Contemporary Advances in Physics, XXXI—Spinning Atoms and Spinning Electrons [1]

By KARL K. DARROW

NO doubt you are all accustomed to thinking of atoms as objects—very small objects, of course—which are endowed with *weight.* I can say that with perfect safety to an audience of engineers and physicists; but indeed it can be said with safety to any audience—I mean, of course, any audience literate enough to attach any meaning at all to such a word as "atom." It may be that philosophers of the past have imagined weightless atoms—I am not historian enough to deny that, nor to affirm it; but if such have ever been invented, they have remained quite outside the currents of modern thought. For us, weight is a property which we attribute to the atom. Since this is, after all, a professional audience, I will now change over to that other word which many people have such difficulty in distinguishing from "weight": I will say that *mass is a property which we attribute to the atom.* In a way, that is a negative statement. It means that we do not hope to explain mass in terms of something more fundamental; it means that we accept mass as being itself so fundamental that even the elementary particles have it. When I say "elementary particles," I am still referring in part to the atoms, though it is a somewhat careless usage to do so; but I am referring also to electrons both positive and negative, to protons, to alpha-particles, to nuclei—to all the particles, in effect, of which the atoms are built up. Also I ought to include the corpuscles of light, but this lecture will be quite long enough if I leave them almost unmentioned. All of these particles, then, are endowed with mass; each of them has a characteristic mass of its own, which we do not attempt to explain, but which we do try to measure as closely as we can.

There is another property, familiar to you though not to everyone, which we accept as equally fundamental and equally unexplainable with mass: it is *electric charge.* We attribute it also to the elementary particles, though not, it is true, to all of them. We assign it to the electrons, of course, and to protons and alpha-particles and all of the hundreds of nuclei which distinguish the elements and the isotopes

[1] A lecture delivered before the American Physical Society at Chicago on November 27, 1936, and before the American Institute of Electrical Engineers at New York on May 6, 1937.

thereof from each other. When I say that we "assign" it, I do not for a moment mean that we are doing something arbitrary or untestable. We *know* that these elementary particles are charged, and indeed we have measured their charges. Atoms do normally appear to us uncharged, but we know that that is because the elementary particles of which they in their turn are made up are some of them positive, some of them negative, and the balance normally happening to be perfect. Some particles, the neutrons and the corpuscles of light, seem permanently chargeless; but perhaps some day we shall find it expedient to regard them as groups of smaller particles having charges which balance one another. Apart from these two cases, we may say with assurance that whenever we penetrate as far into the fine structure of substance as we are able to go, we find the elementary particles invested with mass and with charge.

And now I arrive at the subject of this talk, the *third* property of the elementary particles: the property which is called "angular momentum," or "spin" for short. Now of course I am speaking as to an audience of physicists, for if this were an audience of laymen it would certainly be frightened by such a term as "angular momentum." This is a misfortune, and perhaps a defect of general education; for angular momentum is about as important as mass or charge, not only on the scale of the elementary particles but also on the scale of the visible world. Think what it would mean if there were no such thing as the conservation of angular momentum! the earth might cease from turning, it might cease from providing us with the regular alternation of day and night, and with our standard of the flow of time; it might even cease from traversing its regular orbit, and fly off into space or fall into the sun. Well, I do not wish to scare you with any such dire imaginings—I only want to remark on the fact that the human race has been acquainted for a very long time indeed with angular momentum as something which is unvarying, imperturbable, incessant; for of all the unvarying and imperturbable and incessant things in the world, the rotation of the earth is the most obvious and the most striking. So striking it is, that you might reasonably expect that all the philosophers and all the physicists of the past would have conferred the property of spin on all the atoms which they have invented. Well, they did not; the notions of the spinning atom, the spinning electron, the spinning nucleus are among the newest in physics. I think that some of the reasons for the delay will be evident later on in this talk, but it remains partly mysterious, at least to me. Looking back on the situation with the well-known advantages of hindsight, I do feel a good deal of surprise that the spinning atom did not make an earlier entry upon the scientific stage. Perhaps some of you will remember hearing the words "vortex atom" and "vortex theory" which used to be so prominent in physics, and will take the spinning atom of today for a lineal descendant of those vortices of old. If this were correct, we could trace back the ancestry of the spinning atom for about three hundred years; but I think that it is not correct. The vortices of which Descartes and Malebranche were dreaming three centuries ago were more like whirlpools of streaming particles, and the vortices which were imagined by Helmholtz and Lord Kelvin some fifty years ago were also whirlpools, but they were whirls in an idealized continuous frictionless fluid. Let us pause for a moment to notice how the attitude of physicists has altered in these fifty years! Kelvin and Helmholtz began with the idea of an aethereal fluid pervading the whole of space, and valiantly tried to represent the atoms as whirlpools in that fluid; but we have long since discarded that aether, and our spinning atoms and other elementary particles are small delimited rotating bodies voyaging in a void.

It is nor therefore the vortex which I will introduce to you as the ancestor of the spinning atom, but rather the "Amperian whirl" as it still is sometimes called. You remember, of course, how Ampère in 1820 made a very great achievement which for the purposes of this talk I will divide into three. First, he discovered the fact that an electric current flowing in a circuit is equivalent to a magnet. Next, he worked out the mathematical laws whereby, given a current and the circuit in which that current is flowing, we may calculate the strength or the moment of the equivalent magnet. I will write down the formula for the case of a current i, flowing in a plane loop of area A: the magnetic moment of the equivalent magnet, μ, is given thus:

$$\mu = iA/c.$$

Here c is a factor of transformation which we are now obliged to employ because we habitually use, in atomic physics especially, a unit-system different from Ampère's. The third part of the great achievement was this: Ampère founded what remains to this day the theory of magnetism, by presuming that *the individual atoms of any magnetizable substance are themselves little magnets*, and that *the atoms are magnets because they have little whirls of current in them.*

This notion—the notion that atoms are magnets, and that they are magnets because they have internal circulating currents—is the true forerunner of our present conception of the spinning atom. It is, however, only a very primitive form of the modern conception, and there is much to be added to it. First of all and above all, there is the question of angular momentum. Is angular momentum an attribute of these whirling intra-atomic currents, or is it not? You may think that the answer to this question is self-evidently "yes!" but remember that for many generations of our forefathers electricity was an *imponderable* fluid. Weber, however, did consider the affirmative answer, and Maxwell even attempted to ask the question of Nature by experiment—vainly, as it turned out. Not till the electron was discovered did the mass of electricity become a prominent part of experience. A moment ago I divided Ampère's achievement into three parts; similarly I wish now to divide the discovery of the electron into three. Those who isolated and identified and measured the electron were proving three things: first, that negative electricity consists of corpuscles of a definite charge, e; second, that these corpuscles have a mass, m; and following from these two, the principle which I have called the third part of the discovery, *viz.* that an electron revolving in an orbit has an angular momentum.

I will designate angular momentum in general by the letter p, and now I will show you a formula for the ratio of μ to p in an atom in which an electron is running around in an orbit and constituting an Amperian whirl. The formula, like this other one, for μ, is valid for an orbit of any shape, but to get it quickly I will simplify by postulating a circular orbit. The radius of the circle being r, the area A is πr^2; the current around it is equal to the electron-charge e, multiplied by the number of times per second that the electron runs around the orbit; if I denote the velocity of the electron by v, this number is $v/2\pi r$; hence the product iA/c is equal to $evr/2c$. Now the angular momentum p of the electron, as you all know, is mvr; and hence for the ratio I derive:

$$\mu/p = e/2mc,$$

which is one of the most important formulae in the whole of atomic physics. You notice that it does not involve in any way the size or shape of the orbit or the frequency with which the electron travels around it. It is the same for any or all of the revolving electrons of any atom of any kind.

Now let us see how this formula may be tested. Imagine a rod of some highly magnetizable metal, iron for instance, and imagine it to be unmagnetized at the start. This means, that at the start the little atomic magnets are pointing at random in all directions; that is to say, the vectors which represent their magnetic moments are pointing every way, and so are the vectors which represent their angular momenta, the latter being parallel to the former. Since these atomic vectors of angular momentum are pointing every way at random, they add up to zero, and the rod as a whole possesses no resultant angular momentum; it is just standing still. Now let the rod be surrounded with a solenoid, and by means of a current in the solenoid let it be magnetized to saturation. Now all the arrows representing magnetic moments are pointing parallel to the axis of the rod. But so are all the arrows representing atomic angular momenta! their resultant is no longer zero—suddenly there has arisen a *resultant angular momentum*, belonging to the totality of all the atomic magnets, and quite large enough to be detected, instead of being tiny like the angular momentum of an individual atom. Unless our theory is fundamentally wrong somewhere, we should be able to observe this resultant angular momentum.

The experiment is done by hanging the rod vertically from a fine suspension, and sending the magnetizing current through the solenoid. At the instant of the magnetization, the rod turns sharply on its axis, twisting the suspending fibre. Thus it manifests the angular momentum of which I have just been speaking—though I ought to say that what we observe is of the nature of a recoil, or back-kick: when the totality of the little atomic magnets suddenly acquires its resultant angular momentum, the substance of the rod as a whole acquires an equal and opposite amount (so as to keep constant the total amount of angular momentum in the universe) and it is the latter which we detect. The experiment is quite a delicate one, but its technique has been developed to a remarkable degree since it was first attempted twenty years ago by Einstein and de Haas. What we measure is the ratio of the magnetization of the rod-as-a-whole to the angular momentum of the rod-as-a-whole; and this is just the same as the ratio of μ to p for the elementary atomic magnets. There are not many properties of matter of which we can say that the value measured on a large piece of matter is the same as the value for the individual atom; but there are a few, and this is one of them.

Now in giving you the result, let me first emphasize the general principle that here we have evidence of the spinning of elementary particles, and of the interrelation between spinning and magnetism. Next, I give you the numerical result itself. For iron and nearly all of the other ferromagnetic materials, we find:

$$\mu/p = e/mc$$

or *twice* the theoretical value which I quoted a moment ago.

This cannot be explained by assuming any peculiarity of size or shape or frequency of the electron-orbits in the atoms, for as I just said the theoretical formula is independent of all these things. We are obliged to make some more drastic assumption. If I had unlimited time before me, I might sketch the history of our assumption; but as I don't, I will come straight to the present situation. We assume first, that in the iron atoms in the rod the electron-orbits are so oriented with respect to each other that their magnetic moments kill one another off completely. We then assume that every electron has a magnetic moment and an angular momentum of its own, intrinsic to it and inherent in it, and altogether independent of whether or not the electron is revolving in an orbit. Just as the earth has a rotation of its own in addition to its elliptical course around the sun, so we imagine that the electron has a rotation of its own; this rotation has an angular momentum, and with it there is connected a magnetic moment. (You will remember doubtless that the earth also has a magnetic moment, but this is one of the analogies which it is better not to force too far.) When we magnetize the iron rod, it is the *electrons* which we are turning; the vectors which we cause to point all in the same direction are the magnetic moments and the angular momenta which are inherent in the electrons, and the value of their ratio is the value which is characteristic of the "spinning electron," as we call it. Therefore, amplifying the notation a little, I write:

$$\mu/p = g(e/2mc)\begin{cases} g = 1 \text{ for electron-orbits,} \\ g = 2 \text{ for spinning electron,} \end{cases}$$

and now I leave the spinning electron for a few minutes, in order to turn again to the theory of electrons revolving in their orbits.

You all realized, of course, that when I converted the Amperian whirl of current into an electron running around an orbit, I was adopting the atom-model known by the names of Rutherford and Bohr; for these were the original thinkers who impelled all the rest of us, following in their footsteps, to think of the atom as a positively-charged nucleus around which electrons are revolving like planets around the sun. This is an atom-model in which magnetism is inherent—*a Rutherford-Bohr atom is intrinsically a magnet.* Anyone who did not know the history of the model might well assume that it was designed expressly to account for magnetism, and any such person might also quite reasonably assume that all the physicists of the early nineteenth century thought of it simultaneously as soon as the electron was discovered. Well, it was *not* designed expressly to account for magnetism, and most of the physicists of the early nineteenth did *not* think of it—or if they did, they thought of it only to reject it. At that time, the atom-model with the orbital electrons seemed to be disqualified by a very potent reason; for according to the classical electromagnetic theory, an electron revolving in an orbit ought to radiate all of its energy in a very short time and fall into the nucleus. Bohr was the man who overrode this objection. He overrode it, not in order to construct a theory of magnetism in defiance of it, but in order to construct a theory of spectra in defiance of it. This theory has been extraordinarily successful. Our theory of magnetism is hardly more than a by-product of that theory of spectra; and this, in an odd sort of way, enhances its credit. A theory devised expressly for a certain purpose is always less impressive than one which follows incidentally from a successful theory devised for quite another purpose; and the contemporary theory of magnetism is a wonderful example of this latter and more impressive type.

The main element of Bohr's theory of spectra—if one can speak of one element as the main one, which is really not quite proper—is an assumption about the angular momentum of the electron in its orbit or, let me say, the angular momentum p of the electron-orbit. It was assumed that the electron may revolve, without radiating its energy, in any orbit of which the angular momentum is an integer multiple of $h/2\pi$, — h now standing, of course, for the famous quantum-constant of Planck which is the emblem of modern physics. I write this down as follows:

$$p = (1, 2, 3, 4, \cdots)(h/2\pi).$$

Bohr was thinking at first about the hydrogen atom; but hydrogen is an inconvenient example to use in talking about magnetism, and iron is a very complicated case indeed, so I will talk entirely about the sodium atom.

The sodium atom has a nucleus with a charge of $+ 11e$, and eleven electrons circulating in orbits around it. This certainly sounds formidably complex, but it happens—and I shall later remind you of this fact—that the orbits and also the spins of ten of the electrons are so oriented with respect to one another that their angular momenta and their magnetic moments completely neutralize each other. I shall therefore ask you to imagine these ten inner electrons as a sort of cloud. The eleventh electron of the sodium atom—known technically as the "valence" electron—cruises around this system; sometimes it is traveling in an orbit completely outside the cloud, sometimes in an orbit which cuts across the cloud, but never in an orbit which is entirely or even mainly inside the cloud. The ten inner electrons which constitute the cloud neutralize a part of the force with which the nucleus acts upon the valence-electron; but they make—I repeat —not the slightest contribution to the angular momentum or to the magnetic moment of the atom.

I have given above, the permitted values of the angular momentum of this orbit of the valence-electron. Now I point out that to each of these permitted values of p corresponds a permitted value of the magnetic moment μ, which I obtain by multiplying the former with $e/2mc$:

$$\mu = (1, 2, 3, 4, \cdots)(eh/4\pi mc).$$

However, only one (at most) of these values can be appropriate to the normal state of the sodium atom; all the rest must correspond to abnormal, unusual, or "excited" states. We are going to be interested primarily in the normal state, so we must identify the right one. In the early days of the Bohr theory, the right one was supposed to be the first which I have written down. However, the theory has been greatly remodelled and bettered since those days, with the aid of what is known as "quantum mechanics"; and it now seems quite certain that these lists of the permitted values of p and μ for electron-orbits are both incomplete. I must add to each of them the value *zero*, so that the two lists become

$$p = (0, 1, 2, 3, 4, \cdots)h/2\pi,$$
$$\mu = (0, 1, 2, 3, 4, \cdots)eh/4\pi mc.$$

Moreover, it is precisely this new value *zero* which belongs to the normal state of the sodium atom. So the theory, in this stage, quite definitely prescribes that the sodium atom in its normal state should have no magnetic moment and no angular momentum. But now let us look at the data.

It would do no good in this connection to make measurements on solid or on liquid sodium, for in those "condensed phases" the atoms are crowded so closely together as to be badly distorted. We can, however, experiment on free atoms of sodium, in their normal state, in the way illustrated by Fig. 1. In the upper portion, A represents an "oven," consisting of a small box heated electrically and containing some liquid sodium which is steadily being vaporized. There is a hole in the wall of the box through which free sodium atoms are steadily shooting in all directions, with the distribution-in-speed which we know from the theory of thermal agitation; and beyond, there is a sequence of diaphragms with slits in them which delimit a straight and narrow beam of these fast-moving atoms. Disregarding what the theory has just said, let us suppose that each of these atoms is a magnet—a bar magnet, with a north pole and a south pole. As they emerge from the oven, these atoms must surely be oriented at random in all directions.

Continuing to look at the upper part of Fig. 1, we have two large magnet-poles, and the beam travels between them, having no trouble with molecules of air as it shoots along, for the whole of this apparatus is enclosed in a highly-evacuated tube. It may seem natural to visualize these magnet-poles as the two broad flat extremities of a horseshoe-magnet, with a uniform magnetic field pervading all the space between them. Such an arrangement, however, would make the experiment futile.

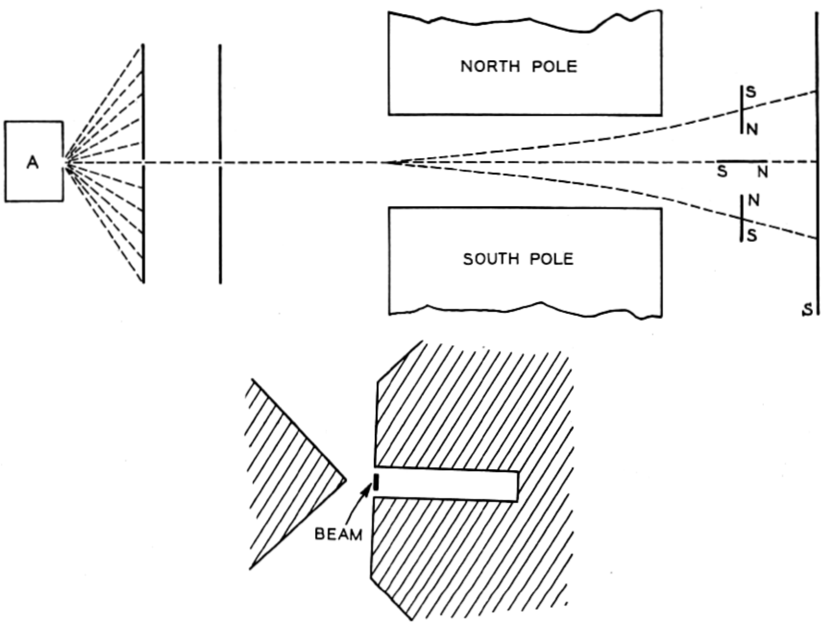

Fig. 1—Longitudinal section and cross-section of apparatus for the Gerlach-Stern experiment.

Nothing would happen to any of the atoms, for in the uniform field the north pole of each atomic magnet would be pressed downward just as hard as and no harder than the south pole is drawn upward, and the net force would be zero. The beam would go on unbroadened and undeflected, and make a small spot on the screen *S*, the spot being just opposite the slits in the diaphragms. Something else must be tried; and what we do—or rather, what Gerlach and Stern did in Hamburg some fourteen years ago—is, to shape one of the magnet-poles in the form of a wedge and hollow out the other, so that the field between the two shall no longer be uniform. The lower part of Fig. 1 represents the cross-section of such a pair of pole-pieces. The field-strength is now much greater near the wedge than near the opposite pole-piece, and there is a vertical gradient of field-strength which may be made fairly constant over most of the interspace. Rabi at Columbia achieves the same result more efficiently by peculiar arrangements of current-carrying wires instead of iron magnets.

Now consider (thinking classically!) a few of the atomic magnets as they shoot across this non-uniform field. Think of one which originally is oriented vertically, with north pole down and south pole up—the south pole will be in a stronger part of the field than its mate; it will be drawn upward harder than the north pole is pushed downward; the atom will sweep in a parabolic arc upward. Think of another which originally is oriented vertically with north pole up and south pole down—it will be swept in a parabolic arc downward. Think of another which originally has its axis pointing horizontally—it will shoot in a straight line across the field, as though the pole-pieces were not there.[2] Now think of all those which are oblique to the vertical; they will describe parabolic arcs of intermediate curvatures, upward or downward as the case may be. One infers that the beam must be spread out into a continuous fan, making a continuous vertical band upon the photographic plate.

Moreover, from the upper edge or from the lower edge of this continuous band, it should be possible to determine the magnetic moment μ of the atoms. For let us consider one of the vertically-oriented atoms, and call its pole-strength M and the length of its magnet r. The upward force on the north pole is MH,—H standing for the field-strength at the point where the north pole is. The downward force on the south pole is $M(H + dH/dz \cdot r)$. The net force is $Mr \cdot dH/dz$, which is $\mu(dH/dz)$, because Mr is the magnetic moment μ of the magnet by definition. The acceleration of the little atom is equal to this force divided by m_a, the mass of the sodium atom. The deflection is equal to half the acceleration by the square of the time during which the atom is exposed to the force. This time is equal to the distance D which the atoms traverse across the field, divided by v the speed which they have when they come out of the furnace. So finally, we have:

$$\text{Deflection} = \frac{1}{2}\frac{\mu(dH/dz)}{m_a}(D/v)^2.$$

We ascertain the deflection by looking at the end of the band on the photographic plate, and we can ascertain all the other things in the

[2] When thinking classically, we must not expect the atomic magnets to turn their axes toward the field-direction as they shoot across the field; the gyroscopic quality of these magnets, due to their angular momentum, inhibits this.

equation excepting μ,—v is the hardest to estimate accurately—and so we can solve the equation for μ.

When the experiment is done there appears, however, a very remarkable thing. Instead of there being a long band upon the plate, there are just two spots. Instead of the beam having been broadened out into a continuous fan, it has evidently been split into two separate pencils. It looks as though the field had acted first of all upon the magnets, by setting them all vertical,—half of them with north pole up, and half with north pole down. Not this phenomenon alone, but many others in Nature show us that this is just what happens. You may perhaps feel for the moment that it is intelligible, after all; the compass-needle turns to the north—why should not the little atomic magnets, as soon as they enter the field, turn their south poles toward the north pole of the magnet which attracts them? Well, this would not account for the magnets which constitute the beam which bends away from the wedge-shaped north pole, instead of toward it; and indeed it does not even account for the beam which bends toward the wedge-shaped pole. Classically the field should have no orienting effect whatsoever upon the atoms, and yet it evidently does.[2] This is one of the phenomena of the atomic world which we cannot properly visualize in terms of the behavior of objects large enough to be tangible and visible. All that I can do is to assert it, and to say that it justifies us in using this formula to calculate μ. When we use it, the value which we find for the magnetic moment of the sodium atom in its normal state is

$$eh/4\pi mc,$$

which happens to be one of the values in the sequence which I just wrote down.

I repeat that according to the theory in its present stage, the electron-orbits in the normal sodium atom have a net magnetic moment of zero. This value $eh/4\pi mc$ is, therefore, the magnetic moment due to the spin of the valence-electron—it is the magnetic moment of the spinning electron. I write it in the appropriate place, and then with the aid of the g-value derived from the gyromagnetic effect I write down the value of angular momentum which we assign to the spinning electron:

$$p = \tfrac{1}{2}(h/2\pi).$$

To this roster of three statements about the spinning electron I now make a final addition. The Gerlach-Stern experiment on sodium shows that a beam of sodium atoms—which for this purpose is the equivalent of a beam of spinning electrons—is divided into two by a magnetic field. I write down "2" to indicate this number of separated beams; but I will call it by preference the "number of orientations in the field," because that is the fundamental point. The spinning electron always sets itself in one or the other of two orientations, with respect to whatever field it happens to be traversing. We call them the "parallel" and the "anti-parallel" orientations, though according to quantum mechanics these terms are a little too strong. Here then is the list of the properties of the electron-spin: g equal to 2—angular momentum equal to $\frac{1}{2}(h/2\pi)$—magnetic moment equal to $eh/4\pi mc$—two permitted orientations in any field.

It has doubtless struck you as rather odd that I began by talking about the angular momenta and the magnetic moments of electron-orbits, and then carefully picked out a couple of special cases in which these neutralized each other altogether and there was nothing left over except what I ascribed to the electron-spin. Is there no point at all, then, in talking about the electron-orbits? Oh, very much so! Indeed there are cases in which the electron-spins neutralize each other altogether, and we have nothing left over except what is attributed to the orbits. To do this I may choose an atom like magnesium, which has a nuclear charge of $+ 12e$, a cloud of ten inner electrons which neutralize one another completely as to angular momentum and magnetic moment (just as in sodium), and *two* valence electrons instead of one. In some of the states of the magnesium atom—not in all of its states, but in *some* of them—the spins of the two valence electrons are oriented opposite to each other in the atom, and cancel each other out. When the atom is in a state of this kind, then nothing is left over except the angular momenta and the magnetic moments of the orbits of the two valence-electrons; and then, all the statements of the orbital theory (page 326) are applicable—g is equal to unity, the angular momentum takes one of the values $nh/2\pi$ and the magnetic moment takes one of the values $n(eh/4\pi mc)$. Moreover, there is another theorem derived from quantum mechanics which turns out to be valid: the number of orientations of such an atom in a field, the number of separated beams which appear in the Gerlach-Stern experiments, is chosen from among the members of this sequence: 1, 3, 5, 7. . . . (It is a most interesting historical fact, that Gerlach and Stern were moved to undertake their difficult experiment by the wish to test this remarkable assertion of quantal theory.) You notice that the number 2 does not appear in the sequence; were it not for the electron-spin, we never could obtain it; it is distinctive of the spinning electron.

I must, however, admit that all these cases of which I have been speaking are special, and comparatively rare. Both the cases in which the spins neutralize each other perfectly, and the cases in which the orbital moments neutralize each other perfectly—both types are unusual. Still more unusual, and yet occurring here and there, is the most special of all cases—that in which *all* of the moments and *all* the momenta, both those of the spins and those of the orbits, balance one another perfectly so that the sums are zero. An atom in such a state is completely devoid both of magnetism and of spin; such atoms are those of helium, of neon, of argon and the other noble gases, when in their normal states. Usually, however, we find ourselves confronted with some example of the general case, in which neither the spins nor the orbits are completely neutralized. The atom has an angular momentum which is a sort of composite or resultant of the angular momenta of the spins and the orbits, and it has a magnetic moment which also is a sort of composite or resultant.

If I were to embark on the description of the general case this lecture might go on interminably, and at its end you would probably not remember anything except what you had already known at its beginning. The laws of the composition of spins and orbits are so foreign to our customary ways of thinking, and the formulæ which express them are so curiously built, that to work once only through them is not sufficient: one has to memorize the derivations and the results alike, and go over them incessantly until they are imprinted on the brain. I think you will agree to this readily enough, when I remind you that this theory is none other than the general theory of spectra; for even quite outside the ranks of physicists, the theory of spectra is beginning to be notorious for its complexity. I shall not venture even to give the formulæ, much less their derivations; it must suffice to fill out the two lists of p-values and μ-values on which I have already begun, and the list of n-values or numbers-of-permitted-orientations.

The spinning atom is a congeries of electrons, all of them always possessing spin, most of them usually possessing orbital motions; and these motions are compounded with each other in such ways, that:

First, the angular momentum of the spinning atom has one of the values,

$$p = (0, \tfrac{1}{2}, 1, \tfrac{3}{2}, 2, \tfrac{5}{2}, 3 \cdots)h/2\pi.$$

Second, the number of beams in the Gerlach-Stern experiment, or the number of permitted orientations of the atom in the field, has one of the values,

$$n = 1, 2, 3, 4, \cdots.$$

Third—and now comes a surprise, for you will probably expect me to say of the magnetic moment that it is $eh/4\pi mc$ multiplied either by an integer or a half-integer; but this is not so. The actual state of affairs is described by a formula which is called the g-formula, because it gives g in terms of the moments, both spin and orbit, of the individual electrons. It was discovered by Landé and interpreted in terms of the spinning electron by Goudsmit. The g-formula gives unity, as of course it must, in the special cases where the electron-spins cancel each other and only the orbital moments are left over, and it gives 2 in the special cases where the orbital moments are neutralized with only the spins left over. In the other cases it may give any one of a large variety of values: mostly one gets simple-looking fractions such as 9/8 and 4/3 and 5/6. The magnetic moments are then computed by multiplying the appropriate g-values times $e/2mc$, into the existing values of angular momentum.[3]

I now turn to that component of the atom of which the spin remains to be discussed—to the *nucleus*.

As I have already intimated, the values of p and n and μ for the electron-family of any atom are mostly ascertained by analyzing their spectra and utilizing the great general theory of spectra. Such magnetic experiments as I used for my examples are relatively few, and feasible for relatively few substances. The reason for making this remark at this late moment is, that by analyzing spectra we may also learn something about the spins of nuclei; for nuclei also are invested with these properties of angular momentum and of magnetic moment. I take in particular the case of the *proton*—that lightest of all nuclei, the nucleus of the lightest known kind of atom which is ordinary hydrogen, so called to distinguish it from "heavy" hydrogen. Analysis of the spectrum of hydrogen shows us that the proton is capable of taking two permitted orientations in a field; thus, our first piece of information about the proton-spin is conveyed by writing $n = 2$.

Now that we have this piece of information, we deduce that as the spinning electron has an n-value of 2 and an angular momentum of $\frac{1}{2}(h/2\pi)$, so the proton with its n-value of 2 must have an angular momentum of $\frac{1}{2}(h/2\pi)$. Continuing along this line of thought, we are further tempted to infer that the proton should have a g-value of 2 and a magnetic moment of $(eh/4\pi mc)$. But what shall we suppose

[3] Should any reader intend to proceed from this article to a thorough study of atomic theory, he should be warned in advance that according to the latest form of quantal theory, the p-values and the μ-values here given are the values of the projections of these vectors upon the field-direction, the magnitudes of the vectors themselves being somewhat greater: I have given details as to this in "Contemporary Advances in Physics, XXIX, . . ." This *Journal*, **14**, pp. 293 ff.(1935).

about m? Formerly it represented the mass of the electron; now we are dealing with a different sort of particle, having a mass which (as many other kinds of experiments show us) is about 1835 times as great as the electron-mass. I denote this mass by M. It seems natural, then, to expect for the magnetic moment of the proton the value $eh/4\pi Mc$, or about 1/1835 of that of the spinning electron.

This is a formidably small magnetic moment to hope to measure, nay even to detect! yet Stern and his pupils undertook to measure it, and they succeeded. Of course, modifications had to be made in the technique which worked so well for sodium. Hydrogen being a gas at room-temperature, no heated oven was required; nevertheless they used an "oven," but it was refrigerated instead of being heated—a sort of super-ice-box; this was in order to obtain slow-moving atoms, for the slower the atoms traverse the field, the more accurately the experiment can be made. I just said "atoms"; but as most people know, the particles of gaseous hydrogen are not atoms, but diatomic molecules—systems composed of two protons and two electrons apiece. This is a circumstance which in many desirable tests of modern theoretical physics is a great inconvenience, for usually our simplest theoretical affirmations refer to hydrogen atoms and we should like to be able to experiment on them directly. Here, however, it turns out to be a great convenience, indeed perhaps the only thing that makes the experiment possible. For if we had an isolated hydrogen atom, the magnetic moment of its electron would so far exceed that of its proton that the latter would be indetectable. (Perhaps it is not superfluous to mention that bare protons could not be used in the experiment either, as the magnetic field would exert so large a force upon their moving charges that the forces upon their magnetic poles would be insignificant by comparison.)

But if in a single hydrogen atom the magnetic moment of the electron swamps that of the proton, how shall this fate be avoided for a system composed of two electrons and two protons? Here enters in, and in a very important and significant way, that law of the permitted orientations. Just as a spinning particle of angular momentum $\frac{1}{2}(h/2\pi)$ can take only two permitted orientations in a field, so it can take only two with respect to another particle of its kind—the parallel and the anti-parallel. It chances—or rather it does not chance, it follows from the underlying laws of Nature—that in the hydrogen molecule the two electrons are oriented anti-parallel to each other. Their magnetic moments cancel each other, and do not trouble the experimenter.

May not, however, the same thing happen in respect to the two protons, so rendering the experiment hopeless? It turns out that for these the spins are anti-parallel in some molecules but parallel in others. Molecules of the former type, which is called para-hydrogen, are indeed useless for the experiment; but molecules of the latter type, which is called ortho-hydrogen, are available, and in them the magnetic moments of the two protons collaborate so that the magnetic moment of the molecule-as-a-whole is twice as great as that of the single proton, a welcome assistance! In ordinary gaseous hydrogen at room temperature, about three-quarters of the molecules are ortho-hydrogen.

When the experiment was at last achieved by the school of Stern, it was found that the foregoing inference as to the μ-value of the proton is roughly but not exactly correct! The latest information is, that the magnetic moment of the proton is close to $2\frac{1}{2}$ times $eh/4\pi Mc$. Measurements with another method by Rabi and his school have confirmed these results; and we are definitively debarred from believing that for the proton and the electron, the magnetic moments stand in the inverse ratio of the masses. Perhaps this signifies that the proton is itself a composite particle, a notion for which there is some support from other sources.

I mention briefly the characteristics of a few other nuclei. After the proton, the next simplest is the deuteron or nucleus of the heavy-hydrogen atom. It is composed of a proton and a neutron, the latter being a neutral particle of about the same mass as the proton. We can observe free neutrons wandering about in space, but we cannot determine their spins nor their magnetic moments. The deuteron, however, has three permitted orientations (this we discern from the spectrum of heavy hydrogen) and consequently an angular momentum of $(h/2\pi)$. It is inferred that the neutron has $\frac{1}{2}(h/2\pi)$ for its angular momentum, and that in the deuteron these two constituent particles—proton and neutron—are oriented with their equal spins parallel to one another. The magnetic moment of the deuteron is less than that of the proton, and accordingly the neutron must have its magnetic moment oppositely directed to that of its companion in the system, even though their angular momenta be similarly directed—a strange complication!

After the deuteron, the next simplest among the nuclei (except two which are much too rare for investigation) is the alpha-particle or helium nucleus. It is composed of two neutrons and two protons. We find that its angular momentum and its magnetic moment are zero, a clear indication that the four spins of its components are cancelling each other two by two, and the four magnetic moments likewise. Next comes a nucleus composed of three protons and three neutrons, belonging to the element lithium. It is found that in angular momentum as in magnetic moment it is practically a duplicate of the deuteron, its six components having disposed themselves into a nearly normal deuteron attached to a nearly normal alpha-particle. I might proceed some distance farther along the list of the known nuclei after this fashion, were there space; but it is best to close this section with a general rule: *nuclei with an even number of constituent particles* (protons and neutrons) *have even spins, nuclei with an odd number of particles have odd spins.* "Even" and "odd" in this formulation mean that the angular momentum is an even or an odd integer multiple of $\frac{1}{2}(h/2\pi)$, respectively. One sees that if any two spins of magnitude $\frac{1}{2}(h/2\pi)$ are allowed to choose only between parallel and anti-parallel orientations, the rule follows inevitably; reversely, from the rule (which is based on experience with some fifty or sixty kinds of atom), we derive extra strength for that theorem about orientations.

Now to come to the conclusion and the climax. Although this property of angular momentum, of being allowed to take only a limited number of permitted orientations—although this strange and wonderful property of angular momentum was introduced in this lecture as though it pertained only to atoms subjected to applied external fields, yet it manifests itself far more broadly. Indeed, it manifests itself universally, and the stability and the character of the world are due to it. I have already mentioned in half-a-dozen places how it manifests itself within the molecule and within the atom: how in the atom, it is responsible for those laws of composition which determine the angular momentum and the magnetic moment of the electron-family—how in the hydrogen molecule, it establishes a difference between ortho-hydrogen and para-hydrogen—how in the nucleus it fixes the angular momenta and the magnetic moments of composite nuclei as sharply as those of their constituents the proton and the neutron. Perhaps these seem to be remote and unimportant qualities; but what has just been said of them may be said with equal truth and equal force of *all* the chemical and physical properties of all the elements, mass alone excepted (and even mass not fully excepted). If this feature of angular momentum did not prevail, there could not be the fixity of properties which characterizes each element by itself and the variety of properties which characterizes the totality of the elements. Gold would not be gold, lead would not be lead, oxygen would not be oxygen, helium would not be helium; for though it is commonly said that each element is distinguished by its nuclear charge and the number of electrons in its electron-family, this is not adequate. It is the law governing angular momentum which imprints upon these electron-families the characters which we recognize as the properties distinctive of the elements. It seems strange indeed that character should depend upon motion, and fixity upon the laws of whirling things; but however strange it may seem, there is no doubt about it. In the construction of houses the builder requires raw material in the form of brick and stone and wood and steel; but he requires also principles of architecture, whereby the raw materials may be parceled off and integrated into the general design. In the construction of the physical world, mass and charge fulfil the role of raw materials, and the laws of angular momentum furnish the principles of the architecture thereof.