Dielectric Constants and Power Factors at Centimeter Wave-Lengths

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The theory underlying the measurement of dielectric constants and power factors, by means of resonant lengths of coaxial transmission line, is developed, apparatus used for such measurements is illustrated and the measurement routine described. A table of typical results is appended together with an "X tan X" table for aid in the calculations.

Introduction

THERE are two instrumentalities available for measuring dielectric constants and power factors at centimeter wave-lengths. These are, coaxial conductor lines and wave guides. Which one is, for any condition, the more favorable one depends a great deal upon the wave-lengths used. Under the conditions encountered in this work the coaxial line appeared to have the practical superiority, down to something like 10 cms. wave-length, anyway. Below this, the wave guide is very manageable and has several advantageous features.

When this work was begun, the most easily available and practicable vacuum tube which would oscillate around 20 cms. wave-length was the W. E. Co. 368A. This could be pushed down to something below 19 cms. wave-length but was undependable there and as a practical compromise 22.5 cms. wave-length was finally chosen. Later another tube became available and as it could be operated down to at least 9 cms. it was used in the more recent work. Thus, while the bulk of the measurements made were at 22.5 cms. wave-length, a good share of the samples investigated were also measured at approximately 10 cms. wave-length.

Any measurements made at these wave-lengths must be made in the form of transmission line measurements and the dielectric must be physically part of the coaxial line. There are various transmission line quantities definable and measurable, such as series impedance per unit length, shunt admittance per unit length, surge impedance, impedance transformation factor, voltage and current step-up factors, resonance selectivity or "Q", etc. The first two are measurable directly only at long wave-lengths, the last two are properties of space resonant line elements. Of these the "Q" was the most advantageous in the present instance.

"Q" Definition

At low frequencies the resonance selectivity factor of lumped circuits is identified as the "Q" and is defined as $\frac{\omega L}{R}$. It is measured by a detuning process. For a length of transmission line with negligible shunt conductance losses this process gives $\frac{\omega L}{R}$ as for a coil; when this process is applied to complex circuits the physical embodiment of the "Q" becomes difficult to realize and it is preferable to define the "Q" in terms of the detuning process itself. This is equally true for the resonant, centimeter wave, line element and we proceed as follows: For this element some current or voltage amplitude, conveniently measurable, is selected and three values of it are measured as the line tuning is varied. This variation may be either in generator frequency for constant line length or in line length for constant generator frequency.

Thus, for example,

$$\left.\begin{aligned} Q &= \frac{f_0}{f_2 - f_1}, \quad \text{where } f_2 > f_0 > f_1 \\ Q &= \frac{\ell_0}{\ell_2 - \ell_1}, \quad \text{where } \ell_2 > \ell_0 > \ell_1 \end{aligned}\right\} A_2^2 = A_1^2 = \frac{A_0^2}{2} \tag{1}$$

with A_0 as the resonant amplitude. For low-loss lines the two definitions will give the same results in practice. Neither is ideal for second order accuracy since there is a variation of line constants with frequency in the first and a variation in total attenuation in the second.

For practical reasons it is usually preferable to excite and observe the line resonance in terms of the current at one end, this end shorted. The elementary line lengths are then the quarter and the half-wave ones, the former with open circuit far end, the latter with shorted far end. The latter is the more nearly ideal unit. In order to short effectively the input end, the input and output couplings must be made as loose as possible. As these couplings are reduced the observed "Q" will asymptotically approach the line "Q". At the present moment the line variation in length is the most convenient process, the chief trouble being the micrometric measurement of the tiny length changes involved. Thus for 10 cms wave-length and a half-wave coaxial line, a "Q" of 1000 involves a plunger movement of .0019 inches.

Theory of Measurement

It is shown in the appendix that the "Q" of a given resonant line segment can be broken up into parts representing the equivalent "Q's" of the terminal impedances and the line itself. Thus

$$\frac{1}{Q} = \frac{1}{Q_q} + \frac{1}{Q_0} + \frac{1}{Q_\ell} \tag{2}$$

where "Q" is the actually measured quantity, Q_q is the part due to the line itself, Q_0 and Q_ℓ the parts due to the near and far end terminations, respectively.

If we now take a quarter-wave line segment, with near end shorted through a movable plunger and far end open, we may make two "Q" measurements without and with the far end loaded with a dielectric segment, and obtain

$$\left.\begin{array}{l} \left\{\begin{array}{l} \dfrac{1}{Q'} = \dfrac{1}{Q_q} + \dfrac{1}{Q_0} = \dfrac{d'}{\lambda/4} \\[2ex] \dfrac{1}{Q} = \dfrac{1}{Q_q} + \dfrac{1}{Q_0} + \dfrac{1}{Q_\ell} = \dfrac{d}{\lambda/4} \end{array}\right. \\[2ex] \text{and} \quad \dfrac{1}{Q} - \dfrac{1}{Q'} = \dfrac{1}{Q_\ell} = \dfrac{d - d'}{\lambda/4} \end{array}\right\} \left\{\begin{array}{l} d' = \ell_2' - \ell_1' \\ d = \ell_2 - \ell_1 \end{array}\right. \tag{3}$$

with d' and d equal to the widths of the resonance curves halfway down in power. These two d's are, of course, directly measurable.

When the line is loaded with a dielectric segment the loaded part of the line can be represented as an impedance Z_ℓ connected to the unloaded remainder of the line. The effect of the loaded segment upon the unloaded line (See appendix, eq. 4) appears in the form $\dfrac{\sqrt{\frac{Z}{D}}}{Z_\ell}$ where $\sqrt{\dfrac{Z}{D}}$ is the surge impedance of the unloaded line, with "Z" and "D" the series impedance and shunt admittance, respectively, for unit length of this line. If we put

$$\tanh \theta = \tanh (a_\ell + ib_\ell) = \frac{\sqrt{\frac{Z}{D}}}{Z_\ell} \tag{4}$$

we have

$$\left\{\begin{array}{l} \dfrac{1}{Q_\ell} = \dfrac{d - d'}{\lambda/4} = \dfrac{4a_\ell}{\pi} \\[2ex] \Delta\ell + t = \dfrac{\lambda}{2\pi} b_\ell \end{array}\right. \tag{5}$$

where $\Delta\ell$ is the measured plunger movement necessary to retune the line, after adding the dielectric loading, and "t" is the length of the dielectric segment.

Now, the power factor of "Z_ℓ" is the same as that of $\frac{\sqrt{\frac{Z}{D}}}{Z_\ell}$, as long as $\sqrt{\frac{Z}{D}}$ is substantially a resistance, and since

$$\tanh\,(a_\ell + i\,b_\ell) = \frac{\sinh 2a_\ell + i \sin 2b_\ell}{\cosh 2a_\ell + \cos 2b_\ell}, \tag{6}$$

we have

$$\text{power factor } Z_\ell = \text{p.f.} = \frac{\sinh 2a_\ell}{\sin 2b_\ell}. \tag{7}$$

Substituting eq. (5) in (7),

$$\text{p.f.} = \frac{\sinh \frac{2\pi}{\lambda}(d - d')}{\sin \frac{4\pi}{\lambda}(\Delta\ell + t)}, \tag{8}$$

which is the power factor of the loaded line segment in terms only of measurable lengths.

This does not complete the theory, however. We are interested in the power factor of the dielectric itself and it is evident that except for very short dielectric segments, the variation of the standing electrical field along the dielectric segment will result in a calculated power factor smaller than the true one. We also wish to determine the dielectric constant.

The impedance of the dielectric line segment, open circuited at the far end, can be written as

$$Z_\ell = \frac{\sqrt{\frac{L}{\epsilon C}}}{\tanh\left(\alpha + i\frac{2\pi\sqrt{\epsilon}}{\lambda}\right)t} \tag{9}$$

where "α" is the attenuation per unit length and "ϵ" is the dielectric constant. Hence $\tanh\,(a_\ell + ib_\ell) = \sqrt{\epsilon}\,\tanh\left(\alpha + i\frac{2\pi\sqrt{\epsilon}}{\lambda}\right)t$ and

$$\text{p.f.} = \frac{\sinh 2\alpha t}{\sin \frac{4\pi\sqrt{\epsilon}}{\lambda}\,t} \tag{10}$$

an alternative expression. Now when "t" is very small the functions of the angles become equal to the angles and we write, for the dielectric power factor itself

$$\text{P.F.} = \frac{2\alpha t}{\frac{4\pi\sqrt{\epsilon}}{\lambda} t}. \tag{11}$$

Dividing this expression by eq. (10)

$$\text{P.F.} = \text{p.f.} \frac{\sin \frac{4\pi\sqrt{\epsilon}\, t}{\lambda}}{\frac{4\pi\sqrt{\epsilon}\, t}{\lambda}} \cdot \frac{2\alpha t}{\sinh 2\alpha t}$$

and as the last term is always very nearly unity we have, if we put $\frac{4\pi\sqrt{\epsilon}\, t}{\lambda} = 4X$,

$$\text{P.F.} = \frac{\sinh \frac{2\pi}{\lambda}(d - d')}{\sin \frac{4\pi}{\lambda}(\Delta\ell + t)} \cdot \frac{\sin 4X}{4X}. \tag{12}$$

Ordinarily the "sinh" is very closely equal to the angle.

The reactance of the dielectric segment of line is necessarily equal to the reactance of the part of the original line which it displaces, since space resonance occurs in both cases. Hence,

$$\tan \pi \frac{\Delta\ell + t}{\lambda} = \sqrt{\epsilon} \tan \pi \frac{\sqrt{\epsilon}\, t}{\lambda} \tag{13}$$

which we can rewrite to

$$\frac{\pi t}{\lambda} \cdot \tan \pi \frac{\Delta\ell + t}{\lambda} = \pi \frac{\sqrt{\epsilon}\, t}{\lambda} \cdot \tan \pi \frac{\sqrt{\epsilon}\, t}{\lambda}.$$

Putting

$$\begin{cases} y = \frac{\pi t}{\lambda} \tan \pi \frac{\Delta\ell + t}{\lambda} \\ X = \frac{\pi\sqrt{\epsilon}\, t}{\lambda} \end{cases} \quad \text{we have} \quad y = X \tan X. \tag{14}$$

"y" is directly determinable by measurement and this gives X from the $X \tan X$ table supplied.[1] The value of $\epsilon = \left[\frac{X}{\frac{\pi t}{\lambda}}\right]^2$ follows and P.F. is immediately calculable. This completes the reduction of the observation.

[1] As no $X \tan X$ table to the necessary subdivision was available, one was calculated from the Hayashi $\tan X$ tables.

X TAN X

X	0	1	2	3	4	5	6	7	8	9
.00	.0000 0000	.0000 0100	.0000 0400	.0000 0900	.0000 1600	.0000 2500	.0000 3600	.0000 4900	.0000 6400	.0000 8100
.01	.0001 0000	.0001 2100	.0001 4401	.0001 6901	.0001 9601	.0002 2502	.0002 5602	.0002 8903	.0003 2403	.0003 6104
.02	.0004 0005	.0004 4106	.0004 8408	.0005 2909	.0005 7611	.0006 2513	.0006 7615	.0007 2918	.0007 8420	.0008 4124
.03	.0009 0027	.0009 6131	.0010 2435	.0010 8940	.0011 5645	.0012 2550	.0012 9656	.0013 6963	.0014 4470	.0015 2177
.04	.0016 0085	.0016 8194	.0017 6504	.0018 5014	.0019 3725	.0020 2637	.0021 1749	.0022 1063	.0023 0577	.0024 0292
.05	.0025 0209	.0026 0326	.0027 0644	.0028 1163	.0029 1884	.0030 2806	.0031 3928	.0032 5252	.0033 6778	.0034 8504
.06	.0036 0433	.0037 2562	.0038 4893	.0039 7426	.0041 0160	.0042 3096	.0043 6234	.0044 9573	.0046 3114	.0047 6857
.07	.0049 0802	.0050 4949	.0051 9298	.0053 3849	.0054 8602	.0056 3557	.0057 8715	.0059 4075	.0060 9637	.0062 5402
.08	.0064 1369	.0065 7539	.0067 3911	.0069 0486	.0070 7264	.0072 4245	.0074 1429	.0075 8815	.0077 6405	.0079 4198
.09	.0081 2194	.0083 0393	.0084 8796	.0086 7402	.0088 6212	.0090 5225	.0092 4442	.0094 3862	.0096 3486	.0098 3315
.10	.0100 3347	.0102 3583	.0104 4023	.0106 4668	.0108 5516	.0110 6570	.0112 7827	.0114 9289	.0117 0956	.0119 2828
.11	.0121 4904	.0123 7185	.0125 9672	.0128 2363	.0130 5259	.0132 8361	.0135 1668	.0137 5181	.0139 8899	.0142 2823
.12	.0144 6952	.0147 1287	.0149 5829	.0152 0576	.0154 5529	.0157 0689	.0159 6055	.0162 1628	.0164 7407	.0167 3393
.13	.0169 9585	.0172 5984	.0175 2591	.0177 9404	.0180 6425	.0183 3653	.0186 1088	.0188 8731	.0191 6582	.0194 4640
.14	.0197 2907	.0200 1381	.0203 0063	.0205 8954	.0208 8053	.0211 7360	.0214 6876	.0217 6601	.0220 6534	.0223 6677
.15	.0226 7028	.0229 7589	.0232 8359	.0235 9339	.0239 0528	.0242 1927	.0245 3535	.0248 5354	.0251 7383	.0254 9619
.16	.0258 2071	.0261 4731	.0264 7602	.0268 0683	.0271 3975	.0274 7479	.0278 1193	.0281 5119	.0284 9256	.0288 3605
.17	.0291 8166	.0295 2939	.0298 7923	.0302 3120	.0305 8529	.0309 4151	.0312 9986	.0316 6032	.0320 2292	.0323 8765
.18	.0327 5452	.0331 2351	.0334 9464	.0338 6791	.0342 4332	.0346 2087	.0350 0056	.0353 8239	.0357 6637	.0361 5250
.19	.0365 4077	.0369 3119	.0373 2377	.0377 1849	.0381 1537	.0385 1441	.0389 1561	.0393 1896	.0397 2448	.0401 3216
.20	.0405 4201	.0409 5402	.0413 6820	.0417 8455	.0422 0307	.0426 2377	.0430 4664	.0434 7169	.0438 9892	.0443 2832
.21	.0447 5991	.0451 9369	.0456 2965	.0460 6780	.0465 0814	.0469 5067	.0473 9540	.0478 4232	.0482 9144	.0487 4275
.22	.0491 9627	.0496 5200	.0501 0992	.0505 7006	.0510 3240	.0514 9696	.0519 6373	.0524 3271	.0529 0391	.0533 7733
.23	.0538 5297	.0543 3084	.0548 1093	.0552 9325	.0557 7779	.0562 6457	.0567 5358	.0572 4483	.0577 3831	.0582 3404
.24	.0587 3201	.0592 3222	.0597 3468	.0602 3939	.0607 4634	.0612 5555	.0617 6702	.0622 8074	.0627 9673	.0633 1497
.25	.0638 3548	.0643 5825	.0648 8330	.0654 1061	.0659 4020	.0664 7207	.0670 0621	.0675 4263	.0680 8133	.0686 2232
.26	.0691 6560	.0697 1117	.0702 5903	.0708 0918	.0713 6163	.0719 1638	.0724 7343	.0730 3278	.0735 9444	.0741 5842
.27	.0747 2470	.0752 9329	.0758 6421	.0764 3744	.0770 1299	.0775 9087	.0781 7107	.0787 5361	.0793 3847	.0799 2567
.28	.0805 1521	.0811 0709	.0817 0131	.0822 9787	.0828 9679	.0834 9805	.0841 0166	.0847 0763	.0853 1596	.0859 2665
.29	.0865 3971	.0871 5513	.0877 7292	.0883 9308	.0890 1562	.0896 4054	.0902 6783	.0908 9751	.0915 2957	.0921 6403
.30	.0928 0088	.0934 4012	.0940 8176	.0947 2579	.0953 7224	.0960 2109	.0966 7234	.0973 2601	.0979 8210	.0986 4060
.31	.0993 0153	.0999 6488	.1006 3065	.1012 9886	.1019 6950	.1026 4257	.1033 1809	.1039 9605	.1046 7645	.1053 5930
.32	.1060 4461	.1067 3237	.1074 2259	.1081 1527	.1088 1041	.1095 0802	.1102 0810	.1109 1066	.1116 1569	.1123 2321
.33	.1130 3321	.1137 4569	.1144 6067	.1151 7814	.1158 9811	.1166 2057	.1173 4554	.1180 7303	.1188 0302	.1195 3552
.34	.1202 7054	.1210 0808	.1217 4815	.1224 9074	.1232 3587	.1239 8353	.1247 3373	.1254 8647	.1262 4175	.1269 9959
.35	.1277 5997	.1285 2292	.1292 8842	.1300 5649	.1308 2712	.1316 0032	.1323 7610	.1331 5446	.1339 3539	.1347 1891
.36	.1355 0503	.1362 9373	.1370 8503	.1378 7893	.1386 7544	.1394 7455	.1402 7628	.1410 8062	.1418 8758	.1426 9716
.37	.1435 0937	.1443 2421	.1451 4169	.1459 6180	.1467 8456	.1476 0997	.1484 3802	.1492 6873	.1501 0210	.1509 3813
.38	.1517 7683	.1526 1821	.1534 6325	.1543 0898	.1551 5838	.1560 1048	.1568 6527	.1577 2275	.1585 8293	.1594 4582
.39	.1603 1142	.1611 7973	.1620 5075	.1629 2450	.1638 0097	.1646 8017	.1655 6211	.1664 4679	.1673 3421	.1682 2437
.40	.1691 1729	.1700 1296	.1709 1140	.1718 1259	.1727 1657	.1736 2331	.1745 3283	.1754 4514	.1763 6023	.1772 7811
.41	.1781 9879	.1791 2228	.1800 4857	.1809 7767	.1819 0958	.1828 4432	.1837 8188	.1847 2227	.1856 6550	.1866 1156
.42	.1875 6047	.1885 1223	.1894 6684	.1904 2430	.1913 8464	.1923 4784	.1933 1391	.1942 8287	.1952 5470	.1962 2942
.43	.1972 0704	.1981 8755	.1991 7097	.2001 5730	.2011 4654	.2021 3869	.2031 3377	.2041 3178	.2051 3272	.2061 3661
.44	.2071 4343	.2081 5321	.2091 6594	.2101 8162	.2112 0028	.2122 2191	.2132 4651	.2142 7409	.2153 0466	.2163 3822
.45	.2173 7478	.2184 1434	.2194 5691	.2205 0249	.2215 5110	.2226 0272	.2236 5738	.2247 1508	.2257 7581	.2268 3960
.46	.2279 0643	.2289 7633	.2300 4929	.2311 2532	.2322 0442	.2332 8661	.2343 7189	.2354 6025	.2365 5172	.2376 4629
.47	.2387 4397	.2398 4477	.2409 4869	.2420 5574	.2431 6593	.2442 7925	.2453 9573	.2465 1535	.2476 3813	.2487 6409
.48	.2498 9320	.2510 2550	.2521 6099	.2532 9966	.2544 4152	.2555 8659	.2567 3487	.2578 8636	.2590 4107	.2601 9902
.49	.2613 6019	.2625 2462	.2636 9227	.2648 6319	.2660 3736	.2672 1481	.2683 9607	.2695 7952	.2707 6680	.2719 5737
.50	.2731 5125	.2743 4843	.2755 4892	.2767 5273	.2779 5987	.2791 6529	.2803 8415	.2816 0132	.2828 2183	.2840 4570
.51	.2852 7295	.2865 0356	.2877 3756	.2889 7495	.2902 1573	.2914 5992	.2927 0752	.2939 5853	.2952 1297	.2964 7084

The above theory applies to the quarter wave line. This is a rather difficult practical one; it is best to add another quarter wave to make a half-wave resonator, shorted at both ends, with the dielectric positioned exactly in the center. From conditions of symmetry we then employ the above equations, taking half of our measured quantities. Or, in terms of the actually measured four lengths which constitute an observation on a half-wave line, $(d-d')$, t, $\Delta\ell$ and λ, we have,

$$\left.\begin{aligned} \text{P.F.} &= \frac{\sinh \frac{\pi}{\lambda}(d-d')}{\sin \frac{2\pi}{\lambda}(\Delta\ell + t)} \cdot \frac{\sin 2X}{2X} \\ \frac{\pi t}{\lambda}\cdot \tan \pi \frac{\Delta\ell + t}{\lambda} &= X \tan X \\ \epsilon &= \left[\frac{X}{\frac{\pi t}{\lambda}}\right]^2 \end{aligned}\right\} \tag{15}$$

which are the expressions used in this work.

In practice the dielectric plug is pushed into the half-wave line and the line is tuned. The line center is then calculated and the plug reset to this. Retuning checks the correct location. Two trials are always sufficient if the plug was nearly centered originally.

There are several shortcomings affecting this theory. The Q of the unloaded line depends partly on metal power loss along the line. When the line is shortened by the dielectric plug, part of this loss disappears and part is transferred to the dielectric plug. Fortunately these losses are small since they are metal losses at a current node, but for long dielectric plugs or plugs of high dielectric constant the need for correction can arise. The necessary calculations have not yet been reduced to a simple form.

Again, the calculation of half-wave results by means of a quarter wave theory is safe only for a high Q situation. It is easy to show, experimentally, that the maximum line shortening results when the dielectric plug is exactly centered in the line but the calculated power factor is not a maximum here, as might be expected. In the meantime, experience shows that results can be duplicated from day to day and at other frequencies and that over a reasonable range of plug thickness no change in dielectric constant and power factor values, greater than the unavoidable errors of measurement, is obtained.

Description of Apparatus

The apparatus can be divided into three parts for purposes of description.

The high frequency generator consists of a small "relay rack" assembly, including 60-cycle power panel, rectifier panel, meter and control panel and centimeter wave oscillator panel with coaxial conductor output jack. All high-frequency connectors are coaxial conductor units with plug tips.

The measuring unit is shown in the two photographs; Fig. 1, assembled and Fig. 2, disassembled. Two combination input-output heads are shown in Fig. 2. These heads and tubing together with center conductor and plunger are of coin silver. While the highest possible conductivity metal is desirable, pure silver is mechanically too poor for spring fingers and bearing surfaces and the alloy must be used. The good sliding contact properties of silver are preserved but the conductivity is no better than that of copper. Both heads are drilled, for input and output connections, flush with the bottom of the cylindrical cavity terminating the tubing.

Head #1, shown attached in Fig. 1 and detached in lower right-hand corner of Fig. 2, has a silicon crystal, mounted and insulated in a small cylindrical holder which carries a tiny pickup loop, one side of which is grounded to the cylinder. The total length of pickup conductor including loop and crystal "whisker" is about one centimeter and no tuning is necessary. The loop pickup can be adjusted by moving the holder in or out. The d-c circuit is from an insulated pin on the holder through crystal to apparatus body.

The current input connection is through a coaxial plug which is tapped across a fraction of a tunable half-wave line. This fraction consists of a $\frac{1}{8}''$ coaxial conductor terminated in a tiny feed loop; the remainder of the line is an ordinary $\frac{1}{4}''$ coaxial with sliding plunger. The line is used, well off tune, as an input current amplitude control. The coupling with the cavity in head is adjusted by moving the feed loop in or out.

By inverting another half-wave coaxial with feed loop, so as to put the crystal where the feed jack was, it is possible to use an externally mounted crystal as in head #2. For this head the input current amplitude control is obtained by using, as a feeder, a short $\frac{1}{8}''$ coaxial tipped with a tiny loop and a coaxial jack, at opposite ends. This coaxial is mounted in a spring clamped bearing so as to permit a rotation of the plane of the loop. All coaxials, except the measuring unit itself, are 72-ohm ones.

There is no essential difference in operation between these two heads; they are interchangeable. However, head #1 is more convenient in manipulation, during the disassembly required to insert the dielectric sample. (This sample is always positioned in the piece of tubing connecting to the head.)

An ordinary model 301 microammeter, low resistance, served as indicating instrument. By replacing the crystal holder of head #2 with a loop tipped coaxial and plug, a conventional double-detection radio receiver with

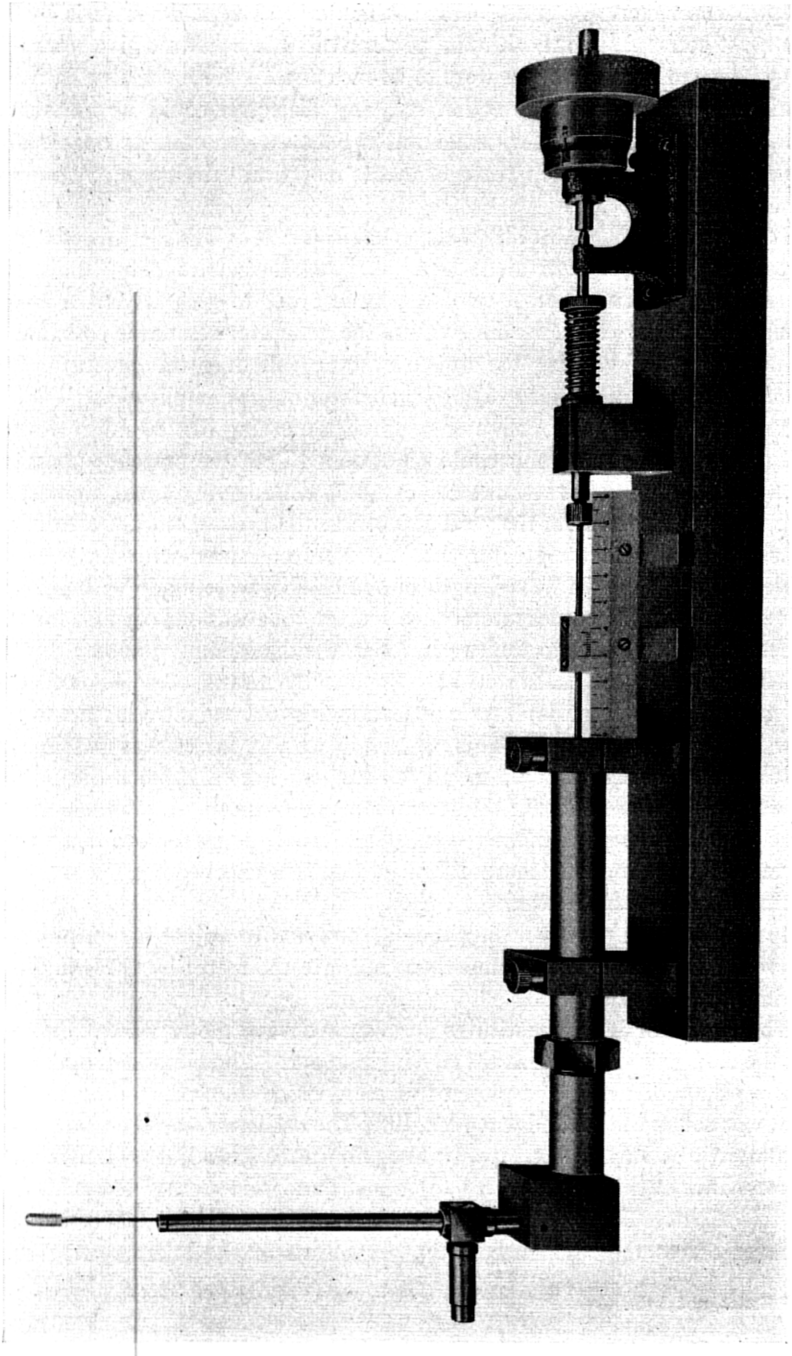

Fig. 1—Measuring unit, assembled

output meter could be used instead. The crystal type detector is by far the most convenient but with the power available wouldn't give workable outputs when bad dielectrics were to be measured. With the amplification available in the double detection set, any dielectric could be measured, while retaining the necessary attenuation between generator-resonator and resonator-receiver to keep these elements electrically independent of each other.

It is necessary to maintain an electrical isolation of this sort to get a high apparatus Q. The equivalent Q of all good dielectrics being high, the measuring apparatus Q must be of the same order to give favorable measuring conditions. And, further, unless the generator-resonator coupling is weak, the act of varying the resonator tune will drag the generator frequency around and will also vary the generator output amplitude.

The crystal plus microammeter required something like 80 millivolts for full scale deflection and this could be obtained with the present apparatus with couplings giving a resonator Q of 1500, while having enough power in reserve to measure any of the good dielectrics. However, most of the dielectrics with power factor greater than .01 were measured with the d.d. receiver. All the 10 cm wave-length measurements were made with this receiver. For the latter measurements a shorter tube was substituted for the tubes shown screwed into the two heads in the disassembly photo.

The crystals were calibrated at 60 cycles by means of a 70-ohm $\sqrt{2}$ attenuation pad.[2] With full scale deflection this pad was introduced and the new scale deflection read. This $\sqrt{2}$ ratio was, as far as was possible to check, maintained in the kilo megacycle range. For calibration the crystal was tapped across 4 ohms in the attenuator pad output. A 15 mf electrolytic condenser was permanently connected across the meter terminals and, by means of a pair of switches, calibration could be checked in a few seconds, during a measurement run.

The calibration process, using the d.d. set, was to adjust the output to a convenient meter deflection and then calibrate the meter by throwing in 3 db in the IF attenuator.

The resonator itself constitutes an accurate wave meter when corrected for the change in diameter at the moving plunger. The method of operation was then as follows. The plunger vernier, which allowed reading to 0.01 cm., was set at the desired wave-length. The oscillator was then turned on and after it had attained temperature equilibrium, was adjusted if necessary to resonance at this value. This adjustment was infrequently necessary and always slight. The apparatus Q was then determined by traversing the plunger across the resonance setting by means of the micrometer. This

[2] Exact, not 3 db.

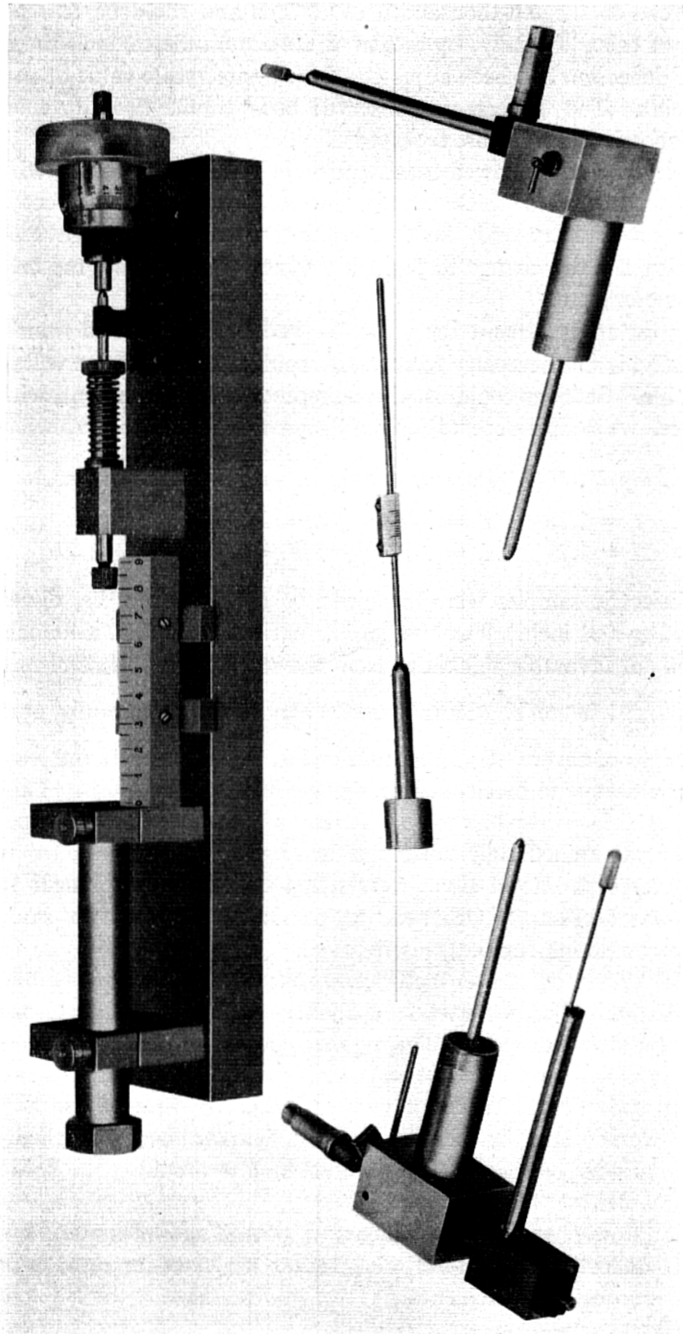

Fig. 2—Measuring unit, disassembled. Two different heads shown

"mike" read to the ten-thousandth of an inch and could be estimated to one-fifth of this. Initially, by means of the amplitude control, the micro-ammeter deflection had been adjusted to the desired scale value at the resonance point. The traverse was observed between the two $\sqrt{2}$ microammeter deflections and was repeated in the opposite direction. When successive round trips showed consistency the value of d' was noted. The dielectric sample, after thickness measurement, was then introduced, centered by cut and try and the Q traverses repeated. This gave d and, after noting $\Delta\ell$, the change in plunger setting for resonance, the measurement was complete.

During the measurement the generator had to be protected from drafts and, usually, it was necessary to traverse rapidly, the power line voltage not being stable. Settings could usually be reproduced to 1 per cent, with adequate care. A sample observation on a good dielectric is the following:

July 28, 1941 Polystyrene plate, all dimensions in cms.

$t = 1.28$	$d' = .0084$	$\lambda = 22.42$
$\Delta\ell = 1.79$	$d = .010$	P.F. $= .00028$, $\epsilon = 2.49$

The dielectric samples were machined on a precision lathe, dimensions being held to .001 inch. The nominal dimensions were O.D. .640 inch, I.D. .174 inch. A favorable thickness, from the standpoint of ease of measurement, is $\left|\frac{\lambda}{10\epsilon}\right|$, in cm's. Cleanliness in handling was carefully observed. After a lapse of several days the interior bearing surfaces of the resonance cavity would have to be cleaned with fine French crocus cloth. The plunger bearing surfaces also had to be smoothed up, fine scratches being polished off. Dirt was immediately noticeable when the plunger contacted it, and when microscopic bits of silver were rolled up under the plunger springs cleaning was necessary. Otherwise no particular treatment or smoothing up of the contacting surfaces was required.

A table of dielectric power factors and constants is a very desirable piece of information. Unfortunately, experience tends to the conclusion that such a table does not exist. The organic plastics in particular, are rather variable from sample to sample and a table of values is merely a table for particular specimens. Where a great number of samples are available "best", "worst" and "most common" values can be established. The accompanying list of observed values must be interpreted in the light of the above statements.

As a large number of measurements of certain special materials had to be made, dielectrics in general were rather neglected and the tabulated values are more or less incidental. It was noted that for the low loss, sub-

TABLE 1

Material	ε		P.F.	
	22.5 cms.	10 cms.	22.5 cms	10 cms.
Ceramic				
BTL F3 Mg. Silicate type	5.83		.00023	
"Dielectene"		3.39		.0038
Glass, Corning				
G1, lead	4.30		.0049	
G8, lime, annealed	6.38		.0102	
G12, lead	6.08		.0035	
199-1	8.70		.0019	
702EJ, Pyrex	6.35		.0067	
702P	4.70		.0053	
704EO	4.42		.0033	
705BA	3.80		.00118	
707DG	4.69	4.8	.0037	.0036
Glyptal	3.38	3.36	.030	.036
Lucite	2.58	2.56	.0090	.0087
Mycalex				
Red	5.91		.0030	
White	5.74		.0033	
Phenolics				
Cast specimen		4.63		.139
Bakelite sheet $\frac{1}{32}''$		3.57		.080
Polyethylene				
Worst			.00229	
Most common	2.26		.00060	
Best			.00031	
Polystyrene				
Worst			.00090	
Most common	2.45		.00070	
Best			.00028	
Polyvinylcarbazole	2.87		.0040	
Rubber				
Hard, brown		2.77		.0041
Hard, black		2.69		.0059
Soft, black	3.15		.0058	
Resin	2.32		.0018	
Styralloy				
No. 10	2.49		.0036	
Desig. Unknown	2.49	2.50	.0019	.00105
No. 22	2.40		.0047	
Styramic				
E1689		2.55		.00087
Tenite II		2.95		.031
Vinylite V	2.78	2.61	.0076	.0068
Wax				
Paraffin	2.17		.00019	
Boler	2.17		.00019	
Superla	2.26	2.26	.00019	.00015

stituted paraffin-type, carbon chain dielectrics no difference, greater than experimental error, exists between the 22.5 and 10 cm. measurements.

Acknowledgement

The measurements by means of the double-detection set were made by my co-worker, Mr. W. E. Eckner, whose valuable assistance I am glad to acknowledge. To Mr. C. F. Mattke, also of the Bell Laboratories, I am indebted for assistance in getting my crude original apparatus into its final finished form.

APPENDIX

The typical ultra high-frequency transmission line can be represented as in Fig. 3

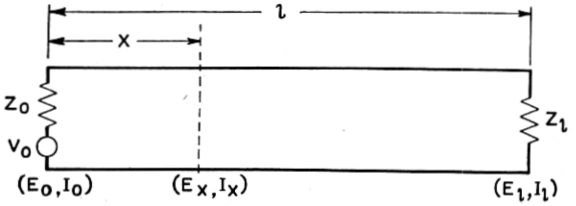

Fig. 3—Equivalent circuit of transmission line

and the equations describing it are

$$\left.\begin{aligned}
E_x &= V_0 \frac{Z_\ell \cosh \sqrt{DZ}\,(\ell - x) + \sqrt{\frac{Z}{D}} \sinh \sqrt{DZ}\,(\ell - x)}{(Z_0 + Z_\ell)\cosh \sqrt{DZ}\,\ell + \left(Z_0 Z_\ell \sqrt{\frac{D}{Z}} + \sqrt{\frac{Z}{D}}\right)\sinh \sqrt{DZ}\,\ell} \\
I_x &= V_0 \frac{\cosh \sqrt{DZ}\,(\ell - x) + Z_\ell \sqrt{\frac{D}{Z}} \sinh \sqrt{DZ}\,(\ell - x)}{(Z_0 + Z_\ell)\cosh \sqrt{DZ}\,\ell + \left(Z_0 Z_\ell \sqrt{\frac{D}{Z}} + \sqrt{\frac{Z}{D}}\right)\sinh \sqrt{DZ}\,\ell} \\
Z_x &= \frac{E_x}{I_x} = \sqrt{\frac{Z}{D}} \cdot \frac{Z_\ell + \sqrt{\frac{Z}{D}} \tanh \sqrt{DZ}\,(\ell - x)}{\sqrt{\frac{Z}{D}} + Z_\ell \tanh \sqrt{DZ}\,(\ell - x)} \\
E_0 &= V_0 - Z_0 I_0, \qquad E_\ell = Z_\ell I_\ell
\end{aligned}\right\} \quad (1)$$

The line constants are $Z = R + i\omega L$, the series impedance per unit length, and $D = G + i\omega C$, the shunt admittance per unit length. From these we have: surge impedance $= \sqrt{\frac{Z}{D}} = S_0$, propagation constant $= \sqrt{DZ}$.

For all lines usable as transmission devices the following approximations hold:

$$\sqrt{DZ} = \alpha + i\beta \quad \alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} \quad \beta = \omega\sqrt{LC} = \frac{\omega}{v} = \frac{2\pi}{\lambda}$$

$$\sqrt{\frac{Z}{D}} = \sqrt{\frac{L}{C}}, \quad C = \frac{1}{S_0 V}, \quad L = \frac{S_0}{V}, \quad v = 3 \times 10^{10} \text{ cm/sec. for air.} \tag{2}$$

For the case of near-end input and output the second of equations (1) can be rewritten as

$$I_0 = \frac{V_0}{\sqrt{\frac{Z}{D}}} \cdot \frac{\cosh \sqrt{DZ}\,\ell + \frac{Z_\ell}{\sqrt{\frac{Z}{D}}} \sinh \sqrt{DZ}\,\ell}{\frac{Z_\ell}{\sqrt{\frac{Z}{D}}} \cosh \sqrt{DZ}\,\ell + \sinh \sqrt{DZ}\,\ell + \frac{Z_0}{\sqrt{\frac{Z}{D}}}\left(\cosh \sqrt{DZ}\,\ell + \frac{Z_\ell}{\sqrt{\frac{Z}{D}}} \sinh \sqrt{DZ}\,\ell\right)} \tag{3}$$

If now we assume a quarter-wave line, the condition of resonance implies that $R_\ell \gg \left|\sqrt{\frac{Z}{D}}\right|$ and $R_0 \ll \left|\sqrt{\frac{Z}{D}}\right|$. The condition of reasonable shortening of the line (or lengthening) by the terminal reactance implies that $X_\ell > \left|\sqrt{\frac{Z}{D}}\right|$, $X_0 < \left|\sqrt{\frac{Z}{D}}\right|$. Hence we shall have $|Z_\ell| \gg \left|\sqrt{\frac{Z}{D}}\right|$, $|Z_0| \ll \left|\sqrt{\frac{Z}{D}}\right|$.

If we put $\frac{\sqrt{\frac{Z}{D}}}{Z_\ell} = \tanh\theta$, we get

$$I_0 = \frac{V_0}{\sqrt{\frac{Z}{D}}} \cdot \frac{\tanh(\sqrt{DZ}\,\ell + \theta)}{1 + \frac{Z_0}{\sqrt{\frac{Z}{D}}} \tanh(\sqrt{DZ}\,\ell + \theta)} \tag{4}$$

We now make the assumption that "Z_0" is a pure resistance (which is no limitation on the measurement to be discussed) and put $\theta = a_\ell + ib_\ell$.

Then,

$$|I_0| = \frac{V_0}{\sqrt{\frac{L}{C}}} \cdot \sqrt{\frac{\tanh^2(\alpha\ell + a_\ell) + \tan^2\left(\frac{2\pi\ell}{\lambda} + b_\ell\right)}{\left[1 + \frac{Z_0}{\sqrt{\frac{L}{C}}}\tanh(\alpha\ell + a_\ell)\right]^2 + \left[\frac{Z_0}{\sqrt{\frac{L}{C}}} + \tanh(\alpha\ell + a_\ell)\right]^2 \tan^2\left(\frac{2\pi\ell}{\lambda} + b_\ell\right)}} \tag{5}$$

This expression cycles, as "ℓ" is varied, and has its maximum or "tuned" value of

$$\frac{V_0 \Big/ \sqrt{\frac{L}{C}}}{\frac{Z_0}{\sqrt{\frac{L}{C}}} + \tanh(\alpha\ell + a_\ell)} \quad \text{for} \quad \tan\left(\frac{2\pi\ell}{\lambda} + b_\ell\right) = \infty$$

$$\text{or} \quad \frac{2\pi\ell}{\lambda} + b_\ell = \frac{(2n+1)\pi}{2} \qquad n = 0, 1, 2, \cdots$$

The resonant length is thus $\ell = \frac{\lambda}{4}\left(1 - \frac{2b_\ell}{\pi}\right)$, for $n = o$. Note that successive resonances differ by a line length of $\frac{\lambda}{2}$; the reactive termination has merely shortened, by the amount of $\Delta\ell = \frac{\lambda b_\ell}{2\pi}$, the first resonant element preceding it. When, therefore, we measure the "Q" of this line segment by line-length tuning we use $\ell = \frac{\lambda}{4}$ in the Q process definition.

The Q process now follows. Putting $\ell = \ell_r \pm \delta\ell$ where ℓ_r is the actual observed resonance length, we have

$$\frac{2\pi\ell}{\lambda} + b_\ell = \frac{2\pi\ell_r}{\lambda} + b_\ell \pm \frac{2\pi\delta\ell}{\lambda} = \frac{\pi}{2} \pm \frac{2\pi\delta\ell}{\lambda}$$

Then $\tan\left(\frac{2\pi\ell}{\lambda} + b_\ell\right) = \tan\left(\frac{\pi}{2} \pm \frac{2\pi\delta\ell}{\lambda}\right) = \frac{1}{\mp\tan\frac{2\pi\delta\ell}{\lambda}}$ and

$$|I_0| = \frac{V_0}{\sqrt{\frac{L}{C}}} \sqrt{\frac{1 + \tanh^2(\alpha\ell + a_\ell)\cdot\tan^2\frac{2\pi\delta\ell}{\lambda}}{\left[\frac{Z_0}{\sqrt{\frac{L}{C}}} + \tanh(\alpha\ell + a_\ell)\right]^2 + \left[1 + \frac{Z_0}{\sqrt{\frac{L}{C}}}\tanh(\alpha\ell + a_\ell)\right]^2 \tan^2\frac{2\pi\delta\ell}{\lambda}}}$$

Forming the current values $|I_{01}| = |I_{02}| = \left|\frac{I_0(\text{resonant})}{K}\right|$, dividing to eliminate $|I_0|$ and discarding squares and products of small quantities in comparison with unity leaves,

$$\sqrt{K^2 - 1} = \frac{\tan\frac{2\pi\delta\ell}{\lambda}}{\frac{Z_0}{\sqrt{\frac{L}{C}}} + \tanh(\alpha\ell + a_\ell)}$$

$$\text{or} \quad \sqrt{K^2 - 1}\,\frac{\lambda}{8\delta\ell} = \frac{\frac{\lambda}{8\delta\ell}\cdot\tan\frac{2\pi\delta\ell}{\lambda}}{\frac{Z_0}{\sqrt{\frac{L}{C}}} + \tanh(\alpha\ell + a_\ell)} \tag{6}$$

which becomes our "Q" when $K = \sqrt{2}$.

In most practical situations the "tan" and "tanh" are equal to their angles. For this condition $Q = \dfrac{\frac{\pi}{4}}{\frac{Z_0}{\sqrt{\frac{L}{C}}} + \alpha\ell + a_\ell}$. If we now put, by definition, $Q_q = \dfrac{\pi}{4\alpha\ell}$ (the Q of the line itself), $Q_0 = \dfrac{\pi}{4}\cdot\dfrac{\sqrt{\frac{L}{C}}}{Z_0}$, $Q_\ell = \dfrac{\pi}{4a_\ell}$, we have

$$\frac{1}{Q} = \frac{1}{Q_q} + \frac{1}{Q_0} + \frac{1}{Q_\ell} \tag{7}$$

the law of Q composition relating the resultant Q to line and terminal Q's.