/s^2}{(2\pi/\lambda)^4}$
6. H.P. $Q \ll 1$	$\dfrac{\sqrt{2/q}}{2\pi/\lambda}$	$\dfrac{1/q}{(2\pi/\lambda)^2}$	$-\dfrac{2(\epsilon-1)/q^2}{(2\pi/\lambda)^2} \to 0$	$\dfrac{\sqrt{2/q^3}}{(2\pi/\lambda)^3}$	$\dfrac{1/q^2}{(2\pi/\lambda)^4}$
7. H.P. $Q \gg 1$	$\dfrac{q/(\epsilon-1)^{3/2}}{2\pi/\lambda} \to 0$	$\dfrac{1/(\epsilon-1)}{(2\pi/\lambda)^2}$	$-\dfrac{2/(\epsilon-1)}{(2\pi/\lambda)^2}$	$\dfrac{q/(\epsilon-1)^{5/2}}{(2\pi/\lambda)^3} \to 0$	$\dfrac{1/(\epsilon-1)^2}{(2\pi/\lambda)^4}$

Where $Q = \epsilon f/2\sigma$, $q = 2\sigma/f = \epsilon/Q$, $r = \epsilon - 1 + \sin^2 \xi_2$ and $s = \sqrt{r^2+q^2}$. See Appendix II for further evaluations of a and b.

where the values of the a's are given in Table I. A similar expression results for horizontal polarization so that the a's are also evaluated for this case.

With the aid of equation (9) this may be expressed as a power ratio between short doublets:

$$\sqrt{\frac{P_r}{P_t}} = \frac{3h_1h_2}{2d^2}\left[1 + a_1\left(\frac{1}{h_1}+\frac{1}{h_2}\right) + a_2\left(\frac{1}{h_1}+\frac{1}{h_2}\right)^2 + a_3\frac{1}{h_1h_2} + a_4\frac{1}{h_1h_2}\left(\frac{1}{h_1}+\frac{1}{h_2}\right) + a_5\frac{1}{h_1^2h_2^2}\right]^{1/2}. \quad (31)$$

For antennas at the heights above the ground that are usual in the ultra-short-wave range, the bracket in equation (31) is unity so we have the useful result that within certain limitations the ratio of received to transmitted power with simple antennas is independent of the wave-length.

When the Q of the ground is large in comparison with unity, equation (30) reduces to the somewhat simpler expression

$$\frac{E}{E_0} = \frac{4\pi h_1h_2}{\lambda d}\sqrt{\left[1+\frac{a_0^2}{h_1^2}\right]\left[1+\frac{a_0^2}{h_2^2}\right]}, \quad (32)$$

where $a_0^2 = \epsilon^2\lambda^2/4\pi^2(\epsilon-1)$ for vertical polarization and $a_0^2 = \lambda^2/4\pi^2(\epsilon-1)$ for horizontal polarization. Likewise when the Q of the ground is small in comparison with unity, equation (30) reduces to

$$\frac{E}{E_0} = \frac{4\pi h_1h_2}{\lambda d}\sqrt{\left[1+\frac{b_0}{h_1}+\frac{b_0^2}{2h_1^2}\right]\left[1+\frac{b_0}{h_2}+\frac{b_0^2}{2h_2^2}\right]}, \quad (33)$$

where $b_0 = -\sqrt{2q}\,\lambda/2\pi$ for vertical polarization and $b_0 = \sqrt{2/q}\,\lambda/2\pi$ for horizontal polarization.

Equations (28), (30), (31), (32) and (33) are valid for all distances beyond those for which the received field strength begins to vary inversely with the square of the distance provided the antennas are not too high. This range of validity contains all practical distances for ultra-short-wave propagation over land and fresh water and the longer distances for ultra-short-wave propagation over sea water. For antennas at greater heights above the ground, equation (27) is required; but usually the range of antenna heights between those for which equation (21) and those for which equation (30) are valid, is small.

The applicability of the approximate equations to the problem in hand may be ascertained as follows. First calculate the parameter x of Fig. 2 to determine if the distance is sufficient for the field strength to be inversely proportional to the square of the distance. The deviation of the attenuation curve from the straight line $E/2E_0 = 1/x$ shows the degree of this approximation for antennas on the ground. If this is satisfactory then equation (27) applies.* An evaluation of the parameters in equation (27) for the greatest antenna height allows a determination of whether equations (28) and (30) apply. If $\boldsymbol{R}$ is within the range where it is a linear function of ξ_2, that is if the curves for $1 - K$ *vs* $\sin \xi_2$ (Figs. 5, 7, 9 and 11) are straight lines for this value of ξ_2, and if $\sin 4\pi h_1 h_2/\lambda d$ is approximately equal to $4\pi h_1 h_2/\lambda d$, then equations (28) and (30) apply. If also Q is very different from unity then either equation (32) or equation (33) applies.

An evaluation of the parameters in equation (27) for the lowest height will allow a determination of whether equation (21) applies. If the second term within the brackets is small compared with the first term, equation (21) applies.

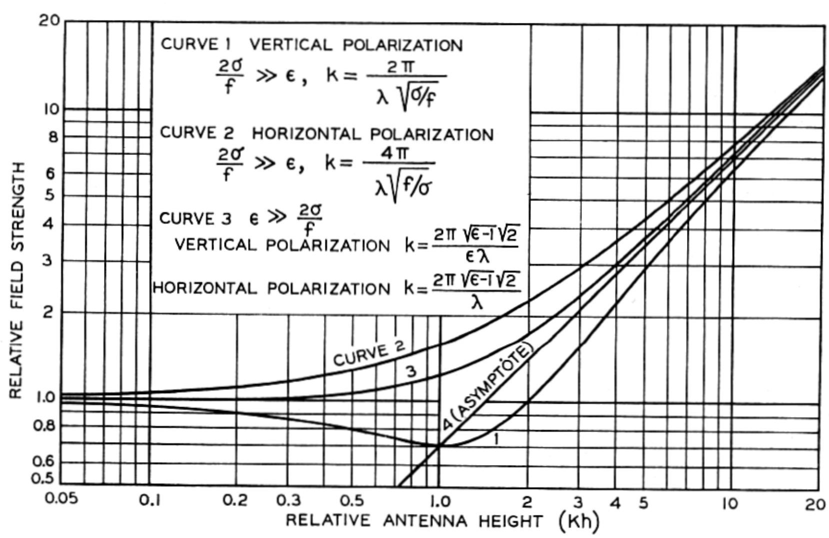


Fig. 13—Variation of received field strength with antenna height.

The variations of the received field strength with antenna height for the four cases of especial interest given by equations (32) and (33) are plotted in Fig. 13. The ordinate gives the ratio of the field strength at the height corresponding to the abscissa to that for zero height. If both antennas are off the ground the product of the ratios corresponding to the antenna heights gives the ratio of the field strength to

* As the antennas are removed from the earth's surface the error introduced because of this deviation is less.

that for both antennas on the ground. The distances between the curves and the straight line labelled " asymptote " give the magnitudes of the factors in equations (32) and (33) by which $(4\pi h_1 h_2/\lambda d)E_0$ must be multiplied to give the field strength.

For transmission over a ground of good conductivity ($Q \ll 1$) with vertical antennas there is a least favorable height for the antennas as indicated by curve 1 of Fig. 13. With both antennas at this height, which is about $1.7\lambda^{3/2}$ meters for ocean water, the received field is one-half what it would be if both antennas were on the ground.

Curve 2 for transmission on horizontal polarization over ground of good conductivity ($Q \ll 1$) shows a steady increase in the received field with increase in antenna height. If curves 1 and 2 were plotted against antenna height in meters for any given ground conditions ($Q \ll 1$), curve 2 for horizontal polarization would not depart appreciably from its asymptote until such small antenna heights were reached that curve 1 for vertical polarization would be substantially independent of antenna height. Hence curves 1 and 4 give a comparison of the received field strength at any height on the two polarizations. At the height for which the field strength is minimum on vertical polarization, the field strength is independent of polarization. For lower antennas vertical polarization gives the greater fields, while for higher antennas horizontal polarization gives the greater fields. The maximum advantage of horizontal polarization over vertical polarization occurs at twice this height and is a factor of two.

As Q increases curves 1 and 2 merge into curve 3 for transmission over a perfect dielectric. While the shape of the curves for the two polarizations is identical and the received field strength is independent of polarization at the greater antenna heights, the field strength is ϵ^2 times as great on vertical polarization as on horizontal polarization with antennas on the ground.

As an example of the use of the curves for the reflection coefficient the relative advantages of different types of ground for low-angle reception (or transmission) on vertical polarization has been calculated. With vertical antennas both the direct and the reflected components are reduced by the factor $\cos \xi_2$ so that the right-hand side of equation (21) must be multiplied by $\cos \xi_2$. The receiving antenna will be assumed to be on the ground.* Figure 14 gives the resulting curves for the indicated ground constants. For very low angles the curves are parallel, indicating that the relative advantages of different types of ground are independent of the angle at these angles. The gain in

* For higher angles of reception the relative advantages of different types of ground may be made approximately the same by properly adjusting the antenna height.

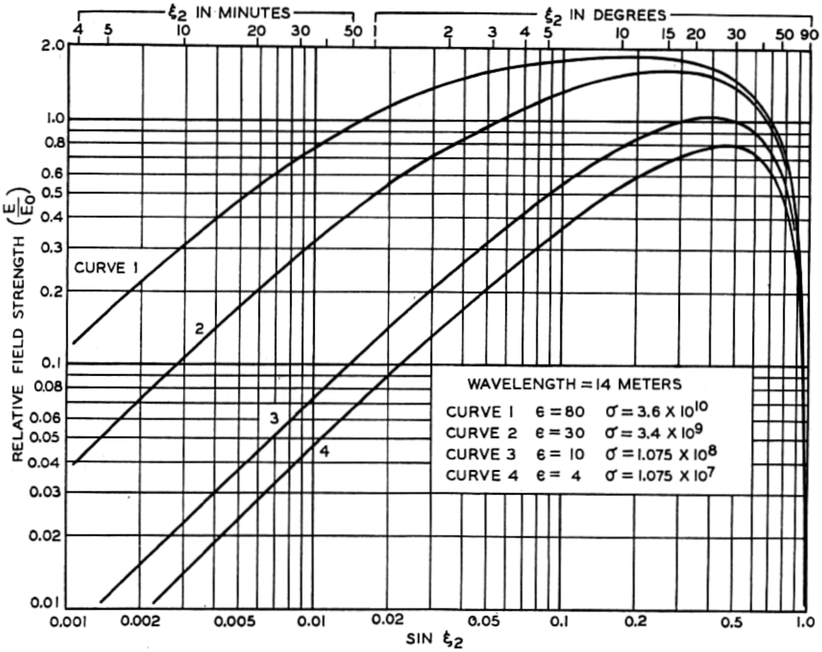


Fig. 14—Relative advantage of different types of ground for low-angle reception on 14 meters with vertical polarization.

locating an antenna above sea water instead of above the following grounds on a wave-length of 14 meters is given in Table II.

TABLE II

	Ground Constants		Gain in db for reception at			
	Dielectric Constant $-\epsilon$	Conductivity $-\sigma$				
	Electro-static Units	Electro-static Units	Small Angles	1°	2°	5°
1. Sea Water . . .	80	3.6×10^{10}	0	0	0	0
2. Salt Marsh . . .	30	3.4×10^{9}	10	7	5	3
3. Dry Ground . .	10	1.075×10^{8}	24	19	15	11
4. Rocky Ground.	4	1.075×10^{7}	28	23	20	14

Acknowledgment

The value of a paper of this type depends to a large degree upon its freedom from errors. With this thought in mind the equations and tables have been checked by my associate, Mr. Loyd E. Hunt, whose cooperation is hereby acknowledged.

Conclusions

For transmission over plane earth with antennas on the ground the received field strength in volts per meter is given by the formula,

$$E = \frac{120\pi HI}{\lambda d} F(x) = \frac{3\sqrt{10}\sqrt{P}}{d} F(x), \tag{34}$$

where HI is the transmitting ampere-meters, P is the radiated power in watts exclusive of ground losses, d the distance in meters, λ the wave-length in meters and $F(x)$ is the factor plotted in Fig. 2. When the Q of the ground is large compared with unity, the factor plotted in Fig. 3 is to be preferred to that plotted in Fig. 2.

When the antennas are not on the ground the received field strength may be calculated by means of equation (27) or its equivalent,

$$\frac{E}{E_0} = 1 + \left[R + \frac{(a - ib)^2\lambda}{4\pi i d}\right] e^{-4\pi i h_1 h_2/\lambda d}, \tag{35}$$

where the reflection coefficient,

$$R = -Ke^{i\psi} = -1 + (a - ib)\sin\xi_2 + \cdots. \tag{36}$$

The quantities K and ψ are plotted in Figs. 5–12.

When the antennas are sufficiently removed from the ground that the second term within the bracket of equation (35) may be neglected the simpler expression given in equation (21) applies.

$$E/E_0 = \sqrt{(1 - K)^2 + 4K\sin^2(\gamma/2)}. \tag{21}$$

When the distance between antennas is sufficiently great that the exponential factor in equation (35) may be replaced by the first two terms in its series expansion, the received field strength may be calculated by equation (30) and Table I. Four special cases are given by equations (32) and (33) and Fig. 13.

Appendix I

The values of the components of

$$W = A + D \approx A - B/2 + F \approx C + F$$

are:

$$A = \frac{1}{1 - \tau^4}\left[1 + \sum_{n=1}^{\infty}\left(\frac{2\pi i\tau^2 d/\lambda}{1 + \tau^2}\right)^n \frac{a_n}{1\cdot 3\cdot 5\cdots(2n - 1)}\right], \tag{37}$$

$$B = \frac{1}{1 - \tau^4}\sqrt{2\pi\sqrt{1 + \tau^2}}\sqrt{\frac{2\pi i\tau^2 d/\lambda}{1 + \tau^2}}\, e^{(2\pi i d/\lambda)(1 - 1/\sqrt{1+\tau^2})}, \tag{38}$$

$$C = -\frac{1}{1-\tau^4}\sum_{n=1}\left(\frac{1+\tau^2}{-2\pi\tau^2 id/\lambda}\right)^n[1\cdot 3\cdot 5\cdots(2n-1)]c_n, \tag{39}$$

$$D = -\frac{\tau^2}{1-\tau^4}e^{(2\pi id/\lambda)(1-1/\tau)}\sum_{n=0}^{\infty} D_n\left(\frac{2\pi id}{\lambda\tau}\right)^n, \tag{40}$$

$$F = \frac{\tau^2}{1-\tau^4}e^{(2\pi id/\lambda)(1-1/\tau)}\sum_{n=1}\left(\frac{1+\tau^2}{-2\pi id/\lambda\tau}\right)^n[1\cdot 3\cdot 5\cdots(2n-1)]f_n, \tag{41}$$

where

$$\frac{1}{\tau^2} = \epsilon - 2i\sigma/f \tag{42}$$

and τ is in the first quadrant. The positive square root of i is to be taken in equation (38).

These expressions follow from those given by Wise [11] when the sign of i is changed so that the implied time factor is $e^{i\omega t}$ in accordance with engineering practice instead of the $e^{-i\omega t}$ employed by Sommerfeld and Wise. Their expressions were derived for an antenna half in air and half in the earth. To obtain the above expressions which apply to antennas on the surface of the earth, Wise's expressions have been multiplied by $2/(1+\tau^2)$. $\boldsymbol{A}$ corresponds to his expression (5), $\boldsymbol{B}$ to his (12), $\boldsymbol{C}$ to his (8) and $\boldsymbol{D}$ to his (6). The quantities, a_n and c_n are substantially unity except when τ is not small.

$$\left.\begin{aligned} a_1 &= \frac{\tanh^{-1}\sqrt{k}}{\sqrt{k}} = \sum_{n=1}^{\infty}\frac{k^{n-1}}{(2n-1)}, \qquad a_2 = \frac{3(a_1-1)}{k}, \\ a_n &= \frac{(2n-1)(2n-3)(a_{n-1}-a_{n-2})}{(n-1)^2 k}, \end{aligned}\right\} \tag{43}$$

$$\left.\begin{aligned} c_1 &= 1, \qquad c_2 = 1-\frac{k}{3}, \\ c_n &= c_{n-1} - \frac{(n-1)^2 k}{(2n-1)(2n-3)}c_{n-2}, \end{aligned}\right\} \tag{44}$$

$$\left.\begin{aligned} D_0 &= 1, \qquad D_1 = \sqrt{l}\tanh^{-1}\sqrt{l} = \sum_{n=1}^{\infty}\frac{l^n}{(2n-1)}, \\ D_2 &= D_1 - l, \quad D_n = \frac{(2n-3)D_{n-1} - lD_{n-2}}{(n-1)^2}, \end{aligned}\right\} \tag{45}$$

where

$$k = \frac{\tau^2}{1+\tau^2} = \frac{1}{\epsilon+1-2i\sigma/f}, \tag{46}$$

$$l = \frac{1}{1+\tau^2} = \frac{\epsilon-2i\sigma/f}{\epsilon+1-2i\sigma/f}, \tag{47}$$

The f_n's are the same functions of l that the c_n's are of k.

Appendix II

The phase angle introduced by the path difference is:

$$\Delta = \frac{2\pi}{\lambda}\left[\sqrt{d^2 + (h_1 + h_2)^2} - \sqrt{d^2 + (h_1 - h_2)^2}\right]$$
$$= \frac{4\pi h_1 h_2}{\lambda d}\left[1 - \frac{h_1^2 + h_2^2}{2d^2} + \frac{3h_1^4 + 10h_1^2h_2^2 + 3h_2^4}{8d^4} - \cdots\right]. \quad (48)$$

The magnitude and phase of the reflection coefficient are given by the following equations.

$$R = -Ke^{i\psi}$$
$$K = \sqrt{\frac{1-\alpha}{1+\alpha}} = 1 - \alpha + \tfrac{1}{2}\alpha^2 - \tfrac{1}{2}\alpha^3 + \tfrac{3}{8}\alpha^4 + \cdots,$$
$$\alpha = \frac{mx}{1+x^2}, \quad \tan\psi = \frac{nx}{1-x^2}, \quad \sin\xi_2 = x/\sqrt{c},$$
$$m = a/\sqrt{c}, \quad n = b/\sqrt{c}, \quad \xi_2 = \frac{\pi}{2} - \theta,$$

where θ is the angle of incidence.

$$q = 2\sigma/f = \epsilon/Q, \quad r = \epsilon - 1 + \sin^2\xi_2,$$
$$s = \sqrt{r^2 + q^2}, \quad \frac{r}{s} = \left[1 + \left(\frac{q}{r}\right)^2\right]^{-1/2},$$

where ϵ and σ are respectively the dielectric constant and conductivity of the ground in electrostatic units and f is the frequency in cycles per second. The values a, b and c depend on the polarization and ground constants as shown in Table III below.

For grazing incidence, $\xi_2 \to 0$, $1 - K \to a\xi_2$ and $\psi \to b\xi_2$. On vertical polarization near normal incidence for $\epsilon^2 + q^2 \gg 1$, $x \gg 1$ and the approximations $1 - K \to a/c \sin\xi_2$ and $\psi \to \pi - b/c \sin\xi_2$ are useful. These coefficients are given in Table III.

For normal incidence, $\sin\xi_2 = 1$, $\alpha = \dfrac{\sqrt{2}\sqrt{s+\epsilon}}{s+1}$ and $\tan\psi = \dfrac{\sqrt{2}\sqrt{s-\epsilon}}{1-s}$. At Brewster's angle, $\cot\xi_2 = \sqrt{\epsilon}$ and $K = 0$ when $q = 0$ for vertical polarization. For vertical polarization the minimum value of K is $\sqrt{\dfrac{2-m}{2+m}}$ and occurs when $x = 1$ and $\sin\xi_2 = \sqrt{\dfrac{s}{\epsilon^2 + q^2}}$. For horizontal polarization the maximum value of b occurs when $q = \sqrt{3}\,r$ and is equal to $-1/\sqrt{2r}$. Under these conditions $s = 2r$ and $n = -1$.

TABLE III

	m	n	$\frac{1}{\sqrt{c}}$	$a = m\sqrt{c}$	$b = n\sqrt{c}$	$\frac{a}{c} = \frac{m}{\sqrt{c}}$	$\frac{b}{c} = \frac{n}{\sqrt{c}}$
1. Vertical Polarization in General	$\sqrt{2}\left[\sqrt{\frac{1+\frac{r}{s}}{1+\left(\frac{q}{\epsilon}\right)^2}}+\sqrt{\frac{1-\frac{r}{s}}{1+\left(\frac{\epsilon}{q}\right)^2}}\right]$	$\sqrt{2}\left[\sqrt{\frac{1+\frac{r}{s}}{1+\left(\frac{\epsilon}{q}\right)^2}}-\sqrt{\frac{1-\frac{r}{s}}{1+\left(\frac{q}{\epsilon}\right)^2}}\right]$	$\sqrt{\frac{s}{\epsilon^2+q^2}}$	$\frac{\sqrt{2}}{s}[\epsilon\sqrt{s+r}+q\sqrt{s-r}]$	$\frac{\sqrt{2}}{s}[q\sqrt{s+r}-\epsilon\sqrt{s-r}]$		
2. V. P., $Q \ll 1$	$\sqrt{2}$	$\sqrt{2}$	$\frac{1}{\sqrt{q}}$	$\sqrt{2q}$ *	$\sqrt{2q}$ *	$\sqrt{\frac{2}{q}}$	$-\sqrt{\frac{2}{q}}$
3. V. P., $Q \gg 1$	2	$\frac{2r-\epsilon}{\epsilon r}q$	$\frac{\sqrt{r}}{\epsilon} = \sqrt{\frac{\epsilon-1}{\epsilon^2-x^2}}$	$\frac{2\epsilon}{\sqrt{\epsilon-1}}$ *	$\frac{(\epsilon-2)q}{(\epsilon-1)^{3/2}}$ *	$\frac{2\sqrt{\epsilon-1+\sin^2\xi_2}}{\epsilon}$	$\frac{7\epsilon-8+8\sin^2\xi}{(\epsilon-1+\sin^2\xi)}\frac{\sqrt{\epsilon-1+\sin^2\xi_2}}{4\epsilon^2}q$
4. Horizontal Polarization in General	$\sqrt{2}\sqrt{1+\frac{r}{s}}$	$-\sqrt{2}\sqrt{1-\frac{r}{s}}$	$\sqrt{s}$	$\frac{\sqrt{2}\sqrt{s+r}}{s}$	$-\frac{\sqrt{2}\sqrt{s-r}}{s}$		
5. H. P., $Q \ll 1$	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{q}$	$\sqrt{\frac{2}{q}}$	$-\sqrt{\frac{2}{q}}$		
6. H. P., $Q \gg 1$	2	$-\frac{q}{r}$	$\sqrt{r} = \frac{\epsilon-1}{1-x^2}$	$\frac{2}{\sqrt{\epsilon-1+\sin^2\xi_2}}$	$\frac{q}{[\epsilon-1+\sin^2\xi_2]^{3/2}}$		

* For vertical polarization, $x \to 0$ requires $\xi_2 \to 0$. The tabulated values of a and b in rows 2 and 3 are true only for $\xi_2 \to 0$. For horizontal polarization x is never large.

Appendix III

In using the equations and curves of this paper to calculate the field, the ground constants appropriate to the location of interest should be employed. The literature on ground constants is already large and is continually increasing. An exhaustive summary of this literature would be out of place here, but as an aid to those who do not have available the ground constants of the locality in which they are interested, the following table is presented.

The first four sets of values have been widely used. The conductivities of grounds 1 and 5 have been accepted by the Madrid Conference as representative of ocean water and average ground. The conductivities of grounds 5 to 8 were obtained from field strength surveys.[12] The conductivities of water 9 to 11 were obtained from sample measurements by Mr. L. A. Wooten of these laboratories at a temperature of 25° C. Both the conductivity and dielectric constant of water vary appreciably with the temperature, approximately in accordance with the relationships

$$\sigma = \sigma_{25^\circ}(1 + 0.02t),$$
$$\epsilon = 80 - 0.4(t - 20),$$

where t is the temperature in degrees centigrade.* The conductivity also varies from place to place in the ocean due to changes in its composition. The constants of grounds 12 to 15 were obtained from measurements on samples by Mr. C. B. Feldman of these laboratories. The constants of grounds 16 and 17 are typical of measurements made by Dr. R. L. Smith-Rose on English soil.[14]

In general, both the conductivity and dielectric constant of the ground vary with temperature, moisture content and frequency as well as location. For a more complete treatment and extensive bibliographies see C. B. Feldman [13] and R. L. Smith-Rose.[14]

Column 6 of Table IV gives the frequency for which $Q = 1$ for each type of ground. At higher frequencies $Q > 1$ and the ground tends to resemble a dielectric; at lower frequencies it tends to resemble a conductor.

Columns 7, 8, 9 and 10 give the values of the parameter x of Fig. 2 for a distance of 1 km. and the indicated frequency. For any other distance, x is equal to these values times the distance in kilometers. When $Q \ll 1$, x is proportional to the distance and to the square of the frequency. When the frequency is small compared with that

* The first equation was obtained from the values given for sodium chloride in the International Critical Tables. The second equation is given in the same source for pure water.

TABLE IV

Type of Ground	Relative Dielectric Constant	Conductivity in			Values of f_0 in $Q=f/f_0$	For Fig. 2 — When $Q \ll 1$, $x=c_1 d f^2/f_1^2$. Values of c_1 in km^{-1} for			For Fig. 2 — When $Q \gg 1$, $x=c_2 d f/f_2$. Values of c_2 in km^{-1} for $f_2=50$ mc.	For Figs. 5–12 — Values of $q=2\sigma/f$ for		
		emu	esu	mhos per meter	Mc.	$f_1=50$ kc. (6000 m.)	$f_1=500$ kc. (600 m.)	$f_1=50$ mc. (6 m.)		$f=50$ kc.	$f=500$ kc.	$f=50$ mc.
1	2	3	4	5	6	7	8	9	10	11	12	13
1. Sea Water	80	10^{-11}	0.9×10^{10}	1	225	2.91×10^{-6}	2.91×10^{-4}	2.91		360,000	36,000	360
2. Fresh Water	80	10^{-14}	0.9×10^{7}	10^{-3}	0.225	0.291			13	360	36	0.36
3. Wet Ground	10	5×10^{-14}	4.5×10^{7}	5×10^{-3}	9.0	0.058	5.8		94	1,800	180	1.8
4. Dry Ground	4	10^{-15}	0.9×10^{6}	10^{-4}	0.45	2.91			196	36	3.6	0.036
5. Middle West U. S.		10^{-13}	0.9×10^{8}	10^{-2}		0.029	2.9			3,600	360	3.6
6. Texas		3×10^{-13}	2.7×10^{8}	3×10^{-2}		0.01	1.0			10,800	1,080	10.8
7. Southeast U. S.		4×10^{-14}	3.6×10^{7}	4×10^{-3}		0.072	7.2			1,440	144	1.44
8. New England		2×10^{-14}	1.8×10^{7}	2×10^{-3}		0.145	14.5			720	72	0.72
9. Atlantic Ocean near N. J.		4.32×10^{-11}	3.9×10^{10}	4.32	975	0.67×10^{-6}	0.67×10^{-4}	0.67	13	1,560,000	156,000	1,560
10. Average fresh water lakes in N. J.		6.7×10^{-14}	6×10^{7}	6.7×10^{-3}	1.5	0.043	4.3		13	2,400	240	2.4
11. Lake Michigan		2.44×10^{-13}	2.2×10^{8}	2.44×10^{-2}	5.5	0.012	1.2		13	8,800	880	8.8
12. Holmdel, N. J.	15–25	$1-2\times10^{-13}$	$1-2\times10^{8}$	$1-2\times10^{-2}$	8–26				40–65			
13. Netcong, N. J.	5–10	$1-3\times10^{-14}$	$1-3\times10^{7}$	$1-3\times10^{-3}$	2–12				94–167			
14. Japan	15	2×10^{-14}	1.8×10^{7}	2×10^{-3}	2.4	0.145	14.5		65	720	72	0.72
15. Philippine Islands	12	3×10^{-14}	2.7×10^{7}	3×10^{-3}	4.5	0.1	10		80	1,080	108	1.08
16. Dry Soil	3–4	1.1×10^{-16}	10^{5}	1.1×10^{-5}	0.06					4	0.4	0.004
17. Moist Soil	30–40	$1-2\times10^{-13}$	$1-2\times10^{8}$	$1-2\times10^{-2}$	5–13							

given in Column 6 these conditions are fulfilled and the proportionality factor is given in Columns 7, 8 and 9 for three frequencies. When the frequency is large compared with that given in Column 6, $Q \gg 1$ and the parameter x of Fig. 2 is proportional to the distance and the first power of the frequency. This proportionality factor is given in Column 10 for a frequency of 50 mc.

The parameter $q = 2\sigma/f$ of Figs. 5–12 is given in Columns 10, 11 and 12 for three frequencies.

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