

## Dial Clutch of the Spring Type\*

By C. F. WIEBUSCH

The mathematical theory is developed for the spring clutch which consists of two coaxial cylinders placed end to end and coupled torsionally by a coil spring fitted over them. Relations are derived whereby it is possible to design spring clutches in terms of the requirements and the constants of the spring material. Experimental verification of the relations is given. The theory of residual and active stresses as applied to the springs is discussed.

THE operation of all present day machine switching telephone systems depends on the use of the telephone dial. The dial originates the current pulses required to operate the step-by-step, panel, or crossbar switching equipment and for the reliable functioning of this equipment the pulses must occur within a closely limited frequency range. The stepping pulses are produced during the unwinding of the dial from the position to which it has been wound by the subscriber and it is this unwinding which must occur at a constant speed. To accomplish the speed control a governor depending on centrifugal force is used. It is not desirable that the governor come into action on the windup of the dial as this would put an extra load on the user's finger and slow up the operation of dialing. A clutch which holds in one direction of rotation and is free in the other direction is therefore interposed between the governor and the finger wheel with its associated circuit interrupting mechanism. In the past, the most commonly used clutch consisted of a pawl and ratchet, but this has now been replaced by the spring clutch because of its quietness and lower cost. A partially assembled dial using a spring clutch is shown in Fig. 1.

The ideal clutch for a dial governor would be one offering zero coupling torque during dial windup and an infinite positive coupling in the other direction. In practice the free torque in the windup direction need only be small compared to the torque of the main spring, while the holding torque in the other direction need only be great enough to withstand the main spring torque plus any helping torque that a

\* Essentially the same material was presented at National Meeting of Applied Mechanics Division of The American Society of Mechanical Engineers, New York, N. Y., June 14-15, 1939, and published in *Journal of Applied Mechanics*, September 1939, under the title of "The Spring Clutch."

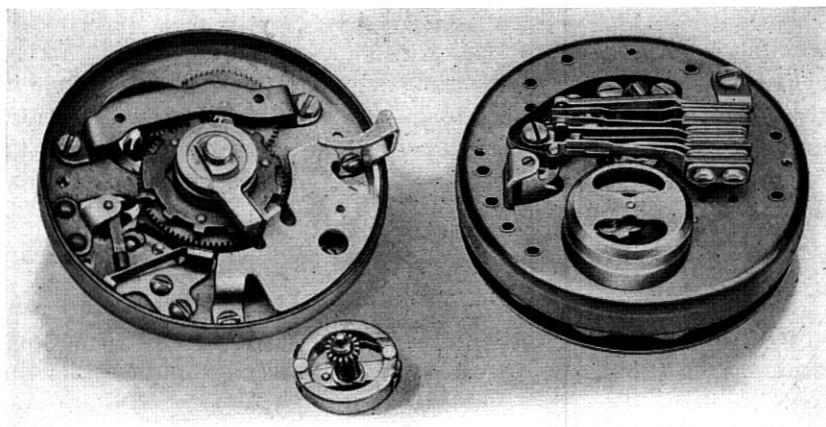


Fig. 1—Dial governor and front and back view of partially assembled dial.

subscriber may impatiently exert. The first limit is easy to set up on the basis of the mechanical constants of the dial; the fixing of the latter limit required measurements on the strength of a considerable number of persons. It was found that the maximum force that an ordinary man can exert with any finger at the finger hole of a dial is about six pounds. When the proper factors of safety have been added to these limits it is possible to specify exactly the requirements on the dial clutch. The remainder of this paper is devoted to the development of the relations and a discussion of the problems involved in designing a spring clutch to meet a given set of requirements.

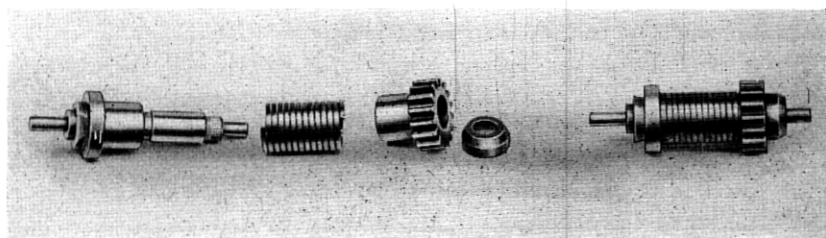


Fig. 2—Telephone-dial clutch.

The type of spring clutch to be discussed here consists of two cylinders placed end to end, rotating on a common axis, and torsionally coupled by the friction between the cylinders and a coil spring fitted over the cylinders. A photograph of such a clutch, assembled and apart, from a telephone dial is shown in Fig. 2. In spite of widespread use there seems to be little theoretical discussion of this device in the literature.

It is obvious that if the driving drum be rotated in the direction to wind up the spring and decrease the diameter, the spring will grip the cylinders and will be capable of exerting more torque than it would in the direction of rotation which tends to unwind the spring. Equations are to be developed which will permit the calculation of these two torque values in terms of the physical dimensions and the material constants of the clutch.

#### TORQUE OF SPRING CLUTCH IN THE FREE DIRECTION

In Fig. 3 assume that the spring is fastened to the left-hand arbor in order that any slipping which may take place must occur on the driving drum on the right. Assume also that the spring is so formed that the

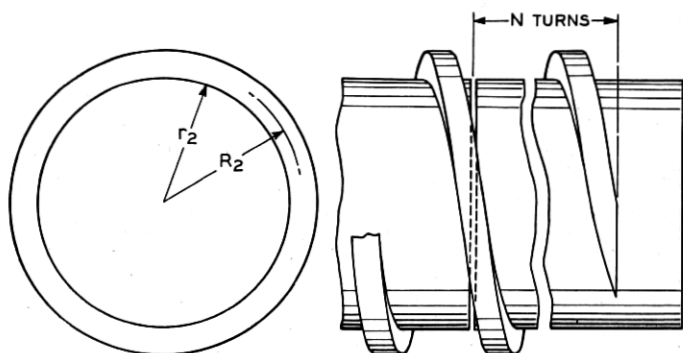


Fig. 3—Diagram of spring clutch.

inward radial force on the drum per unit length of the material is constant when no torque is applied.

#### NOTATION

- $l$  = length along the line of contact of the spring on the arbor, measured from the free end to any point, in.
- $\mu$  = coefficient of friction between the spring and the arbor
- $r_2$  = radius of the arbor, in.
- $R_2$  = radius to the neutral bending axis of the spring when on the arbor, in.
- $N$  = number of turns on the right-hand arbor
- $P$  = compression in the spring wire at any point due to the applied torque; this is not the stress in the material but the resultant force acting across the entire cross section of the wire, lb.
- $f_0$  = radial force of spring on arbor when no torque is applied, lb. per in. of contact line.

As compression exists in the spring wire at any point when the arbor is turned to make the spring unwind, there will be a radial force subtracting from  $f_0$  at every point. This subtracting force is  $P/r_2$ . The increase of compression in the wire along the length of the line of contact due to friction is

$$dP = \mu(f_0 - P/r_2)dl, \quad (1)$$

which upon integration gives

$$l = - (r_2/\mu) \ln [f_0 - (P/r_2)]C,$$

where  $C$  is a constant of integration equal to  $1/f_0$  since  $P = 0$  at  $l = 0$ . Hence

$$P = r_2 f_0 (1 - e^{-\mu l/r_2}). \quad (2)$$

Since  $l = 2\pi r_2 N$ ,

$$P = r_2 f_0 (1 - e^{-2\pi N \mu}). \quad (3)$$

Since the torque is equal to  $Pr_2$

$$T = r_2^2 f_0 (1 - e^{-2\pi N \mu}) \text{ (in.-lb.)}. \quad (4)$$

It will be observed that for any but fractional values of  $N\mu$  the exponential term becomes very small and

$$T = r_2^2 f_0 \quad (N\mu > 1). \quad (5)$$

If  $N\mu = 1.0$  this expression is in error by only 0.2 per cent. It can thus be seen that provided  $N\mu$  does not become too small, variations in  $N$  or  $\mu$  do not affect the torque exerted. The torque will depend only on the radius of the arbor and on the force  $f_0$  which is controlled entirely by the dimensions and the elastic properties of the spring.

#### TORQUE OF SPRING CLUTCH IN THE GRIPPING DIRECTION

If the torque is applied to the clutch in the direction to wind up the spring, instead of unwind it as in the previous case, the force  $P'/r_2$  due to the tension  $P'$  in the spring wire adds to the inward force  $f_0$  and the relation corresponding to equation (1) is

$$dP' = \mu(f_0 + P'/r_2)dl. \quad (6)$$

From which by the same method as before

$$P' = r_2 f_0 (e^{2\pi N \mu} - 1). \quad (7)$$

The corresponding torque is

$$T' = r_2^2 f_0 (e^{2\pi N \mu} - 1) \text{ (in.-lb.)}. \quad (8)$$



In this case the torque increases rapidly with an increase in either  $N$  or  $\mu$  especially for large values of  $N\mu$ . The coefficient of friction is in general a rather variable factor. It is to be expected that the slipping torque will also be variable, but since a lower limit can in general be set for this coefficient it will always be possible to make the number of turns of the spring such as to give any desired lower limit of torque.

#### THE RADIAL FORCE ON THE ARBOR

One method of evaluating the force  $f_0$  occurring in the torque relations depends on equating the potential energy of strain per unit length of the wire when on the arbor to the work done in expanding the spring from its free diameter to the diameter of the arbor.

Let

$E$  = Young's modulus for the spring material, psi

$I$  = the area moment of the wire section, in.<sup>4</sup>

$h$  = the radial thickness of the wire, in.

$R_1$  = free radius to the neutral axis, in.

$r_1$  = free inner radius of the spring, in.

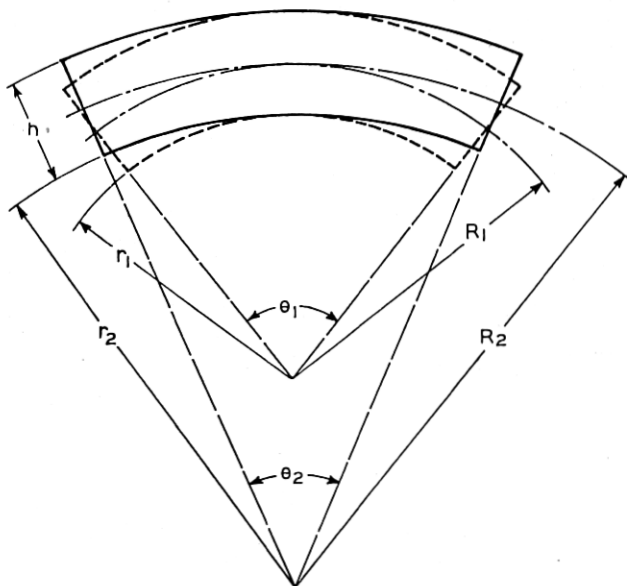


Fig. 4—Element of spring in initial and expanded condition.

Consider the portion of spring wire shown in Fig. 4 straightened out from the initial radius of curvature  $R_1$  to the radius  $R_2$ . The

fibers above the neutral axis will be compressed while those below the neutral axis will be stretched. Let  $y$  be the distance of any given fiber from the neutral axis. The length of the undistorted fiber will be  $L = (R_1 + y)\theta_1$  while after bending this same fiber will have the length  $L' = (R_2 + y)\theta_2$ . The strain or the change in length per unit length is

$$\frac{L - L'}{L} = 1 - \frac{(R_2 + y)\theta_2}{(R_1 + y)\theta_1}. \quad (9)$$

Since along the neutral axis there is no change in length,  $\theta_2 = L_0/R_2$  and  $\theta_1 = L_0/R_1$ . Substituting these values in equation (9) gives

$$\text{Strain} = (y/R_2)(R_2 - R_1)/(R_1 + y). \quad (10)$$

The potential energy per unit volume in a material strained in tension or compression is

$$W/V = (E/2)(\text{Strain})^2. \quad (11)$$

Substituting the value of strain from equation (10) in equation (11) the energy density at any point of the deflected wire will be

$$\frac{W}{V} = \frac{E}{2} \left[ \frac{y(R_2 - R_1)}{R_2(R_1 + y)} \right]^2. \quad (12)$$

Let  $b$  represent the width of the wire at the point  $y$ . Then for a wire symmetrical about the neutral axis the strain energy per unit length of the wire becomes

$$\frac{W}{l} = \int_{-h/2}^{h/2} \frac{E}{2} \left[ \frac{y(R_2 - R_1)}{R_2(R_1 + y)} \right]^2 \frac{R_1 + y}{R_1} b dy, \quad (13)$$

where the factor  $(R_1 + y)/R_1$  represents the ratio of the length of the fiber at the point  $y$  to the length along the neutral axis. If all values of  $y$ , and hence also  $h/2$ , are small compared to  $R_1$  there will be little error made in neglecting the  $y$ , which carries both positive and negative values, in the expression  $R_1 + y$ . Hence

$$\frac{W}{l} = \int_{-h/2}^{h/2} \frac{E}{2} \left( \frac{R_2 - R_1}{R_2 R_1} \right)^2 b y^2 dy. \quad (14)$$

The integral of  $b y^2 dy$  is the area moment  $I$  of the section and therefore

$$\frac{W}{l} = \frac{1}{2} EI \left( \frac{R_2 - R_1}{R_2 R_1} \right)^2. \quad (15)$$

This must be equal to the work done per unit length of the neutral

axis, by the force per unit length  $F(\Delta R)$  working through the distance  $\Delta R$  where  $\Delta R = R_2 - R_1$ . That is

$$\int_0^{\Delta R} F(\Delta R) d\Delta R = \frac{1}{2} EI \left[ \frac{\Delta R}{R_1(R_1 + \Delta R)} \right]^2. \quad (16)$$

Differentiating both sides with respect to  $\Delta R$  gives  $F(\Delta R)$  for the left-hand side, and after simplification

$$F(\Delta R) = \frac{EI}{R_1} \frac{\Delta R}{(R_1 + \Delta R)^3}. \quad (17)$$

Substituting for  $\Delta R$  its value  $R_2 - R_1$  gives

$$F(\Delta R) = EI(R_2 - R_1)/R_1 R_2^3. \quad (18)$$

The equivalent force per unit length measured along the surface of the arbor must be larger than this value in the ratio of  $R_2/r_2$  since the same total force is here distributed over a shorter length. This latter force is  $f_0$ ; hence

$$f_0 = \frac{R_2}{r_2} F(\Delta R) = EI \frac{R_2 - R_1}{R_1 r_2 R_2^2} \text{ lb. per in.} \quad (19)$$

The value of  $f_0$  calculated from this equation in terms of the constants of the spring material and the dimensions of the spring may be used in equations (4) and (8) to calculate the free torque and the slipping torque of the clutch.

#### EXPERIMENTAL CHECK OF THE FREE-TORQUE RELATION

In order to check the validity of the relation for the free torque, equation (4), and that for the radial force on the arbor, equation (19), the free torque of a given spring on arbors of various diameters as well as the torque for different numbers of turns on the same arbor was measured. The spring of  $0.0085 \times 0.022$ -in. phosphor-bronze ribbon was attached to a short vertical shaft suspended by a torsion fiber of measured torsional stiffness. The free end of the spring was placed over a vertical arbor capable of rotation. The arbor was rotated and the angle of twist of the torsion fiber was measured thus giving a measure of the slipping torque. The precision of the measurements of torque was about 0.5 per cent although the sensitivity to small changes was about 0.2 per cent. No measurable increase of torque occurred by increasing the number of turns on the arbor beyond six. This is to be expected if the coefficient of friction exceeds about 0.12.

For all succeeding measurements seven to eight turns were used. As a further check on the independence of the torque and the coefficient

of friction for this number of turns a measurement was made before and after oiling the arbor and spring with a light lubricating oil. There was a decrease in torque of approximately 0.25 per cent.

The inside diameter of the spring as measured by a taper gage was 0.180 in.  $\pm$  0.001 in. The torque for arbors ranging in size from 0.182 to 0.193 in. was determined and is shown in Fig. 5. This curve ex-

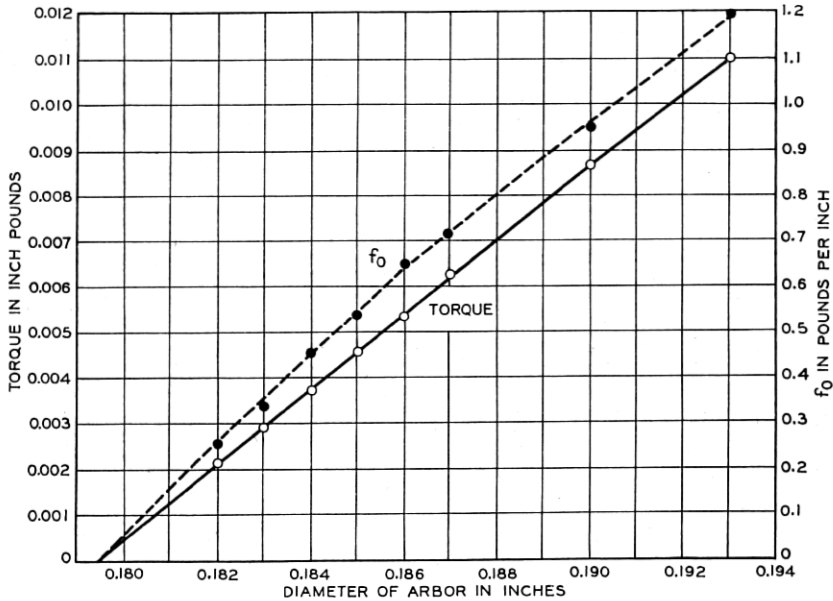


Fig. 5—The free torque and the radial force on the arbor for a phosphor-bronze spring on arbors of different diameters.

trapolated to zero torque gives, for an accurate measure of the inside diameter of the spring, 0.1794 in. Using this value and the quantity  $EI$ , determined by obtaining the resonant frequency of a straight short length of the ribbon, of which the spring was made, vibrating as a fixed free reed, the radial force on the arbor as calculated by equation (19) is shown by the dotted curve as a function of the arbor diameter  $2r_2$ . The points indicate values of  $f_0$  obtained from the measured values of torque by the use of equation (5). The two sets of values agree within about 2 per cent.

As a further check on the validity of the calculations under practical conditions, the free torque of a phosphor-bronze spring on a dial-governor arbor was measured for various numbers of turns of the spring engaging on the slipping arbor. The torque due to bearing friction alone with no spring in place was also measured and found to

be approximately 0.001 in.-lb. This constant value was subtracted from the other measured values of torque. The resulting values of free spring torque are plotted as a function of the number of turns in Fig. 6. The torque values calculated by equations (4) and (19) and

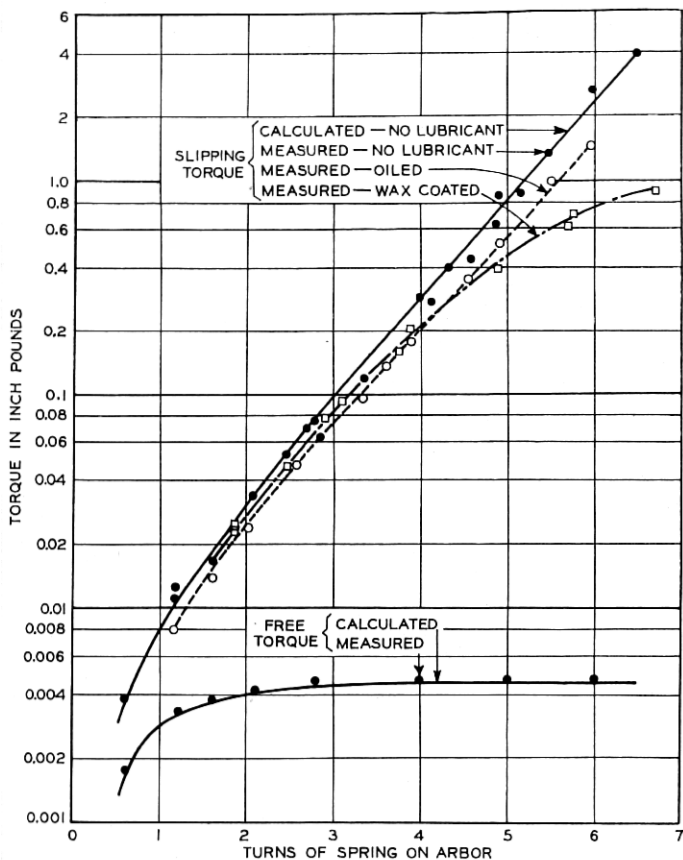


Fig. 6—Dependence of torque on the number of turns of the spring engaging the arbor.

using the value of  $\mu = 0.165$  obtained as described in the next section, are shown on the curve to indicate the agreement.

#### EXPERIMENTAL CHECK OF THE HOLDING-TORQUE RELATION

It is to be expected that any relation for which the coefficient of friction is a controlling factor will be difficult to check accurately. Equation (8) for the slipping torque is of this type. It is possible however by taking a large number of measurements to establish the

validity of the equation and then by determining limiting values for the coefficient of friction, to use this equation as a design relation, especially if only a minimum torque limit is set. Measurements of the slipping torque, as a function of the number of turns, were made on the same dial clutch that was used for checking the free torque.

This slipping torque was not steady as was the case in the free direction but varied as much as  $\pm 20$  per cent. An average value was taken in each case. An uncertainty also existed regarding the number of turns engaging the rotating arbor. Since the crossover from one arbor to the other requires practically one whole turn, slight differences in the arbor diameters may result in a gain or loss of almost half a turn. In addition to these factors there was an end effect due to the fact that the free end of the spring wire was cut off square rather than beveled but this factor although calculable was neglected in view of the other uncertainties. The results of these measurements are shown in Fig. 6.

From equation (8) it can be seen that for large values of  $N$  the plot of  $T'$  versus  $N$  will be a straight line provided  $\mu$  is independent of the force between the spring and the arbor. The slope of this straight line when multiplied by the proper constant, which can be shown to be 0.368, gives the coefficient of friction. For the experimental points shown in Fig. 6 this value of  $\mu$  is 0.165. Using this value of  $\mu$  in equation (6) the calculated curve was plotted. Considering the uncertainties involved the calculated and measured curves are in good agreement.

The dotted curve of Fig. 6 shows the effect of lubricating the clutch with a light machine oil. This resulted in only a small decrease of the coefficient of friction. The curve shown by the dashes illustrates the effect of lubricating a clutch with spermaceti. The coefficient was no longer a constant but decreased with an increase in load.

#### SPRING STRESSES

In determining the load that a spring clutch will withstand, first without stretching which will result in backlash, and second without breaking, initial as well as load stresses must be considered. The initial stresses are made up of the residual stresses due to forming the spring, plus the stresses due to expanding the spring to fit the arbor, that is from an inner radius  $r_1$  to an inner radius  $r_2$ .

Of these limiting load values the easiest to calculate is the torque required to break the spring. This is given by the product of the radius of the spring and the breaking strength of the spring wire. Loads much smaller than this value would stretch some of the fibers of the spring, especially those in which a high initial stress already

existed. As will be shown, such stretching will cause the radius to increase at those portions of the spring where the applied stresses are highest, that is, for those turns near the dividing line of the arbor.

Substituting for  $y$ , in equation (10), the distances from the neutral axis to the extreme inner and outer fibers will give the strain in these fibers. Provided  $R_1$  is considerably larger than  $h/2$ , half the thickness of the material, the distance to these extreme fibers becomes  $h/2$  and the  $y$  in the denominator can be neglected in comparison to  $R_1$ . The maximum fiber stress due to placing the spring on the arbor is this value of strain multiplied by Young's modulus. Then

$$S_0 = \frac{h}{2R_2} \frac{R_2 - R_1}{R_1} E. \quad (20)$$

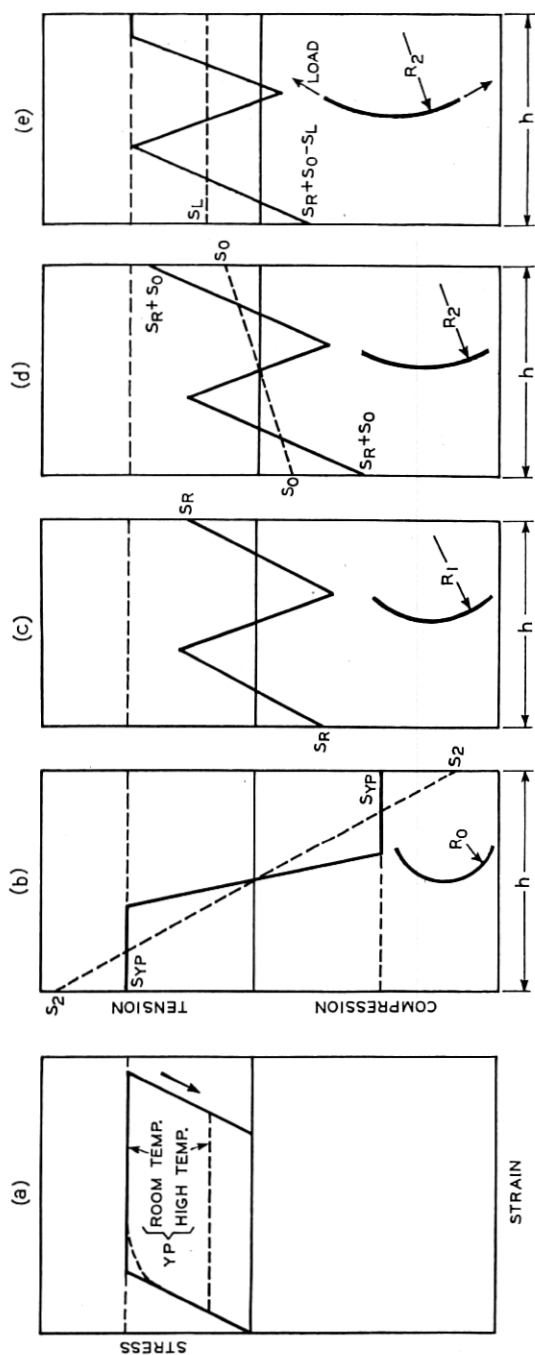
This stress is in the form of compression for the outer fibers and tension for the inner. Since a load on the clutch results in a tension in the spring, the stress given by equation (20) must be added to the load tension stress to get the total stress on the inner fibers of the spring. This initial stress therefore reduces the load-carrying capacity of the clutch.

#### RESIDUAL SPRING STRESSES

When a straight wire is wound upon an arbor to form a coil spring the strain on the inner and outer fibers must exceed that corresponding to the yield-point and plastic-flow results. To simplify the discussion assume the idealized stress-strain curve shown by the heavy lines of Fig. 7(a). The stress distribution across any section of the wire while wound on the winding arbor will be as shown by the heavy lines of Fig. 7(b) where  $S_{YP}$  is the yield-point stress, the maximum stress that the material will sustain. The moment, across the section, required to produce this bending is

$$M = \int_{-h/2}^{h/2} Sbydy \quad (21)$$

where  $b$  is the width of the wire at any point of  $y$  distance from the neutral axis and  $S$  is the stress at the same point. If the spring is released, it expands to a radius  $R_1$  in which condition the external and the internal moments are both zero. It is now possible by applying the same moment as was given by equation (21) to reduce the radius of curvature again to  $R_0$  but without causing additional plastic flow. The added stress distribution produced by this second bending must therefore follow a straight line as shown by  $S_2$ - $S_2$ , which together with the residual stresses in the relaxed condition (radius  $R_1$ ) must





equal the distribution  $S_{YP}-S_{YP}$  resulting from the original forming operation. Therefore the stress distribution in the relaxed condition (radius  $R_1$ ) must be the difference between  $S_{YP}-S_{YP}$  and  $S_2-S_2$  or as indicated in Fig. 7(c). The value of  $S_2$  is of course so determined that the moment as specified by equation (21) is the same for the dotted-line as for the solid-line stress distribution.

The value of  $S_R = S_2 - S_{YP}$  is relatively easy to determine for rectangular and round wire on the basis of the straight-line stress-strain characteristic if the bending has been sufficiently severe to have caused plastic flow almost to the neutral axis. The moment given by the actual stress distribution will then differ but little from that obtained by equation (21) with  $S$  replaced by  $S_{YP}$ , a constant. The values of  $S$  corresponding to the  $S_2-S_2$  distribution are given by

$$S = 2S_2y/h. \quad (22)$$

For a rectangular wire  $b$  is a constant and equating the moments corresponding to the two stress distributions gives

$$\int_{-h/2}^{h/2} S_{YP}bydy = \int_{-h/2}^{h/2} 2\frac{S_2}{h}by^2dy,$$

from which  $S_2 = (3/2)S_{YP}$  or

$$S_R = S_2 - S_{YP} = (1/2)S_{YP} \text{ (rectangular wire)}. \quad (23)$$

Similar integrations in the case of a round wire of radius  $h/2$  for which  $b = 2\sqrt{[(h/2)^2 - y^2]}$  gives

$$S_R = 0.7S_{YP} \text{ (round wire)}. \quad (24)$$

These equations give the residual fiber stresses in the extreme inner and outer fibers under the assumed conditions. In any case where the stress-strain characteristic is known a correct value for the residual stress can be obtained by graphical integration. The values given by equations (23) and (24) will, however, be fair approximations even in cases where the stress-strain characteristic is curved provided it does not exhibit strain hardening to a decided extent. Since a limited amount of plastic flow takes place for any stress above the proportional limit, which is generally far below the yield point, the residual stress may be sufficient to cause a small amount of creep. Any additional stresses will then cause permanent deformation of the spring.

The analysis will be continued on the basis of the idealized straight-line characteristic. Figure 7(d) shows the result of placing the spring

on the clutch arbor. The line  $S_0$ - $S_0$  shows the stresses added by this expansion where  $S_0$  is given by equation (20). The sum of these stresses and those shown in Fig. 7(c) gives the total stresses as shown by the solid line of Fig. 7(d). If now a load be put on the clutch a uniform stress  $S_L$  will be added but this stress for even relatively light loads may be sufficient to cause the total stress on the inner fibers to exceed the yield point as is indicated in Fig. 7(e). The inner fibers are consequently stretched and when the load is released and the spring taken from the arbor it will be found that the center turns of the spring have expanded. Even with the spring on the arbor if the clutch is turned in the free direction it will be noticed that these center turns raise off the arbor. It was shown in the paragraphs on the clutch torque in the free direction that the torque did not increase appreciably after the first few turns. This can be explained by the fact that as soon as the outward radial force due to the compression along the wire is equal to the initial inward radial force of the spring on the arbor the friction on these turns vanishes. The value of the compression will be fixed by a relatively few end turns. Hence if the inward force of some of the center turns decreases due to their stretching this compression will be sufficient to expand the turns to clear the arbor. If  $S_L$  is still further increased the stretch will be sufficient to cause the diameter of the center turns to exceed the arbor diameter even when no torque is applied in the free direction.

Since the yield point of metals decreases at higher temperature it is possible to produce a spring having lower residual stresses by the proper heat-treatment. If the wire is wound on an arbor and then heated, additional plastic flow takes place since the maximum stress that can be sustained at the high temperature is that shown in Fig. 7(a) as the high-temperature yield point. If the spring is then cooled and released the expansion will not be as great as for the untreated spring. The residual stresses will again be given by equation (23) or (24) where  $S_{YP}$  is taken as the lowest yield point reached in the temperature cycle. In Fig. 8, (a), (b), and (c) show the stresses in the heat-treated specimen corresponding to those shown in Fig. 7, (c), (d), and (e), for the untreated spring. Figure 8(c) shows that for the same load stress as for Fig. 7(e) no permanent deformation has taken place. It is of course important that the strain-relieving temperature should not go high enough to lower permanently the strength of the material. This limit<sup>1</sup> for phosphor bronze is about 320° C.

To determine the stress-temperature characteristics of 18-8 stainless

<sup>1</sup> "Better Instrument Springs," Robert W. Carson, *Trans. A.I.E.E.*, vol. 52, September, 1933, p. 869.

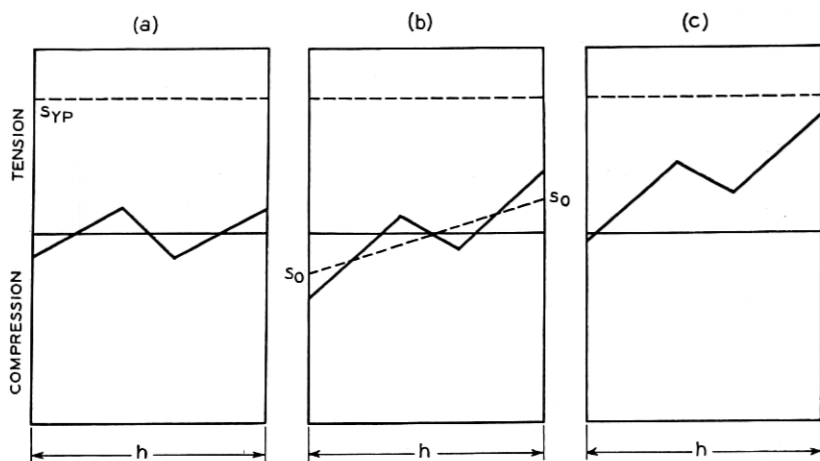


Fig. 8—Stress conditions in a strain-annealed clutch spring.

steel<sup>2</sup> a number of springs of  $0.0068 \times 0.021$ -in. ribbon were wound on 0.1486-in. arbors and given various heat-treatments. They were then cooled, released from the arbors, and measured for inside diameter. Table 1 gives the results of these measurements.

TABLE 1  
EFFECT OF HEAT-TREATMENT ON SPRINGS  
Ribbons of 18-8 stainless steel,  $0.0068 \times 0.021$  in., heat-treated on  
0.1486-in. winding mandrels

Heat-treatment temp., °C.	Time, hr.	Diam. after release, in.	Residual stress, psi
25	..	0.228	111000
100	4	0.206	88000
200	4	0.188	66000
300	4	0.182	58000
400	4	0.175	47000
470	4	0.169	37000
500	4	0.166	33000
400	$\frac{1}{4}$	0.177	51000
400	$\frac{1}{2}$	0.176	49000
400	1	0.176	49000
400	2	0.175	47000
400	3	0.175	47000
400	4	0.175	47000

The residual stresses were calculated by noting from equation (23) that  $S_R = S_2/3$  and then obtaining  $S_2$  from equation (20) rewritten as

$$S_2 = \frac{h}{2R_1} \frac{R_1 - R_0}{R_0} E. \quad (25)$$

<sup>2</sup> 8 per cent nickel, 18 per cent chromium.

Straight pieces of the stainless-steel ribbon were given the same series of heat-treatments as the springs. Young's modulus was determined for each of these samples and a bending test was also applied to determine whether the wire had been permanently annealed. No appreciable effect was noted. The proportional limit and the ultimate tensile strength of this ribbon at room temperature as measured on a tensile-testing machine were 41,600 and 252,000 psi, respectively. It is thus seen that except with the high-temperature anneals the residual stress alone exceeds the proportional limit and any additional stress will cause a permanent deformation.

It is also possible to obtain a favorable residual-stress distribution, that is, an initial compression on the inner and a tension on the outer fibers. If the released spring having the stress distribution shown in Fig. 7(c) is expanded until considerable plastic flow takes place on the inner and outer fibers the stress distribution of Fig. 9(a) is obtained, which on release results in the residual-stress distribution as shown in Fig. 9(b).

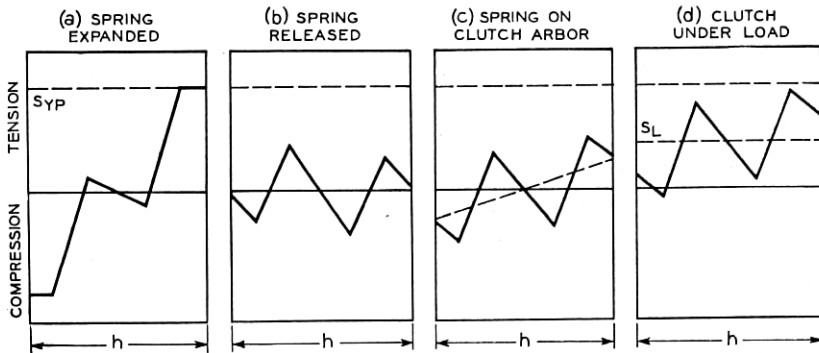


Fig. 9—Stress conditions in an expanded clutch spring.

To verify the validity of these arguments three springs of stainless-steel ribbon with different preliminary treatments and one of heat-treated phosphor bronze were tested for backlash as a function of previous loading. The backlash angle was measured from a point of slipping in the free direction to the point in the holding direction at which it would sustain a load of 0.05 in.-lb. An initial load of 0.5 in.-lb. was then applied and removed and the backlash measured as before. This was repeated for various loads up to the breaking point of the spring. The results are shown in Fig. 10. In the case of the untreated stainless steel the backlash began to increase immediately. Its higher initial value was probably due to the unavoidable stressing occasioned

by assembling the spring on the clutch arbor and to the 0.05-in.-lb. testing torque. The backlash of the other three samples remained constant to slightly above 0.5 in.-lb. and then began to rise. At low loads the phosphor bronze was better while at higher loads the heat-treated stainless steel had the advantage; the breaking load for the latter was also about 30 per cent higher.

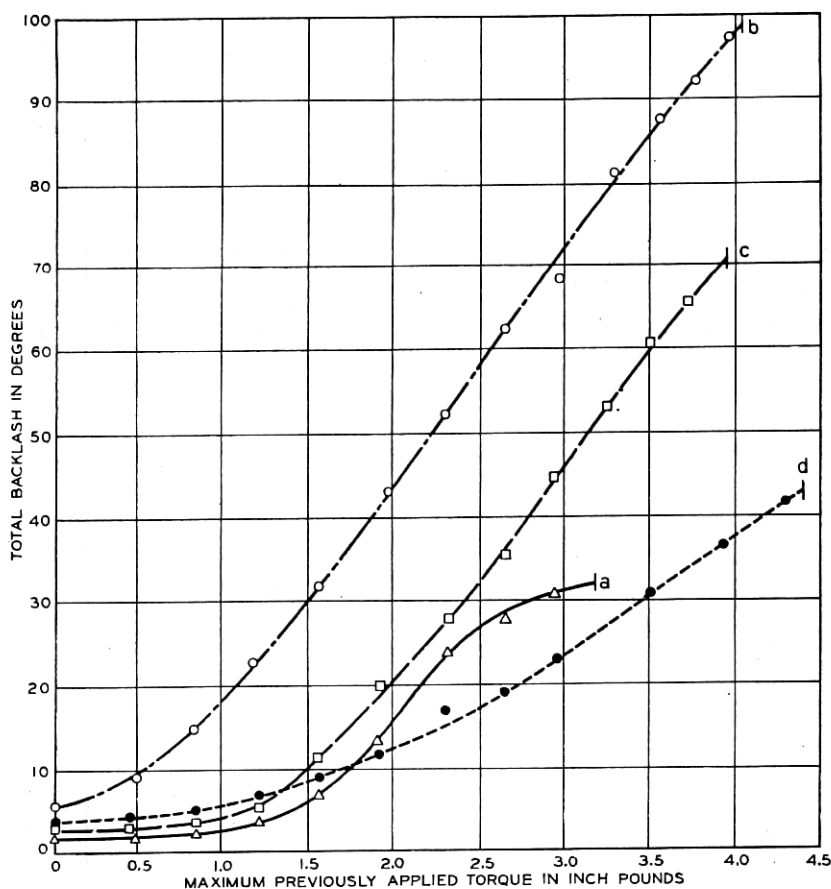


Fig. 10—Effect of overload on clutch backlash. Arbor diameter, 0.190 in.

(a) Phosphor-bronze-ribbon spring,  $0.0085 \times 0.022$  in., heat-treated 4 hr. at  $230^{\circ}\text{C}$ ., 0.1835 in. ID.

(b) 18-8 stainless-steel ribbon,  $0.0068 \times 0.021$  in., wound on 0.127-in. mandrel and released, 0.183 in. ID.

(c) 18-8 stainless-steel ribbon,  $0.0068 \times 0.021$  in., wound on 0.120-in. mandrel and released, 0.170 in. ID, mechanically expanded to 0.183 in. ID.

(d) 18-8 stainless-steel ribbon,  $0.0068 \times 0.021$  in., wound on 0.157-in. mandrel, heated 4 hr. at  $470^{\circ}\text{C}$ ., cooled, and released, 0.182 in. ID.

## CONCLUSION

The relations developed in the preceding sections are sufficient to determine uniquely the correct spring dimensions for a spring clutch of specified free and gripping torque provided the material constants and the cross-sectional shape (round or rectangular) of the wire, as well as the length and diameter of the clutch arbor, are specified. Choice of values for the last two factors is based largely on permissible heating resulting from the slipping in the free direction. Furthermore, as would generally be the case, if only maximum values of the free torque and minimum values of the gripping torque are given a number of solutions can be obtained from which to choose the most convenient. By combining the relations derived, in a manner to permit step-by-step calculation, the design of spring clutches is reduced to a simple routine.