Low Temperature Coefficient Quartz Crystals

By W. P. MASON

In this paper a review and amplification are given of the types and characteristics of existing low temperature coefficient crystals. The principal types are the coupled frequency crystals, the long bar crystals, and the *AT*, *BT*, *CT* and *DT* shear vibrating crystals. The theoretical frequencies for the *AT* and *BT* crystals agree well with those calculated from the Christofel formula for the velocity of propagation in an aeolotropic medium. For a finite plate other frequencies appear which are caused by couplings to the flexure and low-frequency shear modes. It is shown that harmonics of the high-frequency shear mode can be excited and will have low temperature coefficients. They can be made to stabilize the frequency of ultra-short-wave oscillators. The properties of the low-frequency *CT* and *DT* shear vibrating crystals are described. Overtone vibrations of the shear mode of approximately twice the frequency, having zero temperature coefficients, have been found and these have been labeled the *ET* and *FT* cuts.

It is shown that if two or more rotations of the cut are made with respect to the crystallographic axes, a line of zero temperature coefficient high-frequency crystals will be obtained. For the low-frequency shear crystals a surface of zero temperature coefficient crystals should result.

In the last section the variation of frequency with temperature of low coefficient crystals is discussed, and the variation of a new cut, labelled the *GT*, is described. This cut has zero first and second derivatives of the frequency by the temperature, and as a result has a very constant frequency over a wide temperature range. It has been applied to very constant frequency oscillators and frequency standards and has given a constancy of frequency considerably in excess of that obtained by other low coefficient crystals.

I. INTRODUCTION

DURING the past several years a number of crystal plates have been found which have the property that at a specified temperature their frequency will not change with a small change in temperature. These crystals have proved very useful in stabilizing the frequencies of oscillators used in frequency standards, broadcasting stations, radio communication transmitters, airplane transmitters, and for other purposes. In order to bring out their properties and spheres of usefulness a review and amplification of them are given in this paper.

The first types of zero temperature coefficient crystals were the so-called coupled types which obtained their low coefficient by virtue of the interaction between two modes of motion. The first crystal of this type was the "doughnut" crystal invented by W. A. Marrison,[1] which was used in the Bell System frequency standard. In this crystal the principal vibration is a shear and this is coupled to a flexure motion in the ring. The low coefficient is obtained from the fact that the shear has a positive temperature coefficient, while the flexure has a negative coefficient, and due to the coupling there is one region for which the temperature coefficient goes through zero. The next crystal of the coupled type was a Y cut crystal of specified dimensions invented by R. A. Heising.[2] In this crystal a high-frequency shear with a positive temperature coefficient was coupled to a harmonic of a low-frequency flexure, and a zero coefficient resulted at one temperature due to the coupling. Outside of their use in a frequency standard, such coupled types of crystals have not been applied much for commercial purposes on account of the difficulty of adjusting them, the difficulty of mounting them, and the prevalence of spurious frequencies near the desired frequency.

The next low-temperature coefficient crystals were crystals of the long bar type. It has been known for a long time that the temperature coefficient of an X cut crystal with its length lying along the Y or mechanical axis was very low provided the width of the crystal lying along the optic axis is very small compared to the length. This is illustrated by Fig. 1 taken from a former paper[3] which shows that for a crystal whose width is less than 0.15 of its length the temperature coefficient is about 2 parts per million per degree centigrade. Furthermore, it was found by the writer in 1930[3] that if the thickness of the crystal laying along the X or electrical axis was increased the temperature coefficient was decreased and in fact for certain ratios of axes the coefficient approached zero. For a bar of square cross section the zero coefficient occurs when the ratio of width to length is approximately 0.272. This apparently is also the method for obtaining a low-temperature coefficient used in the Hilger resonator. The second harmonic of this vibration has been used in the frequency standards of the Physikalisch-Technische Reichsanstaldt.[4] In their standards

[1] "A High Precision Standard of Frequency," W. A. Marrison, *Proc. I. R. E.*, April 3, 1929.

[2] This crystal is described by F. R. Lack in "Observation on Modes of Vibration and Temperature Coefficients of Quartz Crystal Plates," *Proc. I. R. E.*, July 1929, Vol. 17, pp. 1123–1141, and Patent No. 1,958,620 issued May 15, 1934.

[3] "Electrical Wave Filters Employing Quartz Crystals as Elements," *B. S. T. J.*, July 1934, Pages 411 and 412.

[4] A. Scheibe and V. Adelsberger, *Ann. d. Phys.* 18, 1, 1933.

the length is cut along the X axis and the vibration is excited by fields applied along the length of the bar. Since a rotation about the optic or Z axis does not change the properties of the elastic constants involved in this vibration, this bar should have a zero temperature coefficient at about the same ratio of axes as that given above. The zero angle of orientation is, however, not the most favorable angle of orientation for the fundamental vibration of a long bar, for if the length of the crystal lies at an angle of + 5° with respect to the Y or mechanical axis, the coefficient of a long bar is nearly zero.[5] These long bar type crystals have been used to a small extent to control oscillators and to stabilize the pass bands of filters. Their small use is attributable to the fact that they vibrate at low frequencies and are difficult to excite in an oscillator circuit.

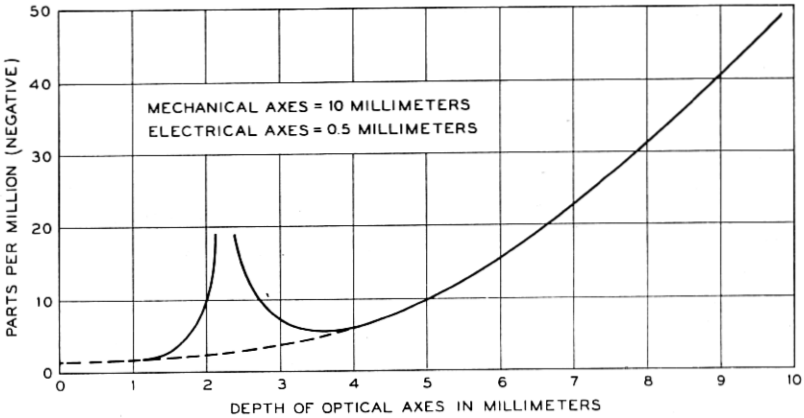

Fig. 1—Temperature coefficient of a perpendicularly cut crystal for varying ratios of width to length.

The AT and BT high-frequency shear crystals and the CT and DT low-frequency shear crystals are other low temperature coefficient crystals and they are discussed in detail in section II. These crystals are cut with their planes at specified angles with respect to the crystallographic axes and all of them involve a single rotation about an axis which is parallel or approximately parallel to one of the crystallographic axes. It is shown in section III that such crystals are not the only zero coefficient crystals of these types that can be obtained, for if we allow three rotations about the crystallographic axes a whole surface of zero temperature coefficient crystals can be found. These crystals

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[5] Matsumara and Kansaki, "On the Temperature Coefficient of Frequency of Y Waves in X Cut Quartz Plates," *Reports of Radio Researches and Works in Japan*, March 1932.

are more difficult to cut than the standard crystals and are more subject to couplings to other modes of motion and hence most of them are probably of more theoretical interest than of practical value.

All of the zero coefficient crystals described above are zero coefficient at a specified temperature only and for temperatures on either side of the specified temperature the frequency usually increases or decreases in a parabolic curve with temperature. This merely expresses the fact that the frequency-temperature curve is not exactly linear, but must be expressed more generally in a series of powers of the temperature. Then for all the crystals considered above, the first derivative of the frequency by the temperature is zero at the specified temperature T_0. The next term of importance is the square term and hence most crystals have a frequency which varies as the square term of the temperature about the zero coefficient temperature T_0. A crystal cut, labelled the *GT* crystal, has recently been found for which both the first and second derivatives of the frequency by the temperature are zero. As a result this "*GT*" crystal has a very constant frequency over a very wide temperature range, and in fact does not vary by more than one part in a million for a temperature range of 100° centigrade. For a temperature range of ± 15° C. it can be adjusted so that it does not vary by more than one part in ten million. This crystal has been applied to portable and fixed frequency standards and has given a constancy of frequency considerably in excess of any other piezo-electric crystal used under the same conditions. It has also been applied in quartz crystal filters to give pass bands which do not vary appreciably with temperature.

II. Standard Zero Temperature Coefficient Crystals

AT and BT Zero Temperature Coefficient Crystals

Crystals which employ the characteristics of a single shear mode of vibration to obtain a zero temperature coefficient are the *AT* and *BT* cut crystals.[6] These crystals vibrate in shear and their frequencies are determined principally by the thickness of the quartz plate. Their mode of vibration is similar to the ordinary *Y* cut, and they obtain their zero coefficient from the fact that the temperature coefficient of the shear mode changes from positive to negative as the angle of cut is rotated about the *X* axis by positive or negative angles from the position of the *Y* cut crystal. Figure 2 shows the method of cutting these plates from the natural crystal. Figure 3 shows the temperature coefficient of these crystals plotted against the angle of

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[6] "Some Improvements in Quartz Crystal Circuit Elements," F. R. Lack, G. W. Willard, and I. E. Fair, *B. S. T. J.*, July 1934, pp. 453–463.

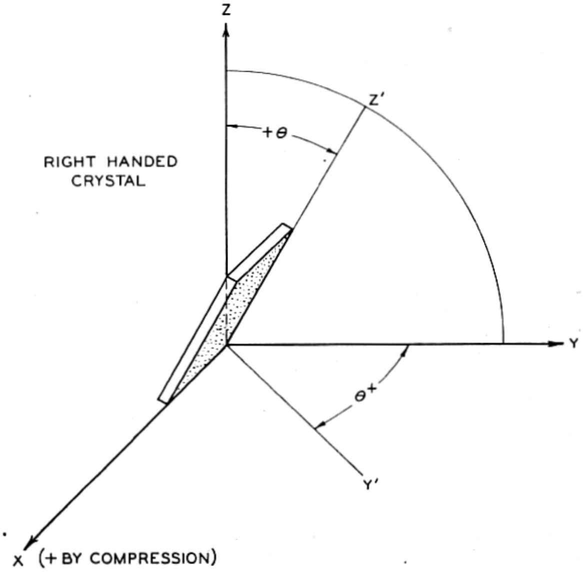

Fig. 2—Diagram illustrating angles used in expressing orientation of *AT* and *BT* plates within the natural crystal.

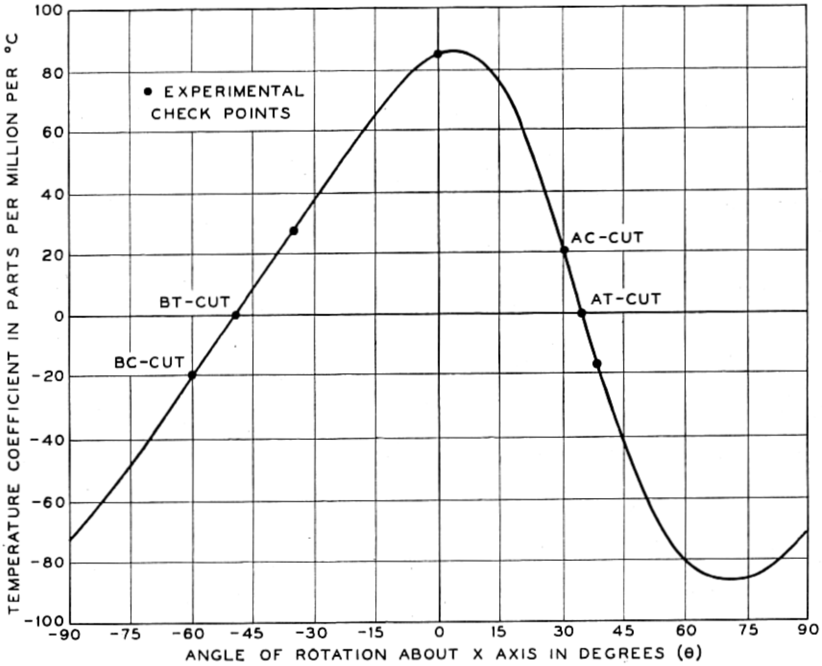

Fig. 3—Temperature coefficient for thin plates plotted as a function of the angle of cut.

cut while Fig. 4 shows the frequency constant of the crystal; i.e., the kilocycles for one millimeter thickness plotted as a function of the angular cut. The *AT* crystal which occurs at an orientation of $+35° - 20'$ has a frequency constant of 1662 kilocycles for one millimeter thickness while the *BT* cut which occurs at $-49°$ has a frequency constant of 2465 kilocycles for one millimeter thickness.

The frequency curve of Fig. 4 agrees very closely with the frequency calculated from the elastic constants used in the formula for the velocity of propagation of an aeolotropic medium given by E. B. Christofel.[7] Christofel showed that for any direction of propagation in an elastic solid, there were three different waves whose velocity of propagation could be obtained from the determinant

$$\begin{vmatrix} \lambda_{11} - \rho c^2, & \lambda_{12}, & \lambda_{13} \\ \lambda_{12}, & \lambda_{22} - \rho c^2, & \lambda_{23} \\ \lambda_{13}, & \lambda_{23}, & \lambda_{33} - \rho c^2 \end{vmatrix} = 0. \tag{1}$$

In this equation ρ is the density, c the velocity of propagation, and λ's are related to the elastic constants of the crystal by the formulae

$$\begin{aligned}
\lambda_{11} &= c_{11}l^2 + c_{66}m^2 + c_{55}n^2 + 2c_{56}mn + 2c_{15}nl + 2c_{16}lm, \\
\lambda_{12} &= c_{16}l^2 + c_{26}m^2 + c_{45}n^2 + (c_{46} + c_{25})mn \\
&\qquad\qquad + (c_{14} + c_{56})nl + (c_{12} + c_{66})lm, \\
\lambda_{13} &= c_{15}l^2 + c_{46}m^2 + c_{35}n^2 + (c_{45} + c_{36})mn \\
&\qquad\qquad + (c_{13} + c_{55})nl + (c_{14} + c_{56})lm, \\
\lambda_{23} &= c_{56}l^2 + c_{24}m^2 + c_{34}n^2 + (c_{44} + c_{23})mn \\
&\qquad\qquad + (c_{36} + c_{45})nl + (c_{25} + c_{46})lm, \\
\lambda_{22} &= c_{66}l^2 + c_{22}m^2 + c_{44}n^2 + 2c_{24}mn + 2c_{46}nl + 2c_{26}lm, \\
\lambda_{33} &= c_{55}l^2 + c_{44}m^2 + c_{33}n^2 + 2c_{34}mn + 2c_{35}nl + 2c_{45}lm,
\end{aligned} \tag{2}$$

where l, m, and n are respectively the direction cosines between the direction of propagation and the x, y, and z axes. For quartz

$$c_{22} = c_{11};\quad c_{24} = -c_{14};\quad c_{55} = c_{44};\quad c_{56} = c_{14};\quad c_{66} = (c_{11} - c_{12})/2$$

and

$$c_{15} = c_{16} = c_{25} = c_{26} = c_{34} = c_{35} = c_{36} = c_{45} = c_{46} = 0.. \tag{3}$$

For a rotation about the x axis for which a positive angle is measured in a counter clockwise rotation for a left handed crystal and a clockwise direction for a right handed crystal when an electrically positive face

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[7] See Love's "Theory of Elasticity," page 298, fourth edition.

(determined by a compression) is up, the values of l, m, and n are

$$l = 0; \qquad m = \cos\theta; \qquad n = -\sin\theta. \tag{4}$$

In this definition, a right handed crystal is taken as one which causes the plane of polarization of light traveling along the Z or optic axis to rotate in the sense of a right handed screw. Substituting the values of (3) and (4) in (2) we find

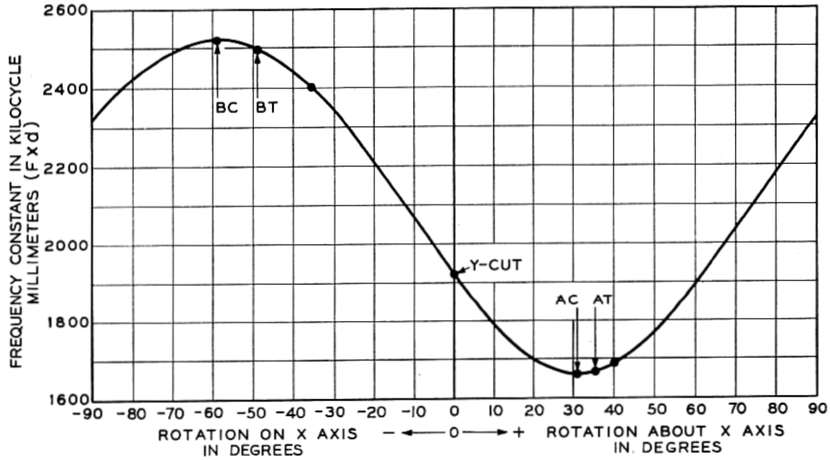

Fig. 4—Frequency constant for thin plates plotted against angle of cut.

$$\begin{aligned}
\lambda_{11} &= c_{66}\cos^2\theta + c_{44}\sin^2\theta - 2c_{14}\sin\theta\cos\theta = c_{66}',\\
\lambda_{23} &= -\,c_{14}\cos^2\theta - (c_{44} + c_{23})\sin\theta\cos\theta,\\
\lambda_{22} &= c_{22}\cos^2\theta + c_{44}\sin^2\theta + 2c_{14}\sin\theta\cos\theta,\\
\lambda_{33} &= c_{44}\cos^2\theta + c_{33}\sin^2\theta,\\
\lambda_{12} &= \lambda_{13} = 0.
\end{aligned} \tag{5}$$

With these values of λ, the three solutions of equation (1) are

$$\begin{aligned}
c_1 &= \sqrt{\frac{\lambda_{11}}{\rho}};\\
c_{2,\,3} &= \sqrt{\frac{1}{2}\left[\frac{\lambda_{22}}{\rho} + \frac{\lambda_{33}}{\rho} \pm \sqrt{\left(\frac{\lambda_{22}}{\rho} - \frac{\lambda_{33}}{\rho}\right)^2 + 4K^2\frac{\lambda_{22}}{\rho}\frac{\lambda_{23}}{\rho}}\,\right]},
\end{aligned} \tag{6}$$

where $K^2 = \lambda_{23}^2/\lambda_{22}\lambda_{33}$.

The frequency of any plate with its edges free to move will be

$$f = \frac{c}{2t}(2n + 1) \qquad n = 0, 1, 2, \cdots, \tag{7}$$

where t is the thickness of the plate. Hence for the A type vibration which corresponds to the first velocity c_1, the frequency will be

$$f_1 = \frac{1}{2t}\sqrt{\frac{c_{66}\cos^2\theta + c_{44}\sin^2\theta - 2c_{14}\sin\theta\cos\theta}{\rho}} = \frac{1}{2t}\sqrt{\frac{c_{66}'}{\rho}}. \quad (8)$$

The solid curve of Fig. 4 shows a plot of this equation while the measured values are shown by dots.

The frequencies of the other two modes of motion are given by

$$f_{2,3}^2 = \tfrac{1}{2}\left[f_A^2 + f_B^2 \pm \sqrt{(f_B^2 - f_A^2)^2 + 4K^2 f_A^2 f_B^2}\right],$$

where

$$f_A = \frac{1}{2t}\sqrt{\frac{\lambda_{22}}{\rho}}; \qquad f_B = \frac{1}{2t}\sqrt{\frac{\lambda_{33}}{\rho}}; \qquad K = \frac{\lambda_{23}}{\sqrt{\lambda_{22}\lambda_{33}}}. \quad (9)$$

This formula is the same as that for the frequencies given by two coupled modes[8] and hence can be interpreted as a mode of vibration, determined by λ_{33}, and a mode of vibration, determined by the constant λ_{22}, coupled together through the coupling compliance λ_{23}. For an isotropic medium one of these modes would be a pure shear and the other a longitudinal mode, but in a crystalline medium the motions are not strictly along or perpendicular to the direction of motion The A type vibration which is an x_y' shear vibration is not coupled to the other two since the coupling elasticities λ_{12} and λ_{13} are equal to zero. For a more general rotation, however, they will not necessarily be equal to zero and hence the general solution of equation (1) will represent two shear like vibrations, the x_y' and y_z', and a nearly longitudinal y_y' vibration all mutually coupled together.

The Christofel formula is only valid for a plate of thickness t which extends to infinity in all other directions and hence this solution does not show the coupled frequencies due to the contour dimensions which occur in a finite plate. In general there are two types of vibration which couple strongly to the A type vibration, the low-frequency shear modes and the flexure modes in which bending occurs in the xy' plane. As pointed out by Lack, Willard and Fair,[9] both the AT and the BT occur near angles of cut for which the coupling to the z_x' low frequency shear mode vanishes. Hence one would expect that these crystals would have fewer subsidiary resonances and this expectation is verified by experiment. A practical result is that the

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[8] "Electrical Wave Filters Employing Quartz Crystals as Elements," W. P. Mason, July 1934, page 444, *B. S. T. J.*

[9] "Some Improvements in Quartz Crystal Circuit Elements," *B. S. T. J.*, July 1934. The problem of couplings is discussed in more detail in the U. S. Patent 2,173,589, Sept. 19, 1939 issued to R. A. Sykes and the writer. In this patent the *AC* cut and the − 18.5° *X* cut crystals are described.

AT and *BT* crystals can control considerably more powerful oscillators without danger of the crystals breaking than can the *X* or *Y* cut crystals. The frequency spectrum of an *AT* cut plate ground down from a large ratio of dimensions to a smaller one is shown on Fig. 5.

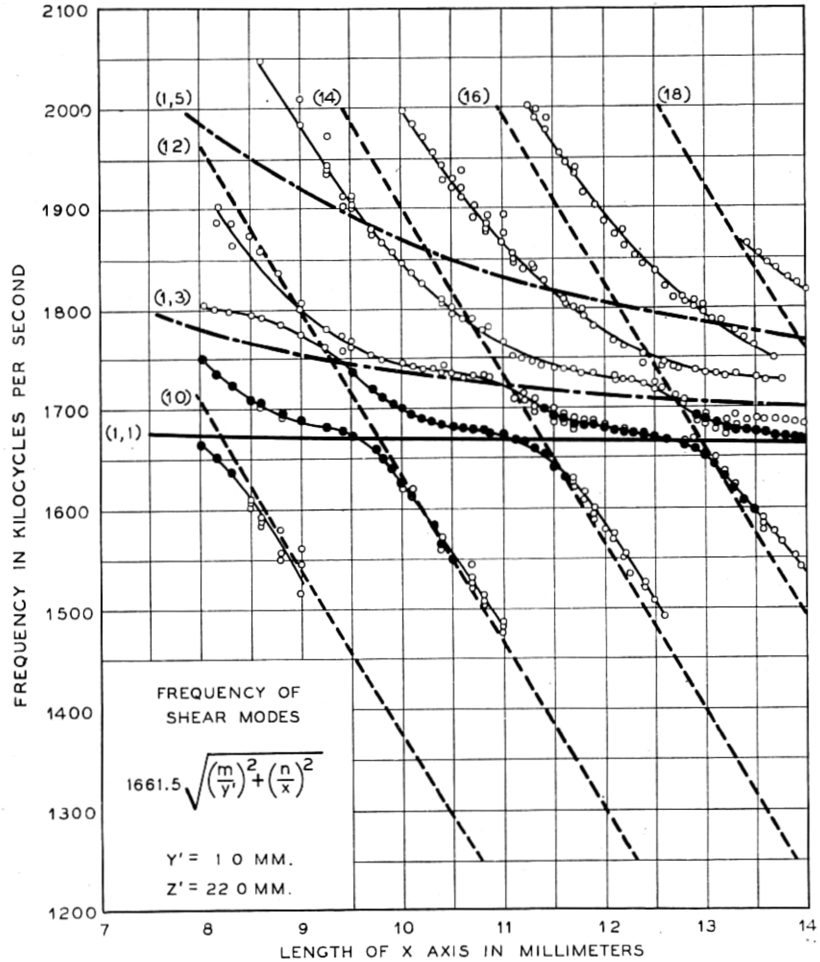

Fig. 5—Frequency spectrum (dots) for an *AT* plate as a function of ratio of length to thickness. The dashed lines represent calculated flexural vibrations. The whole line is the principal shear mode. The dot dash lines are other shear modes.

Most of the prominent frequencies can be identified as shear frequencies of the type discussed in a previous paper[10] and harmonics of flexure

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[10] "Electrical Wave Filters Employing Quartz Crystals as Elements," W. P. Mason, *B. S. T. J.*, July 1934, page 446. The verification was made by R. A. Sykes who kindly supplied Fig. 5.

vibrations. This figure shows clearly that the strongest flexures entering are controlled by the length of the X axis rather than the Z' axis.

As shown by equations (6), (7) and (8) the *AT* and *BT* cut crystals have odd harmonic vibrations which are controlled by the same elastic constants as the fundamental vibrations. Since they are controlled by the same elastic constants, the harmonic vibrations have the same temperature coefficients as the fundamental mode and hence will have nearly zero coefficients. This property has been made use of in oscillators in controlling high-frequency vibrations with crystals whose thicknesses can be obtained commercially.

CT and DT Low-Frequency Zero Temperature Coefficient Crystals

Another set of zero temperature coefficient crystals which are particularly useful for low frequencies has recently been described by Hight and Willard.[11] They are related to the *AT* and *BT* cuts discussed above in that they use the same shearing motion to produce the low coefficient. This relation is illustrated by Fig. 6 which shows

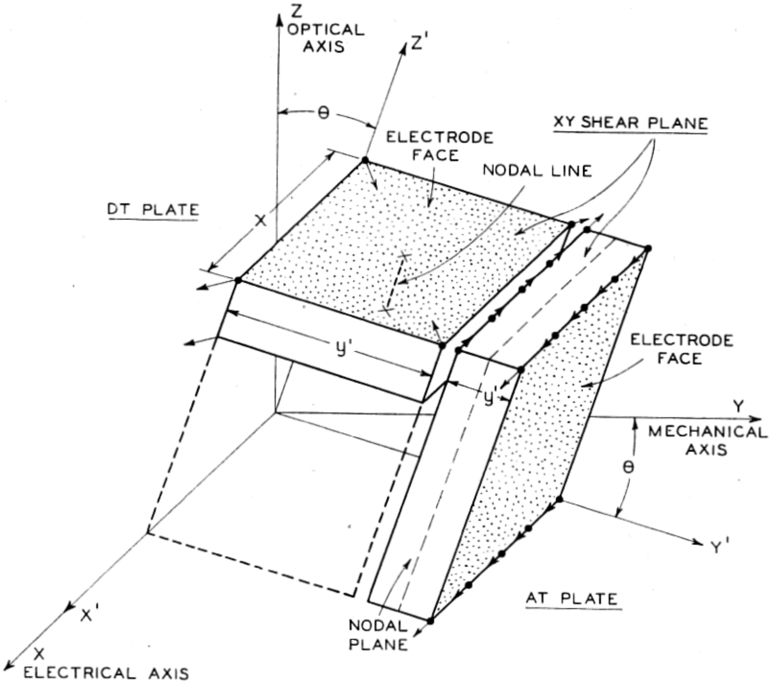

Fig. 6—Relation between the *AT* and *DT* cuts.

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[11] "Presented before the Institute of Radio Engineers, March 3, 1937. Published in *I. R. E. Proc.* May, 1937, p. 549. Similar crystals are also discussed in U. S. Patents 2,111,383 and 2,111,384 issued to S. A. Bokovoy.

the approximate orientations of the *AT* cut and the *DT* cut. In the *AT* plate the x_y' strain is produced by a shear mode of vibration as shown by the arrows which represent instantaneous displacements. In the *DT* plate the x_y' strain is produced by a shear mode of vibration as shown again by the arrows. Two diagonally opposite corners move radially outward while the other two move radially inward. The relatively low frequency of the *DT* plate results from the relatively large frequency-determining dimensions x and y'. The temperature coefficient of frequency of these plates may be made zero, for the proper angles of cut, since it goes from a large positive value at one orientation to a large negative value for an orientation 90 degrees from the first. Actually the angle of cut of the *DT* plate is not exactly 90 degrees from the *AT*. This is due to the fact that the frequency for a square plate involves the s_{66}' constant rather than the c_{66}' constant which controls the frequency of a thin plate. Similarly we find that there is a crystal almost 90° from the *BT* which has a zero coefficient and this has been designated the *CT*.

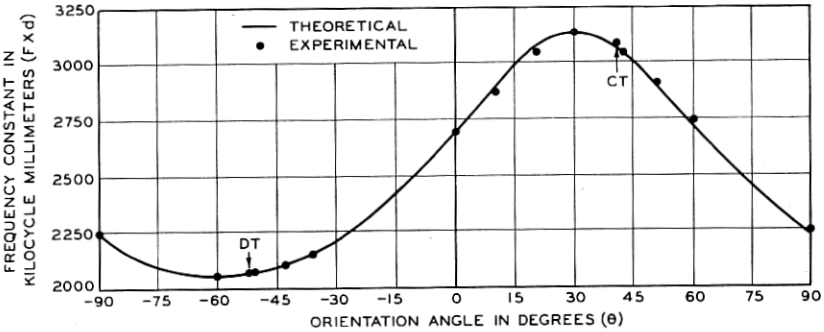

Fig. 7—Frequency constant for low-frequency shear crystal plotted against angle of cut.

Figure 6 shows that the electrode faces of the *DT* crystal are placed on the $z'x$ plane and hence the shear mode generated would ordinarily be called the z_x' mode even though it is similar to the x_y' shear mode in the *AT* crystal at right angles to it. The measured frequency constant of such a series of square plates is shown on Fig. 7. In the absence of a complete theoretical solution [12] taking account of all the elastic couplings for a square plate vibrating in shear, an empirical

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[12] An approximate solution neglecting coupling was given in a former paper "Electrical Wave Filters Employing Quartz Crystals as Elements," page 446. This solution is not complete enough, however, to allow calculations of temperature coefficients with very great accuracy.

formula was developed for the frequency which is

$$f = \frac{1.25}{2d}\sqrt{\frac{1}{\rho s_{55}'}}, \tag{10}$$

where $d = x = z'$ if the plate is square and $d = (x + z')/2$ if only nearly square. The elastic constant s_{55}' depends on the orientation angle θ according to the equation,

$$s_{55}' = s_{44}\cos^2\theta + s_{66}\sin^2\theta + 4s_{14}\sin\theta\cos\theta. \tag{11}$$

Figure 7 shows the measured values of frequency and the values calculated from equations (10) and (11). Agreement is obtained within 2 per cent.

From equations (10) and (11) the temperature coefficient of frequency of a shear vibrating plate should be for a square crystal

$$T_f = -(1/2)\left[T_x + T_z' + T_\rho + \frac{s_{44}T_{s_{44}}\cos^2\theta + s_{66}T_{s_{66}}\sin^2\theta + 4s_{14}T_{s_{14}}\sin\theta\cos\theta}{s_{44}\cos^2\theta + s_{66}\sin^2\theta + 4s_{14}\sin\theta\cos\theta}\right]. \tag{12}$$

The temperature coefficient of length along the optic axis is about 7.8 parts per million (per degree centigrade) while that perpendicular to the optic axis is 14.3 parts per million. For any other direction

$$T_l = 7.8 + 6.5\cos^2\theta, \tag{13}$$

where θ is the angle between the length and the optic axis. Hence

$$T_x = 14.3; \qquad T_z' = 7.8 + 6.5\cos^2\theta,$$

and

$$T_\rho = -36.4 \text{ per degree C.} \tag{14}$$

The temperature coefficients of the six elastic constants were evaluated in a former paper.[13] Since then they have been slightly revised so that the best values now are

$$\begin{array}{lcl} T_{s_{11}} = +12, & & T_{c_{11}} = -54.0, \\ T_{s_{12}} = -1{,}265, & \text{this} & T_{c_{12}} = -2{,}350, \\ T_{s_{13}} = -238, & \text{results in} & T_{c_{13}} = -687, \\ T_{s_{14}} = +123, & & T_{c_{14}} = +96, \\ T_{s_{33}} = +213, & & T_{c_{33}} = -251, \\ T_{s_{44}} = +189, & & T_{c_{44}} = -160, \\ T_{s_{66}} = -133.5, & & T_{c_{66}} = +161. \end{array} \tag{15}$$

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[13] "Electric Wave Filters Employing Quartz Crystals as Elements," W. P. Mason, *B. S. T. J.*, 13, p. 446, July 1934.

Using these values in equation (12) the expected temperature coefficients for the low-frequency vibration are as shown on Fig. 8 The measured points are shown on the curve. The zero temperature coefficients occur at the angles + 38° and − 53°. These crystals have been designated the *CT* and *DT* low-frequency shear crystals. These types of crystals are useful for stabilizing low-frequency oscillators ranging from 50 *KC* to 500 *KC*.

Just as the *AT* and *BT* crystals have harmonics which can be used to control oscillator frequencies, so also do over-tones of the low-frequency shear crystals exist. They do not bear, however, the simple relation to the fundamental that the high-frequency harmonics do. S. C. Hight has found a mode of motion, which is probably related to the second flexural vibration, of nearly twice the frequency of the low-frequency shear mode and which has zero temperature coefficients at angles of + 66°-30′ and − 57°. These crystals have been designated respectively as the *ET* and *FT* crystal cuts. Figure 9 shows a plot of this frequency versus orientation. It will be observed that the frequency constant of this mode of motion is about twice that for the low-frequency shear mode and hence these crystals can be obtained in reasonable sizes for twice the frequencies that the *CT* and *DT* crystals can be obtained.

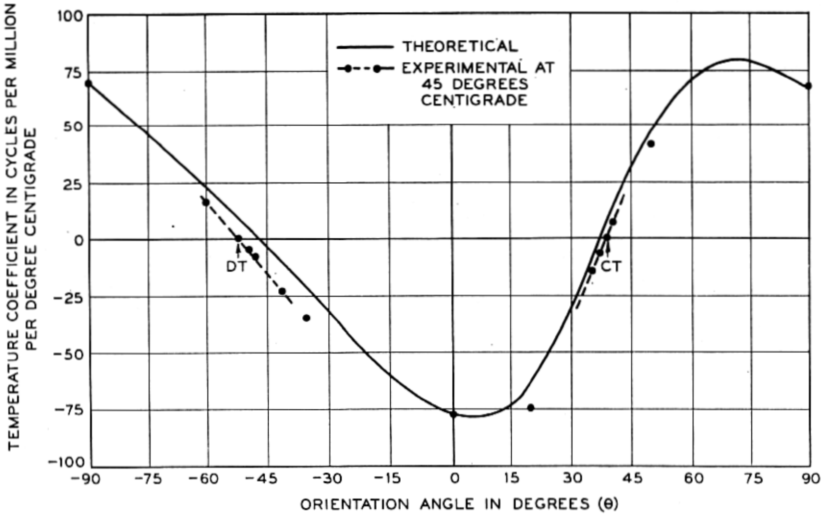

Fig. 8—Calculated and measured temperature coefficients for low-frequency shear crystals.

Practically all the work done has been on square or nearly square plates. Some time ago Bechmann [14] and Koga [15] published work done on crystals which departed from the square shape for which zero coefficients were obtained at somewhat different angles and different frequencies than those given for the *CT* and *DT* crystals. This is due to the fact that when the crystal shape departs from the square, the frequency approaches more nearly the resonant frequency of the crystal vibrating in its second flexure mode and the increased coupling changes the angle for which the coefficient becomes zero. The square crystal is the one which has fewer secondary frequencies and is therefore more desirable.

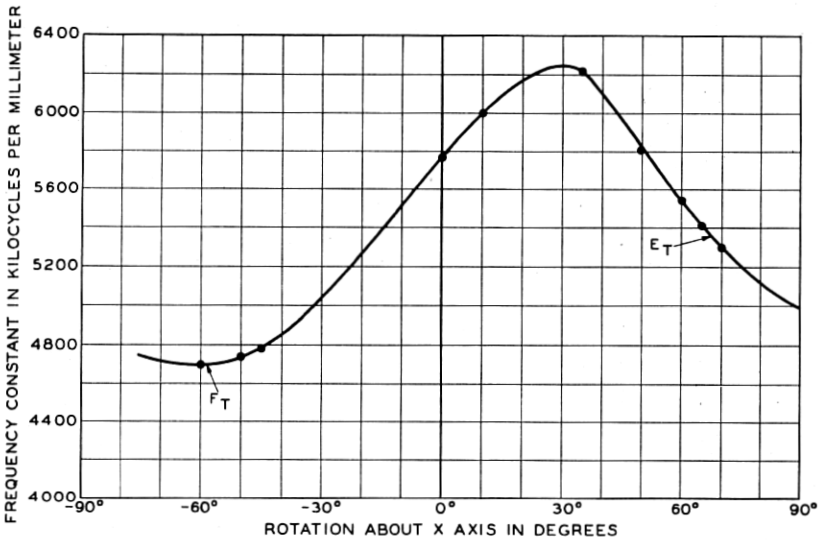

Fig. 9—Frequency constant for *E* and *F* type vibrations.

III. Zero Temperature Coefficient Crystals for More General Orientations

Shortly after the discovery of the *AT* and *BT* crystals it was realized that zero temperature coefficient crystals could be obtained at a variety of angles provided two rotations of the crystal with respect to the crystallographic axes were used. This would allow the direction of the shearing axis to point in any direction with respect to the crystallographic axes. Using the c_{66}' constant as the elastic constant determining the frequency, it was found that there was a whole series of zero

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[14] R. Bechmann, *Hochfrequenstechnik u Elektroakustic*, Vol. 44, No. 5, p. 145.

[15] I. Koga, *Report of Radio Research in Japan*, Vol. IV, No. 2, 1934. See also Patents 2,111,383 and 2,111,384 issued to S. A. Bokovoy.

temperature coefficient crystals whose plot as a function of the two rotations would be a line in which the *AT* and *BT* cuts would be in the region of two points on the line. A few of these crystals whose angles were in the region of the *AT* crystal were measured and were found to have zero coefficients but also had a much more complicated frequency spectrum than the *AT* or *BT* crystals when cut to have their major faces more nearly parallel to the x axis.

Recently Bechmann[16] has made calculations and experiments in respect to double orientation crystals which have zero temperature coefficients. The calculations were made by means of the Christofel formula of equations (1) and (2). Although this gives the same result as that calculated from the constant c_{66}' for rotations around the x axis, it differs from it somewhat for more general rotations. If we expand equation (1) we obtain the cubic equation for the frequency of oscillation.

$$\begin{aligned} f^6 &- f^4(f_A^2 + f_B^2 + f_C^2) \\ &+ f^2[f_A^2 f_B^2(1 - K_{AB}^2) + f_A^2 f_C^2(1 - K_{AC}^2) + f_B^2 f_C^2(1 - K_{BC}^2)] \\ &- f_A^2 f_B^2 f_C^2(1 - K_{AB}^2 - K_{AC}^2 - K_{BC}^2 + 2K_{AB}K_{AC}K_{BC}) = 0, \end{aligned} \quad (16)$$

where

$$f = \frac{c}{2t}; \quad f_A = \frac{\sqrt{\lambda_{11}/\rho}}{2t}; \quad f_B = \frac{\sqrt{\lambda_{22}/\rho}}{2t}; \quad f_C = \frac{\sqrt{\lambda_{33}/\rho}}{2t};$$

$$K_{AB} = \frac{\lambda_{12}}{\sqrt{\lambda_{11}\lambda_{22}}}; \quad K_{AC} = \frac{\lambda_{13}}{\sqrt{\lambda_{11}\lambda_{33}}}; \quad K_{BC} = \frac{\lambda_{23}}{\sqrt{\lambda_{22}\lambda_{33}}}.$$

f_A, f_B, f_C can be interpreted as the three primary frequencies and would correspond to the three solutions of (16) if the couplings K_{AB}, etc., were zero. The three solutions of (16) then will be these three primary modes modified by the coupling between them. If we let

$$\begin{aligned} P = [(f_A^2 + f_B^2 + f_C^2) &- 3[f_A^2 f_B^2(1 - K_{AB}^2) \\ &+ f_A^2 f_C^2(1 - K_{AC}^2) + f_B^2 f_C^2(1 - K_{BC}^2)]]/9, \end{aligned} \quad (17)$$

$$\begin{aligned} Q = [&2(f_A^2 + f_B^2 + f_C^2)^3 - 9(f_A^2 + f_B^2 + f_C^2) \\ &\times [f_A^2 f_B^2(1 - K_{AB}^2) + f_A^2 f_C^2(1 - K_{AC}^2) + f_B^2 f_C^2(1 - K_{BC}^2)] \\ &+ 27 f_A^2 f_B^2 f_C^2(1 - K_{AB}^2 - K_{AC}^2 - K_{BC}^2 + 2K_{AB}K_{AC}K_{BC})]/54, \end{aligned}$$

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[16] "Researches on Natural Elastic Vibrations of Piezo-Electrically Excited Quartz Plates," R. Bechmann, *Zeit. f. Technisch Physik*, Vol. 16, No. 12, 1935, pp. 525–528. This multiple orientation of high-frequency shear crystals is also the basis of the *V* cut crystal of Bokovoy and Baldwin discussed for example in British Patent No. 457,342 issued May 27, 1936.

and set

$$\cos\psi = \frac{Q}{P^{3/2}},$$

the three solutions will be

$$
\begin{aligned}
{}_1 &= \sqrt{2\sqrt{P}\cos\frac{\psi}{3} + \frac{(f_A{}^2 + f_B{}^2 + f_C{}^2)}{3}}, \\
f_{2,\,3} &= \sqrt{-\,2\sqrt{P}\cos\left(\frac{\psi}{3} \pm \frac{\pi}{3}\right) + \frac{(f_A{}^2 + f_B{}^2 + f_C{}^2)}{3}}, \qquad (18)
\end{aligned}
$$

From these equations and equation (2), the frequencies and temperature coefficients of all three modes of motion have been calculated by Bechmann. Based on these calculations the angles of zero coefficient are shown on Fig. 10 for the angular placement of the direction of propagation adopted on Fig. 11.

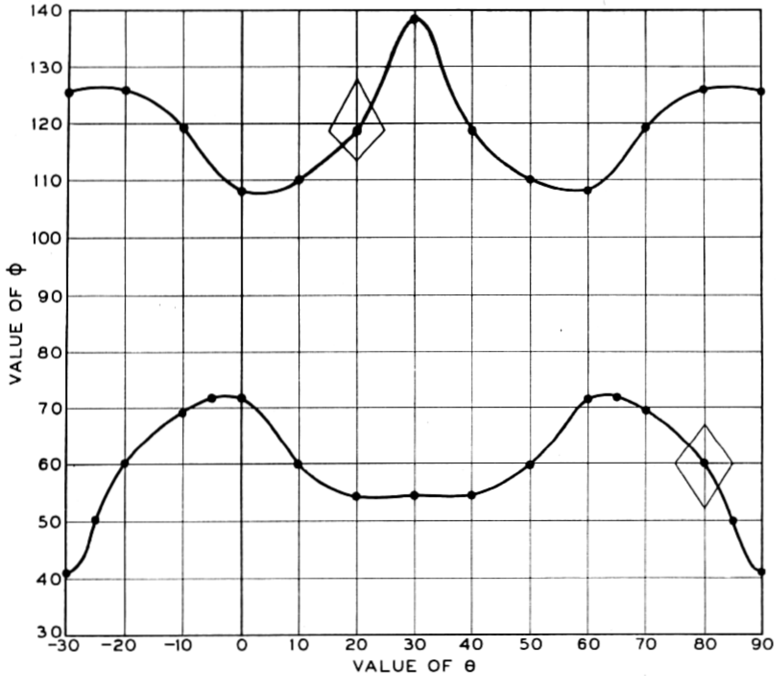

Fig. 10—Angles of cut for zero temperature coefficient high-frequency shear crystals for two rotations.

Using the empirical formula (10) for the low-frequency shear vibration a surface of zero coefficient low-frequency shear vibrating crystals can be calculated.[17] For this crystal three angles are required to

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[17] Multiple orientation low- and high-frequency shear crystals are discussed in British Patent 491,407 issued to the writer on September 1, 1938.

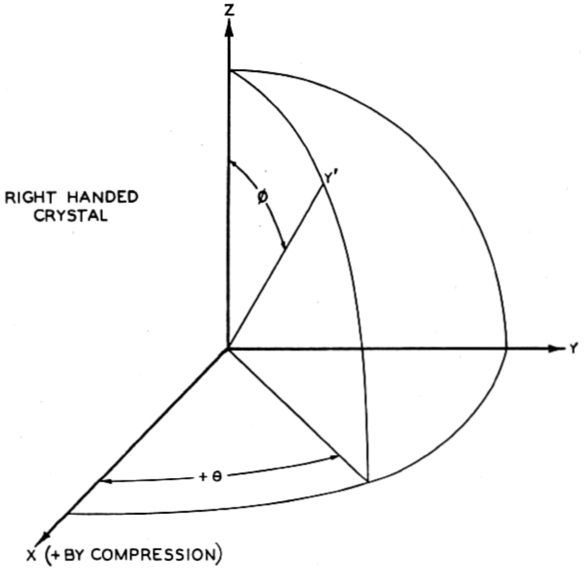

Fig. 11—Angular system for locating the axis of shear of high-frequency crystals with two rotations.

specify the position of the plate since, for a low-frequency shear crystal, rotating the plate around its shearing axis will change the s_{55}' constant and hence the frequency and temperature coefficient of the plate. If we let the position of the plate with respect to the crystalline axes be denoted by the angles, θ, φ and γ, measured as shown on Fig. 12 it can be

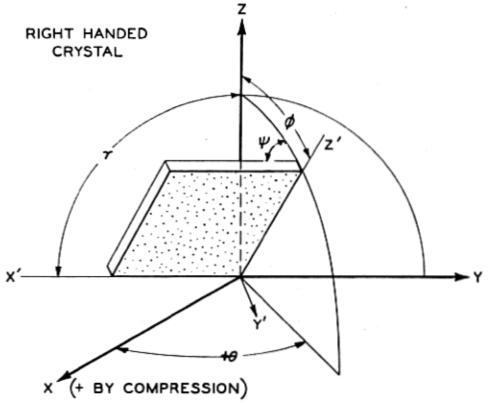

Fig. 12—Angular system for locating low-frequency shear crystals with three rotations.

shown that the s_{55}' constant is given by the equation

$$\begin{aligned} s_{55}' = {} & (s_{11} - 2s_{13} + s_{33}) \cos^2 \psi \sin^2 2\varphi + s_{66} \sin^2 \varphi \sin^2 \psi \\ & + 4s_{14} \sin \varphi \, [\sin 3\theta \cos \varphi \, (\cos^2 \psi \cos^2 \varphi - \sin^2 \psi) \\ & + \cos 3\theta \sin \psi \cos \psi \, (\cos 2\varphi + \cos^2 \varphi)] \\ & \qquad + s_{44} (\cos^2 \psi \cos^2 2\varphi + \sin^2 \psi \cos^2 \varphi), \end{aligned} \tag{19}$$

where the angle γ is given in terms of a new angle ψ and φ by the equation

$$\cos \gamma = \sin \varphi \cos \psi. \tag{20}$$

If we introduce this expression into equation (12) and introduce the numerical values of equation (15), the expression for the temperature coefficient of a low-frequency shear crystal cut at any angle becomes

$$\begin{aligned} T_f = \Bigg[& 4.5 + 2.9 \, (\sin^2 \varphi \cos^2 \psi + \cos^2 \varphi) + \Bigg[\frac{- 5877.5 \cos^2 \psi \sin^2 2\varphi}{195 \cos^2 \psi \sin^2 2\varphi} \\ & \frac{+ 15790 \sin^2 \varphi \sin^2 \psi + 10340 \sin \varphi \, [\sin 3\theta \cos \varphi \, (\cos^2 \psi \cos 2\varphi - \sin^2 \psi)}{+ 292.8 \sin^2 \varphi \sin^2 \psi - 172.4 \sin \varphi \, [\sin 3\theta \cos \varphi \, (\cos^2 \psi \cos 2\varphi - \sin^2 \psi)} \\ & \frac{+ \cos 3\theta \sin \psi \cos \psi \, (\cos 2\varphi + \cos^2 \varphi)]}{+ \cos 3\theta \sin \psi \cos \psi \, (\cos 2\varphi + \cos^2 \varphi)]} \\ & \frac{- 19{,}525 \, (\cos^2 \psi \cos^2 2\varphi + \sin^2 \psi \cos^2 \varphi)}{+ 200.5 \, (\cos^2 \psi \cos^2 2\varphi + \sin^2 \psi \cos^2 \varphi)} \Bigg] \Bigg]. \end{aligned} \tag{21}$$

Figure 13 gives a contour map of the location of the angles of zero temperature coefficient. The dotted lines indicate the paths for which the piezo-electric constant is a maximum and hence for which the crystal is most easily excited.

IV. A NEW CRYSTAL CUT, LABELED THE GT CRYSTAL, WHICH HAS A VERY CONSTANT FREQUENCY FOR A WIDE TEMPERATURE RANGE

All of the zero temperature coefficient crystals so far obtained have a zero temperature coefficient only for a specified temperature, while on either side of this temperature the frequency either increases or decreases in a parabolic curve with the temperature. This is well illustrated by Fig. 14 which shows a comparison of the frequency stability of the standard zero temperature coefficient crystals over a wide temperature range. What is plotted is the number of cycles change in a million from the zero coefficient temperature. These curves show that for a 50° C. change from the zero coefficient temperature the frequency of standard zero temperature coefficient crystals may change from 30 to 140 parts per million. The curves are usually nearly para-

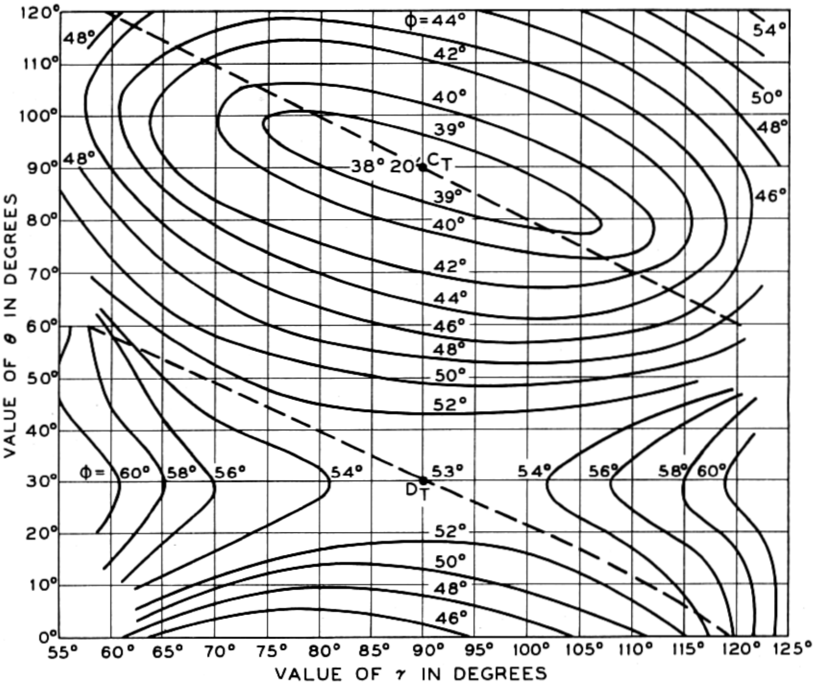

Fig. 13—Contour map of zero temperature coefficient low-frequency shear crystals with three rotations.

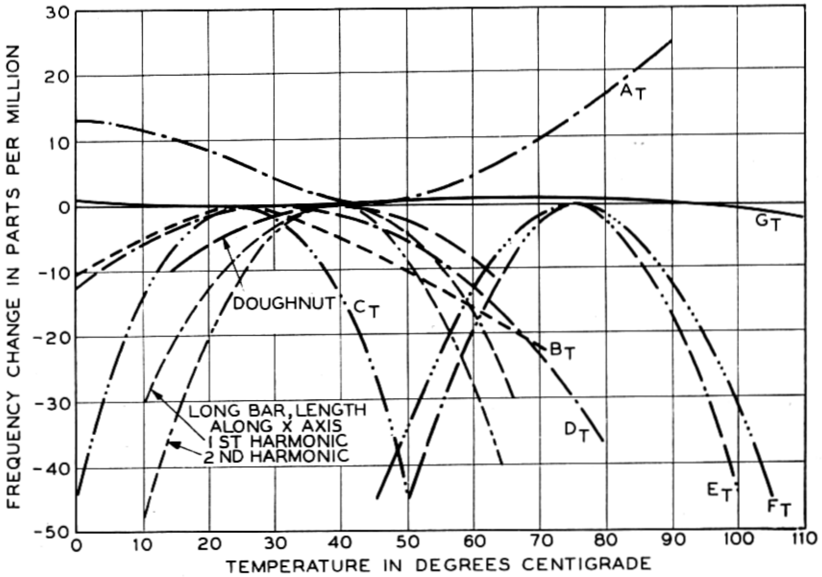

Fig. 14—Frequency temperature relations for zero temperature coefficient crystals.

bolas. This is what would be expected for in general we can write the frequency as a function of temperature by the series

$$f = f_0[1 + a_1(T - T_0) + a_2(T - T_0)^2 + a_3(T - T_0)^3 + \cdots], \quad (22)$$

where T_0 is any arbitrary temperature. Differentiating f with respect to T we have

$$\frac{df}{dT} = f_0[a_1 + 2a_2(T - T_0) + 3a_3(T - T_0)^2 + \cdots]. \quad (23)$$

For a zero coefficient crystal the change in frequency will pass through zero at some temperature T_0. Hence $a_1 = 0$, and the frequency will then be

$$f = f_0[1 + a_2(T - T_0)^2 + a_3(T - T_0)^3 + \cdots]. \quad (24)$$

Since a_2 will ordinarily be much larger than succeeding terms, a parabolic curve will be obtained. If a_2 is positive the frequency will increase on either side of the zero coefficient temperature T_0 and if negative it will decrease.

Recently a new crystal cut, labeled the *GT*, has been found for which both a_1 and a_2 are zero. As a result the parabolic variation with temperature is eliminated and the frequency remains constant over a much wider range of temperature. The variation obtained is plotted on Fig. 14 by the curve labeled *GT*, and, as can be seen, the frequency does not vary over a part in a million over a 100° C. change in temperature.

This crystal, which will be described in a forthcoming paper, has found considerable use in frequency standards, in very precise oscillators, and in filters subject to large temperature variations. It has given a constancy of frequency considerably in excess of that obtained by any other crystal.