Equivalent Modulator Circuits

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Equivalent modulator circuits are developed in the form of linear resistance networks. They are equivalent in the sense that the current magnitude in any mesh of the network is equal to the current amplitude of a corresponding frequency component in the modulator. The elements of the network are determined by the properties of the modulator, while the terminating resistances are those physically existent in the connected circuit.

With this correspondence demonstrated, the operating features of the modulator may be deduced from the known properties of linear networks. Among the properties considered are the transfer efficiency from signal to sideband, and the input resistance to signal as affected by the sideband load resistance.

Equivalent networks are worked out for a number of interesting cases, involving different impedances to unwanted modulation products, together with different non-linear characteristics. The equivalents come out comparatively simple in form under the restrictions noted and followed in the text, which make the carrier large compared to the signal, and the circuit elements purely resistive.

CONSIDERED from the circuit standpoint, a number of modulator performance features are important in any application. Among these features might be mentioned the efficiency of power transfer from signal input to sideband output, and associated with it the question of how the signal input energy is distributed among the different frequency components and dissipated in the modulator itself. Then, too, we need to know how the impedance of the modulator to any component depends upon the modulator structure and upon the connected impedances to other products.

In attempting to get answers to these questions by mathematical analysis, we encounter lengthy and cumbersome expressions in general which do not lend themselves to ready physical interpretation. The physical interpretation of these equations may be facilitated by introducing equivalent circuits of familiar form. One form commonly used in the past replaces the non-linear system by a circuit including a series of generators and linear impedances.

This may be illustrated by reference to a simple non-linear circuit, in which carrier and signal generators are connected through an external resistance to a two-terminal non-linear element such as a diode, or a copper oxide rectifier. The effects of non-linearity show up in the change of modulator resistance with changes in applied potentials and in the appearance of new frequency components. These effects may be reproduced quantitatively if we replace the non-linear element by its equivalent consisting of a linear internal resistance together with a series of internal generators—indicated at the right of the dashed line of Fig. 1-*A*. It is easy to see from this

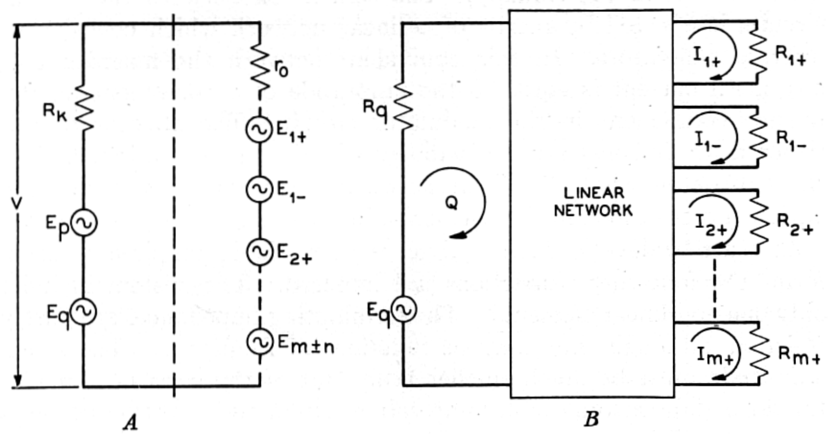

Fig. 1—(*A*) Equivalent modulator circuit in which the modulator is replaced by a fixed internal resistance together with a series of generators. (*B*) Equivalent modulator circuit replacing the modulator by a network of linear elements which serves to couple the signal circuit into the paths followed by the modulation products. In this circuit the mesh currents represent the amplitudes of the various frequency components.

circuit [1] what the amplitude of any current component should be; for the general component

$$I_{m\pm n} = E_{m\pm n}/(r_0 + R_{m\pm n}).$$

Despite the apparent simplicity of this relation, a difficulty arises as soon as we attempt to state the internal generator e.m.f.'s explicitly, since they are found to be tied up with the impressed potentials, the

[1] Here we denote the carrier frequency by $p/2\Pi$ and the signal frequency by $q/2\Pi$; the corresponding generator potentials are E_p and E_q respectively, and the external resistance is R_k where k indicates the frequency at which the resistance is effective.

The new frequencies are usually made up of sums and differences of integral multiples of carrier and signal frequencies. In general, they may be represented by $(mp \pm nq)/2\Pi$, where m and n are integers or zero. It is advantageous to adopt an abbreviated notation for the voltage component, say, of any frequency by which the general component is indicated as E_{m+n}. Further when n is unity it is omitted from the subscript, so that the generator e.m.f. of frequency $(mp \pm q)/2\Pi$ is indicated as E_{m+}. One of the restrictions mentioned further on results in limiting n to unity.

modulator characteristics, and the external circuit impedances. For this reason the equivalent circuit of Fig. 1-*A* reveals only part of the story, and in general the relation between the amplitudes of impressed and generated components remains somewhat obscured.

In a number of cases of practical interest it is possible to represent the connection between the amplitudes of various frequency components by means of a different type of equivalent circuit. Figure 1-*B* is an illustration of this type, in which the paths of the current components are shown individually. The connection between the various circuits is effected by means of a linear network which contains no internal generators. In this equivalent network the magnitude of any mesh current is equal to the amplitude of a corresponding frequency component in the modulator circuit. The purpose of this paper is to demonstrate the validity of this representation, and to show in detail what the linear network looks like when applied to various types of modulating elements, in a variety of interesting cases.

In order to develop such equivalent networks in simple and useful form, the following restrictions are imposed. The system includes only one non-linear element.[2] The terminating impedances are purely resistive, although they may be functions of frequency. The signal amplitude must be much smaller than that of the carrier. Finally, the slope of the modulator current-voltage characteristic never becomes negative. Under these conditions a number of modulating systems can be treated, including variable resistance modulators with a variety of current-voltage characteristics, and the variable resistance microphone.

The section following deals with the modulator as a resistance (or conductance) varying at carrier frequency. Succeeding sections consider the behavior of such variable elements under different circuit conditions.

I. Carrier Controlled Resistance

In setting up equations from which the equivalent networks are obtained, the restriction on signal amplitude permits us to assume the modulator to be a resistance or conductance varying at carrier frequency. This commonly used assumption may be arrived at with the aid of Fig. 2, which shows a typical non-linear current-voltage

[2] Other cases are to be found in a paper by R. S. Caruthers on "Copper Oxide Modulators in Carrier Telephone Systems," presented at the A.I.E.E. Winter Convention, January 1939.

Modulators including a plurality of elements can frequently be replaced by an equivalent structure with a single modulating element. This is true of the rectifier type of modulator. In the double-balanced or ring type, however, under certain conditions the equivalent circuit involves merely an ideal transformer connecting signal and sideband circuits.

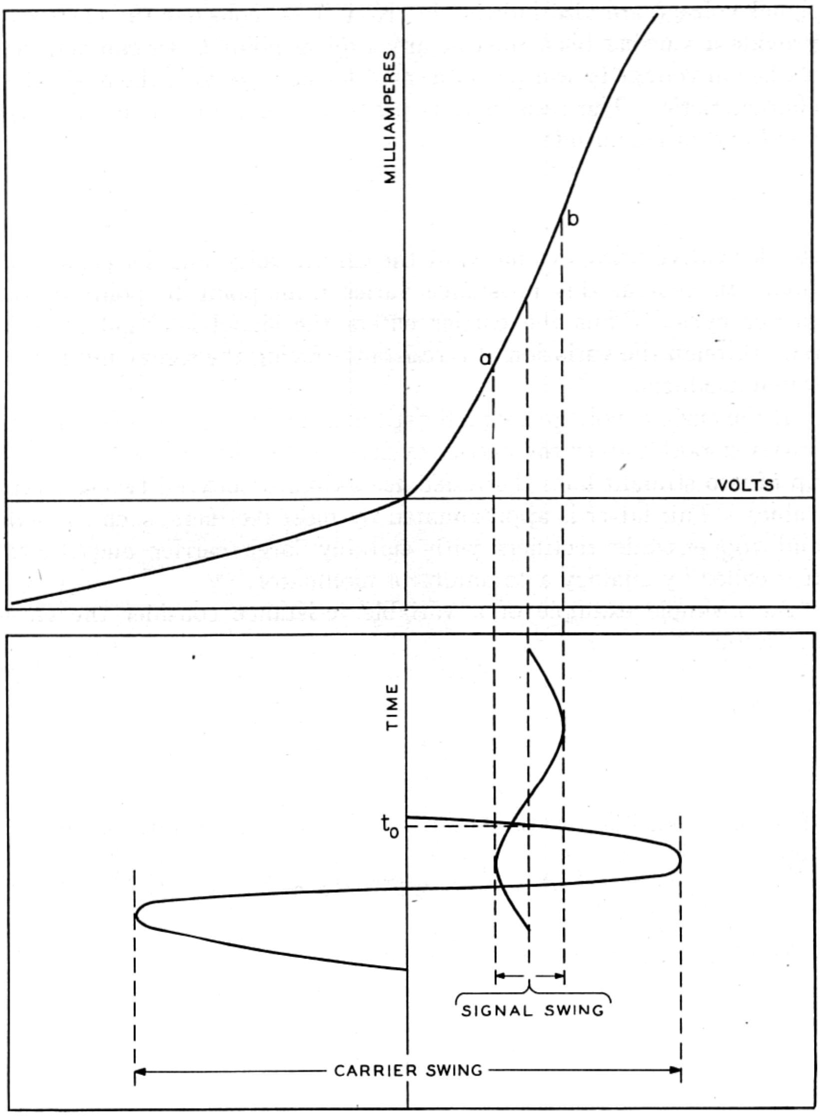

Fig. 2—Non-linear current-voltage relation representative of certain types of modulators. The impressed carrier voltage is large in comparison to the signal, the variation of which is represented in the neighborhood of the potential reached by the carrier wave at time t_0.

curve. The variation with time of a large carrier voltage and a small signal voltage are also indicated. Now if we consider the carrier to provide a varying bias, then at any typical point t_0 we can consider the signal voltage to sweep over a small segment (a, b) of the modulator characteristic. The resistance is practically constant over this segment and of magnitude

$$R = \frac{dv}{di}, \tag{1}$$

the derivative being evaluated at the carrier voltage under consideration. In general this resistance varies from point to point of the carrier cycle. Thus the carrier enters the signal-sideband relation only through the variation of a resistance facing the signal and modulation products.

If the current-voltage characteristic is a smooth curve the resistance varies smoothly over the carrier cycle. If the characteristic is made up of two straight lines the resistance switches between two constant values. This latter is approximated by most rectifiers, such as diode and copper-oxide rectifiers with suitably large carrier amplitudes; it is called by analogy a commutator modulator.

As a simple example of a variable resistance consider the characteristic

$$i = av + bv^2, \tag{2}$$

from which

$$1/R = \frac{di}{dv} = a + 2bv. \tag{3}$$

If the impressed carrier potential, v, is $P \cos pt$, the conductance is

$$G = \frac{1}{R} = a + 2bP \cos pt. \tag{4}$$

More generally, the resistance or conductance for a given characteristic can be expressed as a series

$$R = r_0 + \sum_1^\infty 2r_n \cos npt, \tag{5a}$$

or

$$G = g_0 + \sum_1^\infty 2g_n \cos npt. \tag{5b}$$

Here the coefficients depend only on the modulator characteristic and the carrier amplitude. In special cases some of the coefficients vanish.

Thus an expansion for the linear rectifier includes only those coefficients for which n is odd, whereas one for a modulator exhibiting odd symmetry in its current-voltage relation, such as thyrite, includes only coefficients for which n is even.

The choice between (5*a*) and (5*b*) in any given case is usually a matter of convenience.[3] This will be made clear by the forthcoming examples. In every case use will be made of Ohm's law in one of the two forms

$$v = Ri$$

or

$$i = Gv.$$

For simplicity, we select the relation which leads to the smallest number of terms in the expansion. Thus if, from the form of the terminating impedance, we know that i involves only a small number of significant frequency components, whereas the voltage involves a large number, (5*a*) will be used. If the potential, v, across the modulating element is known to be the simpler, (5*b*) will be used.

In the practical application of modulators to carrier systems the impedance characteristics of the connected selective circuits for taking out the desired sideband energy provide, to a good approximation, just such simplification. Thus a filter is substantially resistive in its pass band and the suppression regions may be designed to have either a very high or a very low impedance. If very high, no currents flow in these frequency regions and (5*a*) applies; if very low no potentials appear across it in these frequency regions so that (5*b*) applies.

II. Single Sideband—High Impedance Outside Band

We will first consider a single sideband modulator involving any variable resistance which can be expressed in the form (5*a*). The terminating resistance is R_q to signal and R_{1+} to the upper second order sideband. Because of the high terminating impedance which we assume to all other products, all current components other than signal (Q) and sideband (I_{1+}) are negligibly small.

The total current flowing in the circuit is then

$$i = Q \cos qt + I_{1+} \cos (p + q)t. \tag{6}$$

The potential across the non-linear element ($v = Ri$) is obtained from (5*a*) and (6) as

$$v = \left[r_0 + \sum_1^\infty 2r_n \cos npt \right] [Q \cos qt + I_{1+} \cos (p + q)t]. \tag{7}$$

[3] Except for those cases in which the occurrence of an infinity in any one of these two quantities prohibits its use.

After multiplying, and separating out the different frequency components, each frequency component of v is equated to the corresponding terminating generator e.m.f. minus the potential drop across the external impedance. Carrying out this process for signal and sideband, respectively,

$$\begin{aligned} E_q &= (R_q + r_0)Q + r_1 I_{1+}, \\ 0 &= r_1 Q + (R_{1+} + r_0) I_{1+}. \end{aligned} \tag{8}$$

If Q and I_{1+} are considered as mesh current amplitudes in a simple linear circuit, it is evident that r_1 represents a mutual resistance, and that Fig. 3 represents an equivalent network. In this system the

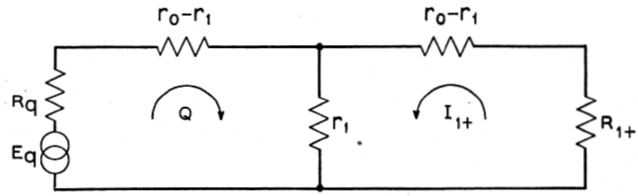

Fig. 3—Equivalent modulator network connecting signal and sideband when other modulation current components are suppressed by high circuit impedances.

signal source is connected to the sideband load by a simple T network.[4] It will be found in subsequent cases similarly, that the connection between signal and sideband circuits may be effected by a network comparatively simple in form. Hence we can make our deductions concerning the performance of modulator circuits by reference to the well known properties of such equivalent networks. In the present case, for example, we can draw the following conclusions.

1. The modulator loss becomes negligibly small if the series arm resistance, $r_0 - r_1$, is very small and the shunt arm resistance, r_1, is relatively large.
2. Considering the modulator network as fixed, maximum power is transferred when the signal and sideband resistances match the characteristic resistance of the network, so that

$$R_q = R_{1+} = \sqrt{r_0^2 - r_1^2}. \tag{9}$$

3. Under matched impedance conditions the power efficiency is

$$\eta = \left[\frac{r_1}{r_0 + \sqrt{r_0^2 - r_1^2}} \right]^2. \tag{10}$$

[4] While the results come out most simply in terms of a T network, the various possible transformations (for example to a π or to a lattice network) are of course equally valid.

The term *power efficiency*—as used here—means the ratio of the power delivered to the load resistance (R_{1+}) to that introduced at the input side of the network by the signal source. The corresponding current ratio of sideband to signal is the square root of η. If the sideband resistance is shorted the current ratio rises to its maximum. The ratio of voltage at the network output to that at the network input when the load resistance is made infinite coincides with this value (r_1/r_0).

If we consider the various possible kinds of resistance variation with time under the restrictions noted, it appears that the greatest attainable value of the power ratio is unity. It is evident from the equivalent circuit that this limit corresponds to no loss from signal to sideband. The closest approach to this no-loss condition is obtained in a modulating element presenting a resistance which, over a carrier cycle, varies between widely different resistances, taking on one extreme value for a small fraction of the cycle and remaining near the other extreme for the remainder of the cycle. Under these conditions the series arm of the equivalent net tends to zero, the shunt arm to infinity. There are practical limitations to the extent to which these conditions can be approached in practical modulators. For example the best attainable values of the two resistance extremes usually depend upon the modulator characteristic, and upon the carrier amplitude employed which may be limited by heat dissipation or by voltage breakdown. Further limitation is imposed by parasitic capacitances, which effectively limit the maximum attainable modulator resistance.

The commutator modulator may be used to illustrate the results of analysis above. Figure 4 shows the variation of the resistance of

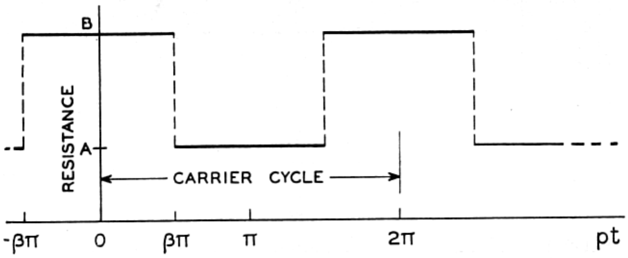

Fig. 4—Variation of resistance with time in a commutator modulator, in which the resistance is switched from A to B, remaining at the higher value B for the fraction β of the carrier cycle.

such a modulator over a carrier cycle. B and A are the two values of the resistance ($B > A$) and β is the fraction of the carrier cycle over which the resistance is B. The coefficients of the resistance expansion are readily shown to be

$$r_0 = \beta B + (1 - \beta)A, \tag{11}$$

$$r_k = (B - A)\frac{\sin k\beta\pi}{k\pi}. \tag{12}$$

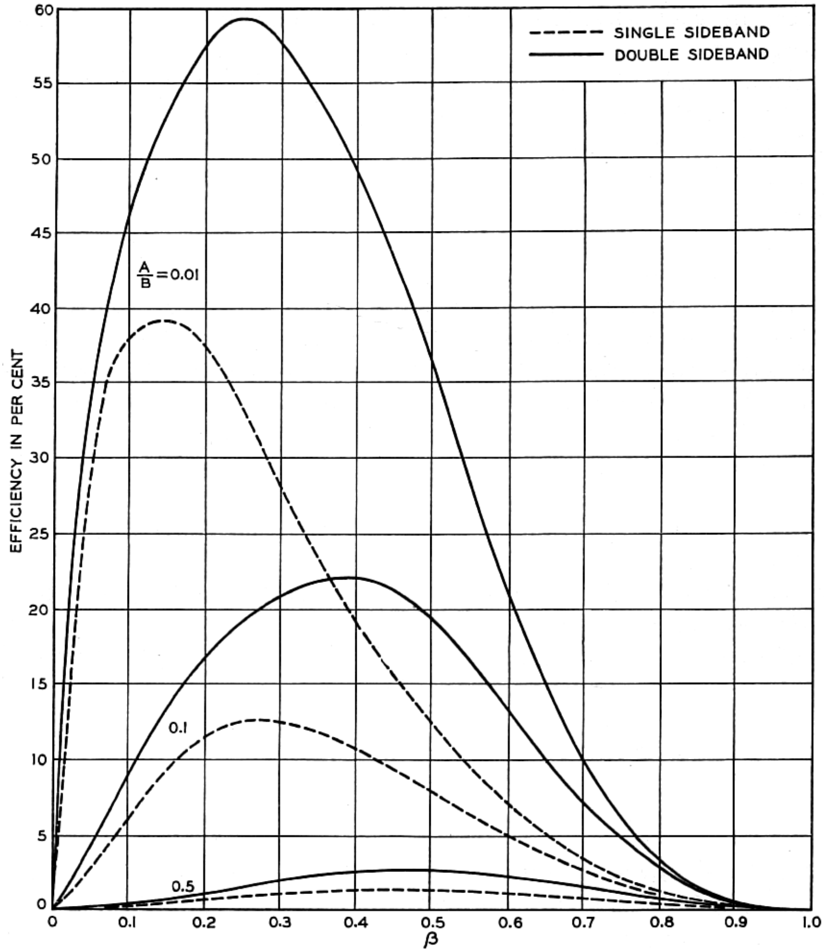

Fig. 5—Efficiency of a commutator modulator shown as a function of the pulse fraction β with the ratio of the two commutator resistances as parameter. Here the resistance terminations to signal and to sideband are optimum and the circuit impedance is made high to unwanted components. Full line applies to double sideband output, dashed line to single sideband output.

The efficiency (with optimum termination) depends only on β and the ratio A/B. The dashed curves on Fig. 5 show the variation of efficiency with these parameters. It is evident from the equivalent circuit that best efficiency is obtained with $r_0 - r_1$ small and r_1 large. In this case r_0 increases linearly with β, and r_1 varies sinusoidally with β—having its maximum at $\beta = 0.5$. The immediate conclusion is that the optimum conditions are obtained when β is less than 0.5. In fact the efficiency approaches the limiting value of 100 per cent when β is very small and B is much greater than A, as may be seen from (11) and (12).

III. Double Sideband—High Impedance Outside Band

In double sideband operation both upper and lower sideband currents flow, but all other modulation products are suppressed as in the previous case. Here the equations for signal and upper and lower sideband respectively are

$$\begin{aligned} (r_0 + R_q)Q + r_1 I_{1+} + r_1 I_{1-} &= E_q, \\ r_1 Q + (r_0 + R_{1+})I_{1+} + r_2 I_{1-} &= 0, \\ r_1 Q + r_2 I_{1+} + (r_0 + R_{1-})I_{1-} &= 0. \end{aligned} \tag{13}$$

Comparing (13) with the equations for a three-mesh circuit, we obtain the equivalent network of Fig. 6. It is obvious from the symmetry of this network that the two sidebands are equal when $R_{1+} = R_{1-}$. Conditions for optimum efficiency may be put in form permitting convenient comparison with the single sideband case when we assume equal resistances to both sidebands.

Efficiency curves of a commutator modulator are shown on Fig. 5 for both single and double sideband cases. They differ primarily in that the utilization of two sidebands gives greater efficiency, except in limiting cases. The outstanding difference is that the unsymmetric network has optimum signal and sideband resistances which are not equal except at three values of β equal to 0, 1/2 and 1. Modulators are often operated with β approximately 1/2, so that in this case the results here check with the common experience that the two terminating resistances should be equal. It may be remarked that only in highly efficient modulators would unequal terminations make an appreciable difference in the efficiency of power transfer.

A comparison of Figs. 3 and 6 gives some light on the difference in efficiency of the single and double sideband cases. The comparison is made when $R_{1+} = R_{1-}$. From the symmetry of the circuit of Fig. 6, I_{1+} then equals I_{1-} and the mutual resistance $(-r_2)$ may be eliminated, leaving a simple T network connecting the input and the load. This is, with two exceptions, the T network of Fig. 3 with all elements doubled in magnitude. The input series arm is decreased ($r_0 - 2r_1$ instead of $2r_0 - 2r_1$) and the output series arm is increased by an element $2r_2$. Since $2r_2$ is generally much smaller than r_0 there is a net gain in efficiency.

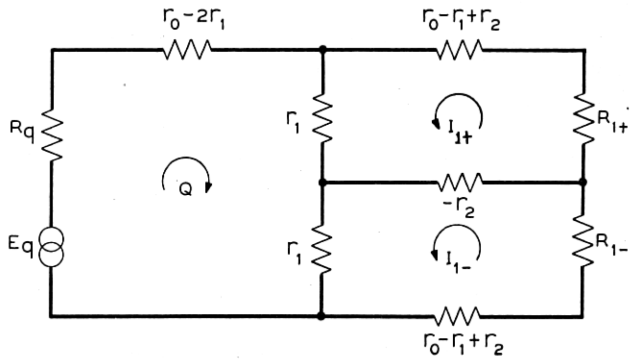

Fig. 6—Equivalent modulator circuit showing the connection between signal and two sideband circuits. All other modulation current components are suppressed by means of high circuit impedances.

IV. Low Impedance Outside Band

The foregoing systems involved high impedances, suppressing the flow of current at all but two or three frequencies. Circuits are as readily obtained in the case of low impedance to all but two or three of the modulation products. The physical systems this approximates are the same as those previously discussed except that the terminating filters must present a low impedance to frequencies outside the band.

All the external potential drops across the modulating element are taken as negligible except components at the signal and one or two sideband frequencies. The analysis, corresponding to that of the previous sections, uses Ohm's law in the form $i = Gv$. Thus this analysis, and the equivalent circuits, involve the expansion of a conductance instead of a resistance. The equations corresponding to (8) and (13) are equations in V_q, V_{1+} and V_{1-}. In the single sideband case a resulting equivalent circuit is that of Fig. 7. This is a simple symmetric π network. From its well known characteristics the optimum terminating conductance and maximum efficiency are immediately available:

$$G_{1+} = \frac{1}{R_{1+}} = \sqrt{g_0^2 - g_1^2}, \tag{14}$$

$$\eta = \left[\frac{g_1}{g_0 + \sqrt{g_0^2 - g_1^2}} \right]^2. \tag{15}$$

These expressions are identical in form with corresponding ones obtained for the high-impedance single-sideband case, in which conductance components replace resistance components. This con-

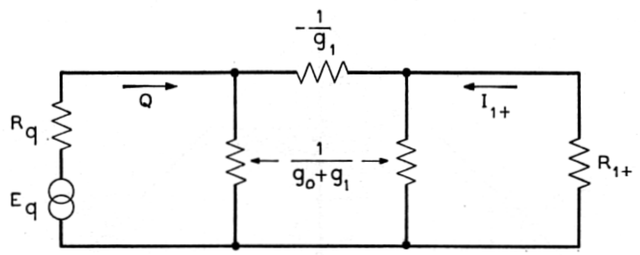

Fig. 7—Equivalent modulator circuit showing connection between signal and a single sideband. All other modulation voltage components are suppressed by means of low circuit impedances. The use of a Π network and the specification of element values as conductances are both matters of convenience.

firms what has been observed in several special cases: that there is no theoretical advantage of either impedance extreme over the other in the general case.[5] Which one to use in any particular case depends upon the special characteristics of the modulator or upon the practicability of obtaining required impedance conditions.

The equivalent network for the corresponding double sideband system is shown in Fig. 8. Similarly to the previous double sideband case, the symmetry shows that when $R_{1+} = R_{1-}$ the sideband current amplitudes are equal and there is no potential across the coupling resistance $-(1/g_2)$. Thus it may be shorted, reducing the circuit to a simple unsymmetric π. The matching resistances and maximum efficiency may be obtained as before.

The results again are identical with the high-impedance case, if resistances are replaced by conductances. The comment made on the single-sideband case still holds—that there is no general theoretical advantage of either a high- or low-impedance system over the other as far as maximum possible efficiency is concerned.

The curves of Fig. 5 are evidently immediately applicable to the low-impedance circuits provided all resistances are replaced by conductances.

There are, of course, practical advantages of the high- or the low-impedance circuit in particular cases. For example, it is commonly easier to make the terminating impedance to unwanted frequency

[5] The form of the equations in corresponding high- and low-impedance cases suggests that the impedance and efficiency relations for one case could be deduced from those of the other through the principle of duality. See Guillemin, "Communication Networks," Vol. 2.

components very small rather than very large. The impedance matching may be simpler in one case than in the other since, for the same efficiency, the matching resistances are quite different in the two cases.

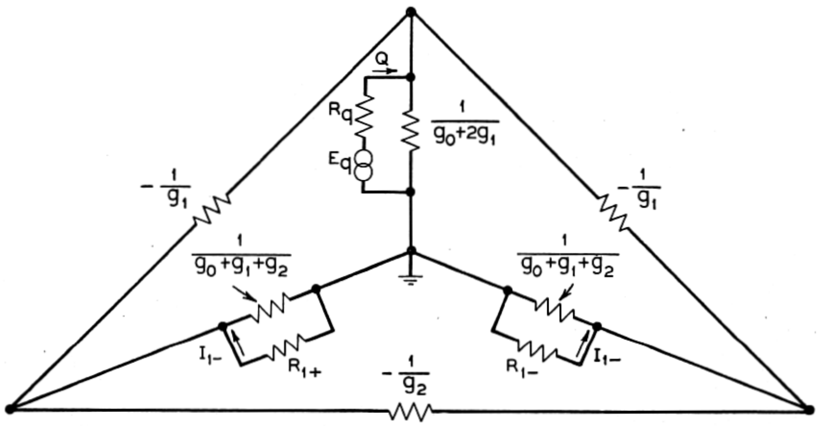

Fig. 8—Equivalent modulator circuit showing connection between signal and two sidebands. All other modulation voltage components are suppressed by low circuit impedances. When one of the sideband paths is shorted, the network reduces to that of Fig. 7.

V. Finite Resistance to Distortion Products

One more extension may be made, without excessively complicating the equivalent circuit. This is an extension to the case of a constant resistance R to all unwanted products, which yields information unobtainable from the limiting cases previously treated of $R = \infty$ and $R = 0$.

This problem can be handled in a simple way by the artifice of incorporating R within the modulator proper. In that case the external impedances to signal and sideband must be reduced by R to keep the total circuit resistance at its correct value, while the external resistance to any other modulation product then becomes zero. This brings the situation down to that considered and solved in the section immediately preceding.

By this manipulation the equivalent circuit is obtained as that of Fig. 9 in the single sideband case. The primes indicate coefficients in the expansion of the modified characteristic. These coefficients are immediately available in the case of the commutator, since the sole change there is an increase in both values of the variable resistance by the amount R. Comparing the efficiency of transformation with that obtained with extreme values of R as in the cases preceding, it appears that the use of extreme values of R results in the maximum obtainable power.

The impedance conditions specified above permit the flow of any generated products, so that an infinite network would be required in

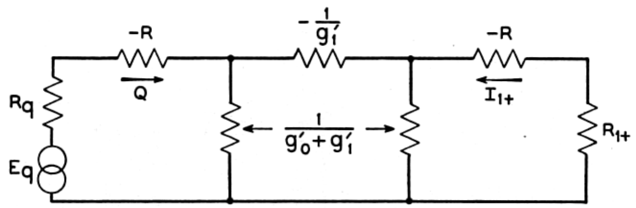

Fig. 9—Equivalent modulator circuit showing connection between signal and single-sideband circuits when a constant resistance R is effective to all other modulation products. The primed coefficients (g_k') apply to the conductance coefficients when the series resistance R is included in the original modulator.

general to bring out the relation of any one product to the others. Discussion was limited, however, to signal and upper sideband; the amplitudes of other components did not appear explicitly.

The section following deals with a different case involving an infinite network in which individual meshes are treated explicitly.

VI. Equivalent Network for the Idealized Resistance Microphone

Other variable resistance systems may be put into equivalent form. For example, from the electrical side, an idealized variable resistance microphone actuated by a sinusoidal acoustic wave can be represented by the resistance

$$r = R_0 + R \cos qt.$$

This is exactly the form of the variable resistance already discussed, with $r_0 = R_0$, $2r_1 = R$, and $r_n = 0$ for $n > 1$.

If one were interested in the modulation products with other frequency components impressed electrically, the systems would be of the same type as those of the previous section. In the case of the microphone the d-c. voltage impressed leads to current components which are d-c. and harmonics of the signal q. In this case, the equations are

$$\left.\begin{aligned} V_0 &= E_0 - R_bI_0 &&= R_0I_0 + RI_1/2, \\ V_q &= \quad\; - R_qI_1 &&= R_0I_1 + RI_0 + RI_2/2, \\ V_{2q} &= \quad\; - R_{2q}I_2 &&= R_0I_2 + R(I_1 + I_3)/2, \\ &\cdots \\ V_{nq} &= \quad\; - R_{nq}I_n &&= R_0I_n + R(I_{n-1} + I_{n+1})/2, \\ &\cdots \end{aligned}\right\} \tag{16}$$

in which R_b represents the external d-c. resistance, and R_{nq} represents the external resistance to the nth harmonic.

Figure 10 shows the equivalent circuit for this system in the form of an infinite ladder structure. From this circuit relative magnitudes

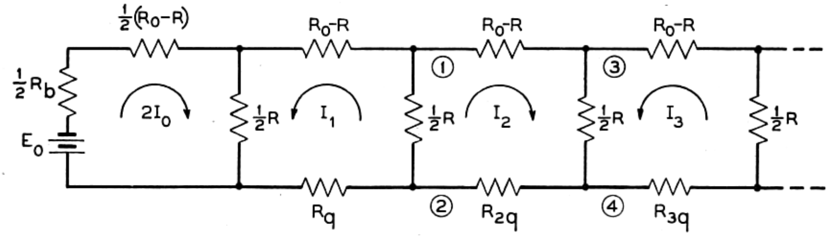

Fig. 10—Equivalent circuit of idealized variable resistance microphone. Here mesh currents represent amplitudes of the various frequency components. R_0 represents the fixed and R the variable internal microphone resistance, while R_{nq} represents the external circuit resistance to the nth harmonic of the signal.

of the various frequency components are readily perceived; evidently the successive harmonic current components decrease progressively in magnitude. A large value of R_{nq} makes the nth harmonic and all successive harmonics small. A case in which quantitative information is obtainable in simple form is that in which the resistances to all harmonics are equal ($R_{nq} = R_t$, $n > 1$). In this case the equivalent network beyond terminals (1) and (2) is a simple recurrent structure and the resistance is obtainable by the customary methods of handling infinite recurrent networks. The admittance looking in at the terminals (1, 2) is the iterative admittance of the latter and is given by [6]

$$\frac{1}{R_i} = \frac{2}{R} + \frac{1}{R_0 - R + R_t + R_i}. \tag{17}$$

Solving for R_i gives

$$R_i = \frac{R}{1 + \sqrt{1 + \dfrac{2R}{R_0 - R + R_t}}}.$$

As far as d-c. and fundamental current components are concerned, the network beyond (1), (2) may be replaced by R_i and the system reduces to a two-mesh circuit from which Q, I_0 and power relations are obtainable very simply. If harmonic current amplitudes are

[6] In an infinite recurrent structure, the resistance R_i must be the same looking in at successive point pairs (1, 2) (3, 4) etc. This is stated in (17).

desired, they may be obtained from equations (16), using the known values of I_0 and I_1.

The two-mesh circuit for I_0 and Q can immediately be put in the form of an unsymmetric T terminated at one end by the battery and its internal impedance and at the other end by R_q. The optimum terminating resistances and corresponding efficiencies are obtainable as in previous cases but it is evident from the network, without further computation, that losses are minimized (with suitable termination) if $R_0 - R$ and R_i are small compared with R. R_i is decreased by decreasing R_{nq} $(n > 1)$. These conditions mean that the best electrical efficiency is obtained when the resistance variation is large and the unwanted signal harmonics are short circuited.

VII. Extensions and Summary

Equivalent networks can be obtained in some cases when the restrictions on the relative amplitudes of signal and carrier are removed. It is evident from Fig. 2 that the value of the variable resistance at any instant then depends not only on the carrier amplitude but also on the signal amplitude. Thus the equivalent networks are no longer made up of constant resistances, but depend upon the magnitudes of both signal and sideband components. Further, new components appear involving multiples of the signal frequency. The equivalent for this case lacks the simplicity of those discussed here, a simplicity which appears when one of the two input components is much greater than the other.

The reason for the restriction to pure resistances becomes evident when one attempts to generalize the results. The current components will then have phase angles differing from zero in general. Consideration of lower sidebands then shows that the phase angles must have their signs reversed in certain circumstances, which leads to obvious complexities. Again in purely resistive circuits it is possible to determine the instantaneous current-voltage relation and hence to specify the resistance variation as a function of time. In a reactive circuit, however, additional difficulty arises in that the relations are much more complex and in general impossible to specify in simple terms.

To summarize, the presentation has been limited to the simplest circuits used for modulation by means of a variable resistance. In each example, the inter-relations between modulation product amplitudes, terminating resistances, and types of modulator characteristics are shown in terms of familiar linear resistance networks. From these, qualitative information concerning the properties of the system is more readily obtained than from the equations and, in some cases, the solutions for effective impedances and current and voltage amplitudes are obtainable without further recourse to the equations.

ACKNOWLEDGMENT

The writers are indebted to several of their associates in the Bell Telephone Laboratories for the use of unpublished material in this paper. In particular, acknowledgment is due to Mr. R. S. Caruthers, Mr. J. M. Manley, Dr. G. R. Stibitz, and Mr. R. O. Wise, who originally obtained some of the impedance and efficiency relations.