A Multiple Unit Steerable Antenna for Short-Wave Reception *

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This paper discusses a receiving system employing sharp vertical-plane directivity, capable of being steered to meet the varying angles at which short radio waves arrive at a receiving location. The system is the culmination of some four years effort to determine the degree to which receiving antenna directivity may be carried to increase the reliability of short-wave transatlantic telephone circuits. The system consists of an end-on array of antennas, of fixed directivity, whose outputs are combined in phase for the desired angle. The antenna outputs are conducted over coaxial transmission lines to the receiving building where the phasing is accomplished by means of rotatable phase shifters operating at intermediate frequency. These phase shifters, one for each antenna, are geared together, and the favored direction in the vertical plane may be steered by rotating the assembly. Several sets of these phase shifters are paralleled, each set constituting a separately steerable branch. One of these branches serves as an exploring or monitoring circuit for determining the angles at which waves are arriving. The remaining branches may then be set to receive at these angles. The several receiving branches have common automatic gain control and thus provide a diversity on an angle basis. To obtain the full benefit of the angular resolution afforded by the sharp directivity, the different transmission times, corresponding to the different angles, are equalized by audio delay networks, before combining in the final output.

The experimental system, located at the Bell Telephone Laboratories' field laboratory near Holmdel, New Jersey, is described. This system comprises six rhombic antennas extending three quarters of a mile along the direction to England. Two receiving branches, in addition to a monitoring branch, are provided. Experience obtained with this system since the spring of 1935 is discussed. The benefits ascribable to it are (1) a signal-to-noise improvement of seven to eight decibels, referred to one of the six antennas alone, and (2) a substantial quality improvement due jointly to the diversity action and the reduction of selective fading.

While a three-quarter-mile short-wave antenna system is an unusually long one, the steerability feature permits the employment of considerably more directivity, afforded by further increasing the length. A system two miles long is believed to be practicable and desirable. It could be expected to perform more consistently better than the three-quarter-mile trial installation, and should yield a signal-to-noise improvement of twelve to thirteen decibels

* Presented before Silver Anniversary Convention of the Institute of Radio Engineers, New York City, May 10, 1937. Published in *Proc. I. R. E.*, July, 1937.

referred to one rhombic antenna. With the object of predicting the performance of larger systems, the performance of the experimental system is examined in great detail and compared with theory.

I. Introduction

FOR more than a decade, point-to-point short-wave radio services have employed directional antennas both in transmitting and receiving. Transmitting antenna directivity results in increased field intensity at the receiving location and receiving antenna directivity discriminates against noise. Both directivities improve the signal-to-noise ratio of a given circuit and permit operation under more adverse transmission conditions. Arrays of simple antennas as well as extensive configurations of long wires have been used to produce these directivities in both the vertical and the horizontal planes.

Antennas in present use on the longer circuits, such as the New York-London telephone facilities, represent about the limit of fixed directivity. Further increase or "sharpening" of the directivity would seriously encroach upon the angular range of directions which are effective in the propagation of waves from transmitter to receiver. The vertical angle range useful in transmitting and receiving short waves is considerable. The horizontal range is appreciable although considerably less than the vertical range. To confine the principal antenna response to only a portion of these ranges penalizes the circuit when that portion is ineffective.

Much experience and considerable statistical data have been obtained which determine this useful range of directions for the New York-London circuits, and antennas have been designed in conformity with these results. However, too much weight must not be given to statistical results which indicate, for instance, that ninety per cent of the time the effective angles are, say, in the range from ten to twenty degrees. For, if the remaining ten per cent includes much of the time that has been lost with existing facilities, an antenna designed for a ten- to twenty-degree response may really be of no value, or even detrimental as a means of extending the usefulness of the circuit. Owing to the great variability in conditions on the north Atlantic path and to the relatively small amount of significant data which has been accumulated during times when gain is most needed it might be detrimental to carry fixed directivity further than present practice has adopted.[1]

[1] One way of attacking the problem of obtaining increased antenna gain has been proposed by John Stone Stone in U. S. Patent 1,954,898. This patent relates to fixed antennas but has certain features, such as delay equalization, in common with the system to be described in this paper.

If, however, the directivity can be varied or "steered" to meet the various conditions imposed by nature, a new field is opened in which a new order of antenna sharpness and gain is possible. In addition to the gain in signal-to-noise ratio afforded by directivity, a reduction in selective fading is possible if the sharpness is increased to the point where a separation of differently delayed waves is achieved. As early as 1927, Edmond Bruce [2, 3] found remarkable reductions in short-wave fading by using a receiving antenna having an extremely sharp directional pattern. The successful employment of sharp directivity is, of course, predicated upon considerable stability of wave directions. The experiments reported by R. K. Potter [4] in 1930 suggested that short waves are propagated in a more or less orderly manner and that stable wave directions might exist. Later experiments,[5] made in co-operation with the British Post Office, using pulse transmission to resolve angles in time, gave confirming data and demonstrated clearly the physical facts upon which is based the system to be described in the present paper. These fundamental facts, outlined in the paper describing the experiments just mentioned, are recapitulated here because a clear understanding of their nature and significance is an essential introduction to the subject in hand. In the pulse tests it was found that:

"1. To the extent that we have been able to resolve the propagation into separate (vertical) angles, the separate angles are found not to be erratic; they vary slowly.

"2. There appears to be at least a qualitative relation between angle and delay; the greater the delay the greater the angle above the horizontal.

"The existence of the many waves of different delay, which is known to make fading selective with respect to frequency, greatly impairs the quality of a short-wave radio telephone circuit. . . . The experimental facts, tentatively established, that individual wave angles are fairly stable and that waves of different delay invariably possess different vertical angles, make this problem hold considerable promise.

"The simple antennas described . . . are suitable for angle determination because of their ability to reject a single wave but they are not

[2] E. Bruce, "Developments in Short-Wave Directive Antennas," *Proc. I. R. E.*, vol. 19, pp. 1406–1433, August, 1931; *Bell Sys. Tech. Jour.*, vol. 10, pp. 656–683, October, 1931.

[3] E. Bruce and A. C. Beck, "Experiments with Directivity Steering for Fading Reduction," *Proc. I. R. E.*, vol. 23, pp. 357–371, April, 1935; *Bell Sys. Tech. Jour.*, vol. 14, pp. 195–210, April, 1935.

[4] R. K. Potter, "Transmission Characteristics of a Short-Wave Telephone Circuit," *Proc. I. R. E.*, vol. 18, pp. 581–648, April, 1930.

[5] Friis, Feldman, and Sharpless, "The Determination of the Direction of Arrival of Short Radio Waves," *Proc. I. R. E.*, vol. 22, pp. 47–78, January, 1934.

in general suitable for quality improvement. For such studies it would be preferable to construct a more elaborate antenna whose directional pattern has a single major lobe which is steerable in the vertical plane. Such an antenna would aim to select a narrow range of angles in which occur waves of substantially the same delay."

The present paper describes a steerable antenna receiving system of the general character suggested by the above quotation, and which has been in experimental operation at the Holmdel, New Jersey, field laboratory of the Bell Telephone Laboratories for the past two years. Certain other important features are incorporated in the system, notably an arrangement whereby individual wave groups arriving at different vertical angles are received separately and, after separate delay equalization, combined, thereby incorporating a unique form of diversity. Another important feature possessed by the system is its frequency range which permits operation on all of the frequencies used in short-wave transatlantic services.

II. Principles of Steering Antenna Directivity

An old and elemental type of steering of receiving antenna directivity is found in direction finders. The steering of a directional lobe as distinguished from the steering of a null has been accomplished in recent years. Schelleng [6] reported a moderate degree of horizontal plane steering, accomplished by means of phase shifters. Jansky [7] has obtained horizontal steering by bodily rotating an entire broadside array. Bruce and Beck [3] obtained vertical steering by varying the shape of a rhombic antenna by means of ropes, and demonstrated the value of steering in the reduction of selective fading. The present authors [5] have employed rotatable phase shifters to steer the nulls in the directional patterns of two spaced antennas. In that work the value of the rapid adjustments possible with phase shifters was very apparent. In the linear end-on MUSA [8] system to be described rotatable phase shifters are again employed to steer the vertical response.[9]

In Fig. 1 is shown a schematic representation of a linear end-on array of N equally spaced unit antennas in free space. The antennas are indicated by the numbered points. For simplicity it is assumed, in the following preliminary analysis, that the antennas are spaced far

[6] J. C. Schelleng, "Some Problems in Short-Wave Telephone Transmission," *Proc. I. R. E.*, vol. 18, pp. 913–938, June, 1930.

[7] K. G. Jansky, "Directional Studies of Atmospherics at High Frequencies," *Proc. I. R. E.*, vol. 20, pp. 1920–1932, December, 1932.

[8] The word MUSA is coined from the initial letters of "multiple unit steerable antenna."

[9] U. S. Patent No. 2,041,600.

enough to be substantially isolated from each other. Choosing antenna No. 1 for reference and considering a plane wave arriving at an angle δ with the axis of the array, it is clear that the output of No. 2 will add in phase with that of No. 1 if the phase advance ϕ is made equal to $2\pi ac/v - 2\pi a \cos \delta$, where c = velocity of light and v = the phase velocity of the transmission lines. Similarly, the output of No. 3 will add to that of No. 1 and No. 2 if its phase is advanced 2ϕ, etc. If the spacing, $a\lambda$, is sufficient there will be other angles for which the N outputs add in phase; at intermediate angles the outputs interfere with the result that zeros and minor maxima occur. By properly designing the unit antenna the undesired maxima may be suppressed.

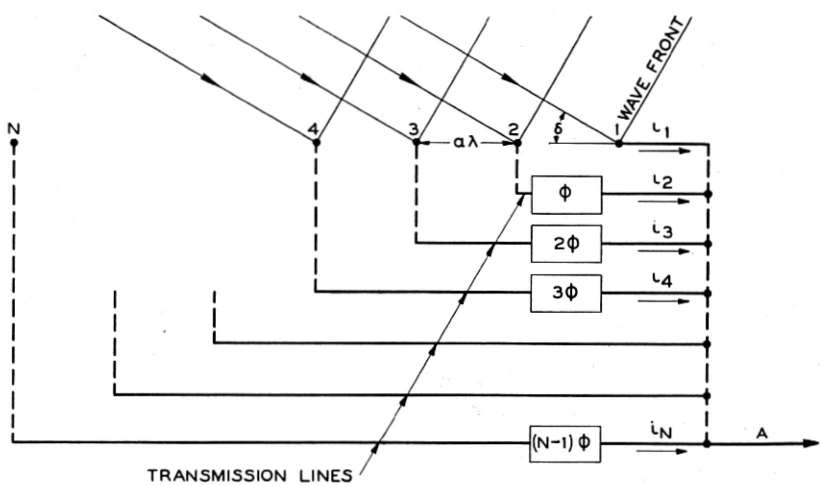


Fig. 1—A steerable antenna array using variable phase shifts ϕ, 2ϕ, 3ϕ, etc. The transmission lines indicated by broken lines are assumed to be of zero length; a is the spacing in free space wave-lengths.

In the Holmdel experimental system the unit antennas are of the rhombic type. An aerial view of the six antennas, which are located on the great circle through England, is shown in Fig. 2. These six antennas, combined as in Fig. 1, yield polar directional patterns such as those shown at the top of Fig. 3. The solid line pattern and the dashed line pattern correspond to different values of the phase shift ϕ. The multiple phase shifts of Fig. 1 are obtained by gearing the phase shifters to a common shaft which enables the directional pattern to be steered simply by rotating the shaft.

Thus far we have discussed the problem of sharp steerable directivity from the point of view of a single plane wave, whereas it is well known that multiple ionosphere reflections usually produce several more or less discrete waves, or bundles of waves, having different vertical angles and different transmission delays. To obtain the maximum advantage, however, requires that all of the several wave bundles be separately received and suitably combined after the transmission delays have been equalized. The achievement of this objective

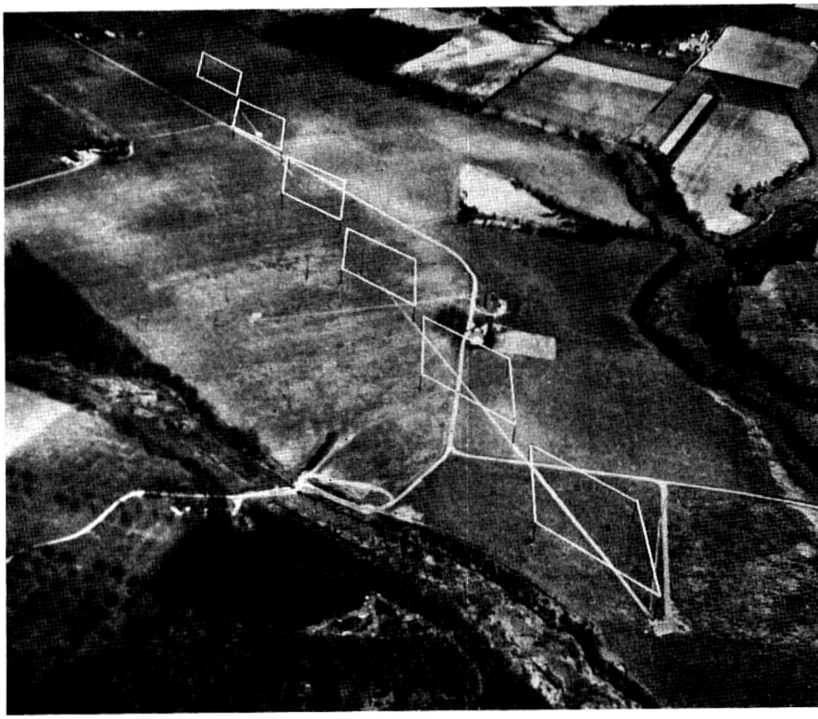


Fig. 2—Airplane view of the three-quarter-mile experimental MUSA on the receiving laboratory site located near Holmdel, New Jersey. The white line beneath the antennas is the newly filled trench in which coaxial transmission lines are buried. The building appearing in the right-hand foreground houses the receiving apparatus. The ground is flat to within ± 4 feet.

not only yields the ultimate gain in signal-to-noise ratio but at the same time *reduces the distortion associated with selective fading.*

The method of obtaining sharp steerable directivity by combining the output of fixed antennas through phase shifters makes it possible to use the same antennas and transmission lines to provide several separately steerable lobes each of which is in effect an independent MUSA.[10] In the experimental system, shown schematically in Fig. 3,

[10] R. K. Potter, U. S. Patent No. 2,030,181.

the antenna outputs are combined at intermediate frequency, and the separately steerable lobes are obtained by branching each of the six first detectors into three phase shifters and combining the outputs of

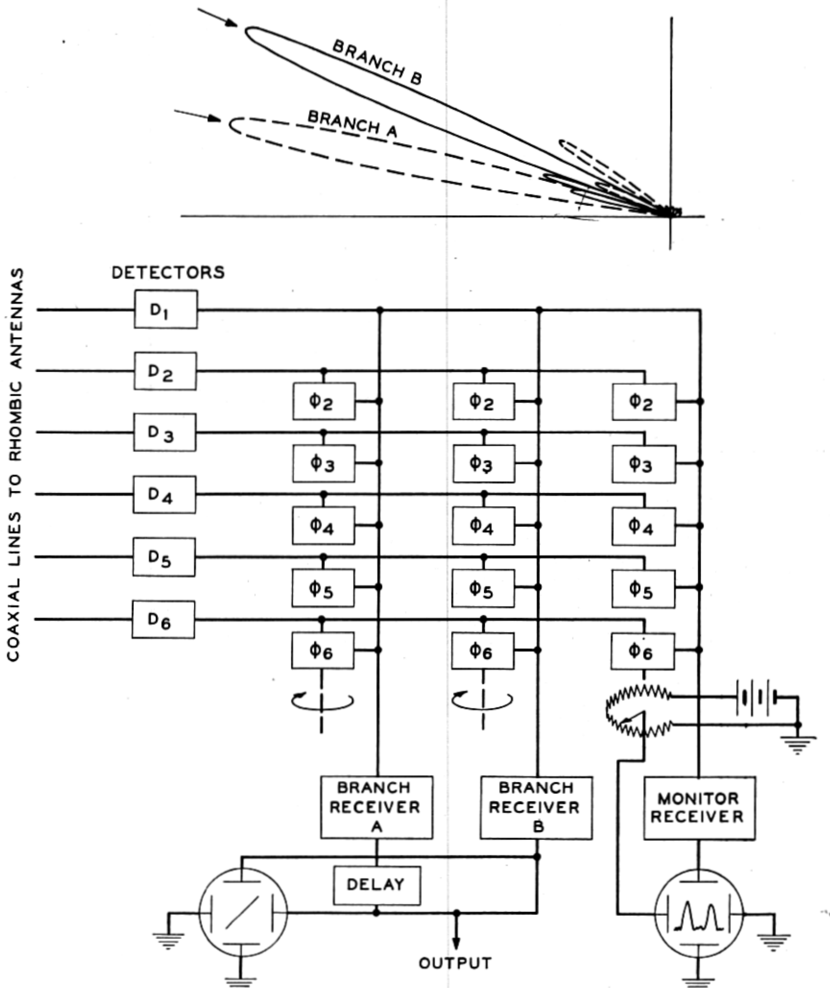


Fig. 3—Schematic diagram of the experimental MUSA receiver. The five phase shifters ϕ_2, ϕ_3, etc., of each branch, are geared to a shaft to provide the phase shifts ϕ, 2ϕ, 3ϕ, etc., of Fig. 1. The inset at the top shows the directional patterns of the two branches when steered at angles of 12 and 23 degrees, at a wave length of 25 meters.

the phase shifters to form three steerable branches. One branch is used continuously to explore the angle range to determine at which angles the waves are arriving. The other two branches are set accordingly and their outputs are "received" by conventional receivers, with common automatic gain control. The demodulated audio outputs are equalized for difference in transmission time and then combined. A cathode-ray oscilloscope displays the output of the exploring or monitoring branch. It plots amplitude (provided by a linear rectifier) as the ordinate, against phase shift ϕ_2 (corresponding to ϕ in Fig. 1). The screen of the oscilloscope is of the retentive type and thus displays several consecutive sweeps at once. A pattern corresponding to two waves is illustrated. The other cathode-ray oscilloscope is used in the adjustment which equalizes the delay of the two waves. Delay is added to the low angle branch until the oscilloscope shows a line (or compact elongated figure) which oscillates between the two axes as the two waves fade differently. This means that all of the audio frequencies of one branch are combining in phase with those of the other.

The above brief description was introduced to acquaint the reader with the essentially simple features of the MUSA system. Before describing the details and the results obtained with the experimental system, a more comprehensive analysis of steering principles will be given.

Returning to Fig. 1, it is assumed, of course, that the transmission lines are terminated in their characteristic impedance at the receiving terminal [11] (the phase shifters of Fig. 1) so that the phase is distributed linearly along the lines. Neglecting line loss (or equalizing it), the N currents, equal in magnitude and different in phase, are

$$\left.\begin{aligned} i_1 &= I\epsilon^{j\omega t} \\ i_2 &= I\epsilon^{j\{\omega t+\phi-2\pi a(\upsilon-\cos\delta)\}} \\ i_3 &= I\epsilon^{j\{\omega t+2[\phi-2\pi a(\upsilon-\cos\delta)]\}} \\ &\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ i_N &= I\epsilon^{j\{\omega t+(N-1)[\phi-2\pi a(\upsilon-\cos\delta)]\}} \end{aligned}\right\} \tag{1}$$

where i = instantaneous current in exponential notation
ω = angular frequency
N = total number of unit antennas
a = spacing in free space wave-lengths
$\upsilon = c/v$ = the ratio of the velocity of light to that of the transmission line.

The sum of the N currents is

$$A = I\epsilon^{j\omega t}\{1 + \epsilon^{j[\phi-2\pi a(\upsilon-\cos\delta)]} + \cdots + \epsilon^{j(N-1)[\phi-2\pi a(\upsilon-\cos\delta)]}\}. \tag{2}$$

[11] Non-characteristic terminations at the receiving ends of the lines are permissible if all terminations are identical and if the antennas are matched to the characteristic line impedance. Conversely, characteristic terminations at the receiving ends suffice if the antenna impedances are merely identical.

This exponential series may be evaluated with the aid of the identity [12]

$$1 + \epsilon^{j\theta} + \epsilon^{j2\theta} + \cdots + \epsilon^{j(n-1)\theta} \equiv \frac{\sin \frac{n\theta}{2}}{\sin \frac{\theta}{2}} \epsilon^{j\frac{(n-1)\theta}{2}}.$$

Using this summation we have

$$A = I \frac{\sin \frac{N}{2} [\phi - 2\pi a(v - \cos \delta)]}{\sin \frac{1}{2} [\phi - 2\pi a(v - \cos \delta)]} \epsilon^{j\{\omega t + (N-1)/2[\phi - 2\pi a(v - \cos \delta)]\}}. \quad (3)$$

The amplitude of A in (3) is the array directional pattern or array factor. It is zero when the numerator alone is zero, i.e., when

$$\frac{1}{2}[\phi - 2\pi a(v - \cos \delta)] \neq 0, \pm \pi, \pm 2\pi \cdots$$

and simultaneously

$$\frac{N}{2}[\phi - 2\pi a(v - \cos \delta)] = 0, \pm \pi, \pm 2\pi \cdots.$$

It attains its maximum value of NI when the denominator and numerator are zero simultaneously, i.e., when

$$\frac{1}{2}[\phi - 2\pi a(v - \cos \delta)] = 0, \pm \pi, \pm 2\pi \cdots$$

and

$$\frac{N}{2}[\phi - 2\pi a(v - \cos \delta)] = 0, \pm \pi, \pm 2\pi \cdots.$$

Plots of (3) for ten unit antennas ($N = 10$) spaced five wave-lengths ($a = 5$) are shown in Fig. 4 for two arbitrary values of ϕ. The same array used at twice the frequency ($N = 10$, $a = 10$) has the directional patterns shown in Fig. 5. The abscissas are labeled earth angle although nothing has been said thus far concerning the disposition of the N antennas with respect to the earth. In order that the simple multiple phase shifts of Fig. 1 shall suffice to steer the array, reflection from the ground must affect the phase of all antenna outputs identically. This is assured by constructing the array over, and parallel to, a flat expanse of ground. Since the angle δ measures the direction of

[12] This identity may be deduced by substituting $\epsilon^{j\theta}$ for r in the well-known formula for the sum of a geometrical progression

$$1 + r + r^2 + r^3 + \cdots + r^{n-1} \equiv \frac{r^n - 1}{r - 1}.$$

the wave referred to the direction of the array axis the array factor represents a surface of revolution. Figures 4 and 5 show merely axial cross sections which, for a horizontal array, may be considered vertical plane patterns.

Equation (3), as well as Figs. 4 and 5, shows that the sharpness of the principal lobe depends upon the total length of the array in wave-

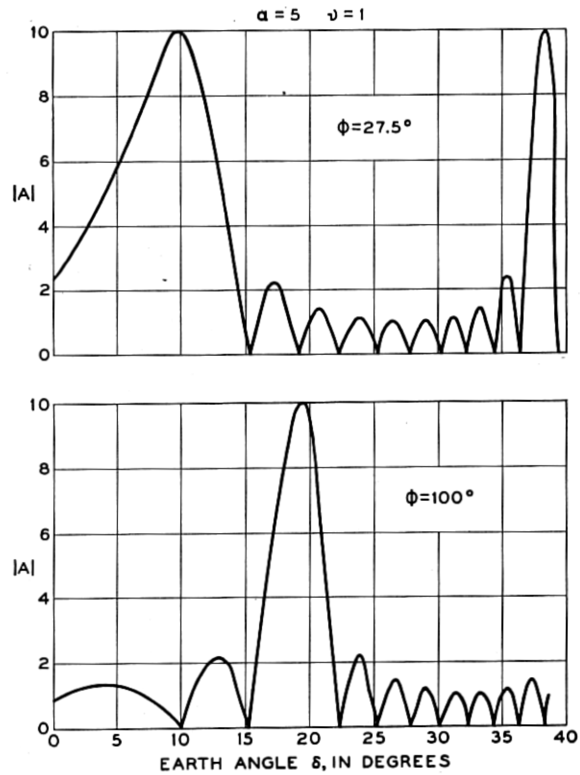


Fig. 4—Plots of the array factor for a 45-wave-length horizontal end-on array.

lengths, i.e., upon Na, while the angular spacing of adjacent principal lobes depends inversely upon the spacing "a." Thus, a single lobed pattern results if the array consists of a large number of closely spaced units.

A single lobed pattern is desirable, but to obtain it by using a large number of unit antennas [13] with separate transmission lines and phase

[13] The reader may observe that the reduction of the spacing would, if carried so far as to make "a" a fraction of a wave-length, violate the assumption that there is negligible reaction or coupling between unit antennas. As stated, this assumption is made in the interest of simplicity. It is theoretically possible to compensate for coupling between antennas so that (1), (2), and (3) still hold.

shifters would be a rather extensive undertaking. Provided a restricted range of steering is permissible, a simpler solution is to employ comparatively few large unit antennas and to let their directional pattern suppress the undesired principal lobes of the array pattern. Useful angles for transatlantic circuits are confined to the range from zero, or some low undetermined limit, to some higher limit. In what follows let δ_m represent an angle a little above the useful range so that a

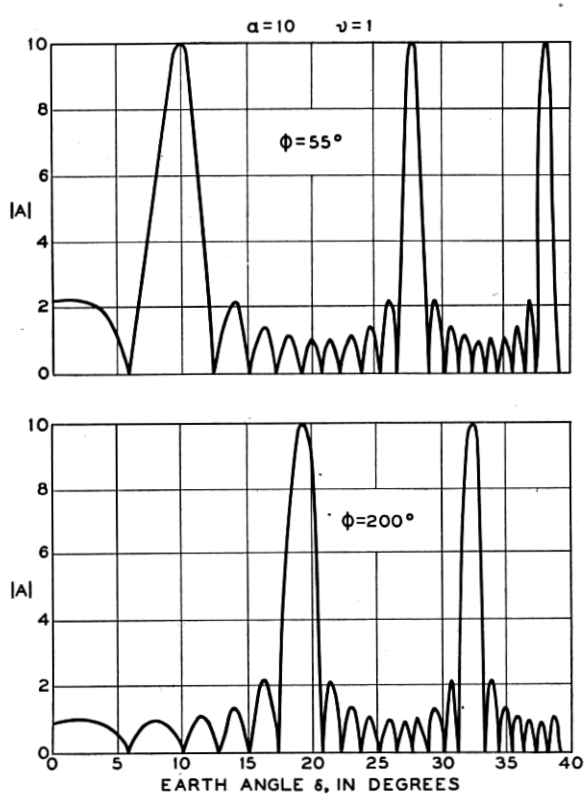


Fig. 5—Plots of the array factor for a 90-wave-length array; that of Fig. 4 used at twice the frequency.

null may be located at δ_m without imposing an excessive loss. The array may then be designed so that when the first principal lobe is steered at zero angle the second falls at δ_m or beyond. The question of whether the array design permits the construction of a suitable unit antenna in the length $a\lambda$ allotted to it is considered in the following paragraph. As a matter of fact, this analysis closely follows the actual steps in the development of the MUSA system.

Turning back to the ideal system comprising a very large number of closely spaced unit antennas, which yields the single lobed pattern, let us divide the antennas into N groups with n antennas in each group. Calling the group spacing "a" and the phase shift between adjacent antennas ϕ the application of (3) gives, dropping the exponential factor,

$$A = \frac{\sin \frac{nN}{2}\left[\phi - 2\pi \frac{a}{n}(\upsilon - \cos \delta)\right]}{\sin \frac{1}{2}\left[\phi - 2\pi \frac{a}{n}(\upsilon - \cos \delta)\right]}. \tag{4}$$

Multiplying numerator and denominator by

$$\sin \frac{n}{2}\left[\phi - 2\pi \frac{a}{n}(\upsilon - \cos \delta)\right]$$

results in

$$A' = \frac{\sin \frac{n}{2}\left[\phi - 2\pi \frac{a}{n}(\upsilon - \cos \delta)\right]}{\sin \frac{1}{2}\left[\phi - 2\pi \frac{a}{n}(\upsilon - \cos \delta)\right]} \times \frac{\sin \frac{N}{2}[n\phi - 2\pi a(\upsilon - \cos \delta)]}{\sin \frac{1}{2}[n\phi - 2\pi a(\upsilon - \cos \delta)]}. \tag{5}$$

Equation (5), which appears as the product of two array factors, is merely another way of writing the array factor for the large number (Nn) of unit antennas. The first factor represents an array of n "sub-unit" antennas of spacing a/n and phase shift ϕ. The second represents the array of these arrays with a spacing of $a\lambda$ and a phase shift $n\phi$. We now proceed to treat these two factors independently and assign the values ϕ_f and ϕ_v to replace ϕ and $n\phi$, respectively. Figure 6 depicts such an array of arrays. If now we regard the array of n sub-units as constituting a fixed unit antenna and adjust it to receive at zero angle (by putting $\phi_f = 2\pi a/n(\upsilon - 1)$) in accordance with the lower limit of the useful range, we obtain

$$A'' = \frac{\sin \frac{n}{2}\left[2\pi \frac{a}{n}(1 - \cos \delta)\right]}{\sin \frac{1}{2}\left[2\pi \frac{a}{n}(1 - \cos \delta)\right]} \times \frac{\sin \frac{N}{2}[\phi_v - 2\pi a(\upsilon - \cos \delta)]}{\sin \frac{1}{2}[\phi_v - 2\pi a(\upsilon - \cos \delta)]}. \tag{6}$$

The first factor in (6) represents the pattern of the unit antenna. It is a relatively broad single lobed pattern with maximum response at

$\delta = 0$. It drops to zero at $\delta = \cos^{-1}(1 - 1/a)$. For higher angles nothing but minor maxima occur since "a" is small and n is large. The second factor represents the steerable array pattern of the N unit antennas. With ϕ_v adjusted for maximum response at zero angle (this makes $\phi_v = n\phi_f$) this system is identical with the array of Nn subunits. In this case, all principal lobes of the array factor for the N units, excepting the first, coincide exactly with nulls of the array factor for the n subunits, and the familiar tapered distribution of minor maxima associated with the array of Nn subunits results. As ϕ_v is varied to steer for other angles than $\delta = 0$, the coincidence of nulls and undesired principal lobes no longer occurs. Since, however, the

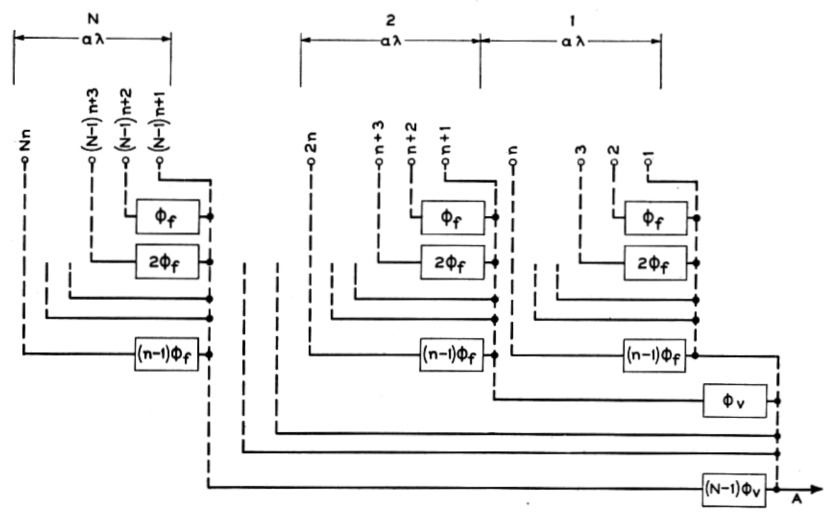


Fig. 6—A steerable array formed by dividing the antennas of Fig. 1 into N groups of n each. The subscripts "f" and "v" refer to fixed and variable phase shifts.

fixed unit antenna has only minor response beyond its first null, those undesired principal lobes are adequately suppressed, and the array may therefore be steered anywhere within the range from $\delta = 0$ to $\delta = \cos^{-1}(1 - 1/a)$, with single lobed response. As the principal lobe is steered away from $\delta = 0$ the maximum amplitude falls off in comparison with that of the array of Nn subunits. This represents a loss of signal-to-noise ratio and is to be regarded as a penalty for compromising to the extent of using fixed arrays as unit antennas. The loss is appreciable, however, only if the array is steered near the upper cutoff angle of the unit antenna. It remains but to select "a" so that $\cos^{-1}(1 - 1/a)$ represents the upper limit of the range, δ_m.

For a fixed physical spacing, "a" varies inversely with the wave-length, which results in an increasing steering range with increasing wave-length. Since the critical angle of reflection from the ionosphere increases with wave-length the upper limit of the range of useful angles can be expected to increase also. By selecting the proper spacing, the steering range and the critical angle can be made to agree satisfactorily. Figure 7 shows a plot of $\delta = \cos^{-1}(1 - 1/a)$ against wave-length for the unit antenna spacing of 200 meters which was

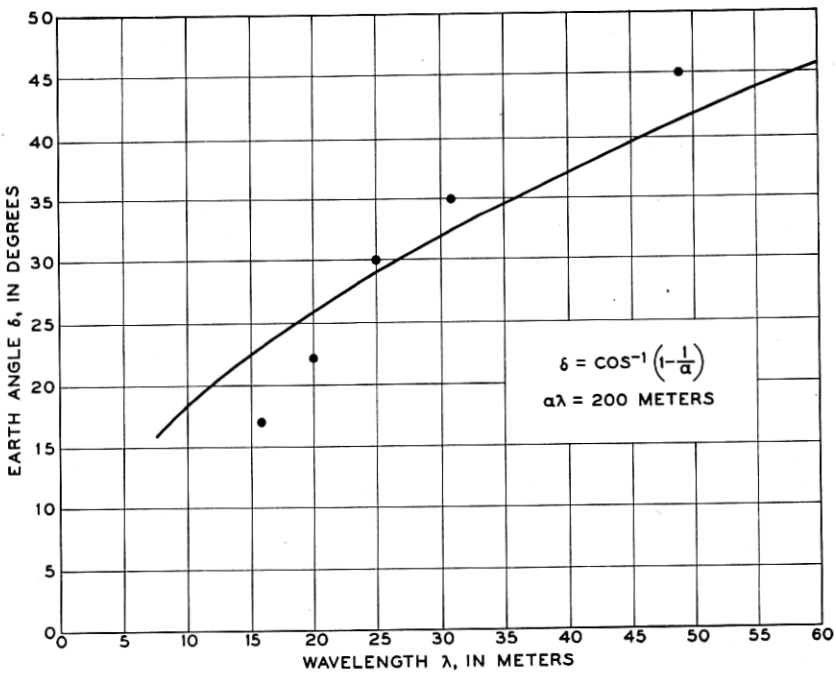


Fig. 7—Measured upper limits of vertical angles as a function of wave-length, compared with the upper limit of the array depicted in Fig. 6. The measured values represent the highest angles observed; usually stronger waves of lower angles predominate.

adopted for the experimental MUSA system to be described. The points denote upper limits of earth angles obtained from measurements made during the years 1933–1936 on signals from Rugby [5] and Daventry, England.

The foregoing analysis shows that

(1) A MUSA system may be so proportioned that the upper limit of its steering range follows, with fair accuracy, the upper limit of the range of useful angles, as the wave-length is varied.

(2) It is theoretically possible to construct a suitable unit antenna in the space provided for it when (1) is satisfied.

III. Description of the Experimental MUSA System

Antennas and Transmission Lines

Any type of unit antenna whose directional pattern suppresses the undesired principal lobes over the required wave-length range is basically suitable for use in a MUSA system. The rhombic antenna [14] does not fulfill this requirement as well as the linear array of subunits discussed in the preceding section. It was, however, selected on account of its advanced state of development. The manner in which it fits into the MUSA array factor will be discussed later.

The coupling or "cross talk" between antennas need not be of negligible magnitude in a MUSA system. For, to a first approximation,

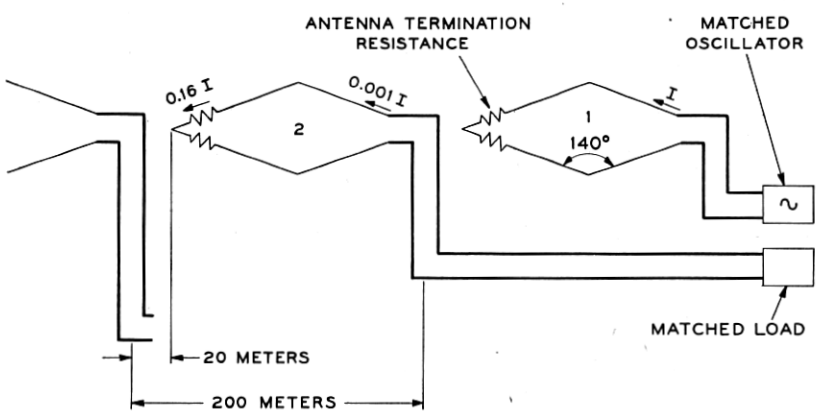


Fig. 8—Measurements of cross talk between adjacent antennas in the MUSA as made from the transmitting point of view.

the coupling is confined to adjacent antennas and is similar for all pairs so that only the *end* antennas could be expected to fail to combine properly with the others. At the ends, "dummy" antennas, not connected with the receiver but terminated like the others, could be erected to supply the coupling necessary to make all antennas alike. Measurements made on the experimental MUSA (Fig. 2) indicated that the cross talk is small enough to be neglected, however, so that dummy antennas ahead of or behind the six regular ones were considered unnecessary. The performance of the system in subsequent tests corroborates this conclusion.

The crosstalk measurements yielded the results indicated on Fig. 8. The small amount of crosstalk current ($0.001I$) measured at the trans-

[14] Bruce, Beck, and Lowry, "Horizontal Rhombic Antennas," *Proc. I. R. E.*, vol. 23, pp. 24–46, January, 1935; *Bell Sys. Tech. Jour.*, January, 1935.

mission line end of the forward antenna (No. 2) and the larger current ($0.16I$) at the other end reflect the fact that the rhombic antenna is "unidirectional." To a first approximation the current in such an aperiodic antenna accumulates progressively towards the output end. Therefore, the "effective" cross talk current is probably less than $(0.16I + 0.001I)/2 = 0.08I$; i.e., the effect upon the field radiated in the principal lobe will be altered by less than ten per cent due to the parasitic excitation of the antenna ahead. Antennas farther ahead as well as those behind contribute relatively nothing.

Since by the reciprocal theorem the directional pattern of any antenna is the same for transmitting and receiving, the crosstalk should likewise result in less than 10 per cent effect in the receiving case.

The measurements of Fig. 8 were made at 18 megacycles. At this frequency the rhombic antennas are proportioned to give maximum radiation approximately end-on. At lower frequencies the crosstalk is probably less.

The coaxial transmission lines are constructed of 60-foot lengths of one-inch copper plumbing pipe spliced with screw type plumbing unions. The inner conductor is one-fourth inch in diameter and is supported by isolantite insulators. The characteristic impedance of the lines is 78 ohms. The lines extend up the poles where they are connected to the antennas through balanced-to-unbalanced matching transformers.[14] At the receiving building the lines terminate on a special jack strip. Nitrogen pressure is maintained in all lines to exclude moisture.

In order to operate the MUSA system it is not essential that the velocity of the transmission lines be known. The velocity must be known accurately, however, in order to determine the angle of the waves as they are selected by the steerable lobe. Accordingly, the velocity was calculated (taking the insulators into account) and also measured. The calculated ratio of the line velocity to the velocity of light is 0.941; measurements yielded 0.933 ± 0.004. Using the value of 0.933, angles less than zero have occasionally been measured. A value of 0.937 would have made the lowest indicated angle just zero.

The longest line is about 1000 meters in length. Its impedance measured at one end when the other end is terminated by a resistance of 78 ohms shows some variation as the frequency is varied. In Fig. 9 are shown the results of impedance measurements made by substituting for the line an equivalent parallel combination of resistance and reactance. The two notable variations occurring at approximately 7.7 and 15.4 megacycles are believed to be caused by a slight irregularity at each joint, which adds a shunt capacitance of the order of 1.8 micro-

microfarads. When spaced regularly at 60-foot intervals these capacitances have a somewhat cumulative effect at frequencies for which 60 feet (18.3 meters) is a multiple of the half wave-length. Sixty feet, when increased by the line velocity ratio, corresponds to 7.7 and 15.4 megacycles. Clearly, line sections which are not short compared with the shortest wave-length should be made unequal so that joint irregularities will not be harmful. The smaller variations of the order of ± 10 ohms may be due to random eccentricities produced by slight buckling of the inner conductor between insulators. With the possible

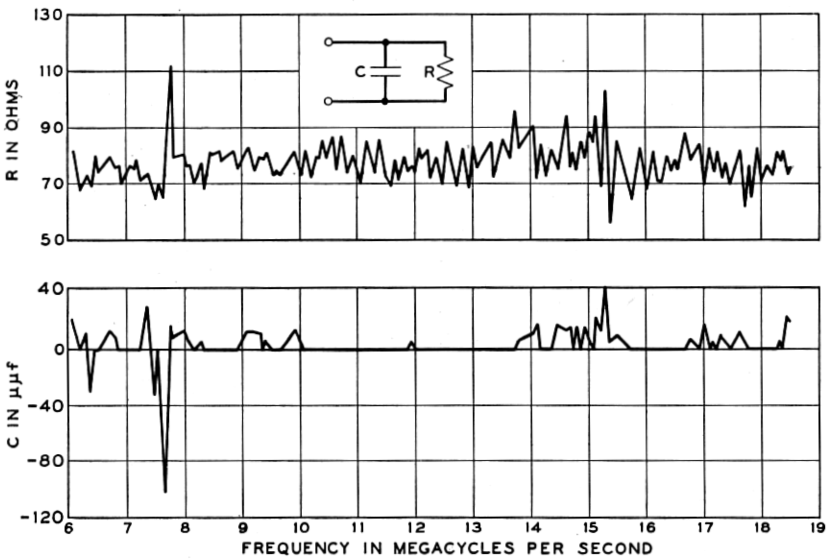


Fig. 9—Impedance measurements made upon the 1000-meter line terminated in a resistance of 78 ohms. The reactance is expressed as shunt capacitance, negative values meaning an inductive reactance numerically equal to the corresponding capacitive reactance.

exception of the two large variations this line is sufficiently smooth for use in a MUSA, as both theory and subsequent experience indicate.

Input Circuit and First Detectors

The MUSA system imposes requirements upon the input circuits and detectors which do not apply to conventional receivers. These requirements are as follows:

(1) The circuits must suppress standing waves on the transmission lines.[15]

[15] This requirement was more easily met than the alternate requirement mentioned in footnote (11).

(2) The phase shift from the transmission line to the phase shifter stage must be alike in all six circuits, independent of wave-length.

In order to simplify the experimental job it was decided to dispense with the selectivity afforded by high-frequency amplifiers and to use the simple circuits shown in Fig. 10. The capacitive coupling to the transmission line is a convenient means of matching the low-impedance lines to the high-impedance circuits. Plug-in coils (L) are used to cover the range from 4.5 to 22 megacycles.

The first detectors are of the two-tube balanced type which suppresses interference from two signals differing by the intermediate

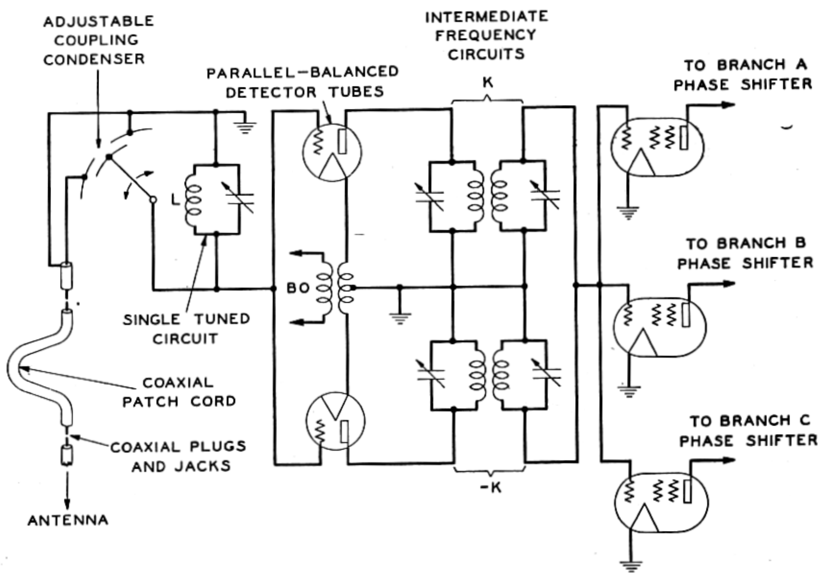


Fig. 10—Input circuit, first detector, and first intermediate-frequency tubes.

frequency and isolates the beating oscillator supply from the input circuits. The latter prevents crosstalk between the six inputs, and assures independence in the tuning of the input circuits. The beating oscillator voltage is introduced, at low impedance, between cathodes [16] by means of the distributing system of equal length coaxial lines shown in Fig. 11. This distributing system gives equiphase beating oscillator inputs to all detectors and makes requirement (2) attainable by having nominal similarity in the remaining parts of the six circuits.

Requirement (1) is met by feeding a test oscillator of 78 ohms impedance into the first circuit jack and adjusting the tuning condenser

[16] W. A. Harris, "Superheterodyne Frequency Conversion Systems," *Proc. I. R. E.*, vol. 22, pp. 279–294, April, 1935.

and the coupling condenser (Fig. 10) alternately until the maximum signal voltage appears on an indicating meter in one of the three intermediate-frequency branches. The three-terminal coupling condenser is an aid in this procedure since varying the coupling imposes only a slight variation in the capacitance across the coil. When the indicating instrument is a square-law vacuum tube voltmeter with the main

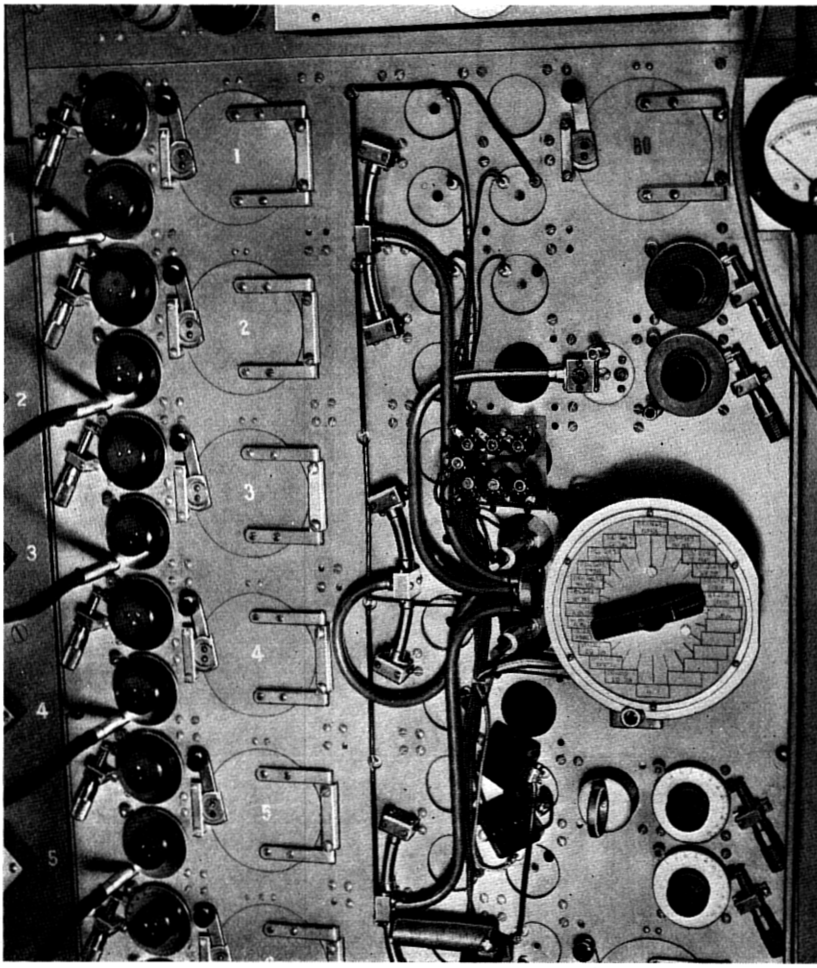


Fig. 11—Close-up view of high-frequency panel with cover removed. The beating oscillator supply line originates in the upper right-hand corner. It supplies the six detectors with equiphase and equiamplitude voltages. Plug-in coils fit into the compartments covered by the six circular doors. Micrometer heads which are used to adjust the six tuning condensers appear. The coaxial patch cords appear at the extreme left.

current balanced out and the remainder indicated by a 30-microampere meter, the sensitivity is more than sufficient to tune the circuits correctly.

The criterion of correct tune is the degree of suppression of standing waves on the transmission lines. To determine whether or not the maximizing adjustment insures an adequate standing wave suppression, a standing wave detector was incorporated in the experimental design. This is shown in Fig. 12. It consists of about 16 meters of 78-ohm coaxial line arranged in a coil and terminated by the

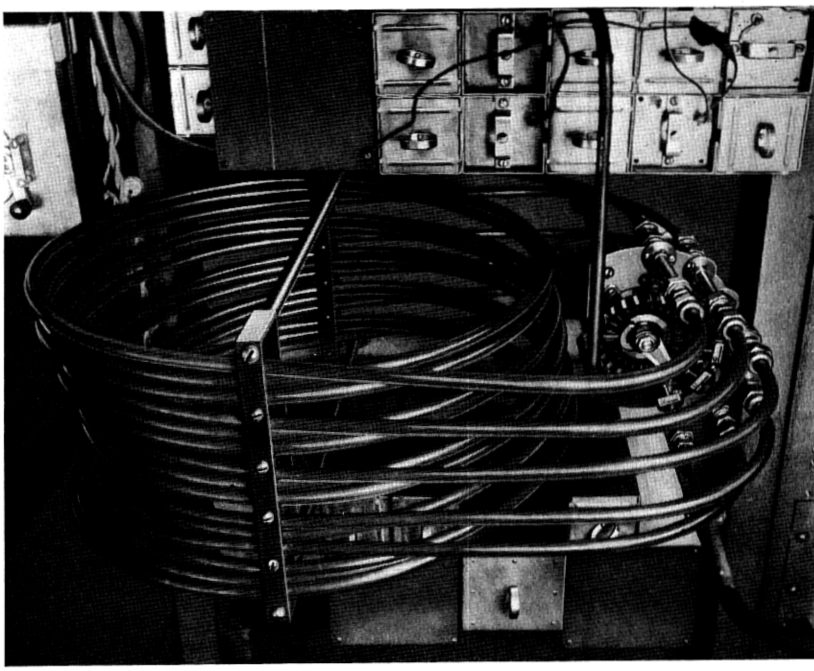


Fig. 12—The standing wave detector comprising 50 feet of 3/8-inch coaxial line, which may be used to test the correctness of the input circuit adjustment.

first circuit to be tested. It is fed at the other end by a test oscillator. Six capacitively coupled taps are brought to the low-capacitance switch shown in the photograph. The selector arm connects the taps to an auxiliary receiver with a high-input impedance. The absence of standing waves is shown by equal readings at the six positions. It was found that the maximizing adjustment results in a standing wave with less than ten per cent total variation, which represents nearly as much suppression as the smoothness of the line allows.

With nominally correct resistance termination standing waves of five per cent usually occur. For standing waves not exceeding ten per cent the accompanying phase distribution along the line does not depart more than a few degrees from the desired linear distribution. The use of the standing wave detector in routine operation was therefore not required.

Phase Shifters

Of the numerous methods of shifting phase the method [17] illustrated in Fig. 13 is the one chosen for the 18 circuits (3 branches, 6 antennas) of the experimental MUSA. Here points *a*, *b*, *d*, and *c* have voltages

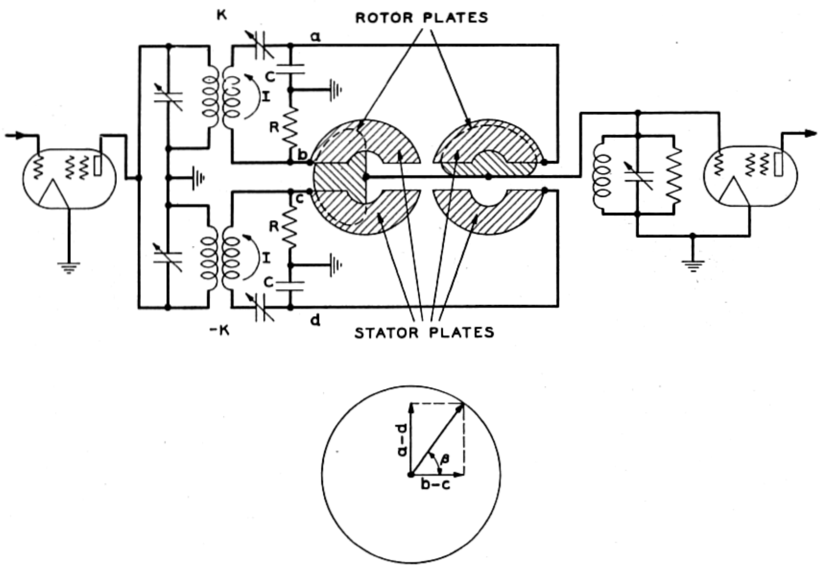


Fig. 13—Circuit diagram and vector diagram of the phase shifter. The rotor plates are especially designed to give a phase shift proportional to shaft angle.

to ground 90 degrees apart. The potential of point b is IR; that of c is $-IR$; that of a is $jI/\omega C$; that of d is $-jI/\omega C$. The resistance R and reactance $1/\omega C$ are made equal at the mid-band frequency so that four equal voltages, distributed equally over 360 degrees of phase, appear on the four stators of the special condenser. A photograph of this condenser appears in Fig. 14. Two specially shaped eccentric rotors mounted in quadrature to each other on the same shaft comprise the output terminal. It will be noted that voltages of opposite phase are connected to adjacent stators. Thus, with the rotors in

[17] L. A. Meacham, U. S. Patent No. 2,004,613.

the position shown dotted in Fig. 13 the output comes from point a since d is not coupled and b and c cancel each other. By shaping the two rotors so that the difference in exposure to opposite stator plates is proportional, respectively, to the sine and cosine of the angle of shaft rotation, the total current flowing from the two rotors will be constant and of phase proportional to the shaft angle. This is illustrated by the vector diagram in Fig. 13 in which β is the shaft angle and vectors $a - d$ and $b - c$ are the quadrature rotor outputs proportional to $\sin \beta$ and $\cos \beta$.

These phase shifters vary in output by less than $\pm$ 5 per cent as the shaft is rotated. The departure from linearity of phase shift is correspondingly small; i.e., less than $\pm$ 5 degrees.

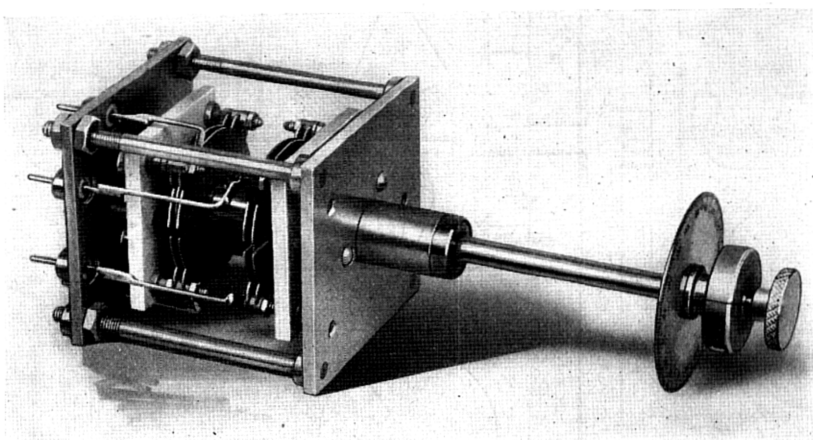


Fig. 14—The phase shifting condenser.

The useful band width of this type of phase shifter is fundamentally limited by the fact that $1/\omega C$ varies with frequency while R does not. However, this limitation does not appear in the Holmdel MUSA in which the percentage band width is small because the phase shifters operate at the intermediate frequency of 396 kilocycles.

The phase shifters are connected to the steering shaft with helical gears of multiple ratios as shown in Fig. 15. The phase shifter shafts may be slipped with respect to the main shaft. After they have been aligned so that locally supplied equiphase inputs to all detectors add in phase at the point where the phase shifter outputs are combined they are locked. This adjustment is independent of signal frequency. Provision is made for adjusting the gain of each of the six phase shifter circuits so that the differences in transmission-line loss may be compensated and any other desired amplitude adjustments made. The photograph of Fig. 15 shows the monitoring or exploring branch whose steering shaft is motor driven at one revolution per second.

Before leaving the subject of phase shifting it may be well to distinguish between phase shift and delay as here used. All electrical net-

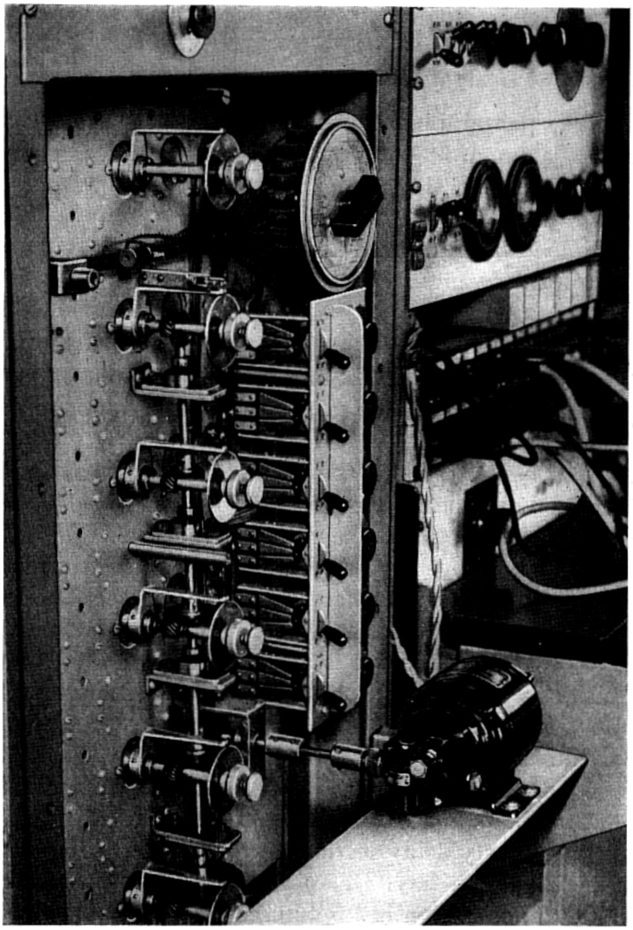


Fig. 15—Phase shifting panel of the monitoring branch. Only five of the six phase shifters are rotated for steering purposes. They are geared to the steering shaft in ratios of 1 : 1, 1 : 2, 1 : 3, 1 : 4, and 1 : 5.

works, except for certain highly distortive ones, possess a phase-frequency characteristic which is such that higher frequencies have their phases retarded with respect to lower frequencies. The ratio of the ncrement of phase retardation to the increment of frequency, i.e., the slope of the phase characteristic, is the delay. It is sometimes called the group delay or group transmission time as distinguished from the "phase time." [18] The delay is the only time which can be measured. It does not determine the phase shift of a particular frequency nor is it determined by the phase shift. A phase shifter applied to the network merely moves the phase curve intact up or down on the phase axis.

General Description of the System

The preceding paragraphs have described features which distinguish the MUSA system from conventional receiving systems. There remain to describe several auxiliary features and to present a unified picture of the whole.

The experimental system was designed for double side-band reception and all of the results reported in this paper refer to double side band. There has recently been completed equipment which may be substituted for the double side-band equipment for the reception of reduced carrier single side-band signals. The new equipment may also be used to select, with crystal filters, one side band of double side-band signals.

The delay to be inserted in the low-angle branch as indicated in Fig. 3 is obtained electrically from an audio-frequency delay network. The delay could theoretically be provided at the intermediate frequency but no advantage would result. The audio-frequency delay network is a special artificial line composed of forty sections and terminated by its characteristic impedance. Each section has a delay of 68 microseconds. A special switch is arranged to tap a high impedance output circuit across any desired section, thus providing a delay of 2.7 milliseconds variable in 0.068-millisecond steps. A special equalizing network [19] which makes the transmission loss the same for all steps and which also equalizes the frequency-loss characteristic so that the response is flat to 5000 cycles for all steps is automatically controlled by this switch. The forty delay sections appear in Fig. 16 just under the shelf on the right-hand bay. The maximum delay which has been required in actual operation is 2.5 milliseconds.

Both linear rectifiers and square-law detectors are provided for final demodulation and either may be switched into service as desired. The

[18] This distinction is brought out by J. C. Schelleng in a "Note on the Determination of the Ionization of the Upper Atmosphere," *Proc. I. R. E.*, vol. 16, pp. 1471–1476, November, 1928.

A general discussion of delay distortion (phase distortion) is to be found in three papers appearing in the *Bell Sys. Tech. Jour.*, vol. 9, July, 1930.

[19] This network and the delay sections were designed by P. H. Richardson of Bell Telephone Laboratories, Inc.

automatic gain control for use with either demodulator is obtained from linear rectifiers but a different diversity connection is made for each type of demodulator, in the interest of output volume constancy. A choice of time constants of 0.06, 0.5, and 4 seconds is provided.

Keys are provided, the ganged manipulation of which makes it possible, among other things, to compare (1) the MUSA output versus any one of the six antennas connected to one branch receiver, and (2) any pair of antennas in ordinary diversity using both branch receivers, versus one antenna using one receiver.

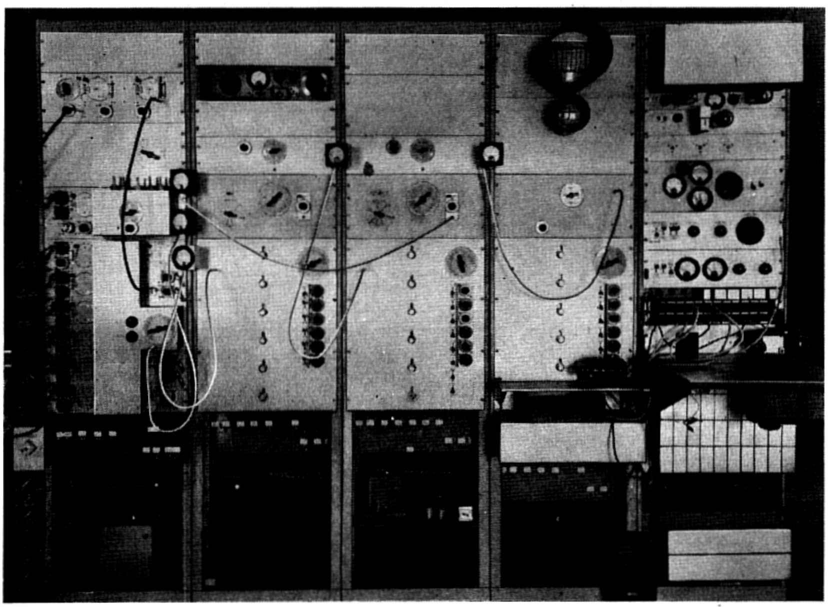


Fig. 16—Front view of the MUSA receiving equipment. The high-frequency bay is at the left and the audio-frequency bay at the right. The branch receivers are the panels directly above the phase shifting panels. The pulse receivers appear above these. At the top of the bay containing the monitoring branch equipment are the two oscilloscopes referred to in Fig. 3. The large tube with the ruled face is the monitoring oscilloscope.

In addition to the regular branch receivers with a 12-kilocycle band width and the monitoring branch receiver with a 2.5-kilocycle band width, two other receivers are provided in the experimental system. These receivers have a 30-kilocycle band width and are used for pulse reception. They are bridged across the inputs of the two regular branch receivers and are connected to a cathode-ray oscilloscope through a commutator.[5]

Various photographs of the MUSA receiver appear with explanatory captions in Figs. 16, 17, and 18.

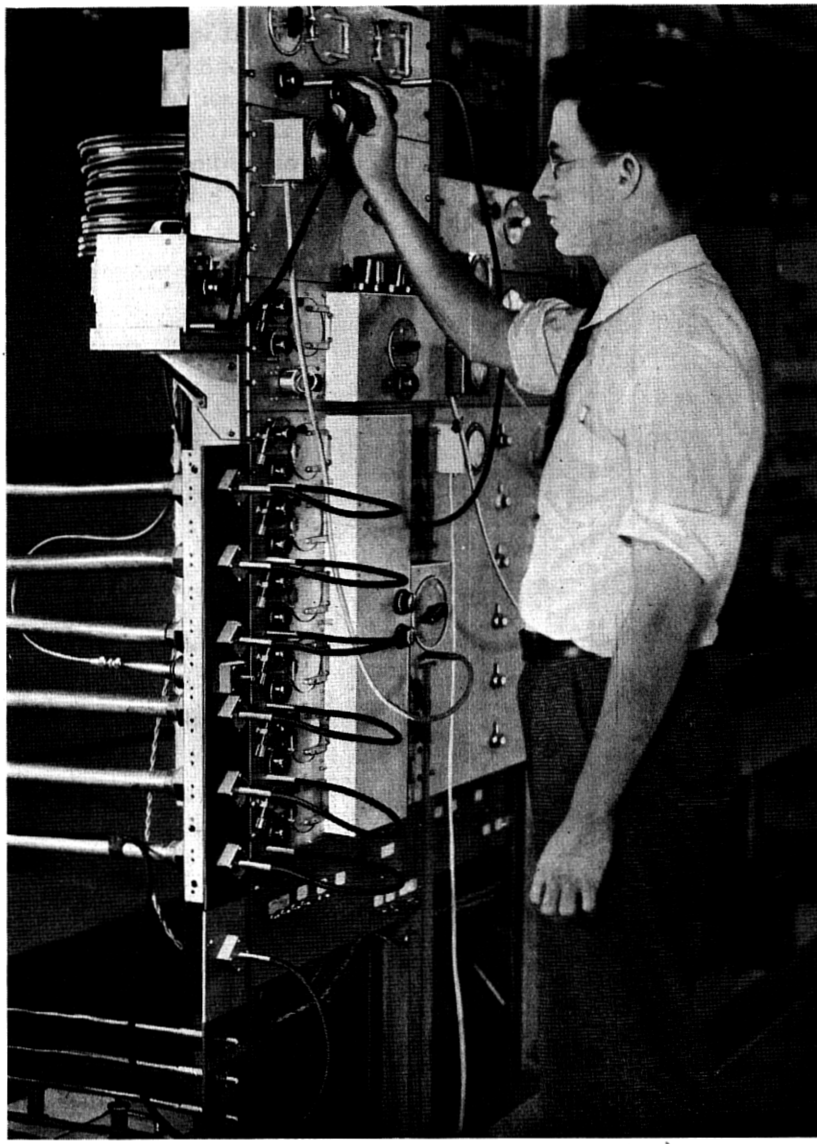


Fig. 17—View showing the six transmission lines and coaxial patch cords. The beating oscillator is mounted upon the shelf and is connected to the power amplifier (which is being adjusted by Mr. Edwards) at the top of the bay.

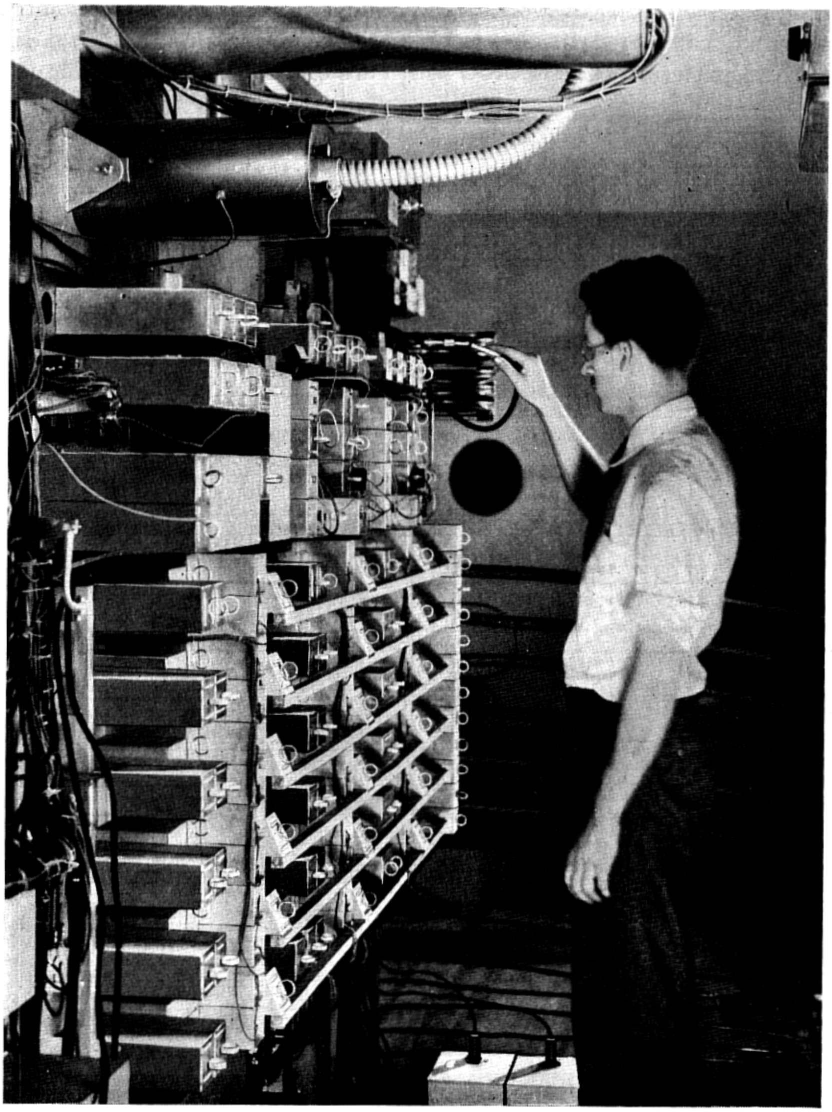


Fig. 18—Rear view of the receiving equipment. The six detector outputs feed the three branches via the square transmission lines.

A family of calculated directional patterns of the experimental MUSA is shown in Figs. 19 and 20. At the top of each column is shown the principal lobe of the vertical directional pattern of the unit rhombic antenna, calculated in the median plane. Beneath are shown six vertical patterns of the MUSA, which are obtained by multiplying the

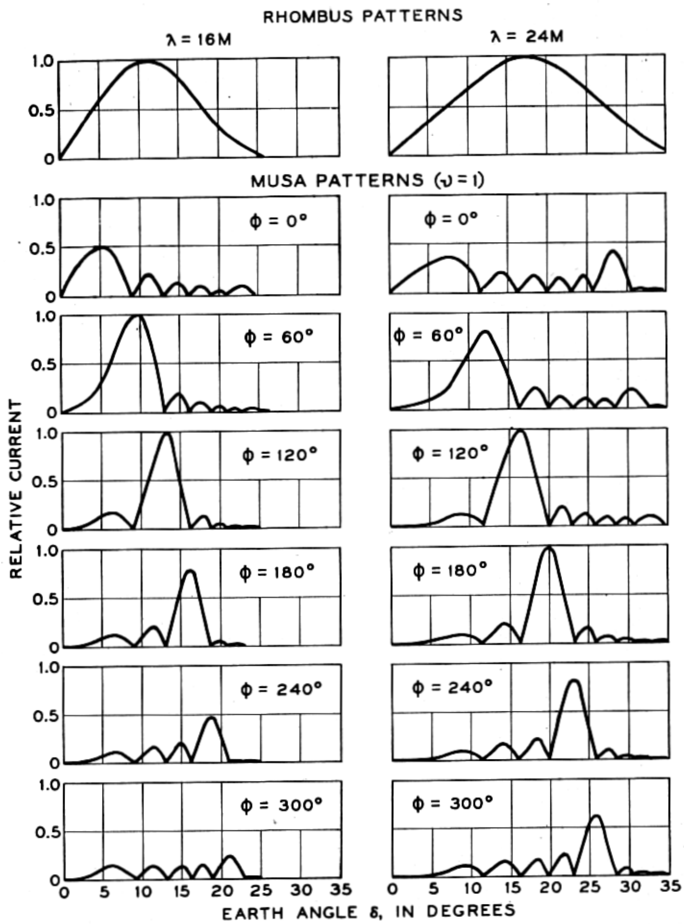


Fig. 19—Vertical directional patterns of the experimental MUSA.

array factor[20] by the unit antenna pattern. The upper pattern corresponds to phasing for zero angle. The remaining ones are plotted for increments of 60 degrees of phase.

These patterns fall short of the "ideal," which the reader may have visualized while reading Section II, in two ways. First, the unit an-

[20] Calculated from (3) putting $v = 1$.

tenna does not suppress the second lobe of the array factor as well as could be desired. By design, it does so for the short waves but inherently fails to do so for the longer waves. Second, the principal lobe of the unit antenna shifts bodily towards higher angles with increasing wave-length, whereas it is desirable to have only the upper cutoff move

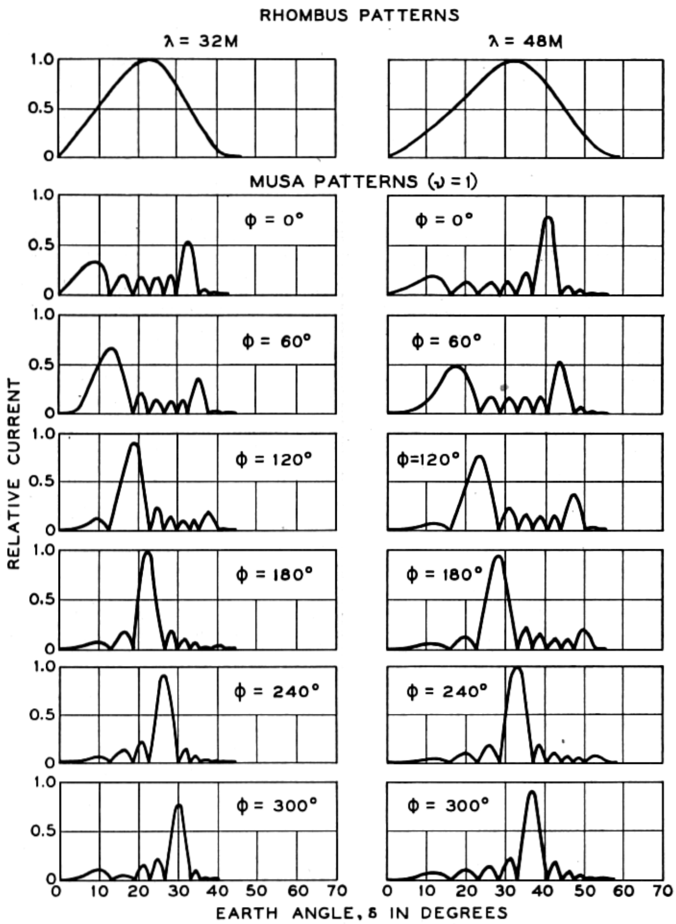


Fig. 20—Vertical directional patterns of the experimental MUSA. Note that the angle scale is half of that in Fig. 19.

upward while retaining low-angle response. Both of these shortcomings restrict the steering range through which the MUSA is essentially single lobed. The upward shift with wave-length of the unit antenna response is in fair agreement with the way in which the mean angle of arrival has been found to vary with wave-length and is, on that account, not altogether objectionable. The Holmdel MUSA employing such unit antennas represents, however, a considerable departure from present antennas of fixed directivity designed from statistical data, and approaches the ideal MUSA steerable over the entire useful angle range.

The curves as plotted assume that the differences in transmission line loss for the various line-lengths have been equalized in the intermediate-frequency circuits. By slightly tapering the amplitudes so that the antennas in the middle of the array contribute more than those near the ends a reduction of the minor lobes has been obtained at the cost of slightly widening the principal lobe. As a result of this, the directional discrimination of the experimental MUSA has been improved. All data and photographic records reported in this paper, however, were obtained before this improvement was introduced.

IV. Tests and General Evaluation [21]

Tests and Experience

Numerous experiments and tests had been carried out on the various parts of the MUSA system before it was first tuned to a transatlantic signal. Despite the fact that all tests concurred in predicting that the system would perform as designed, it was with considerable gratification that a pattern was observed on the monitoring oscilloscope, during one of the early trials, which was almost exactly as calculated for a single wave. Patterns corresponding to two or more waves in various degrees of resolution were observed from time to time. To increase the angle resolution, for test purposes, pulses were transmitted by the British Post Office on several occasions. Turning the steering shaft during these tests clearly showed the principal lobe sweeping through the angle range. When fairly discrete pulses were received the minor lobes could be readily identified. In Fig. 21 is shown a sample of motion picture oscillograms of pulse reception. Two principal waves or, more accurately, wave bundles occurred and were separated by the two MUSA branches as shown. For details of the pulse technique employed in these tests the reader is referred to a previous publication.[5]

Before exhibiting sample motion pictures of typical patterns displayed by the angle monitoring oscilloscope and the delay indicator oscilloscope, further discussion of the former is desirable. The photographs of Fig. 22 show the monitoring oscilloscope pattern with a locally produced equiphase, equiamplitude input supplied to each

[21] The theory and test results of the signal-to-noise advantage are considered together in Part V.

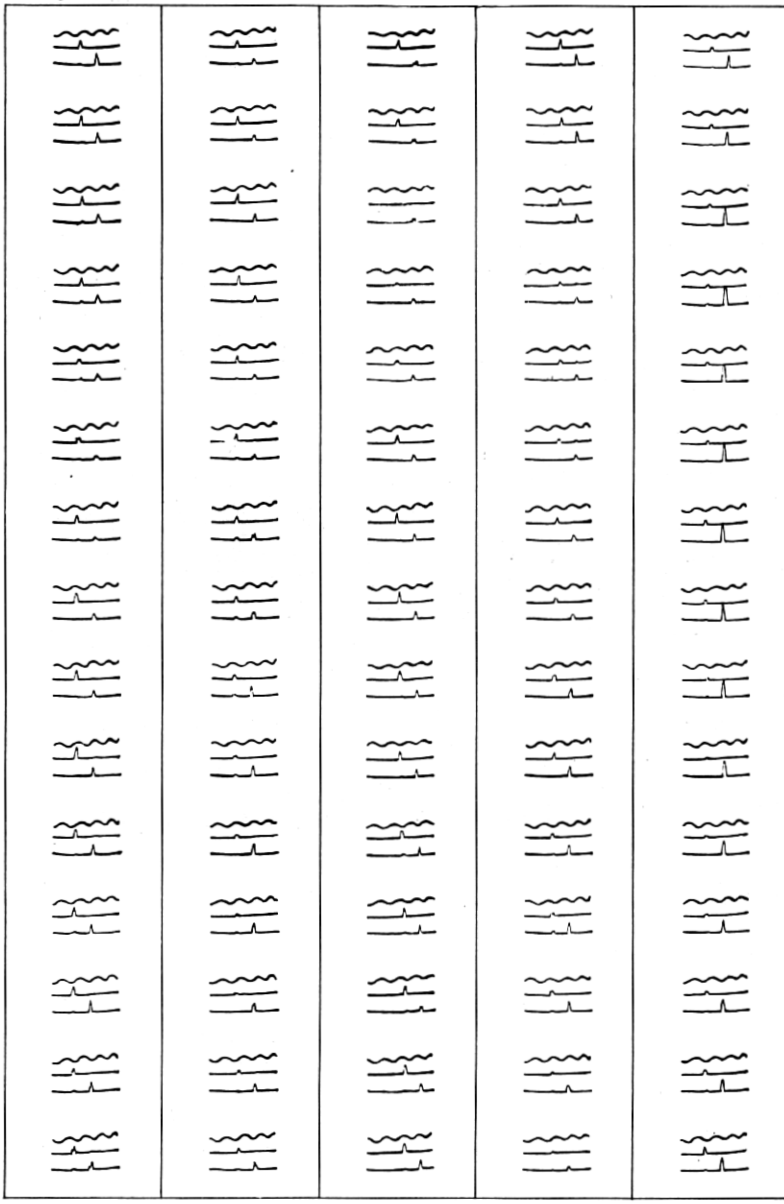


Fig. 21—This retouched plate shows pulses received with MUSA. On each frame time advances from left to right and is measured by the thousand-cycle timing wave. The center trace shows the output of one MUSA branch steered at 25 degrees. The bottom trace shows the output of the second MUSA branch steered at 32 degrees. Wide band amplifiers for pulse reception are bridged in parallel with the speech band intermediate-frequency amplifiers. The transmitted pulses are about 200 microseconds long. GCS (9020 kilocycles) Rugby, February 25, 1936, at 4:11 P.M., E.S.T.

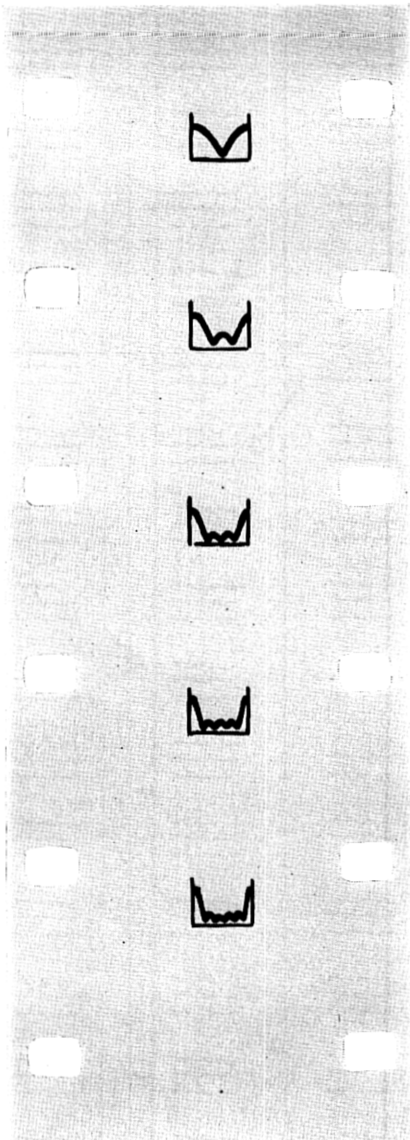


Fig. 22—These five frames show the angle monitoring pattern when a local signal is used to simulate a wave. The bottom frame shows the ideal MUSA pattern for one wave. The remaining frames show the effect of reducing the number of antennas from six (1-2-3-4-5-6) to five (2-3-4-5-6) to four (3-4-5-6) to three (4-5-6) to two (5-6). These films were taken before the amplitude tapering was introduced. Tapered amplitudes reduce the minor lobes to about half of the amplitudes shown.

detector. This figure illustrates the manner in which the pattern is built from the six components after the manner of a Fourier synthesis. The vertical and horizontal axes visible on the monitoring oscilloscope in Fig. 16, but which do not appear naturally on the photographs, were drawn in Fig. 22. As mentioned previously, the oscilloscope sweep axis represents one revolution of the "fundamental" phase shifter so that the beginning and end of the sweep represent the same condition. The ends of the sweep are arbitrarily fixed to represent zero (or 360) degrees of phase shift referred to the output of the first antenna, whose phase is not varied. Consequently equiphase inputs result in a principal lobe half of which appears at each end. This would correspond to a wave of zero angle if the velocity of the transmission lines was equal to that of light. For a lesser velocity, zero angle may occur at any point on the phase axis, depending upon the wave-length. (See Fig. 28 for a sample angle calibration curve.) The principal lobe as well as the four minor lobes of the monitoring oscilloscope represents the output from one wave as the MUSA is steered through its entire range. The oscilloscope pattern, unlike the directional pattern, does not appear sharper for short wave-lengths than for long wave-lengths; the principal lobe is always 120 degrees wide and the minor ones 60 degrees wide on the phase axis. One degree of phase difference, however, represents a difference in steering angle which depends upon the wave-length and the earth angle.

The samples of motion picture film shown in Fig. 23 represent fairly typical "two-path" patterns. The camera was focused to include both oscilloscopes and was manipulated by means of a special step-by-step crank. The operator endeavored to expose each frame during one sweep of the monitoring tube. The delay indicator tube shows a continuous pattern produced by the audio frequencies. A correct delay setting is indicated by a straight line. Here, with the two branches steered at the indicated angles of 8.5 and 20.5 degrees, a delay of 950 microseconds was required to produce the straight line. The diversity action is apparent in the tilting of this line. When the low angle wave, which corresponds to the left-hand peak on the monitoring tube, is predominant the delay indicator line becomes horizontal and, conversely, when the high angle wave is predominant the line approaches the vertical axis. Automatic gain control is used on the branch receivers supplying the speech outputs but is not used on the monitoring branch.

Figure 24 shows, in samples 1 and 2, reception of two waves which are just separable by the directivity present in the Holmdel MUSA. The angles are 15 and 22 degrees and the wave-length is 31.6 meters. The

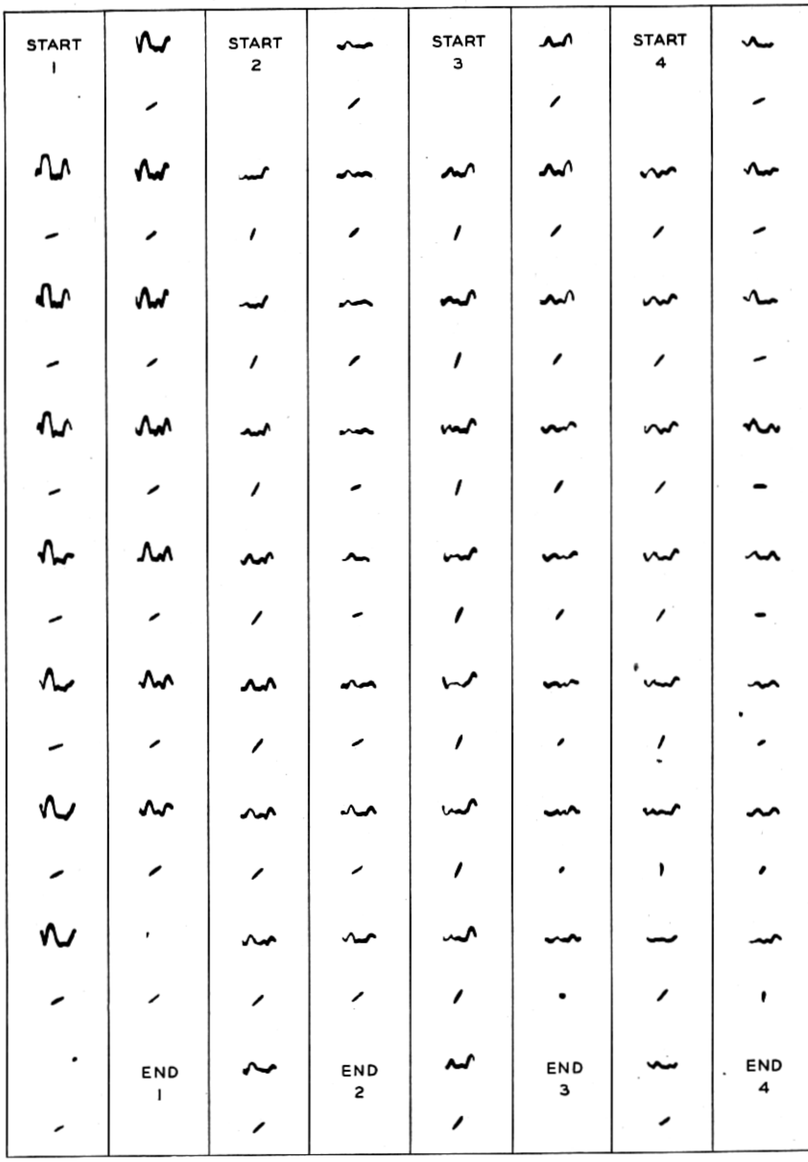


Fig. 23—Pictures of the angle monitoring oscilloscope and the delay indicator tube. The right-hand end of the sweep of the monitoring tube represents zero phase. The indicated angles are 8.5 and 20.5 degrees. The MUSA branches were set at these angles ($\phi_A = 240°$, $\phi_B = 30°$) using 950 microseconds delay. GSE (11,860 kilocycles) Daventry, February 21, 1936, 11:05 A.M., E.S.T. Musical program.

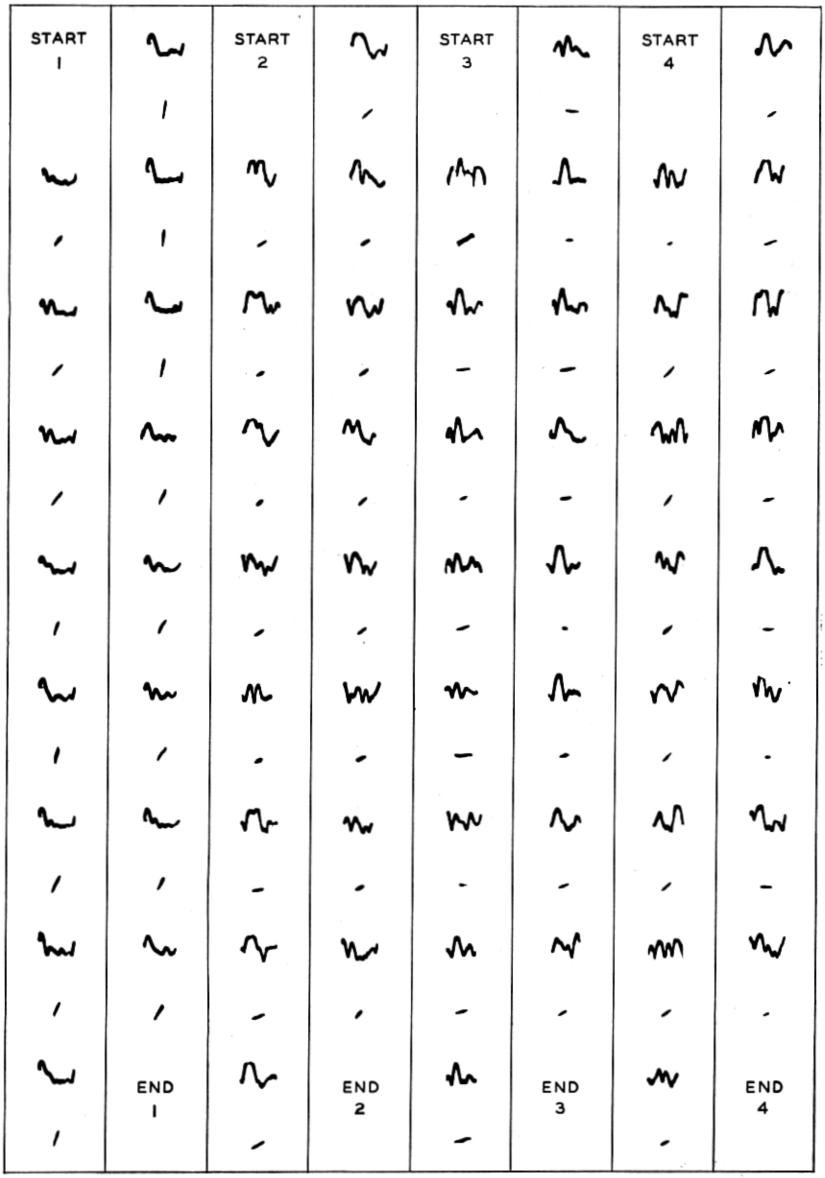


Fig. 24—Pictures of the angle monitoring oscilloscope and the delay indicator tube. In films 1 and 2 the indicated angles are 15 and 22 degrees. The MUSA branches were set at these angles ($\phi_A = 240°$, $\phi_B = 340°$) and a delay of 400 microseconds was used. The time was 3:20 P.M., E.S.T. Films 3 and 4 were taken at 3:15 when a third wave of 26 degrees was present. One branch was steered at this wave ($\phi_B = 50°$); the other at 16 degrees ($\phi_A = 250°$). A delay of 1000 microseconds was required. GSB (9510 kilocycles) Daventry, March 10, 1936.

delay used is 400 microseconds. In samples 3 and 4 a third wave of 26 degrees is present. One branch was steered at this wave; the other was steered at the 15-degree wave and a delay of 1000 microseconds was used.

It is of interest to compare these samples showing the manner in which the MUSA branch outputs combine, with the samples in Fig. 25 which were obtained with a two-antenna space diversity setup. Six antennas were retained in the monitoring branch but five were cut out of each receiving branch, leaving one antenna to supply each branch. In samples 1 and 2, antennas 1 and 6 (1000 meters apart) were retained. In samples 3 and 4, adjacent antennas (Nos. 1 and 2) 200 meters apart were used. These records were obtained about 15 minutes later than those of Fig. 23 and show the same two waves at 8.5 and 20.5 degrees. No delay was used. Note that the outputs combine in phase only when one wave predominates. Inserting delay in either branch is, of course, not effective in improving the audio combination. To do so would impair the addition when one wave is predominant and would not be beneficial when both waves are comparable.

Figure 26 shows, in samples 1, 2, 3, and 4, how the delay indicator tube pattern is affected by the delay adjustment. The two branches were steered at the same angle, thus making both branch outputs identical so that perfect delay adjustment occurs with zero delay. This is the condition depicted in sample 1. In samples 2, 3, and 4 the delays are 340, 680, and 2700 microseconds, respectively.

A number of tests were carried out with the cooperation of the British Post Office in which twelve tones were transmitted. These tones were nonharmonically related. They were separated at the output of the receiver by means of filters, and commutated to appear successively on an oscilloscope. The reader is referred to a paper [4] by R. K. Potter describing this technique. Figure 27 shows a sample of motion pictures made of the oscilloscope patterns. Two receiving systems are compared; the right-hand pattern shows the output of the MUSA while the left shows the output of a conventional receiver connected to a horizontal half-wave antenna. The tones trace the horizontal lines in sequence from top (425 cycles) to bottom (2125 cycles). After one pattern is executed the commutator switches from one receiver to the other. The twelfth tone is omitted to provide time for the switching. The complete double pattern is traced in about one-sixth of a second and the camera is operated at a speed which exposes each frame a little longer than one-sixth of a second.

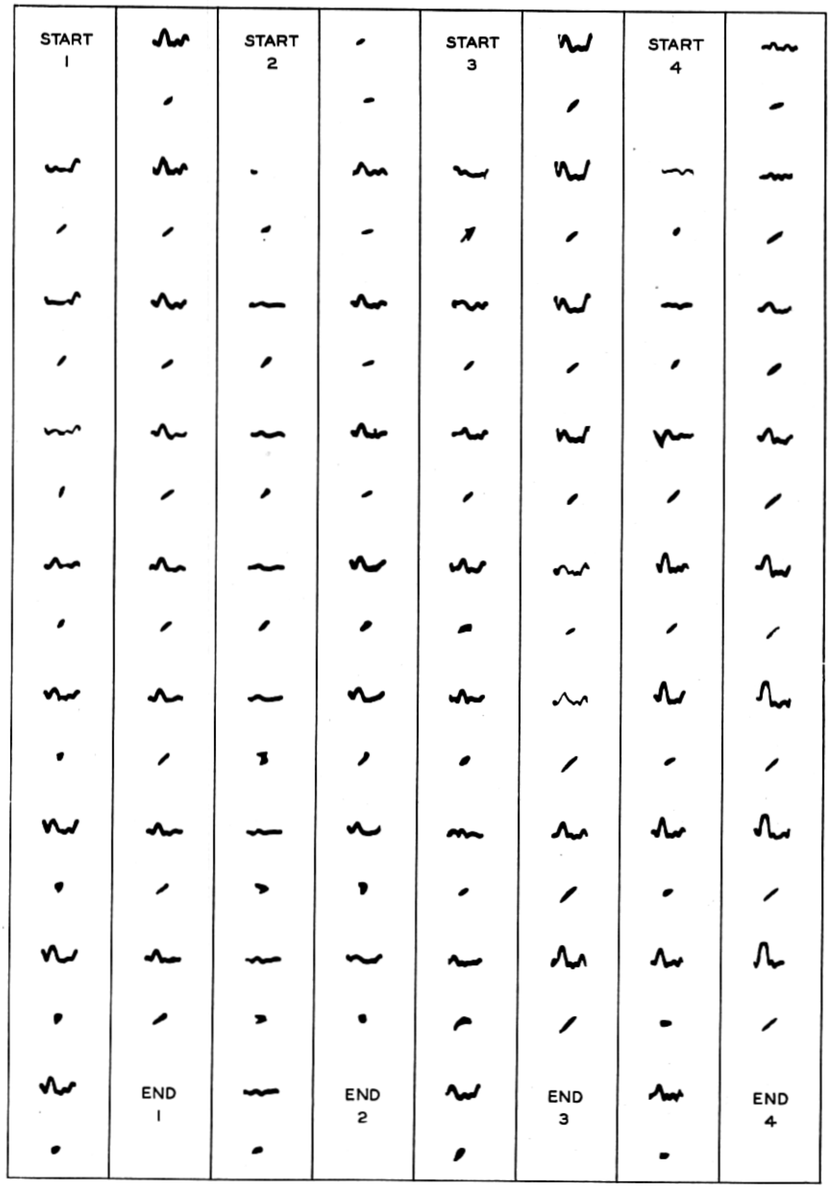


Fig. 25—This plate, made immediately following that of Fig. 23, shows for comparison the manner in which the audio outputs add in two-station space diversity. The angle monitor shows the 8.5- and 20.5-degree waves as before. Films 1 and 2 taken at 11:15 A.M., E.S.T., were obtained with rhombic antennas 1 and 6 (40 wave-lengths apart). Films 3 and 4 taken at 11:20 were obtained with antennas 1 and 2 (8 wave-lengths apart). Note the second harmonic in film 2 particularly. GSE (11,860 kilocycles) Daventry, February 21, 1936. Musical program. Zero delay.

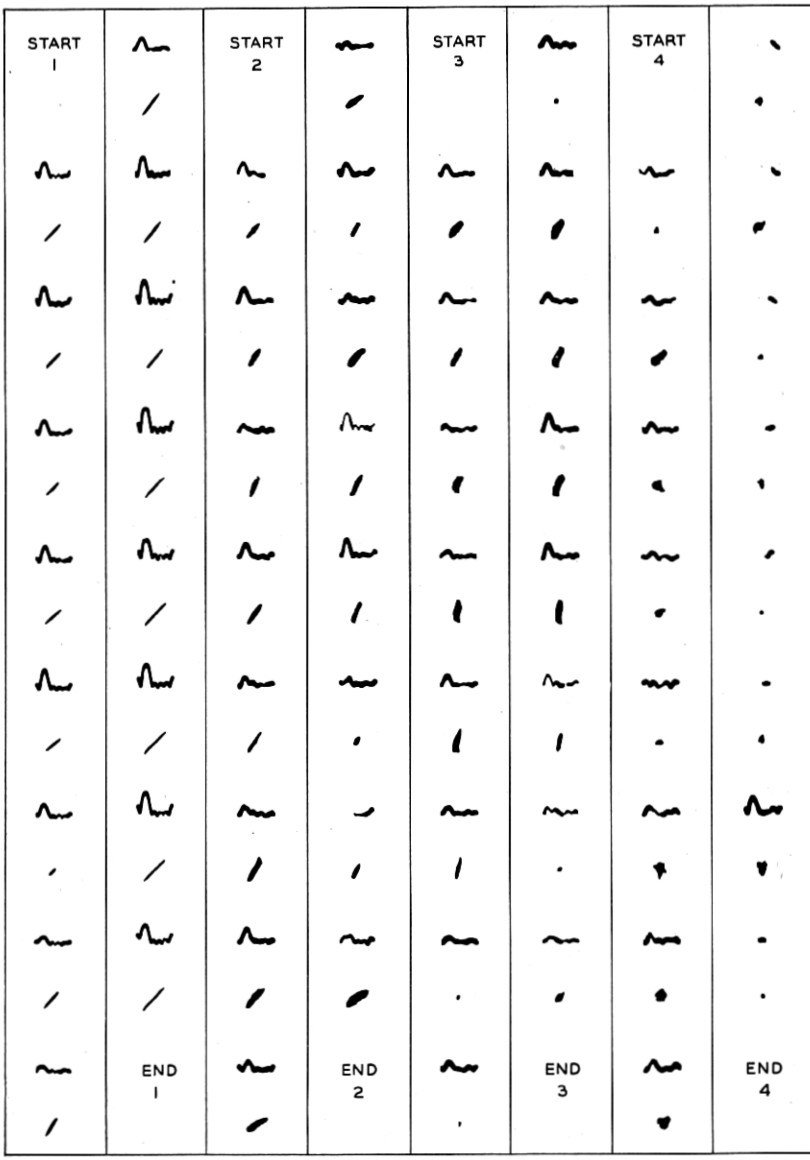


Fig. 26—Film showing the effect upon the audio addition of unequalized delay. Both MUSA branches were steered at the same angle,—that of the major wave shown. Film 1 shows no delay added and since each branch receives the same wave the audio outputs add perfectly. Films 2, 3, and 4 show the effect of adding 340, 680, and 2700 microseconds delay, respectively.

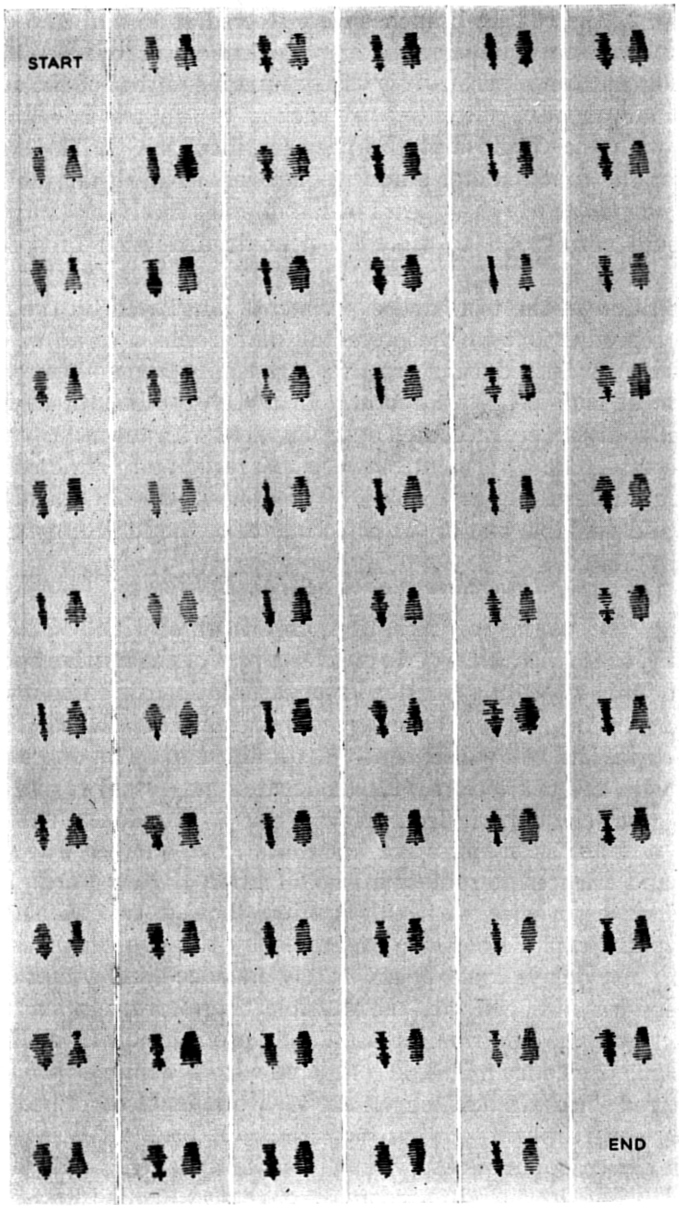


Fig. 27—This is a cathode-ray "multitone" record comparing the MUSA output with that of a horizontal half-wave antenna. The tones on the right-hand side of each frame are the MUSA output; those on the left are the output of the horizontal half-wave antenna. The two MUSA branches were steered at 15 and 22.5 degrees. A delay of 470 microseconds was used to equalize the transmission time. GCS (9020 kilocycles) Rugby, February 24, 1936, at 3:54 P.M., E.S.T.

In Fig. 27 the MUSA branches were steered at 15 and 22.5 degrees and employed an equalizing delay of 470 microseconds. While the MUSA output is not perfect it is vastly superior to that of the doublet. The tone frequencies and filters are such as to suppress harmonic distortion with the result that the patterns show mainly the selective fading of the fundamental audio frequencies. Note that the fundamental output nearly disappears in the doublet receiver. In practice this would correspond to violent harmonic distortion of speech or music.

In addition to the tests and experiments illustrated by the motion picture reproductions in the preceding paragraphs a series of experiments were conducted using broadcast transmission on 49 meters from a station at Halifax, Nova Scotia. In these experiments angles and delay differences were measured and compared with the multiple reflection theory. The agreement between measured and predicted values is not only interesting as a study of the ionosphere but constitutes a unique and valuable test of the performance of the MUSA system.

Observations on VE9HX, Halifax

During the course of reception experiments with GSL (BBC, Daventry, 6110 kilocycles) performed as a part of the routine operating program for the MUSA system, a broadcast station appeared on GSL's frequency. This station carried the programs of CHNS, Halifax, Nova Scotia, and was subsequently determined to be an experimental station with the call letters VE9HX located hear Halifax and nearly on the great-circle path from New York to London. The transmitting antenna is a half-wave horizontal, one-quarter wave above ground and oriented to radiate in the direction of New York.

The first experience with this station showed two stable transmission paths capable of being separated by the two branches of the MUSA. The delays could be accurately equalized and rather definite correlation was obtained with the multiple "hop" propagation picture. This fact and the additional reason that propagation from England on the same frequency might be compared with the simpler phenomena encountered with Halifax led to the measurements described in the following paragraphs.

About eleven hours of observation, distributed over fifteen days, are included. The log aimed to record all changes which occurred during an observation period. The procedure was as follows: The two branches of the receiver were steered at the angles indicated by the monitoring oscilloscope. Delay was added to the lower angle branch until the two audio outputs added. The delay setting was usually critical to one section of the network (67.5 microseconds) and always to two sections. The angles were determined from the calibration curve reproduced in Fig. 28. The phase readings observed on the monitoring oscilloscope were recorded to within ± 10 degrees and the earth angles determined by them are liable to be in error by one degree (possibly 1.5 degrees) apart from the ambiguity due to the multiple lobe characteristics of the MUSA. At this wave-length, the major lobe of the unit rhombic antennas is broad, the first null occurring at 58 degrees, so that two angles had to be considered possible.

The multiple hop picture is illustrated in Fig. 29. Here the delay

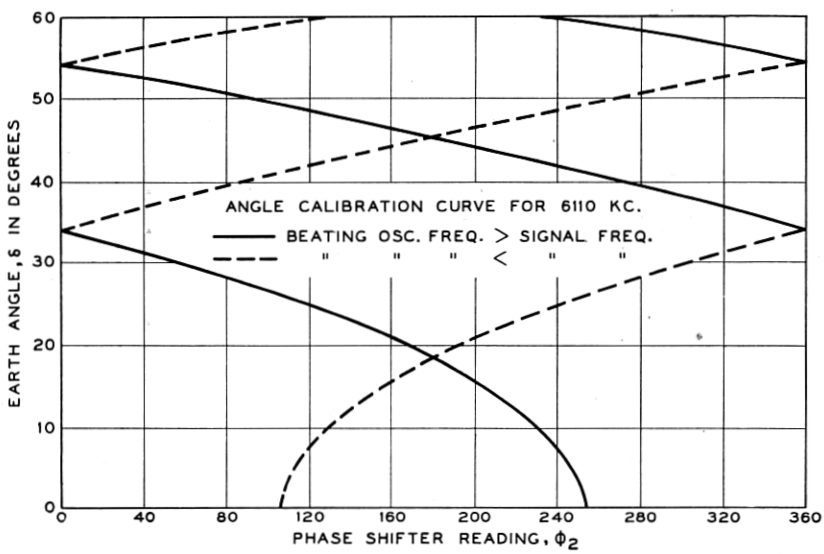
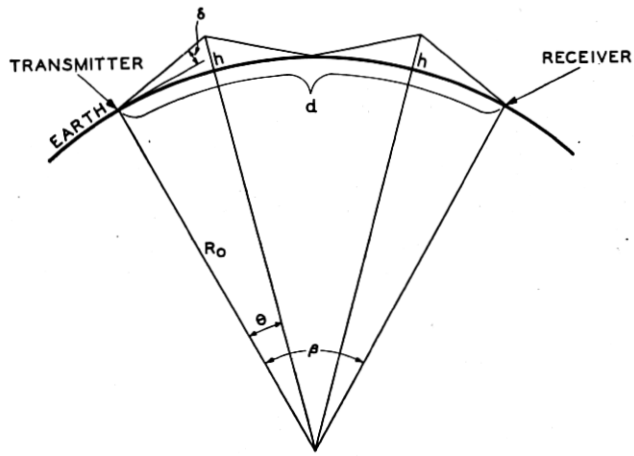


Fig. 28—Calibration curves of the Holmdel MUSA for 49.1 meters, giving the angle of the principal lobe as a function of phase advance ϕ_2 (Fig. 3). Note that the sense of the phase shift depends upon the beating oscillator frequency. The curves are calculated for a velocity ratio $v = 1/0.933$.

referred to the ground wave is expressed in terms of earth angles δ and n, the number of hops or ionosphere reflections. The height h and angle δ are also related through n as shown in Fig. 29. Using the first relation, the curves of Fig. 30 were drawn; using the second relation, points corresponding to various heights were located on the curves. For the Holmdel-Halifax circuit d is 643 miles (1030 kilometers) making $\beta = 9° 21'$. Corresponding to each measured angle there is a delay (referred arbitrarily to the ground wave which, of course, was not received) and a layer height, for each of the modes or orders. Both angles together yield a delay difference which is to be compared with the measured value.



$$\text{DELAY} = \frac{2nR_0 \sin\theta}{C \cos(\delta+\theta)} - \frac{d}{c}\,;\; 1 + \frac{h}{R_0} = \frac{\cos\delta}{\cos(\delta+\theta)}\,;\; 2n\theta = \beta$$

Fig. 29—Delay and angle relations for multiple reflection from a uniform reflecting surface. The number of ionosphere reflections is designated by n.

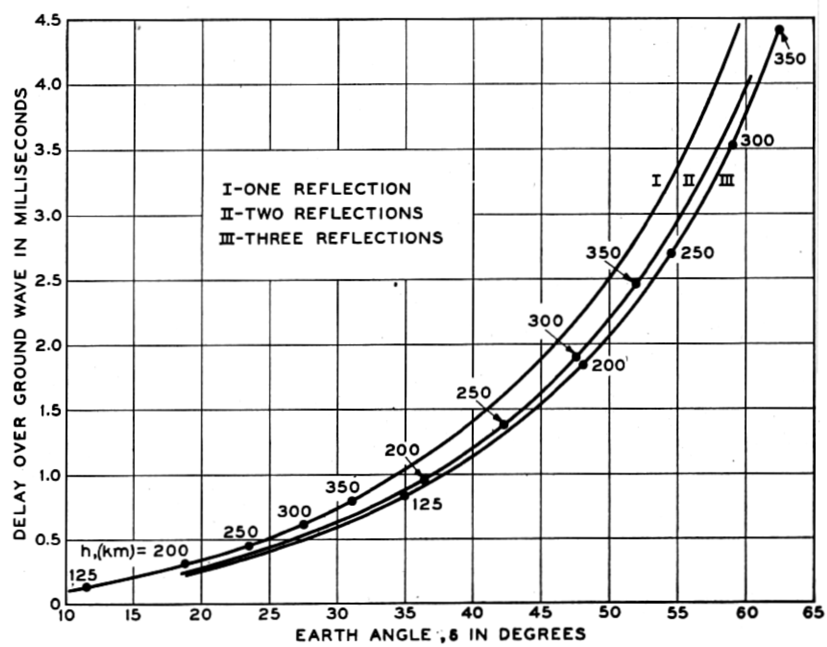


Fig. 30—Curves giving the delay-angle relations for multiple reflection on the Halifax-to-Holmdel path.

TABLE I
OBSERVATIONS ON VE9HX, HALIFAX, NOVA SCOTIA
6110 kilocycles 49.1 meters

	E.S.T.	Date	$\delta_1°$	$\delta_2°$	Relative Delay milliseconds		Virtual Height kilometers			Estimated		Field decibels above 1μv/m.
					meas.	calc.	1st	2nd	3rd	1st	2nd	
		1935										
	P.M. 5:00	11–25	18.2	38	1.01	0.77	195	215				
	P.M. 4:45	11–26	24.5	42	1.01	0.85	260	250				
	P.M. 4:40	12–17	26.4	44	0.95	0.97	290	270				27
	P.M. 4:41	12–17	24.5	43	0.95	0.95	265	255				
	P.M. 5:01	12–17	24.5	42	0.95	0.85	265	250				
	P.M. 4:20	12–18	25.5	44	0.95	1.05	275	270				27
	P.M. 4:25	12–18	24.0	42.5	0.95	0.90	250	250				
A	P.M. 4:40	12–18	23.0	42	—	0.90	245	250				
	P.M. 4:30	12–23	24.5	44	0.95	1.05	265	270				25
	A.M. 10:29	12–26	28.0	44	0.95	0.90	305	270				−14
	A.M. 10:46	12–26	28.0	44	0.81	0.90	305	270				
	A.M. 10:52	12–26	29.5	44	0.78	0.85	330	270				
	P.M. 4:33	12–26	24.5	44	0.88	1.05	265	270		242	245	18
	P.M. 4:49	12–26	24.5	42	0.95	0.85	265	250				
	P.M. 4:58	12–26	24.0	42.5	0.88	0.90	250	250				
	A.M. 10:07	12–27	25.5	43	0.95	0.95	275	255				−14
	A.M. 10:57	12–27	29.5	42	0.68	0.65	330	250		130	243	
	A.M. 11:03	12–27	31.0	42	0.88	0.70		250	100			
	A.M. 11:20	12–27	<8.0	45.5	1.28	1.6+	<120	280				
B						1.5+	<120		185			
	A.M. 11:48	12–27	8.0	47.5	1.28	1.8+	<120	300				
						1.7	<120		195			
	A.M. 11:56	12–27	8.0	45.5	1.28	1.6+	<120	280		130	247	
						1.5+	<120		185			
C	P.M. 4:32	12–27	25.5	44	0.88	1.05	275	270				14
	P.M. 4:59	12–27	27.0	43	0.88	0.85	295	255				
D	A.M. 10:45	12–31	31.0	42	0.50	0.55	350	250				0
	A.M. 11:45	12–31	8.0	35.5	0.71	0.8+	<120	195				
		1936										
	A.M. 10:30	1–2	24.5	42	0.88	0.85	265	250				− 2
	P.M. 6:05	1–14	20.5	42	1.01	1.00	215	250				8
	P.M. 6:35	1–14	18.4	38	1.01	0.77	200	215				
E	P.M. 6:15	1–15	23.0	42	1.08	0.90	245	250				22
	P.M. 6:20	1–15	24.0	42	1.11	0.85	250	250				
	P.M. 6:40	1–16	25.5	42	1.08	0.85	275	250				14
	P.M. 7:21	1–16	24.5	43	1.18	0.95	265	255				
F	P.M. 8:39	1–16	31.5	37	0.27	0.20	355	205				
	P.M. 9:35	1–16	26.4	37	0.47	0.42	290	205				27
	P.M. 5:50	1–21	24.5	42	0.95	0.85	265	250		267	247	22
G	P.M. 6:10	1–21	26.4	44	1.01	0.97	290	270				
	P.M. 6:16	1–21	22.0	40	0.95	0.80	235	230		232	245	
	A.M. 10:40	1–22	24.5	43	0.95	0.95	265	255				− 2
	A.M. 11:05	1–22	24.5	34	0.41	0.35	265	185				
						0.30	265		120			
H	A.M. 11:09	1–22	34.0	43	0.60	0.60	185	255				
						0.65		255	120			
	A.M. 11:30	1–22	24.5	43	0.95	0.95	265	255				
	A.M. 11:35	1–22	24.5	34	0.41	0.35	265	185				
						0.30	265		120			
I	P.M. 6:45	1–24	18.4	38	0.74	0.77	200	215				2

In Table I the virtual heights are deduced from the curves for the assumed hop orders. The calculated relative delay is the delay difference corresponding to these heights. All angles below 60 degrees were considered and all combinations of hop orders were considered for each angle, subject to the experimental knowledge of the sense of the delay. The values shown in the table are the ones which give the best agreement with the measured delay. In most instances there was no question concerning the interpretation; in a few doubtful cases two possibilities are presented (December 27 and January 22).

Examination of the table shows that except near noon, the propagation comprises the first and second reflections from the F region of the ionosphere. Groups A, C, E and G illustrate this. In the majority of instances the agreement is excellent; these cases constitute strong evidence that the MUSA performs correctly.

The discrepancies in the table between layer heights for the first and second hops and between measured and calculated delay are not entirely experimental error. Assuming errors in measured angles sufficient to make the delays agree will, in some cases, increase the discrepancy in heights. An interpretation one might make of this is that the ionosphere is not uniform over the circuit and the regular reflection basis of calculating is not strictly in accord with facts. However, there are other theoretical explanations for discrepancies in height. Under usual conditions, the second reflection height should be slightly greater than the first but for certain ionizations in the E region, the first F reflection may be retarded more than the second F reflection in passing through the E region. Thus the heights may differ in either direction without demanding horizontal non-uniformity. The discrepancies between measured and calculated delay may be explained by horizontal non-uniformity in the ionosphere. For an essentially non-dissipative atmosphere of ions having any vertical distribution but no horizontal gradient, and neglecting the earth's magnetic field, the group delay is identical with that calculated from triangular paths coinciding with the initial earth angles. Breit and Tuve showed this in their 1926 paper. With horizontal variations in the ionosphere such as tilting layers, no kind of agreement could be expected; the waves might even travel via other than great circle routes.

During three days of our observations W. M. Goodall made measurements of virtual height and of critical frequency which enabled him to predict the results we might be expected to observe. His estimates are shown in the next to the last column of the table.

The data for December 27 (B) are interesting in that after 11 o'clock the first F reflection apparently disappeared. Instead, a first reflection from the E layer is indicated. This was predicted by Mr. Goodall on the basis that the E region ionization at noon became so great that 24-degree waves should be reflected. For completeness the table shows an alternative interpretation of a first E reflection and a third reflection from a 185- to 195-kilometer height. The first E reflection and second F reflection are perhaps more likely. The 11:03 record is not explained.

Something similar appeared to happen on December 31 (D). On January 22 (H) normal first and second F reflections occurred with angles of 24.5 and 43 degrees. In addition a third wave of 34 degrees appeared. Two interpretations of this are shown but neither seems very plausible.

As a general rule propagation from Halifax is simpler than from Daventry on the same wave-length. In particular GSL waves received by the two MUSA branches are definitely less discrete and include sufficient delay differences in themselves to prevent the nicety of equalization possible with VE9HX. If multiple reflection takes place, which we have no reason to doubt, it is generally so distorted by non-uniformity over the path or by other factors as to be unrecognizable. In view of the occasional complexity of the Halifax circuit, only one-sixth as long, this is perhaps to be expected.

The absence from these observations on Halifax of any third reflections from the F layer is likely due to the fact that they would fall in the neighborhood of the first null of the rhombic antenna and would have to be much stronger in space in order to appear comparable with the second or first. There have been momentary appearances of waves which might have been third reflections but they did not persist long enough to work with.

When single waves were present, which was not unusual in the later evening hours, the angle more often corresponded with the first F reflection rather than the second.

Additional Numerical Data on Reception with the MUSA

The data shown in Fig. 31 are submitted to supplement the rather meager numerical data on transatlantic reception thus far presented. Here, relative delays and angles taken from the MUSA operating log are shown in plots A, B, and C. Only the end points of the lines are significant; they denote by their abscissas the angles at which the two receiving branches were set. The ordinates of the upper end points denote the equalizing delay. The lines merely connect coexistent points. The data shown were selected from the rather extensive log to present a fair cross section of conditions, omitting, however, all cases in which both branches were steered at the same wave bundle. They cover winter and summer and were obtained with frequencies appropriate to the time and season. Most of the observations were made on transmission from Daventry, the remainder on transmission from Rugby. In D are shown the results of pulse measurements made before the MUSA was in use. Here the angles were measured by the two antenna null method and the delays were observed directly on the oscilloscope time axis.[5] Although as many as five points, each denoting a wave bundle, are shown, generally not more than three were

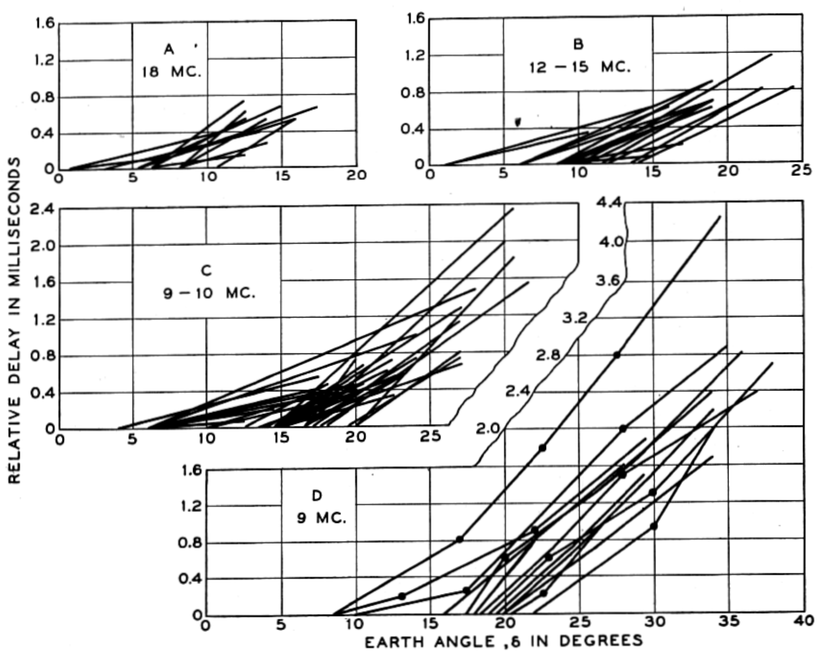


Fig. 31—Pairs of measured angles and relative delay denoted by the end points of the line segments. The data in *A*, *B*, and *C* were obtained with the MUSA; that of *D* was obtained by the use of pulses.

important at once. These measurements were made on transmission from Rugby.

It will be noticed that all four groups of data show that the relative delay per degree of angle difference is small at low angles and increases with the angle, roughly as the multiple reflection theory indicates. (This characteristic is distinctly favorable to the performance of the MUSA since its angle resolving power falls off at very low angles.) The scattering of the data indicates that an equalizing delay determined by the angle settings would not be successful; i.e., the delay must be capable of adjustment to meet the various transmission conditions.

Quality Improvement with the MUSA

The distortion of speech and musical quality which characterizes short-wave circuits is due entirely to the interference of differently delayed waves each of which individually is fundamentally free from all kinds of distortion except non-selective fading. This conclusion is almost self-evident and is corroborated by the results of several years of pulse investigation [5] made in cooperation with the British Post Office.

A MUSA system can be expected to select one out of several multiple reflections. However, these reflections suffer more or less scattering with the result that they appear as bundles of waves of various degrees of compactness. These bundles possess a small spread of both angle and delay. The delay interval included in a bundle of waves is rarely less than 100 microseconds. Double refraction or "magnetoionic splitting" occurring in the ionosphere doubtless accounts for the existence of a small minimum delay. A delay interval of fifty microseconds or so may be detected even in the unusually compact bundles represented by the pulses of Fig. 21. Transmission from Halifax appears to include a delay interval of this order, also. With transatlantic propagation it is not uncommon to have a bundle containing numerous weaker components extending over several hundred microseconds. On rare occasions these have extended over two milliseconds, masking any multiple reflections which may have been present.

The quality associated with one MUSA branch which selects one out of several bundles of waves is thus not perfect. The effect of a delay interval of a few hundred microseconds is scarcely noticeable, however, except during deep carrier fades. Therefore, if diversity action between two branches steered at the low and high angle parts of the same bundle is employed, deep fades are avoided to a large extent, and the quality is almost perfect. When more than one wave bundle is present diversity action between branches steered at the principal bundles accomplishes this escape from deep fades. It is desirable to utilize all of the principal bundles in diversity in order to preserve the discrimination of the MUSA. For, one of the bundles, if not provided with a branch to receive it, would cross talk into the other branches when it momentarily became strong and those provided with branch receivers became weak. Signal-to-noise ratio considerations discussed in Section V constitute an equally important (and related) reason for utilizing all principal bundles.

As distinguished from selective fading, which is greatly reduced by the rejection of all but one wave bundle, general fading is by no means eliminated. The reader may expect, however, that when the MUSA selects one wave bundle from several it restricts the waves accepted to those which have traveled more nearly a common path, and for a given degree of turbulence in the ionosphere, the fading should be slower, since only relative changes among the several waves result in interference fading. Such a tendency no doubt exists and has been noticed occasionally in the operation of the MUSA but rarely has there been a marked effect (excepting certain cases of flutter fading to be described later). This will be understood when it is recalled that even a fifty-microsecond delay interval means that a difference of 500 wave-lengths is involved for a wave-length of thirty meters. In order that the fading rate be sharply reduced it is required that the ionosphere shall preserve this difference, to within a half wave-length, more effectively than it does if larger differences are involved. Since a half wave-length is only 0.1 per cent of 500 wave-lengths a rather high degree of balance is thus required. Evidently, the turbulence of the ionosphere usually prevents such a balance.

Using broadcast signals (double side band) from Daventry a thousand or more comparisons were made of the MUSA versus a single antenna and receiver, using the switching arrangement mentioned in Section III. Remarkable improvements were sometimes observed and some improvement was almost always noted. The exceptions were the instances when distortion was not detectable using one antenna, and the rare occasions when particularly violent flutter fading occurred.

Space diversity reception using two antennas showed a substantial improvement, usually, but failed ever to show the order of improvement demonstrated by the two-branch MUSA when two or more wave bundles of comparable amplitude occurred. Figures 23 and 25 suggest, by the way in which the audio outputs are seen to combine, that the distortion with MUSA reception is slight compared to that with diversity reception.

The increased naturalness which results from reducing the distortion is, of course, pleasing to the ear and has some value in telephone circuits on account of the subscribers' satisfaction. In addition, it increases the intelligibility particularly when considerable noise is present. It is impossible to evaluate the increased intelligibility definitely but, in certain cases at least, it permits the signal-to-noise ratio to be two or three decibels lower. From the point of view of picking up short-wave broadcasts for rebroadcasting, a more substantial value can be attached to the MUSA quality improvement.

To a considerable extent, the magnitude of the quality improvement ascribed to the MUSA in the preceding paragraphs depends upon the fact that double side-band signals were employed. For, with double side-band signals the selective fading caused by the interference of the differently delayed waves results not only in selective fading of the audio output, but also produces non-linear distortion when the carrier fades selectively. This non-linear distortion sounds much like over-modulation, and when it occurs in its more violent forms it completely ruins the quality and intelligibility. With single side-band transmission it is possible to demodulate with such a strong carrier that non-linear distortion is virtually eliminated. The fading of the audio output is sometimes more selective than with double side-band but the resulting quality is substantially better.

Single side-band transatlantic signals were not available during the trial of the MUSA system. However, as mentioned in Section III, receiving equipment was available which rejects one side band and reduces the percentage modulation by a factor of ten or more. It was found that this equipment, applied to the one-antenna system, resulted in substantially reduced non-linear distortion and that the quality could be still further improved by the reduction of selective fading afforded by the MUSA. With MUSA reception there was apparently no quality improvement in going from double to single side band.

Summarizing Discussion

In this section the general performance of the experimental MUSA has been described in a necessarily qualitative manner. Motion picture oscillograms were shown to illustrate the performance under fairly typical transatlantic conditions. An investigation of propagation from Halifax in which the MUSA was employed to identify ionosphere reflections was included to supplement the rather fragmentary evidence available in motion picture oscillograms. The improved quality obtained with MUSA reception was discussed from several points of view. The evaluation of the MUSA has been general; it serves partly to introduce the following section which deals specifically with the signal-to-noise ratio evaluation.

Before closing this section it is appropriate to discuss conditions with which the experimental MUSA could not adequately cope.

On numerous occasions the fact that only two branches are provided has definitely handicapped the performance. More often, however, the need for greater angular discrimination or resolving power has been apparent. Except on infrequent occasions a MUSA two to three times the length of the experimental one and equipped with three branch receivers could be expected to perform as well as the experimental one now performs at its best. The occasions when it might not are the infrequent times when violent flutter fading occurs.

At least one type of flutter fading appears to be associated with a pronounced scattering which results in a kind of shower of erratic waves arriving over a wide range of directions. Receiving antenna directivity has been found definitely helpful in all except the most violent cases. Apparently when improvements due to directivity occur they occur principally by selecting a more or less normally propagated wave bundle and rejecting the shower of erratic scattered waves. When, in the most violent cases, no reduction of the flutter can be achieved the reason may be that the unit antenna accepts too wide a horizontal range to permit the MUSA to discriminate sufficiently against the shower. (It will be remembered that the MUSA array factor is of the form of a semiconical shell and thus the MUSA will, in general, accept as wide a horizontal range as the unit antenna permits.)

V. The Signal-to-Noise Improvement of the MUSA Receiving System

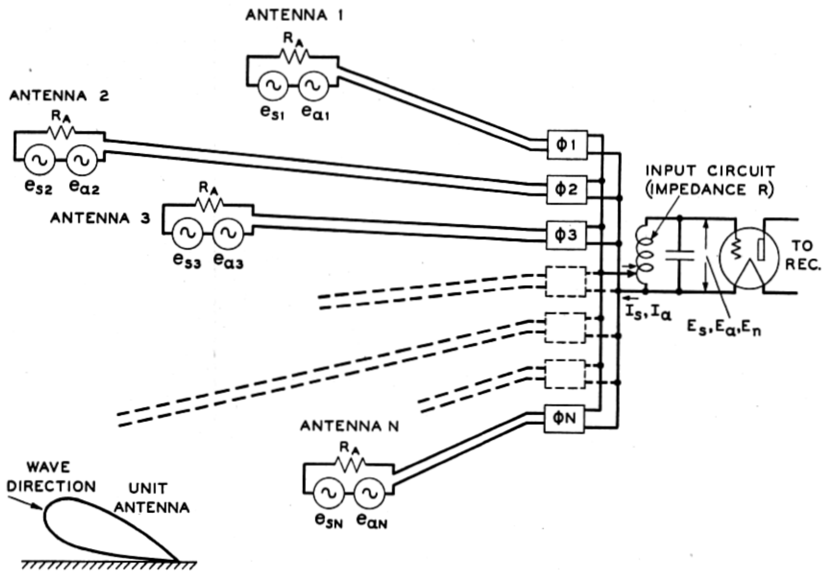
Because of the complicated nature of short-wave transmission and also because of the uncertain state of noise measuring technique, it is not a simple matter to give a satisfactory answer to the question: "What is the signal-to-noise improvement of a MUSA system?" In this section an attempt has been made to simplify the problem by separating the various factors involved. The section begins with an analysis of the problem assuming simple types of wave transmission. This is followed by experimental studies and discussions.

In discussing the signal-to-noise advantage of a MUSA it is understood that a reference receiving system must be adopted, and for this purpose one of the unit antennas connected to an automatic gain controlled receiver was chosen. Other types of antennas as, for instance, a simple vertical or horizontal doublet might have been used but other factors not significant to the MUSA would then have been involved.

Simple Analysis of the Signal-to-Noise Ratio Improvement

The MUSA differs from other directional antennas in that it is an array of antennas between which there is negligible electromagnetic coupling. This allows (but does not require) a different point of view, not explicitly involving directivity, in considering the signal-to-noise advantage of the array. The following analysis is made from this point of view. In Figs. 32 to 34 antennas are represented by signal generators, e_s, static generators, e_a, and resistances R_A. The input circuits of the receivers are matched to the antennas.

In Fig. 32, N spaced antennas are shown connected in parallel. The root-mean-square noise voltage, E_n, at the input to the receivers represents the thermal noise originating in the receiver input circuits.



FIRST ASSUMPTION:
ALL LINES MATCHED
(LINE IMPEDANCES = R_A
PHASE SHIFTER IMP. = R_A
INPUT CIRCUIT MATCHED
TO IMP. R_A/N)

$$I_s = \sum_{K=1}^{K=N} \frac{e_{sK}}{2R_A}, \quad E_s = I_s\sqrt{\frac{RR_A}{N}},$$

$$I_a = \sum_{K=1}^{K=N} \frac{e_{aK}}{2R_A}, \quad E_a = I_a\sqrt{\frac{RR_A}{N}},$$

$$E_n = \text{CONST} \times \sqrt{R}$$

SECOND ASSUMPTION:
SINGLE WAVE. SIMILAR ANT.
($e_{s1} = e_{s2} \cdots\cdots e_{sN} = e_s$
$e_{a1} = e_{a2} \cdots\cdots e_{aN} = e_a$)
SIGNAL CURR. PHASED
STATIC CURR. RANDOM

$$I_s = \frac{N}{2R_a} e_s, \quad E_s = \frac{1}{2} e_s \sqrt{N}\sqrt{\frac{R}{R_a}}$$

$$I_a = \frac{\sqrt{N}}{2R_a} e_a, \quad E_a = \frac{1}{2} e_s \sqrt{\frac{R}{R_A}}$$

$$E_n = \text{CONST.} \times \sqrt{R}$$

$$\frac{E_s}{E_a} = \sqrt{N}\frac{e_s}{e_a}$$

$$\frac{E_s}{E_n} = \sqrt{N}\frac{e_s}{\sqrt{R_A}} \text{CONST.}$$

Fig. 32—Simple signal-to-noise analysis of a system of N spaced antennas. Signal currents are phased and combined at the incoming frequency. The summation signs include addition on the power basis.

For the matched condition this noise is constant and independent of the number of antennas. A single wave is assumed and the signal outputs of the antennas are phased by means of the phase shifters ϕ. The maximum signal power obtainable from N antennas obviously is N times that obtainable from one antenna. In terms of receiver noise,

e_n, the improvement in signal-to-noise ratio is 10 log N decibels referred to one antenna. If, instead of receiver noise, static is the predominating noise, the signal power received is not significant but the same improvement is realized for the general case in which the static is distributed randomly among the N antennas.[22] In that case the N signals are phased to add on a current basis while the N noise sources

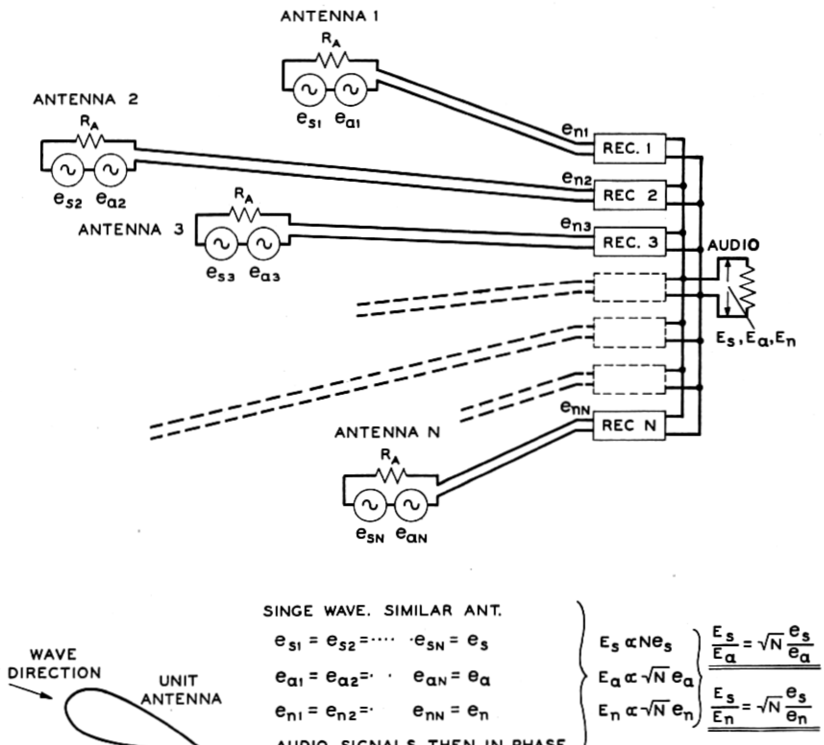


Fig. 33—Simple signal-to-noise analysis of a system of N spaced antennas. Signal currents are combined at audio frequency.

add on a power basis. Analogous arguments apply to a series connection of N antennas and result in the same improvement of 10 log N decibels.

[22] If static comes from all directions simultaneously, its distribution is random among the ideal unit antennas discussed in Section II. This is deduced from calculations which show that gain (signal-to-noise ratio) is proportional to the length of the system; i.e., to the number of unit antennas. The assumption of randomness requires that the spacing of unit antennas having a certain angular discrimination must be equal to or greater than the antenna length required to produce that discrimination in the simple linear end-on type of unit antenna.

That static is, on the average, distributed randomly among the rhombic antennas of the experimental MUSA is shown by measurements described later in this section.

The system described above has been shown mainly to introduce the system shown in Fig. 33. This diagram shows the audio addition of the outputs of N receivers fed by N antennas. Note that this system has no high-frequency phase shifters in the transmission lines. It is in fact similar to the diversity receiving system described by H. H. Beverage and H. O. Peterson.[23] For a *single wave* this is seen to be equivalent to the phased addition at carrier frequency shown in Fig. 32.

The signal-to-noise improvements shown on Figs. 32 and 33 were easily calculated because a single non-fading wave was assumed. In actual practice several fading waves are involved and it is then difficult, if not impossible, to make significant calculations. Later in this section, however, some of the general features of the system shown in Fig. 33 will be discussed from the point of view of several waves.

The MUSA system is characterized by the ability to separate waves and it is therefore possible to analyze it in a simple manner for cases of more than one wave. The arrangement in Fig. 34 corresponds to the Holmdel MUSA. The signals from the equally spaced antennas are here phased at the intermediate frequencies. Since random static and first circuit noise give identical results the analysis is given for static only.

As shown in Case I, if only one wave is present and both branches are phased for it the system functions as in Fig. 33 and it yields the same improvement of 10 log N decibels. If as shown in Case II the second branch is not phased for it (i.e., if the wave falls upon a minor lobe or a minimum of the MUSA directional pattern) less than the full improvement occurs. On the basis of linear audio detectors the reduction of improvement is 20 log x where x lies between 2 and $\sqrt{2}$. This quantity refers to the manner in which the noise from sources 1, 2, $\cdots$ N in Branch A adds with the noise *from the same sources* after having been phased differently and perhaps delayed differently in Branch B.[24] This involves the audio-frequency band width and method of noise measurement. As will be shown later x is usually not much different from $\sqrt{2}$. Taking $x = \sqrt{2}$ the loss in Case II is three decibels. If an audio detector is used which does not demodulate noise when the signal is absent (a square-law detector accomplishes this for practical purposes) this loss disappears, and branches may be phased for temporarily non-existent waves without incurring a penalty. Case III is the important one. It assumes two equal waves. Branch A is

[23] "Diversity Receiving System of RCA Communications, Inc.," *Proc. I. R. E.*, vol. 19, pp. 531–561, April, 1931.

[24] The case of $x = 2$ (in-phase addition) arises only when the phasing and delay of the two branches are alike.

phased for one; Branch B for the other. Again taking $x = \sqrt{2}$, the improvement referred to e_s/e_a is $10 \log N + 3$ decibels. Here e_s/e_a denotes the signal-to-noise ratio in each antenna due to one wave. Referring the improvement to the signal-to-noise ratio of one antenna

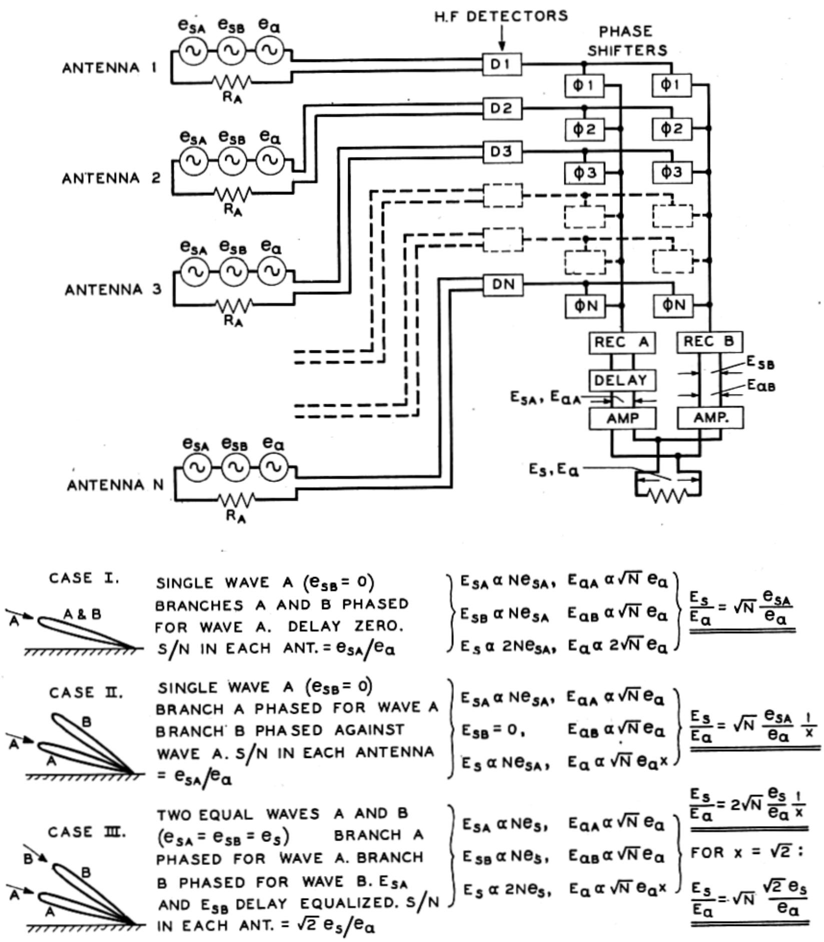


Fig. 34—Simple signal-to-noise analysis of the MUSA system.

receiving both waves, assumed to add randomly, increases the reference by three decibels and reduces the improvement to 10 log N decibels.

This analysis may be expanded to include K waves in which case K *branches* would be required to obtain the gain of *10 log N decibels* referred to one unit antenna.

It has been tacitly assumed in the foregoing analysis of Fig. 34 that the audio outputs of the several branches are delay equalized to add and that there is no diversity action (all of the waves are assumed to remain equal). The influence of fading is difficult to predict and will be discussed later in connection with experimental results.

Some readers, not concerned with details, may omit reading the following subsections and find it sufficient to read only the Summarizing Discussion of this section.

Test Method

From a practical point of view the best way of testing a MUSA system would seem to be to operate it on transatlantic telephone signals and compare its output with that of the reference system. Speech volume and noise could then be measured in the conventional manner. So far as the signal-to-noise improvement is concerned it would be a laborious and lengthy task to get satisfactory data because so often, during the test period,[25] static and receiver noise is masked by transmitted noise, interfering signals, and other man-made noise. To test the experimental MUSA, therefore, a different method was selected which gave significant data in a shorter time.

Since the success of the MUSA is related so fundamentally to the nature of the arriving signal the important thing to be determined by the measurements is how well the MUSA is able to cope with the various conditions of wave arrival. For instance, in the case of a single bundle of arriving waves how close does the actual signal-to-noise improvement come to the 10 log N decibel calculated for Case I (Fig. 34) in which a single non-fading wave was assumed? Likewise, for the case of two-wave bundles do the calculations of Case III agree with measurements?

For these purposes the signal-to-noise measurements would have to be free from directional static, interference, and transmitted noise; otherwise the measured improvements would be distorted. To insure uniform and desirable noise conditions it was decided to use thermal noise originating in the receiver input circuits [26] instead of whatever noise might be present on the radio channel. This was accomplished by inserting resistance pads in the antenna transmission lines to reduce the signal (and external noise) to a level where thermal noise greatly exceeded other noise. Signal-to-noise ratios in the range between fifteen and forty decibels were obtained in this manner, free of interference and directional static, and of transmitted noise.

[25] Transmission conditions during 1935 were comparatively undisturbed.

[26] A portion of the noise originates in the plate circuit of the first detector. For the present purposes this is equivalent to first circuit noise.

Substituting thermal noise for external static may at first seem far-fetched. Except for the fact that static is sometimes sufficiently directional to be received with different intensity as the MUSA is steered differently, the substitution is sound. In general, the static output does not vary with steering, as the measurements described later indicate but to avoid the distortion of results which would occur when this is not so, it was desired specifically to substitute non-directional noise. Studies of the characteristics of static and thermal noise have shown that both are alike so far as the effect of band width upon average and effective values is concerned, and have indicated that both consist of extremely short, randomly distributed pulses which overlap when received and detected by receivers of ordinary band widths. In a given band width, the envelope of the currents produced by static sources is highly irregular in comparison with that produced by thermal agitation. It appears, however, that the *character* of either envelope is not sensibly affected by the number of antennas combined nor by the manner in which the branch outputs are combined, so that both give the same improvement figures using any arbitrary noise measuring method.

There were several possibilities with respect to the signal to be employed in these tests. A single tone, a large number of tones distributed throughout the audio band and other special signals were considered. A simple method requiring no modulation was finally adopted. It consisted in alternately connecting the output of the antenna to be tested and that of the reference antenna to the same receiver. Assuming that the automatic gain control of this receiver would maintain a constant audio output level the signal-to-noise advantage is the ratio of the noise levels. The automatic gain control of the MUSA receiver did not, of course, hold the output level absolutely constant but a correction was easily made for the small variations in level.

The circuits of the measuring equipment are shown in Fig. 35. The rectified carrier appearing in the linear speech rectifier is taken to be proportional to signal and is measured simultaneously with the noise demodulated in the rectifier. When the keys are thrown to position 2 (by a gang arrangement) the signal meter shows the sum of the two rectified carriers and the noise meter reads the combined noise in the output of the diversity mixing amplifiers. Using the sum of the two rectifier currents to represent the signal implies that actual audio outputs from the two branches could be delay equalized to add arithmetically. As applied to a MUSA system this assumption is justified, in general. When the keys are switched to position 1, the rectified carrier of branch B alone appears on the signal meter and noise from branch B appears alone on the noise meter. At the same time the diversity connection is broken and all except one of the six-phase shifter amplifiers in branch B are biased to cutoff; i.e., only one unit antenna is used. The pad "L" is adjusted to give the same audio gain from rectifier B to the noise meter for connection 1 as for connection 2.

By manipulating the keys which control the cutoff biases on the phase shifter tubes the "1" to "2" switchover may also be used to compare one antenna (one receiver) with two antennas in ordinary space diversity or one antenna with all six in a single branch.

The use of receiver noise as a noise source depends upon (1) having the noise equal in all six circuits and (2) upon having it originate ahead of the point where the gain is varied. In well-designed receivers the noise should approach the thermal noise limit of the first circuit. It was found possible to have the signal-to-noise ratio, for a given signal level, of all six high-frequency input circuits equal to within $\pm$ 0.5 decibel and within a few decibels of the thermal limit.

The first tests were made with a local oscillator supplying the signal. They really constituted tests of the measuring set up. All six input circuits were fed simultaneously through 80-ohm pads giving equiphase and equiamplitude signals on each detector grid. This corresponds to receiving a single steady wave, and one branch was "steered" as if to receive such a wave. When the multiple switch was manipulated as to compare one antenna with the steered branch the indicated signal-to-noise improvement was usually between seven and eight decibels, compared with the theoretical value of 7.8 decibels (10 log 6).

Such a local test using the switchover with all associated equipment was made before and after every transatlantic test. Corrections based upon 7.8 decibels were made to the data in cases where the local tests showed a slightly different improvement factor. In all of the work the gains of the phase shifters were adjusted to equalize the difference in line loss. The effect of this is, however, trivial.

In measuring on transatlantic waves with automatic gain control the noise variation, corresponding nearly to the reciprocal of fading, rendered visual noise readings too rough to be suitable. A Weston high-speed db meter (copper-oxide bridge type) having a calibrated range of 16 decibels was used as a noise meter. To this instrument was added a fluxmeter (Fig. 35) of low restoring torque which automatically averaged the variations of the meter pointer over the 15-second periods of observation used in these tests. The fact that the noise meter rectifier is linear means that the noise current is averaged arithmetically

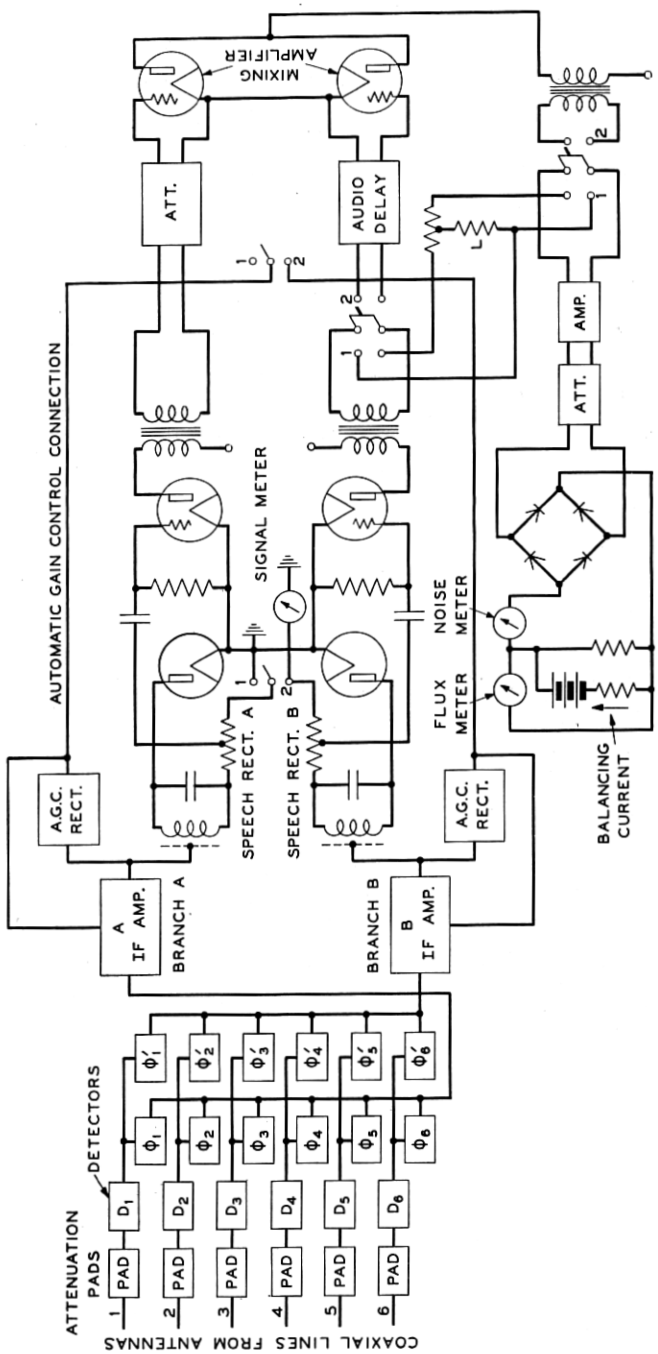


Fig. 35—Circuit used in measuring the signal-to-noise improvement of the experimental MUSA system.

along the time axis. (A discussion of other ways of averaging will be given later.) To permit the maximum time interval during which the restoring torque had negligible effect a balancing current was applied. A telechron motor marked time intervals of 15 seconds with a bell. The switchover between "1" and "2" (Fig. 35) was made and the fluxmeter restored to zero reading at the ring of the bell. The signal meter reading (calibrated in decibels) was maintained fairly steady with the automatic gain control, and could be averaged accurately by eye.

Test Results

The first measurements on transatlantic signals were made on 16 meters in September, 1935. Tests were made on the British Post Office Station GAU (18,620 kilocycles) between noon and 2 P.M., E.D.S.T., on September 16, 17, 18, and 19. At our request the transmitter operated on reduced power, presumably fifteen decibels less than normal. The propagation during these tests was characterized by low angles and slow fading. The angular discrimination of the MUSA for these low angles is so slight that it was decided to use only one branch and defer the question of diversity in these first tests.

Owing to the power reduction and to the fact that the period from September 15 to 18 inclusive was disturbed some of the data had to be obtained without antenna pads. Table II summarizes the results of the measurements.

When thermal noise is a contributing or predominating factor, the signal-to-noise ratio of the six-antenna branch must be compared with both No. 1 and No. 6 antennas in order that the line loss may be accounted for. With thermal noise predominating, the difference between the improvements referred to No. 1 and No. 6 should of course equal the line loss of No. 6. It may be shown that the arithmetic means of the two improvement ratios (voltage) should give very closely the improvement corresponding to random static ($10 \log 6 = 7.8$ decibels). In Table II the arithmetic means of the improvement ratios are, therefore, called the equivalent improvement.

During these tests the indicated angle of arrival was from one to three degrees (such low angle determinations are not trustworthy within perhaps two degrees) and the receiving branch was set correspondingly and not altered during a test. The fading on No. 1 and No. 6 antennas was usually but not always unlike. Adjacent antennas always showed substantially synchronous fading.

A sample of the plots from which the figures in the table were obtained is shown in Fig. 36. This shows the noise readings reduced to a constant signal meter reading for Test 13. Each horizontal line segment represents one 15-second fluxmeter period (the actual period was 14 seconds, one second being required for switching). The arithmetic averages of the segments are shown by dashed lines. It is to be emphasized that the scattering of improvements taken from adjacent readings is not experimental error but is due to the fact that pairs of adjacent readings were obtained at different stages of the fading cycle. Two separate systems (receivers and antennas and measuring equipment) permitting simultaneous measurements would not help this

TABLE II

ONE BRANCH

GAU 18,620 kilocycles

Date	Test No.	Pads in Ant. db	Reference Antenna	Number of Readings	S/N Improvement db	Group Average db
1935						
9–16....	1	20	1	19	3.7	
9–17....	6	20	1	18	3.0	
9–17....	10	20	1	16	3.0	
						3.3
9–16....	2	20	6	20	8.8	
9–17....	7	20	6	20	11.0	
						10.0
9–16....	4	0	1	17	6.2	
9–17....	9	0	1	11	6.5	
9–19....	11	0	1	18	6.0	
9–19....	13	0	1	28	5.6	
9–19....	15	0	1	20	8.0	
						6.4
9–16....	5	0	6	30	6.4	
9–17....	8	0	6	17	7.9	
9–19....	12	0	6	20	5.0	
9–19....	14	0	6	29	6.1	
						6.4

The apparent line loss is 6.7 db (4.3 calculated).

The equivalent improvement obtained from the data with pads is 7.3 db; without pads, 6.4 db. Average 6.9 or 7 db.

situation, since fading would not be synchronous on two systems. Evidently a considerable number of readings must be taken to insure that all stages of fading are equally represented in both of the systems being compared.

Returning to Table II the discrepancy between the apparent line loss and the calculated value (the loss could really be five decibels perhaps) suggests that insufficient data were obtained with the pads. There is a possibility that the measurements without pads involve directional static. However, taking the data as they stand yields an average of seven decibels which is less than one decibel below the value 10 log N

= 7.8 decibels calculated for the non-fading wave of Case I, Fig. 34. (The fact that only one branch was used in the measurements instead of the two branches of Case I does not affect the situation since the idle branch was disconnected and so contributed no signal and no noise.) A reduction of the order of one decibel from the calculated improvement could be expected since all of the waves of one bundle

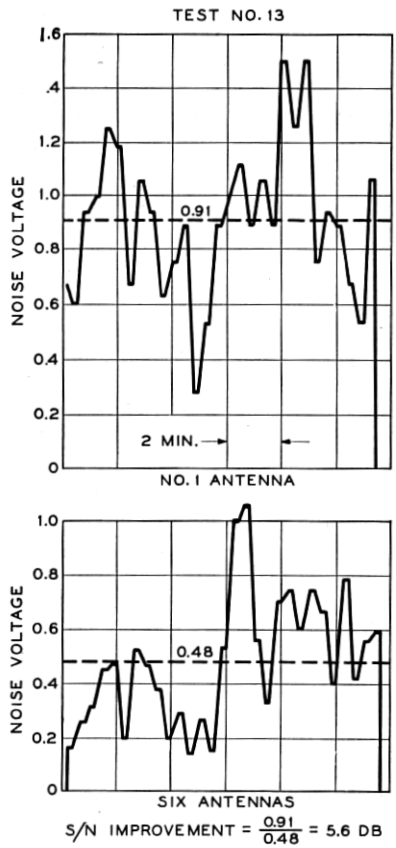


Fig. 36—Sample of measured noise plotted against time. GAU (18,620 kilocycles) Rugby, September 19, 1935.

cannot fall on the apex of the directional pattern. It was encouraging to find that no more loss occurred; i.e., to find that the waves in a single bundle may be phased so effectively.

Before leaving these tests, the results for September 18 should be mentioned. On this day the signal-to-noise ratio was so low, even without antenna pads, that measurements could not be made. The noise on this day was first taken to be thermal noise but was found during the course of experimentation to be external noise [27] some ten decibels higher than thermal noise, as received on a single rhombus. At the end of the test the operator at Rugby keyed the transmitter with tone, advising us that the schedule was completed and wishing us "good night." With one antenna the signal was hopelessly lost in noise; with the six antennas the code was readable.

TABLE III

Two Branches

GBW 14,440 kilocycles

Date	Test No.	Pads in Ant. db	Reference Antenna	Number of Readings	S/N Improvement db	Group Average db	Number of Wave Bundles
1935							
10–2 ...	21	40	1	20	2.7		1
10–2 ...	23	40	1	16	5.3		1
10–2 ...	24	40	1	16	4.4		1
10–8 ...	29	40	1	20	6.3		1
10–10 ...	37	40	1	20	4.4		1
						4.7	
10–2 ...	20	40	6	19	6.9		1
10–2 ...	22	40	6	20	8.4		1
10–2 ...	25	40	6	16	9.0		1
10–8 ...	28	40	6	17	10.9		1
10–9 ...	30	40	6	38	8.0		
						8.5	
9–30 ...	17	40	1	22	3.4		2
9–30 ...	19	40	1	14	4.1		2
10–2 ...	26	40	1	16	4.6		2
10–10 ...	33	40	1	19	2.0		2
						3.5	
9–30 ...	16	40	6	17	8.0		2
9–30 ...	18	40	6	17	7.9		2
10–2 ...	27	40	6	20	6.7		2
10–10 ...	34	40	6	20	7.3		2
						7.4	

The apparent line loss is 3.8 and 3.9 db for the one-wave and two-wave groups, respectively. The calculated loss is 3.8 db.

The equivalent improvement for one-wave group is 6.8 db to which may be added the later determined correction of 1.2 db for the effect of delay, giving *8.0 db*.

The equivalent improvement for two-wave groups is 5.7 db to which may be added 0.8 db for the effect of delay, giving *6.5 db*.

More comprehensive measurements were made on GBW (14,440 kilocycles) and a few on GCW (9790 kilocycles), using the two branches. Since an unmodulated carrier was used, rectified carrier being taken to represent signal, there was no criterion for setting the audio delay. Accordingly, it was kept at zero and a correction introduced later. The results are shown in Tables III and IV.

[27] This noise, which was directive to the extent that four-decibel variation occurred with steering the MUSA, was doubtless a sample of the "star static." It was encountered also on 31 meters in October. See footnote (32).

TABLE IV
TWO BRANCHES
GCW 9790 kilocycles

Date	Test No.	Pads in Ant. db	Reference Antenna	Number of Readings	S/N Improvement db	Group Average db	Number of Wave Bundles
1935							
12–13...	39	40	1	15	4.8		2
12–13...	40	40	1	14	4.7		2
						4.7	
12–13...	38	40	6	14	7.9		2
12–13...	41	40	6	13	6.7		2
						7.3	

The apparent line loss is 2.6 db. The calculated loss is 3.1 db.
The equivalent improvement is 6.1 db to which may be added 0.9 db for the effect of delay, giving *7.0 db.*

The data in the tables are classified roughly according to whether two bundles or one bundle of waves was present. In the latter case the two branches were steered, one on each side of the bundle, a few degrees apart. During these tests slight adjustments in steering were made when indicated by the angle monitoring tube, as in normal operation of the system. The large amount of data taken with GBW makes the results in Table III particularly reliable. This is reflected in the close agreement between measured and calculated line loss. Before discussing the results further the effect of delay needs to be analyzed.

Correction Due to Delay

The effect upon the noise, of delaying one audio output, is shown in Fig. 37. The curves were obtained with the circuit shown in Fig. 35

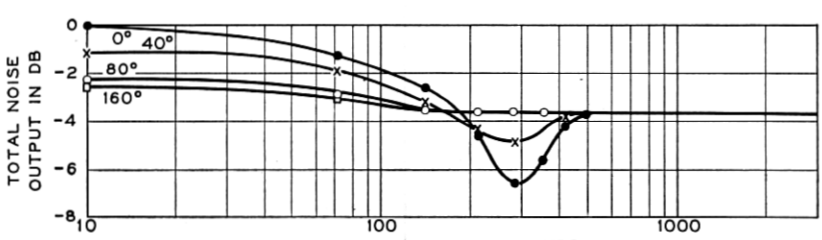


Fig. 37—The effect of delay and phasing upon noise output of the MUSA receiver.

using thermal noise. The equiphase, equiamplitude signal source mentioned previously was used to supply the inputs. Branch A was kept phased so that the six signals added, and branch B was varied. The curves show the effect of delay upon noise in a 250- to 2750-cycle audio band, as measured with the Weston db meter, for various differences in steering. The curves are labeled in terms of the difference in the phase settings of the two branches. The 40-degree or 80-degree curves correspond in practice to steering on each side of a single bundle of waves. The 160-degree curve typifies steering at two separate wave bundles. The use of 100 microseconds (or more) delay is generally advantageous for the audio addition when steering at one bundle. Since this amount of delay makes the audio noise addition nearly random and for widely different steering the addition is also random the assumption that the noise from the two branches adds on a power basis, made in reference to Fig. 34 ($x = \sqrt{2}$), is justified.

The effect of delay is to produce an interference pattern in the audio noise spectrum. This accounts for the dip in the noise curve for a delay of 300 microseconds which locates the first interference minimum at about the center of the audio band. The asymptotic approach to 3.5-decibel reduction corresponds more nearly to a reduction ratio of $2/\pi$ than to $1/\sqrt{2}$ due to the fact that the Weston db meter is nearer linear than square law in response.

In obtaining these curves it was desired to simulate the reception of two waves for which the corresponding branches were phased to add. It was not convenient to set up locally such a two-wave case but the single wave input should give identically the same results provided phases were avoided which resulted in a signal at the second detector too low to demodulate the noise. A signal level so high that further increase did not affect the noise output was used for all points. The real purpose of the signal was to insure that the demodulated noise was not dependent upon the intermediate-frequency bands and that the results would be unaffected by possible differences in intermediate-frequency bands.[28]

As mentioned, noise has been measured with an unweighted 250- to 2750-cycle frequency band. Had a weighting network [29] which emphasizes frequencies in the vicinity of 1000 cycles been used the dips in the curves marked 0° and 40° would have been deeper and would have occurred in the vicinity of 500 microseconds delay.

Returning now to Tables III and IV the measured improvements were corrected to correspond to the effect of the delay which would probably have been used to obtain the best audio addition for the signal. The 1.2-decibel correction for the one-bundle case represents

[28] This precaution was subsequently found to be unnecessary; i.e., similar results were obtained with no input signal.

[29] Barstow, Blye, and Kent, "Measurement of Telephone Noise and Power Wave Shape," *Elec. Engg.*, vol. 54, pp. 1307–1315, December, 1935. Technical Digest published in *Bell Sys. Tech. Jour.*, January, 1936.

the reduction of noise obtained with 60-degree phase difference with the addition of 100 or 150 microseconds delay. The 0.8- or 0.9-decibel correction for the two-bundle case corresponds to any large delay and a large phase difference, say 160 degrees. These corrections would not have been much different if a weighting network had been used.

The measured difference of 1.5 decibels, shown in Table III, between the one-wave bundle and the two-wave bundle measurements is probably real and due to the fact that when the branches are steered at two separated bundles some loss is incurred when one wave disappears for a few minutes. This loss could have been at least partly recovered by using square-law detectors. Tests showing the advantage of square-law detectors over linear detectors are described later in this section. Employing square-law detectors would justify a correction of about one decibel to be added to the two-bundle improvement measurements in Tables III and IV. Applying this correction we summarize the results in Table V.

TABLE V

SUMMARY OF SIGNAL-TO-NOISE MEASUREMENTS

One Bundle of Waves		*Two Bundles of Waves*
One Branch only	*Two Branches*	*Two Branches*
7 db (Table II)	8 db (Table III)	7.5–8 db (Tables III and IV)

These improvement figures for two branches as they stand are approximately equal to 10 log 6 = 7.8 decibels as calculated for non-fading waves, and leave nothing to be ascribed to diversity action. Since a loss of perhaps one decibel occurs in the case of one branch (Table II), the recovery of that one decibel with two branches is to be ascribed to diversity action. Originally, considerably more was expected of the angle diversity. It appears however from theoretical and experimental evidence that one decibel is about what should be expected for the case in hand. It seems appropriate to include this study of diversity here.

Diversity Action

The first attempt to analyze diversity action was made with a graphical approach to the problem. On Fig. 38 is shown a schematic diagram of the system to be analyzed. Two receivers A and B with linear audio detectors may be regarded as fed from two angle branches of a MUSA. The noise generators e_{nA} and e_{nB} are assumed to be of equal power but of random phase. The signal generators e_{sA} and e_{sB} are assumed to fade according to the equations shown beneath the diagram. They represent the carrier amplitude but since fading is assumed to be essentially non-selective in each branch they also repre-

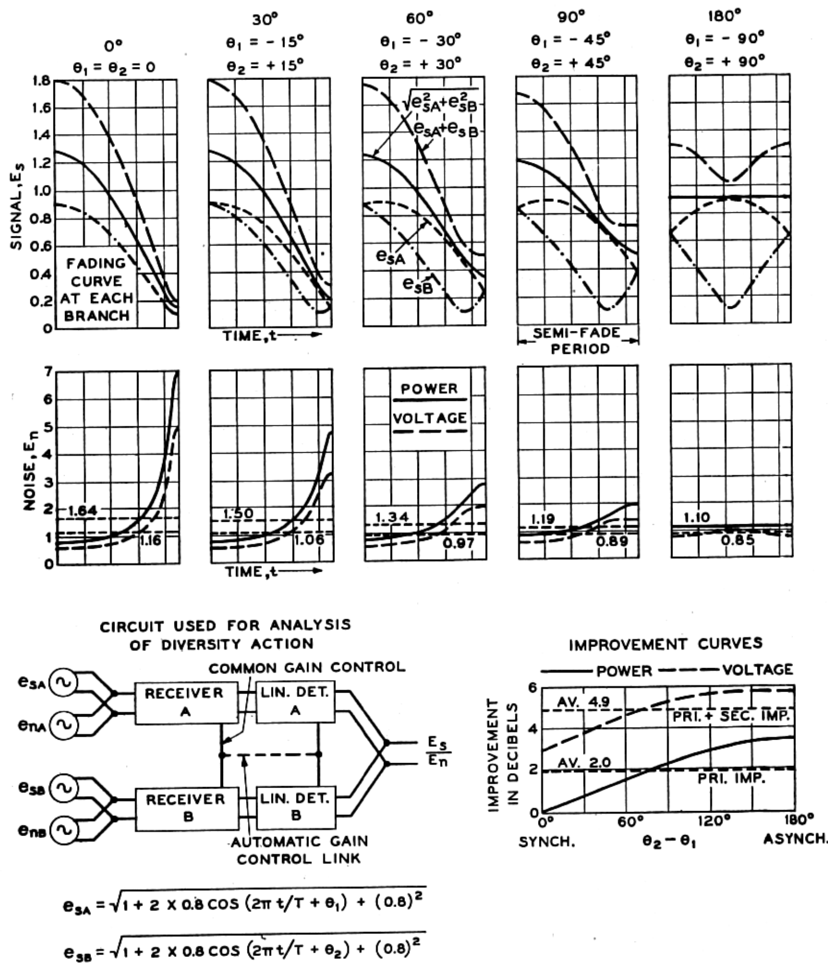


Fig. 38—Graphical analysis of diversity action as it relates to signal-to-noise ratio.

sent the side bands. This type of fading might result from interference between two waves of small relative delay whose amplitude ratio is 0.8, such a pair being received by each branch. The automatic gain control is assumed to be perfect; i.e., the audio output, E_S, is maintained constant.

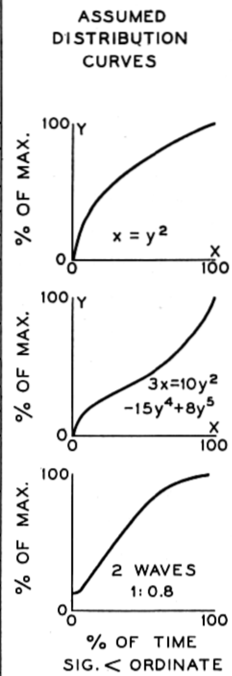
By definition, diversity in fading occurs when the fading of the two branches is not synchronous. If all degrees of asynchronism are equally probable the diversity is random. This is the case considered. In Fig. 38 five stages in the cycle of variation from synchronous to asynchronous fading are used for calculation. In each of these, fading curves corresponding to the two assumed waves are shown displaced from each other by 0, 30, 60, 90, and 180 degrees. With "ideal" automatic gain control, the two receiver gains, always equal, will be proportional to the reciprocal of the resultant of e_{SA} and e_{SB}. For "ideal" linear detectors the noise output of the receivers will be proportional only to the gain. The noise curves plot the noise variation on this basis. Two cases of signal addition are considered—voltage and power addition. The corresponding noise curves differ only as the reciprocals of the resultant signal curves differ. These noise curves are averaged (with a planimeter) and the resulting average signal-to-noise ratios are used to plot the improvement curves shown in the figure. The improvement curve for power addition of signal is located on the improvement axis so that zero improvement is shown for synchronous fading. The curve for voltage addition of signal is located three decibels higher at synchronous fading. These curves are again averaged over the cycle from synchronism to asynchronism (by averaging noise voltage). The improvements are 2.0 decibels and 4.9 decibels.

Power addition of the two signals corresponds in practice to the case in which the delay is unequalized and sufficient to cause the audio outputs of each branch to combine on a power basis like noise. The two-decibel improvement might appropriately be called the primary improvement since it is due solely to the diversified fading. The additional improvement of 2.9 decibels found with voltage addition of the signals is due to favorable discrimination in the addition of signal and of noise and might be called the secondary improvement. The secondary improvement occurs in reception with the MUSA; it has already been included in the 10 log N decibel improvement calculation.

In practice, it would be undesirable to use the "ideal" automatic gain control assumed in the above analysis; the action must be smoothed out with, for instance, a capacitance-resistance network. The effect of this is to reduce the primary gain since the noise peaks, whose avoidance by diversity action results in the primary improvement, are reduced. An analysis of diversity action without automatic gain control was made. In this case the signal was averaged while the noise remained constant. The results are included in the table shown in Fig. 39 which is introduced later.

This treatment of diversity action has been made from the point of view of MUSA reception but is applicable to ordinary space diversity using two stations or antennas. In the case of a single bundle of waves no modification of the analysis need be made; the signal generators then represent the spaced antenna outputs which are fading randomly. In this case voltage addition of the audio outputs may be

NO. OF STATIONS OR BRANCHES	$E_s = \sqrt{e_{sA}^2 + \cdots e_{sK}^2}$ $E_n = \sqrt{e_{nA}^2 + \cdots e_{nK}^2}$ PRIMARY IMPROVEMENT		$E_s = e_{sA} + \cdots e_{sK}$ $E_n = \sqrt{e_{nA}^2 + \cdots e_{nK}^2}$ PRI. + SEC. IMPROVEMENT	
K	NO AGC	AGC*	NO AGC	AGC
QUADRATIC DISTRIBUTION				
1	2.5	0	2.5	0
2	2.8	2.2	5.5	4.8
3	2.9	2.5	7.3	6.8
FOUR WAVE DISTRIBUTION				
1	3.4	0	3.4	0
2	—	—	6.4	5.2
3	—	—	8.2	7.5
TWO WAVE NEAR-SINUSOIDAL DISTRIBUTION 19 DECIBEL FADE				
1	2.8	0	2.8	0
2	3.2	2 0	5.8	4.9
3	—	—	—	—
AVERAGED RESULTS				
1	3	0	3	0
2	3	2	6	5
3	3	2.5	8	7



* AUTOMATIC GAIN CONTROL

Fig. 39—Summary of results of diversity analysis.

expected to occur since a single bundle will typically include only a small delay interval. In the general case of two or more wave bundles the signal generators must be interpreted to represent not the carrier but the side-band average, for fading will then be essentially selective and the audio output will not be proportional to carrier as was assumed in the analysis. If this interpretation is made the signal-to-noise ratio in each receiver becomes proportional to the generator amplitudes e_{sA}

and e_{sB}, and the analysis of Fig. 38 is applicable. In this case voltage addition does not occur since the audio outputs are essentially different owing to the selective fading.[30] They add, in general, to a value intermediate between the power and voltage sum, although for the more complicated conditions they combine on a power basis.

The above analysis has been based upon simple two-wave interference and the results might not be applicable to the more complicated and changing conditions of actual transmission. Accordingly, R. L. Dietzold has made a statistical analysis for other types of fading and for three stations as well as for two. The results appear in Fig. 39 together with the results of the above graphical analysis for two-wave interference fading. The time sequence of amplitude in the more complicated types of fading encountered in practice is not significant; the percentage distribution determines the results. The "four-wave" distribution curve corresponds to four equal waves of random phase. The quadratic distribution curve was deduced experimentally by R. S. Ohl. Except that these different distributions were assumed, the assumptions were the same as those of Fig. 38. The improvements are expressed in decibels referred to the signal-to-noise ratio for one station or branch with ideal automatic gain control.

The small effect upon the results of assuming different time distributions lends significance to these calculations. The averaged round numbers are probably about right.

With no automatic gain control (or with one which acts slowly compared with the fading) there is little or no primary gain. With infinitely fast and stiff gain control action there is a 2- and 2.5-decibel primary gain for two and three stations, respectively.

A few measurements were made at Holmdel on two-station diversity (antennas 1 and 6 of the MUSA). The thermal noise and rectified carrier technique was used. The results appear in Table VI.

The measuring technique was exactly the same as used in obtaining the data for Tables III and IV in which voltage addition of the signal was assumed. A 3-decibel improvement is therefore included in the 3.6-decibel figure. This leaves only 0.6 decibel (possibly one decibel or even 1.5 decibels since the measurements are too meager to be reliable to better than one decibel) for primary gain compared with a possible 2.0 decibels. We are inclined to use about one decibel for primary gain. The time constant on the automatic gain control was of the order of 0.06 second in this and all signal-to-noise comparisons. That this time constant was not fast enough to produce the high noise

[30] This refers to speech signals; in the case of telegraph signals the frequency band is so narrow that fading is always essentially non-selective, and voltage addition occurs.

peaks corresponding to the inverse of fading is shown by the transcribed motion picture record of the signal and noise meter variations, shown in Fig. 40. Note the signal fades.

A secondary improvement of 3 decibels is too high for two-station space diversity; i.e., the signals do not add on a voltage basis.[30]

TABLE VI

ANTENNAS 1 AND 6 IN DIVERSITY
GBW 14,440 kilocycles

Date	Test No.	Pads in Antennas db	Reference Antenna	No. of Readings	S/N Improvement db	Average
1935						
10–9 ...	32	40	1	19	1.7	
10–10 ...	35	40	1	20	2.2	
						2.0
10–9 ...	31	40	6	14	4.9	
10–10 ...	36	40	6	20	5.2	
						5.1

The apparent line loss is 3.1 db. The calculated loss is 3.8 db.
The equivalent improvement is 3.6 db.

Oscilloscope observations of the diversity combinations of the audio outputs of two spaced antennas (No. 1 and No. 6 of the Holmdel MUSA) indicate, however, that on the average the secondary improvement is appreciable and probably about 2 decibels for two antennas and 3 decibels for three antennas. This improvement depends upon

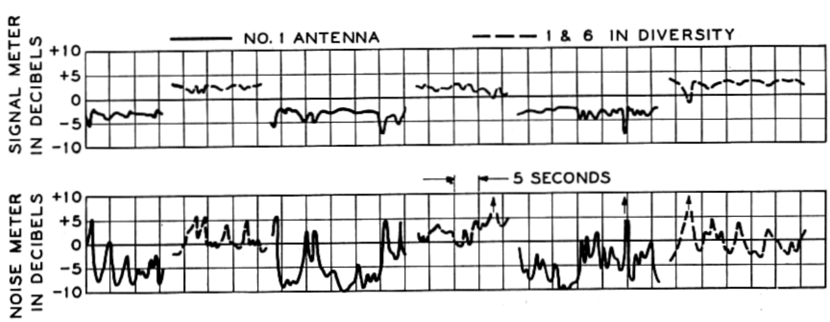


Fig. 40—Sample of signal and noise variations occurring in diversity tests. The arrows indicate noise levels beyond the scale of the meter. GBW (14,440 kilocycles) Rugby, October 10, 1935. 1920 G.M.T.

the number of wave bundles, their angular separation, their relative delays, the spacing of the antennas and the frequency band occupied by the signal. For a single compact wave bundle the secondary improvements will be nearly 3 decibels and 4.8 decibels for two- and three-antenna systems, respectively, but for several bundles of large relative delay the secondary improvement may disappear.

The results of some recent tests of a three-antenna diversity system on trial at Netcong, N. J., carried out under the direction of F. A. Polkinghorn, showed a signal-to-noise improvement of 3 to 3.5 decibels. Assuming a 3-decibel secondary improvement there remains something of the order of 0.5 decibel for the primary improvement. This is plausible in view of the time constant of one second used on the automatic gain controls. Linear audio detectors were used in these tests. As will be discussed later the employment of square-law detectors could be expected to add 0.5 decibel to this figure. Linear detectors are to be preferred, however, on the basis of quality distortion. Table VII is based upon the theoretical and experimental study of diversity action and gives typical results for space diversity systems.

TABLE VII

Summary of Space Diversity Improvements

Number of Antennas	Primary Improvement in db Automatic Gain Control		Secondary Improvements in db Number of Wave Bundles		
	0.06 Sec.	1 Sec.	1	2	3–5
2	1	0.5	3	2	1
3	1	0.5	4.5	3	1.5

Add 0.5 db to primary improvement when square-law detectors are used.

Table VII shows that on the average the secondary improvement is larger than the primary improvement. In other words, the advantage which accrues from the similarity of the antenna outputs exceeds that which accrues from their diversification. This result had not been expected.

It should be emphasized here that the improvements summarized in Table VII for space diversity systems and in Table V for a MUSA system refer to signal-to-noise ratios only; i.e., quality improvement is not included.

An important advantage of a MUSA system over a space diversity system is its ability to maintain its improvement when more than one wave bundle occur, and since two or more bundles are common, the advantage is distinctly real. A further advantage not discussed thus far relates to interfering signals as distinguished from static. Unless the interfering signals fall upon the principal lobe of the MUSA array pattern when it is steered to receive the desired signal, important directional discrimination against the interference occurs. Little or no discrimination against interference can occur in a space diversity system since it lacks the phasing which produces directional discrimination.

The Time Constant of the Automatic Gain Control

Thus far no comments have been made on the improvement figures relating to "no automatic gain control" shown in the table of Fig. 39. This table shows that the signal-to-noise ratio for one antenna ($K = 1$) is from 2.5 to 3.5 decibels higher when no automatic gain control is used; i.e., perfect automatic gain control penalizes the signal-to-noise ratio to that extent.[31] The advantage of automatic gain control is a constant output volume. In practice, a compromise is effected by retarding the action of the control. A time constant of 0.5 or one second is usually used. (This compromise is influenced by quality considerations as well as noise considerations.)

In the MUSA system signal-to-noise ratio measurements the time constant of the automatic gain control circuit (0.06 second) was not changed during the switchover from the MUSA to the single antenna. If a time constant of 0.5 second had been used with the MUSA and a one-second time constant with the reference receiver, the measured improvement would probably have been reduced by a little less than one decibel.

Method of Averaging Noise

In all of the signal-to-noise measurements and in the diversity analysis noise voltage has been averaged arithmetically along the time axis. Owing to a rather marked reduction of noise peaks with the MUSA compared with a unit antenna different improvements would result if different ways of measuring it had been adopted. To investigate this, motion pictures were made of the signal meter and noise meter variations for the MUSA and for the single antenna. The transcribed records appear in Fig. 41. Some calculations have been carried out for the noise distributions marked A, B, and C in Fig. 41. If the noise ratio of B/A measured by arithmetically averaging noise voltage is called 0 decibels, it becomes $+ 2.4$ decibels by averaging power arithmetically. The corresponding figures for B/C are 0 and $+ 2.7$ decibels. Thus, if noise power is averaged instead of noise voltage the measured primary diversity improvement is substantially increased.

[31] The action of the automatic gain control does not change the instantaneous signal-to-noise ratio. Interpreting signal-to-noise ratio as *average signal* divided by *average noise* rather than the average of the *signal divided by the noise* results in this difference.

From the point of view of the interfering effect upon speech it is not clear which method of averaging is more significant. This matter is probably related too closely to the distortion which incidentally accompanies the noise peaks to be considered alone.

In the light of the discussion presented in the preceding pages it appears that the signal-to-noise improvement of the experimental MUSA can be expressed as 8 ± 1 decibels.

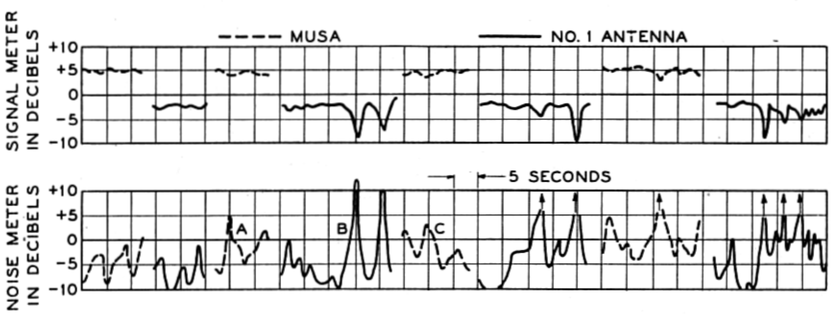


Fig. 41—Sample of signal and noise variations occurring in MUSA tests. The arrows indicate noise levels beyond the scale of the meter. This record was obtained five minutes before that in Fig. 40.

Square-Law Detectors

In the discussion of the signal-to-noise measurements it was stated that the measured improvements would have been higher had square-law audio detectors been used instead of linear rectifiers. For the two-bundle MUSA measurements one decibel was allowed for this and for the three-antenna diversity measurements at Netcong 0.5 decibel was allowed. These figures are based upon tests to be described in the following paragraphs. First, however, an analysis will be made of the effect upon the signal-to-noise ratio of various types of detectors in a MUSA system. Figure 42 is a schematic representation of the system to be analyzed, comprising K branches. The K signal generators e_{sA}, e_{sB}, $\cdots$ e_{sK} represent the various wave bundles as received by the steerable branches. The noise generators e_{nA}, e_{nB}, $\cdots$ e_{nK} are equal in amplitude but random in phase. The detectors are generalized to the extent that the audio output is proportional to the u power of the input.

Assuming that $e_s \gg e_n$ the audio outputs are proportional to $e^u{}_{sA}$, $e^u{}_{sB}$, $\cdots$ $e^u{}_{sK}$. The noise outputs are then proportional to $e_{nA}e_{sA}{}^{u-1}$, $e_{nB}e_{sB}{}^{u-1}$, $\cdots$ $e_{nK}e_{sK}{}^{u-1}$ since the signal-to-noise ratio in each branch must be independent of u. Assuming the signals to be delay equalized,

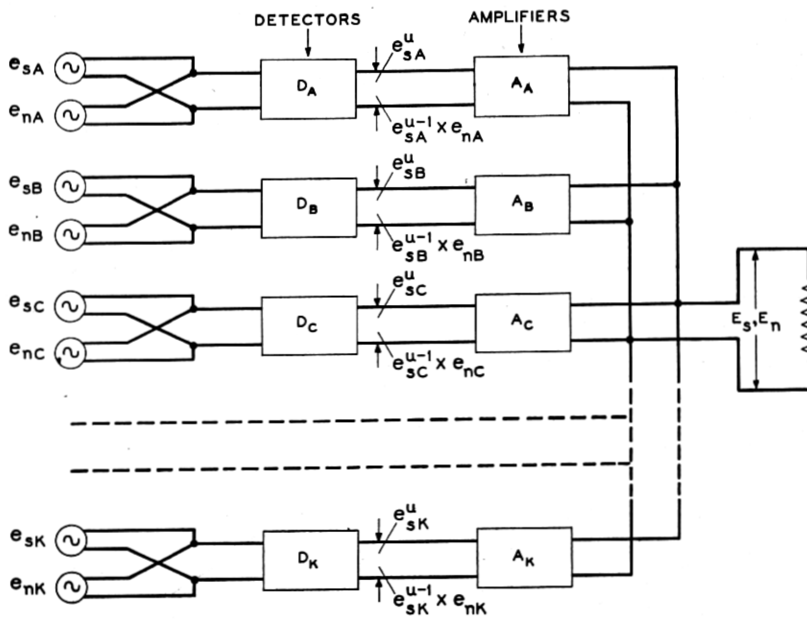


Fig. 42—Circuit employed in the analysis of the effect of detector characteristics upon signal-to-noise ratio.

the signal-to-noise ratio of the final output is

$$\frac{E_s}{E_n} = \frac{e^u{}_{sA} + e^u{}_{sB} + \cdots + e^u{}_{sK}}{e_n\sqrt{e_{sA}{}^{2u-2} + e_{sB}{}^{2u-2} + \cdots + e_{sK}{}^{2u-2}}}. \tag{7}$$

Maximizing this expression with respect to u shows that the maximum occurs for $u = 2$ (square law) and is

$$\frac{E_s}{E_n} = \frac{\sqrt{e^2{}_{sA} + e^2{}_{sB} + \cdots + e^2{}_{sK}}}{e_n}. \qquad (u = 2) \tag{8}$$

That this expression represents the maximum signal-to-noise ratio may also be concluded by observing that it is proportional to the square root of the total energy.

For linear detectors $u = 1$ and the signal-to-noise ratio becomes

$$\frac{E_s}{E_n} = \frac{e_{sA} + e_{sB} + \cdots + e_{sK}}{e_n\sqrt{K}}. \qquad (u = 1) \tag{9}$$

If the branch signals are all equal, i.e., if $e_{sA} = e_{sB} = \cdots = e_{sK}$, (8) and (9) give the same result, but for unequal amplitudes there is an advantage in using square-law detectors.

This analysis shows that square-law detection introduces just the correct amount of emphasis upon the stronger waves and that any additional expansion or contraction of the differences among the waves is detrimental. This means that the gains in all branches should be equal. It also indicates that any arrangement in which the stronger of the several waves is automatically switched in and the remaining ones switched out is inferior.

The experimental MUSA receiver is equipped with both linear and square-law detectors, and some signal-to-noise ratio comparisons were made using locally generated signals. Figure 43 shows schematically the essential parts of the test circuit. The noise generators represent the thermal noise originating in the receiver input circuits. The input signal e_s was modulated with a tone. The calculated curves shown in the figure are obtained from (8) and (9) which reduce to

$$\frac{E_s}{E_n} \propto \sqrt{1 + \left(\frac{e_{sB}}{e_{sA}}\right)^2} \qquad (10)$$
$$u = 2$$

and to

$$\frac{E_s}{E_n} \propto \left(1 + \frac{e_{sB}}{e_{sA}}\right) \cdot \qquad (11)$$
$$u = 1$$

The equation for the square-law detector is sound and was verified by the measurements. The equation for the linear detector should apply only over a certain range of signal and noise levels. The measurements indicate this.

Automatic gain control was not used in these tests since the two gain controls could not be relied upon to "track" sufficiently well. To make the measurements significant manual gain control was used to maintain the receiver gains equal and the output normal. It may be pointed out here that in receiving actual radio signals with linear detectors accurate equality of gains is not required. Moderate differences in gain (of a few decibels) can be depended upon to be beneficial as often as detrimental. With square-law detectors no departure from equality can be beneficial.

The curves of Fig. 43 show that 10- and 20-decibel differences in signal level give the square-law detector an advantage of one and two decibels, respectively. In receiving two bundles of waves the branch outputs commonly fade in and out with the result that their average ratio is of the order of 10 decibels. Such were the conditions, as well as could be estimated, during the two-bundle tests in Tables III and IV. The one-decibel correction applied to those results in Table V is therefore justified. In the one-wave bundle measurements, the percentage of time during which the branch outputs were substantially different was so small that no correction was applied in Table V. In the case of space diversity the correction is also small; about 0.5 decibel seems reasonable.

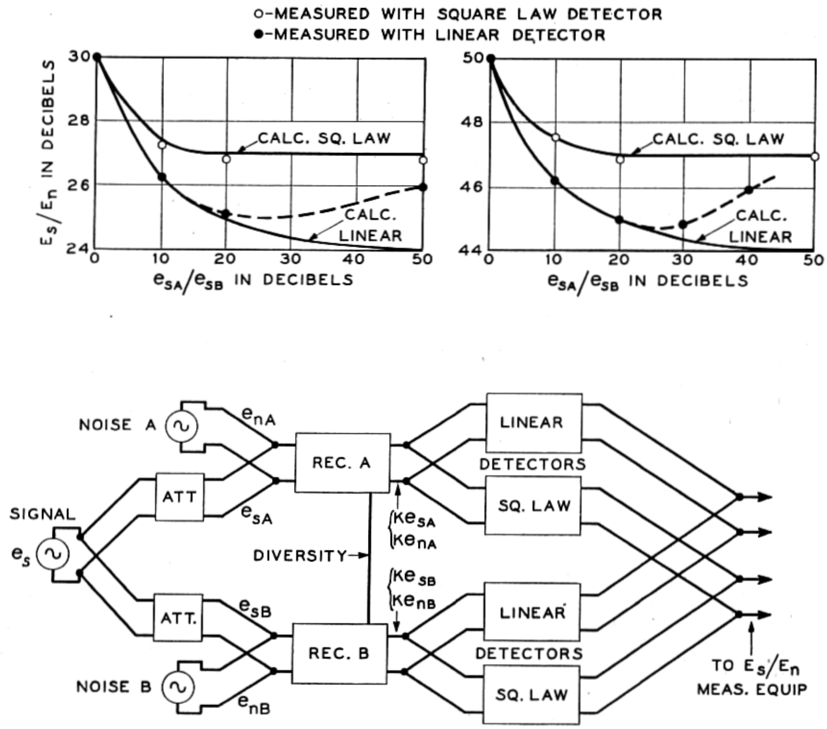


Fig. 43—Test circuit and experimental results of the study of detector characteristics.

From the point of view of distortion in receiving double side-band signals linear detectors are superior to square-law detectors and this compensates their inferiority in signal-to-noise ratio. The principal reason for using linear detection in the signal-to-noise ratio tests of the MUSA was, however, their experimental advantage in simplicity and accuracy (Fig. 35).

Random Addition of Static

In analyzing the spaced antenna systems at the beginning of this section it was assumed that the static outputs of the antennas add on a power basis. An experimental study of this was made by measuring the static output of one unit antenna and comparing it with the static output of the six antennas combined as one MUSA branch. The circuit shown in Fig. 35 was used for these experiments. The results are tabulated in Table VIII.

TABLE VIII

Date	GMT	f_{mc}	Type of Static	Addition Max. db	Addition Min. db	Thermal Noise db	QRM	Method
1935								
9–19	1530	18.6	star	8.5		−12	light	Rdg. DB Meter
10–15	1500	9.51	star	8.0				Rdg. DB Meter
10–16		9.51	distant	7.5		− 6		Rdg. DB Meter
10–22	1500	9.51	crash	8		−20	none	Vary I-F gain
10–23	1500	9.51	distant	8		− 9	light	Vary I-F gain
	1820	11.86	distant	8.5			light	Vary I-F gain
10–24	1500	9.51	star	11.4	5.4	−12	light	Vary I-F gain
	1510	9.51	star	11.0	6.0	−12	light	Vary I-F gain
	2045	9.51	crash	7.5		−30	light	Vary I-F gain
11–1	1450	9.51	distant	9.0		− 8	none	Vary I-F gain
	1830	9.51	distant	8.0		− 7	—	Vary I-F gain
1936								
1–7	1500	9.51	distant	7.5		− 8	light	Vary I-F gain
	1505	9.51	distant	8.0		− 8	light	Vary I-F gain
1–14	0300	4.82	crash	8.2	7.3	−20	none	Fluxmeter
1–15	0300	4.82	crash	6.8	3.0	−30	none	Fluxmeter
		Average		8.0				

The column headed f_{mc} indicates the frequency to which the receiver was tuned. The MUSA was tuned to a desired station which possessed a comparatively clear channel, and, following the sign-off of the station, static was measured without a demodulating carrier. The receiver was, of course, operated with manual instead of automatic gain control. Care was taken to insure that overloading did not occur. The column headed "addition" gives the ratio in decibels of the static output of the six-antenna branch to that of one antenna. Where figures are entered in the middle of the column no effect of steering was noticed. (Effects of the order of one decibel could have been overlooked, however.) The column headed thermal noise gives the ratio in decibels of receiver noise originating in one of the first circuits (measured with a resistance replacing the antenna) to the total noise measured with the unit antenna connected. It shows that thermal noise was negligible.

The star static [32] was steady and therefore accurately measurable. The crash static on 4.82 megacycles was so intermittent that it required the use of the fluxmeter to obtain a satisfactory measurement.

[32] K. G. Jansky, "Electrical Disturbances Apparently of Extraterrestrial Origin," *Proc. I. R. E.*, vol. 21, pp. 1387–1398, October, 1935.

The average of all those measurements showing no effect of steering is 8.0 decibels compared with the theoretical figure of 10 log 6 = 7.8 decibels, for random addition. The random assumption employed in the analysis is thus justified on the average.

Summarizing Discussion

The aim of the signal-to-noise study described in this section has been not so much to evaluate the intrinsic merit of the experimental MUSA system as to compare its behavior with a simple theory. The element of research has been to find out how well the transatlantic waves fit into the background of the simple theory. To this end somewhat artificial devices, thermal noise and rectified carrier, were substituted for static and speech signals.

The study included an analysis of diversity action in which the effect upon the signal-to-noise ratio of (1) delay equalization, (2) detector characteristics, and (3) automatic gain control action was displayed prominently. Although those effects, taken together, are important, they are individually small and could be separately evaluated only by locally controlled test methods.

We propose now to review the results of the tests and studies. The slight decrease in improvement which would have occurred had the speed of the automatic gain control been reduced, and the increase in improvement which would have resulted had noise *power* been averaged do not affect the fundamental considerations and will be neglected here.

One wave bundle received with one branch (Table V) yielded an improvement of 7 decibels which is about one decibel less than 10 log N = 7.8 decibels calculated by simple theory (Fig. 34, Case I). Comments relating to this have been given.

One wave bundle received with two branches steered on each side of the center of the bundle yielded eight decibels improvement (Table V). Of this, one decibel is due to primary diversity action and three decibels are due to the secondary diversity gain. This three-decibel gain was assumed in the simple theory although it was not then designated as secondary diversity gain. It accrues by virtue of voltage addition of the signals and power addition of the noise. Both of these conditions are satisfied in Table V. There remain four decibels which represent the signal-to-noise improvement in each branch, referred to one antenna. This indicates a loss of three decibels as compared with a single branch steered at the center of the bundle, which gives seven decibels; this is reasonable when it is remembered that the branches were steered apart by a phase difference of about 60 degrees ($\phi_A - \phi_B$

= 60°). A loss of three decibels is about what one would estimate upon inspecting the directional pattern for $\phi = 60$ and 120, say, on Fig. 20. The procedure in which two branches are steered at one bundle as in the above is frequently employed and is an important factor in the operation of a MUSA.

The case of two wave bundles tabulated in Table V also yields an improvement of 7.5 to 8 decibels. Of this, one decibel and three decibels are due to primary and secondary diversity action, respectively, as in the case of one bundle. This leaves 3.5 to 4 decibels for the signal-to-noise improvement in each branch referred to a unit antenna. But the unit antenna has the advantage of two bundles, whereas the MUSA branch excludes one of them, a three-decibel difference. In comparison with a unit antenna receiving only one bundle, the improvement to be ascribed to one branch thus is increased to 6.5 or 7 decibels. This result compares favorably with the seven decibels yielded by one branch steered at one bundle. It is in this case of two bundles that square-law detectors are most important. Their advantage, amounting to an estimated one decibel, has already been included in Table V, it will be remembered.

The measurements which have permitted the above analysis of the MUSA signal-to-noise improvement were of course supplemented by aural observations made over the course of a year and a half. The listening tests corroborate the analytical results as well as can be expected of such observations. Not infrequently they showed somewhat less than the full eight-decibel improvement. The indications are, however, that a larger MUSA with three (or possibly four) branches would have yielded more nearly its full gain of 10 log N decibels. For, a MUSA receiving system does not perform its functions properly unless it is sharp enough to separate the waves sufficiently to permit effective delay equalization; also, to obtain the full gain, enough branches must be provided to utilize all of the important wave bundles. The Holmdel experimental MUSA is really a conservative approach to the field of steerable directivity. There is, of course, an upper limit to the size of a MUSA, beyond which (1) technical difficulties in phasing, etc., will occur, (2) the cost of the improvement may be less if introduced at the transmitter, and (3) the directional sharpness becomes too great to permit practical operation with waves of the stability encountered in transatlantic transmission. At present, a system about three times the length of the experimental MUSA comprising eighteen antennas and equipped with three branches seems practical. It should yield an improvement of 10 log 18 = 12.5

decibels more consistently than the present MUSA yields eight decibels.

It may be worth while here to point out that as the number of antennas in a MUSA system is increased there is no tendency for static to become subordinate to thermal noise (set noise) or vice versa when static, like thermal noise, adds on a power basis. Only to the extent that transmission-line loss increases with the number of antennas will the ratio of thermal noise to static increase.

A type of transmission sometimes occurs for which the experimental MUSA gives only small signal-to-noise improvement. We refer to the highly scattered propagation associated with flutter fading, discussed at the close of Section IV. In such cases signal-to-noise improvement is not highly significant, however, since at least in the worst cases, the distortion renders the circuit worthless. Thus, increasing the transmitting power is likewise ineffective. On the other hand the experimental MUSA can accomplish something by rejecting some of the scattered waves which appear to be responsible for the flutter fading. This is accomplished without a corresponding *loss* of signal-to-noise ratio since, of course, noise is rejected, too. Fortunately, flutter fading does not seem to be associated prominently with greatly depressed field intensity so the failure to secure signal-to-noise improvement with flutter fading does not appreciably penalize the MUSA as a means of extending operation through periods of depressed field conditions.

VI. Recapitulation

The MUSA receiving system described in this paper is the culmination of some four years effort to determine the extent to which receiving antenna directivity may be carried to increase the reliability of short-wave transatlantic telephone circuits.[33] Fundamental experimental studies of wave propagation were made with particular emphasis upon how the waves arrive. Based upon the results of these studies a system was evolved in which a new technique of phasings was required. The result is a steerable antenna whose signal-to-noise advantage is seven to eight decibels compared with the largest fixed antenna that can be employed effectively. By analyzing this improvement and comparing the various contributing factors with theory, it is possible to estimate that a system three times larger than the experimental one will yield an additional four to five decibels, and will perform better consistently. In addition to the signal-to-noise improvement a

[33] Potter and Peterson, "The Reliability of Short-Wave Radio Telephone Circuits," *Bell Sys. Tech. Jour.*, vol. 15, pp. 181–196, April, 1936.

substantial improvement in quality is obtained by reducing the distortion associated with selective fading. It is both interesting and important to note that whereas so often one advantage is gained only at the expense of another, in the MUSA system the best quality improvement and the greatest signal-to-noise advantage are obtained together, without compromising.

The system developed is expensive and might be thought to illustrate the law of diminishing returns. As a part of a point-to-point radiotelephone system, however, it has certain compensating features not mentioned thus far. One of these is the broad frequency band feature.

With essentially aperiodic unit antennas the MUSA possesses a broad frequency range; i.e., the directional pattern, despite its sharpness, is substantially the same over a band of a hundred or more kilocycles provided the terminal equipment is made sufficiently broad. (See Appendix I.) The broad-band feature is important for its possibilities in multiplexed operation of telephone circuits; i.e., it makes possible, insofar as the antenna system is concerned, the adaptation of some of the carrier telephone methods to radio circuits. It is to be expected that, excepting certain critical cases, fairly large percentage frequency bands will follow virtually the same paths. This assumption was verified by a few experiments in which pulses were received simultaneously from GBS (Rugby, 12,150 kilocycles) and GBU (Rugby, 12,290 kilocycles) 140 kilocycles apart. These tests showed that, although the pulse fading was, of course, not synchronous, the angles involved were alike.

Another compensating feature of the MUSA receiving system is that, with suitable terminal equipment, reception may be carried on from several points at once provided they lie within the horizontal angular range of the unit antenna. Some sacrifice in vertical angular selectivity occurs but this is confined to low angles where it is least important.

Certain features of the system make for economies in plant cost. The fact that a great many components are identical permits manufacturing economies. Also, spare units need be provided only for a few vital functions, since the failure of one of the many similar parts does not disrupt service.

The development of steerable directivity has thus far been concerned with receiving antennas. In receiving, one has the obvious advantage of having, in the monitoring branch, a criterion to dictate the steering adjustments. The lack of such a direct criterion for adjusting transmitting directivity does not, however, rule out the possibility, at some future time, of a MUSA transmitting system. That horizontal steering of transmitting directivity may be decidedly important is strongly suggested by observations made on transmissions from Daventry in which significant effects upon flutter fading have been found to be associated with the orientation of the directional transmitting antennas.

Acknowledgment

The experiments described in this paper necessarily involved the coordinated effort of many individuals, in both the British Post Office and the Bell System, and their help has been appreciated. Mr. E. Bruce had charge of the design of the rhombic antennas and transmission lines, and Messrs. L. R. Lowry and W. M. Sharpless had important parts in the various phases of the work.

The authors are particularly indebted to Mr. R. K. Potter who contributed much through his keen interest throughout the entire work.

Appendix I

Broad-Band Characteristic of the MUSA

The frequency characteristic of the MUSA may be calculated from (3). Frequency and angle appear only in the form $2\pi a(\upsilon - \cos\delta)$ where a is inversely proportional to frequency. By writing the equation

$$\frac{2\pi Df}{c}[\upsilon - \cos\delta] = \frac{2\pi D(f + \Delta f)}{c}[\upsilon - \cos(\delta + \Delta\delta)]$$

we express the angular shift, from δ to $(\delta + \Delta\delta)$, of a given point on the directional pattern as the frequency is varied from f to $(f + \Delta f)$. This equation may be rewritten as

$$1 + \frac{\Delta f}{f} = \frac{\upsilon - \cos\delta}{\upsilon - \cos(\delta + \Delta\delta)}.$$

As an example consider $\Delta f = 200$ kilocycles, $f = 10$ megacycles, $\delta = 30$ degrees, and $\upsilon = 1.05$. Then $\Delta\delta = -0.4$ degree. For lower values of δ, $\Delta\delta$ becomes still smaller.

The frequency characteristic expressed in terms of percentage band and angular shift given by the above equation is independent of the size of the MUSA. It relates to the over-all length of the system, however, by the fact that for greater lengths a given angular shift has more effect.

The broad band of the MUSA reflects the fact that, with the terminal at the "leeward" end as assumed heretofore, the delay of the space paths is nearly the same as that of the transmission-line paths so that if the antenna outputs are phased to add at one frequency they will nearly add at other frequencies. If the terminal is located at the center of the MUSA to economize on transmission line, the frequency range is greatly reduced. The broad band may be regained, however, by delays introduced in the receiving equipment. With a center location, the antennas in the forward and rearward sections of the MUSA must have their phases shifted oppositely, and, unless certain other compensating networks are provided, the two phase shifts must be coupled in different phase relations for different wave-lengths.