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Indicial Response of Telephone Receivers

By E. E. MOTT

A method of analyzing telephone receiver characteristics by indicial response is discussed and illustrated by oscillograms. The indicial response of a telephone receiver is the instantaneous response of the receiver to a suddenly applied electromotive force. This type of response is of particular fundamental interest because it furnishes a key to the solution of transient problems such as are involved in the response to speech waves.

Oscillograms of indicial response, together with the more familiar steady-state frequency response characteristics, are shown for different types of receivers. The relationships existing between the two types of measurements are discussed.

From the standpoint of most faithfully reproducing transients, indicial response data indicate that a receiver having a limited range of frequency response should have a frequency response characteristic which droops gradually rather than abruptly near the upper end of the range.

Introduction

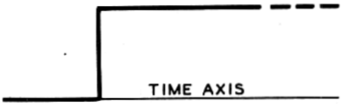
THE use of indicial response analysis as an outgrowth of the Heaviside operational calculus[1] has been extended to a number of different fields. The indicial admittance as defined by J. R. Carson[2] in his analysis of the submarine cable and other transmission problems has been an effective tool in the study of transients. More recently, a similar type of measurement has been used as an indication of performance of amplifiers[3], television equipment[4], and audio frequency transformers[5].

In the field of telephone receivers[6] an analysis by means of impressed square waves has been found useful as a measure of transient response. In the transmission of speech, so much emphasis has been placed upon steady-state frequency response as an indication of performance, that it seems in order to consider the possible advantages of a transient method of analysis, as obtained by measuring the indicial response. Only recently has the technique of such measurement been made feasible by the improvement at low frequencies of amplifiers and related apparatus.

The Indicial Response

The indicial response of a telephone receiver may be defined as the instantaneous sound pressure generated by the receiver in a closed air chamber due to a suddenly-applied unit voltage. This term differs from Carson's indicial admittance only in that sound pressure rather than current response is used. The sound pressure in an air chamber of pure stiffness is a measure of the volume displacement, and as such it is proportional to the transfer displacement admittance of the system. When we are interested in the charge rather than in the current, the admittance takes the form of a displacement admittance, related to the ordinary admittance by a factor of the frequency ω. That Carson's original equations apply to such a system with little if any change may be easily demonstrated. The term $A(t)$ may be used to denote any of these forms of indicial admittance or indicial response.

The form of the applied voltage assumed is shown by Fig. 1. This form, defined by Heaviside as the *unit function*, is a function of time equal to zero before, and unity after the time $t = 0$. More properly, however, it may be regarded as an increment in voltage closely analogous to Isaac Newton's concept of infinitesimal elements of rectangular area, the summation of which forms the basis of the integral calculus. The successive application of small increments of voltage likewise forms the basis of the operational calculus, or more particularly, the basis of the Carson extension theorem.



THE UNIT FUNCTION

Fig. 1

The Carson Extension Theorem

Having obtained the indicial response, either experimentally or theoretically, we have the key to the more general problem where the applied voltage $e(t)$ may be of any form, such as that of speech waves. Let $e(t)$, Fig. 2, be any arbitrary voltage wave corresponding to speech[7]. Let a series of consecutive increments of voltage, differing in time by $\Delta\tau$ be applied, of such magnitude as to build up the form of the curve $e(t)$. By analyzing each of these components in terms of the indicial admittance $A(t)$, and synthesizing them again, the instantaneous sound pressure may be related to the voltage producing it and the indicial admittance $A(\tau)$ by the Carson extension equation[2]:

$$p(t) = \frac{d}{dt}\int_0^t A(\tau)e(t - \tau)\,d\tau$$

When the above integration is carried out, the term τ disappears and is replaced by t. The above sound pressure $p(t)$ represents the sound pressure generated by the receiver in a closed coupler due to an applied voltage $e(t)$.

$$p(t) = \int_0^\infty A(\tau)\, e(t-\tau)\, d\tau$$

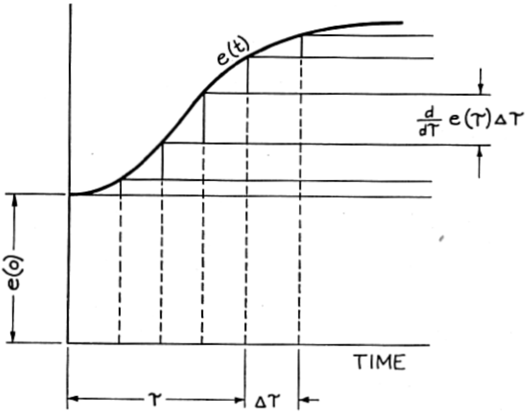


Fig. 2—Method of derivation of Carson's extension formula.

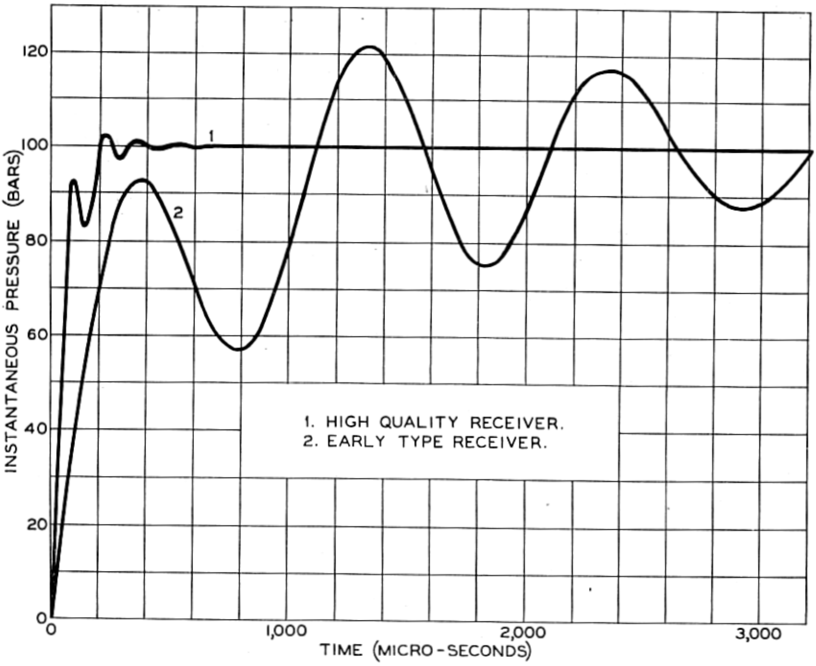


Fig. 3—Indicial admittance of two types of telephone receivers.

From the above, it is evident that the ideal form of receiver response to a suddenly-impressed voltage would be a copy of the unit function shown in Fig. 1, and that any deviation from this form will cause distortion. If the building blocks of the curve $e(t)$ are undistorted, the curve itself will likewise be reproduced free from distortion of wave form. Thus, the more closely the indicial response can be made to approach the form of the unit function, the more closely the receiver sound pressure $p(t)$ will be a copy of any arbitrary speech wave $e(t)$. Curve 1, Fig. 3, shows the indicial response of a receiver having a frequency range of 8000 cps, which comes rather close to this ideal. On the other hand, the further the indicial response departs from this ideal form, the more it will deviate from any impressed transient, such as speech waves. Thus curve 2, Fig. 3, corresponds to a receiver of narrow range, which contains resonant oscillations, and rises much later in time than the other receiver.

Conversion Formulae

The indicial response is as fundamental in character as frequency response, and may be converted into frequency and phase response if the proper integrations are carried out for any particular system, as follows:

$$\text{Indicial Response } A(t) \rightleftarrows \begin{bmatrix} \text{Frequency Response} \\ + \text{ Phase Response} \end{bmatrix} A(\omega) = P(\omega) + jQ(\omega)$$

where $A(\omega)$ is the transfer admittance of the system. In order to carry out these conversions, certain integrations must be performed, either mechanically or theoretically. The following are conversions[7] which may be used to carry out this process:

$$A(t) = \frac{2}{\pi} \int_0^\infty \frac{P(\omega)}{\omega} \sin \omega t \, d\omega$$

$$A(t) = P(0) + \frac{2}{\pi} \int_0^\infty \frac{Q(\omega)}{\omega} \cos \omega t \, d\omega$$

$$\frac{P(\omega)}{\omega} = \int_0^\infty A(t) \sin \omega t \, dt$$

$$\frac{Q(\omega)}{\omega} = \int_0^\infty [A(t) - A(0)] \cos \omega t \, dt$$

Where $P(\omega)$ and $Q(\omega)$ are the real and imaginary parts of the frequency reponse, $A(\omega)$ is expressed in terms of pressure reponse[8], while the indicial response $A(t)$ is expressed as an instantaneous sound pressure. The integrations are difficult to carry out, but serve to show how the two systems of measurement are related, and how they may theoretically be converted one into the other, provided in the case of frequency response the magnitude and phase are both known.

General Applications

The use of indicial response as a tool in telephone receiver studies is particularly adapted to the study of transients. Since all voice and sound transmission, particularly that of orchestral music, may be regarded as essentially a transient problem, it is appropriate that we visualize the effects on the complex wave forms of any distortions which may be present in the transmission apparatus. The indicial response will, in general, depart from the ideal square form, and the amount of this departure may be regarded as indicative of the relative faithfulness of wave form reproduction by apparatus having different frequency characteristics. An examination of these departures should therefore be helpful as a supplementary method of appraising the relative merits of different frequency response characteristics. The effect, for example, of small resonance peaks or dips upon transients is very forcefully shown in the form of the indicial admittance. The departure from squareness of a particular system may often be improved by use of the proper shape of frequency characteristic.

The use of a closed coupler when measuring telephone receivers is particularly adapted for such studies, because the disturbing effects of deficiencies at the low frequencies due to leakage may thus be eliminated. Interpretation by inspection then becomes a matter of observation of the various types of departures at the higher frequencies from the ideal form.

Since listening tests do not always agree with interpretations of physical measurements of steady-state frequency response, it often becomes a matter of interest to obtain different criteria of judgment in which the weight given to the various frequencies may be judged by the relative effects of irregularities in various parts of the frequency spectrum upon the indicial response.

Apparatus and Method of Testing

Various forms of apparatus may be used for receiver testing with square waves. Square-wave generator circuits have been published both for audio[5] and video[3] frequency use, involving vacuum tube circuits which overload at low voltages. For low speeds using low-frequency waves of the order 60 cps, a simple mercury switch operated by an oscillator gives very satisfactory results.

The square-wave voltage is introduced across a small part of the resistance termination as shown in Fig. 4, the whole resistance termination being matched to the magnitude of the receiver impedance at 800 cps. The receiver is then operating from an idealized resistance source having an impedance which matches that of the receiver approximately, over the range of interest.

The receiver is coupled acoustically to a small-diameter condenser microphone by means of a closed coupler[8]. The condenser microphone has a substantially uniform characteristic up to a frequency of 10 kc. The

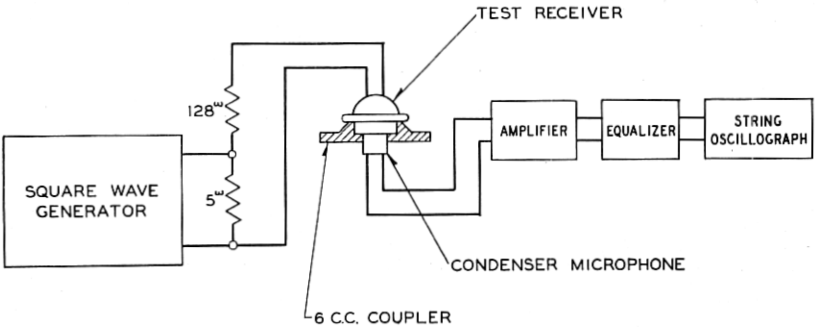
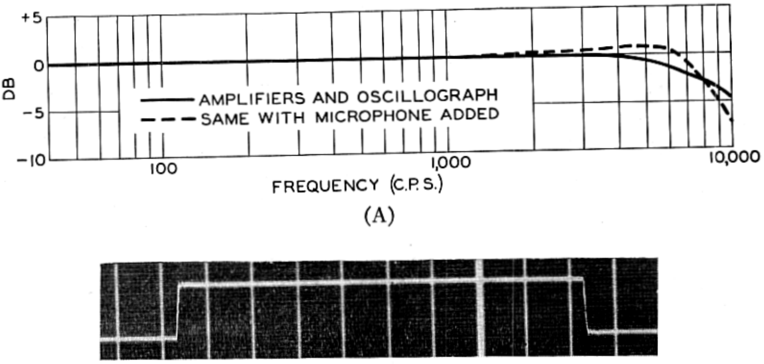


Fig. 4—Circuit diagram of apparatus for indicial response measurements.



(B)

Fig. 5—Frequency response (A) and indicial response (B) of measuring apparatus.

microphone voltage is then amplified to the point where it can be measured by an oscillograph.

Either the cathode-ray oscilloscope or a rapid-recording string oscillograph[9] may be used, but in the latter case it is necessary to equalize the string oscillograph to a frequency of about 10 kc in order to cover the audio frequency range. The choice of these instruments depends somewhat upon whether a permanent record is desired or whether a visual indication is sufficient.

The amplifier must be compensated at low frequencies in order to maintain a strictly square-wave output. The entire system characteristic is shown in Fig. 5 and covers a range of 1 to 10,000 cps with a substantially uniform frequency response. The indicial response of the system is also shown to be reasonably free from irregularities. Such irregularities as do exist are due largely to the sharp cut-off of the system at 10 kc which was necessitated by the limitations of the string oscillograph.

Indicial vs. Frequency Response

The calculated pairs of curves for telephone receivers in Fig. 6 show the relations between the frequency response and the indicial response. Since the characteristics of receivers measured on a closed coupler of known volume are readily amenable to calculation if the constants of the receiver are known, such a procedure is often useful in predetermining the design of a receiver.

The upper three curves, Fig. 6, are the characteristics of a moving coil receiver calculated for three different frequency ranges, being otherwise similar in shape, the curve being shifted in frequency by an arbitrary factor K. The effect on the indicial admittance is to shift it in time by the same factor without change of shape, if the plot is logarithmic as shown. In general, if the cut-off frequency is divided by the factor K, the corresponding time delay will be increased by the factor K. This is an application of a theorem by Carson[2] that:

$$\frac{1}{pZ(kp)} = \int_0^\infty A(t/k)\epsilon^{-pt}\,dt$$

where $p = j\omega$ is proportional to frequency, and t is the time, $\frac{1}{Z(kp)}$ is the frequency response, and $A(t/k)$ is the indicial response. In other words, the curve may be shifted in frequency by a simple transformation and the effect on the indicial admittance curve is very similar except that the shift is in a direction opposite to the change in frequency, and is inversely proportional to the change in frequency scale.

The second group of curves, Fig. 6, relates to the effect of damping on an early magnetic type of receiver, showing the freely resonant condition, a moderately damped, and a highly damped receiver. The curves of indicial response show the effects of free resonance to be very detrimental, and the ringing of the diaphragm is sustained over such a long period that any speech waves would have superposed on them a continual train of sine waves. If the rate of decay of these waves is increased, as shown by the damped curves, a noticeable improvement results. By using critical damping as in the highly damped curve, all oscillations can be eliminated, but the time of pickup is degraded and the departure from a square wave is somewhat greater than for the moderately damped condition.

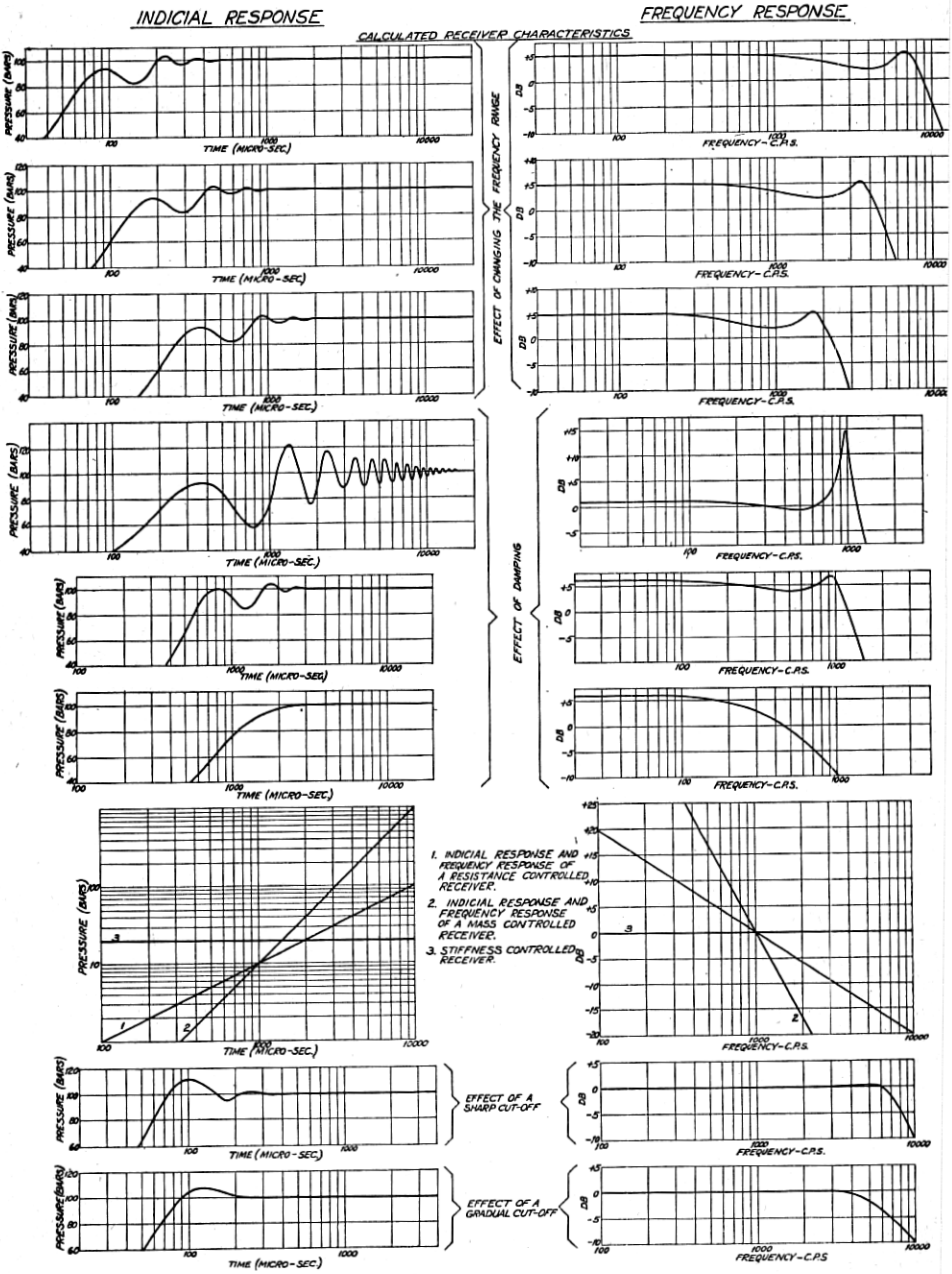


Fig. 6—Calculated indicial response versus calculated frequency response of various types of telephone receivers.

The indicial response shows more emphatically than frequency response, the importance of damping and the oscillations which are to be avoided, or reduced to a minimum. It also shows that the effect of delay is closely related to attenuation of the higher frequencies, and that frequency of cut-off is inversely proportional to the time delay, for a given type of receiver circuit.

There is a noticeable similarity between the appearance of the frequency response and the indicial response curves, and in many cases one curve is approximately the image of the other. As an example of this, the three pairs of linear curves show the similarity of indicial and frequency response for constant velocity, constant acceleration, and constant amplitude devices, as depicted by the three curves denoted by 1, 2, and 3 in which the three moving-coil instruments are assumed to be controlled by (1) a predominance of acoustic resistance behind the diaphragm, (2) a mass controlled system, and (3) a stiffness controlled system. In either case, the fundamental shape of the curves is such that the indicial response is the image of the frequency response in its general character.

The two lower curves, Fig. 6, indicate the effect of a sharp cut-off versus a gradual one. In terms of indicial response, the gradual cut-off appears to be the better of the two, a principle which is widely accepted in television and telegraph transmission.

Experimental Measurements

The oscillographic measurements of indicial response, together with corresponding frequency response measurements of telephone receivers, are shown in Figs. 7, 8, and 9. The oscillograms on the left, Fig. 7, show the type of data which constitute indicial response as compared with the more familiar frequency response on the right.

Curve 1, Fig. 7, represents a moving-coil receiver similar to that calculated in Fig. 3, and constitutes the standard of performance which can be obtained by this particular system of measurement. Each division of the oscillogram represents .001 second, a somewhat faster film speed than is usual for the string oscillograph.

Curve 2 shows the characteristics of a magnetic bipolar type of receiver having a frequency range of 3000 cps with a fairly sharp cut-off at this frequency. The acoustic circuits of this receiver serve to damp the resonance of the diaphragm and extend the range from 1600 up to 3000 cps. The oscillogram shows a partially damped but still somewhat oscillatory condition which is due to the receiver.

With all damping circuits removed, we obtain the characteristic of curve 3, a simple diaphragm resonance, which is similar to the earlier type of receivers of the magnetic type. Curve 2 represents a real improvement over curve 3, both as regards introduction of damping and extending the frequency range.

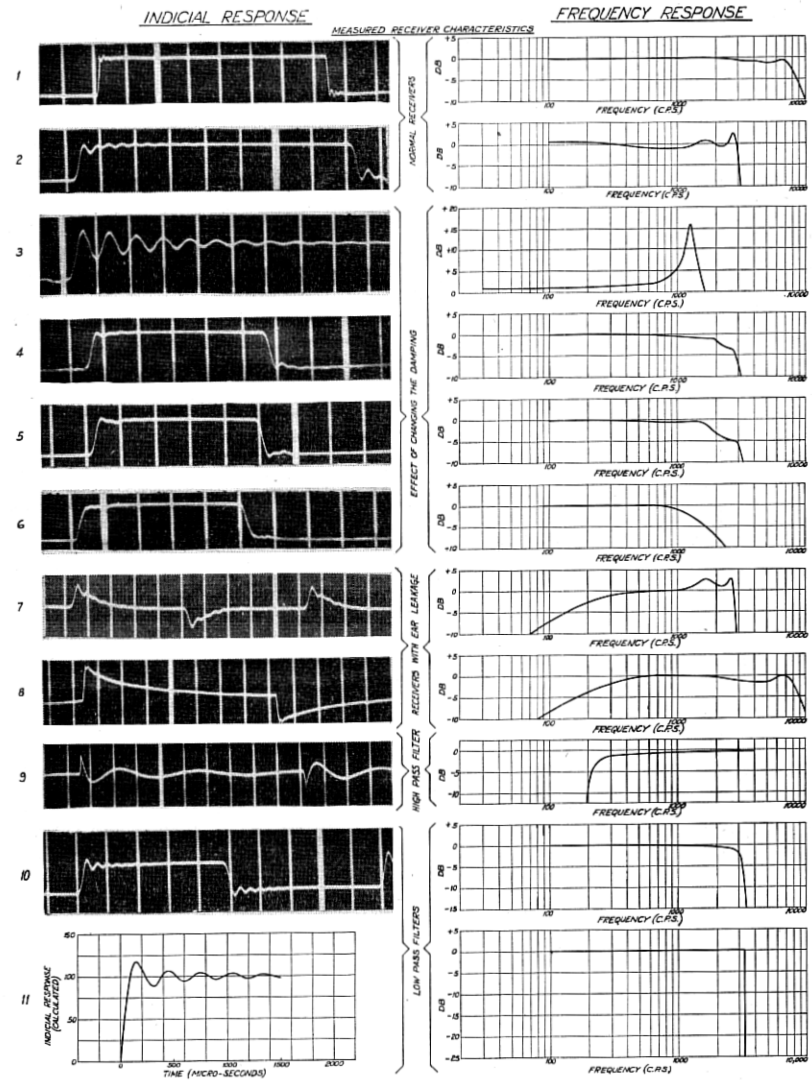


Fig. 7—Measured indicial response versus measured frequency response of various types of telephone receivers and electrical filters.

The effects of further increases in damping are shown by curves 4, 5, and 6. Such changes in the shape of the curve are brought about by relatively simple changes of the constants of the acoustic circuits. The oscillograms indicate a marked improvement as regards oscillations, which is to be expected with increased damping. The time delay is eventually degraded with further increases of damping, however, and the optimum damping is a matter of compromise.

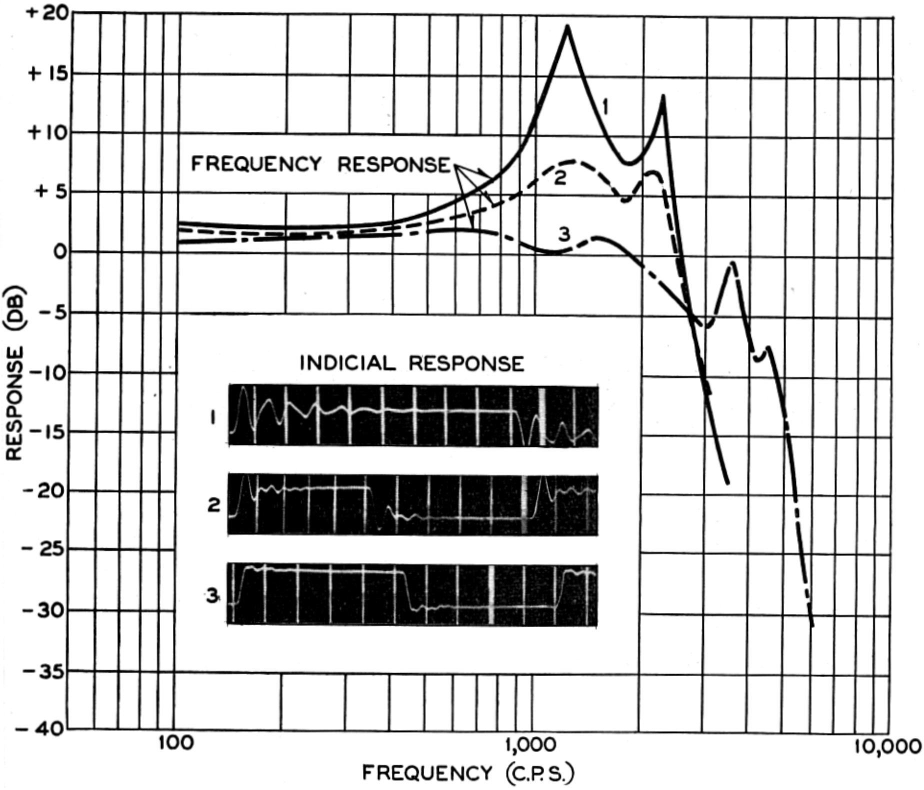


Fig. 8—Three types of hearing aid receivers—frequency response and indicial response.

The effects of a low-frequency cut-off characteristic are shown by curves 7, 8, and 9, Fig. 7. The absence of a *d-c* component makes these curves very difficult of interpretation.

Curve 7, taken with the same receiver as curve 2, except with coupler leakage, shows a loss at low frequencies which is typical of cases where the receiver cap does not make a perfect seal with the ear. The effect on the indicial response is that of a large pulse followed by a few oscillations at the frequency of the leak circuit.

Curve 8 is a similar condition except taken on a high-quality receiver circuit. This also shows a similar effect. The initial pulse contains most of the receiver characteristic, while the curve which follows is mainly dependent on the leakage constants.

Curve 9 is taken on a high-pass filter of the characteristic shown. It may be proved that this curve is the inverted image of the corresponding low-pass filter characteristic, of which a similar curve is shown as curve 10.

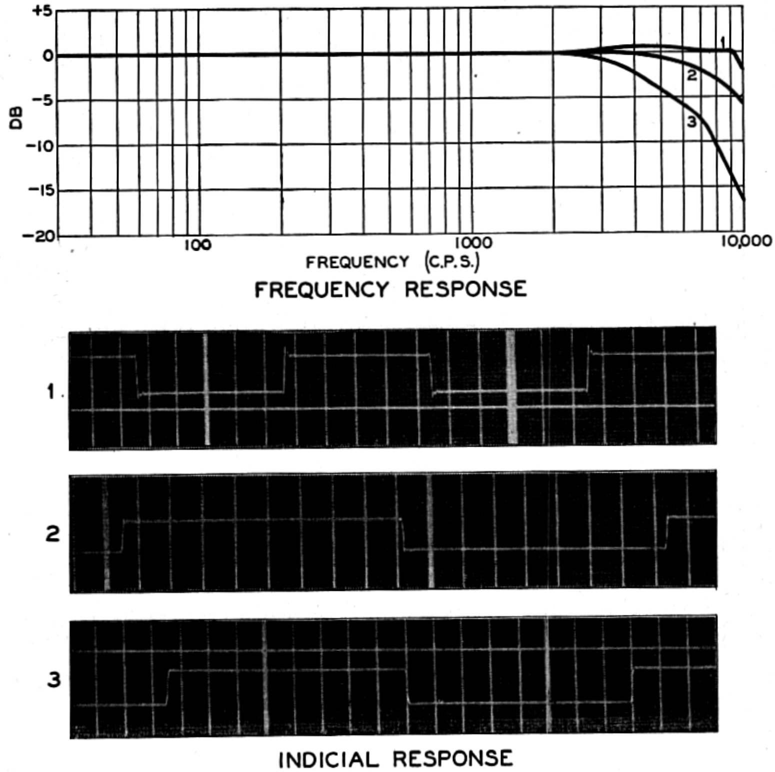


Fig. 9—String oscillograph characteristics—frequency response and indicial response with different amounts of damping.

The curves 7, 8, and 9 show that when the low frequencies are absent, the indicial response becomes too difficult to interpret. We must restrict our measurements to systems which are ideal at the low frequencies in order to interpret the indicial admittance by inspection.

Curves 10 and 11, Fig. 7, are low-pass filter characteristics, the former being a measured curve of a typical filter, while the latter is a calculated curve for an ideal filter. The two curves check reasonably well and indicate the effect of a very sharp cutoff as compared to those of the receivers shown above. This indicates the oscillatory nature of any system having a sharp cutoff at the upper frequencies.

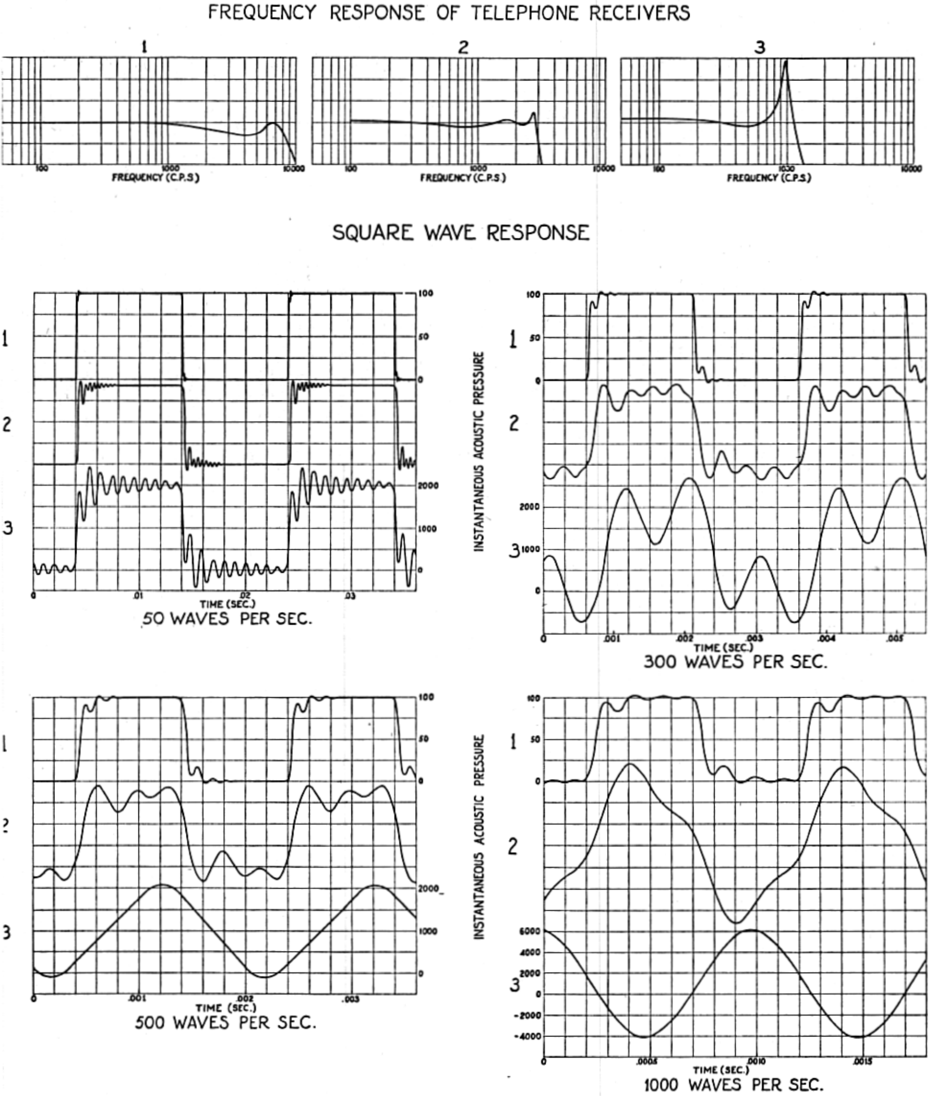


Fig. 10—Transient response to square waves of three different types of telephone receivers denoted by Nos. 1, 2 and 3, whose frequency response characteristics are shown above. Note the change in each type of pattern as the frequency of the square waves is increased.

Figure 8 shows a group of curves of the frequency response and indicial response of a group of receivers used as hearing aids. Curve 1 shows a very efficient but resonant receiver. Curve 2 is somewhat damped but still contains oscillations. Curve 3 is comparatively much better than either of the others from an indicial response viewpoint, and has a drooping frequency response characteristic, and demonstrates the advantages of this form of curve.

Figure 9 shows the effect of adding damping to the system of the string oscillograph when subjected to an ideal square wave. Curve 1, which has a virtually flat characteristic from 1 to 10,000 cps, is characterized by a sharp oscillatory peak in the indicial response. Curve 2 contains some oscillations, while curve 3 is substantially free from oscillations. The trend of these curves also shows the more faithful reproduction of transients obtained with a drooping frequency response.

Figure 10 shows the response to square waves of three receivers having different frequency response characteristics. The low-frequency waves of 50 cps are similar to the indicial response of the three receivers whose frequency characteristics are shown at the top, Fig. 10. As the frequency of these waves is increased to 300 cps, a noticeable departure from the square form is apparent in receiver No. 3. Receiver No. 2 shows a slight departure, while No. 1 is virtually a perfect reproduction.

As the frequency of the square waves is increased to 500 cps, the receiver No. 1 still shows very little departure from the original form. Receiver No. 2 maintains a fair approximation, while receiver No. 3 has lost all resemblance to the square form.

At a frequency of 1000 cps, only the first receiver maintains an approximately square form. Receivers Nos. 2 and 3 have both lost their identity and have become practically pure sinusoids. For all higher frequencies of the square waves, these two receivers will exhibit practically pure sinusoidal forms, due to the relatively sloping character of the frequency response at these frequencies, and the absence of harmonics. The same will be true of receiver No. 1 beyond a frequency of 3000 cps.

It will be realized, of course, that the patterns were obtained with square waves repeated at frequencies of 50, 300, 500 and 1000 cycles per second. While some speech waves approximate square waves in character such waves, when they occur, are repetitive only at the lower range of these frequencies. The above patterns were therefore obtained under conditions much more severe than are involved in the reproduction of speech waves and are included primarily for the purpose of illustrating the sensitivity of this form of analysis when applied to repeated square waves.

Conclusions

To summarize these data, it seems evident that square wave analysis may be applied in some fields of acoustics for both theoretical and practical applications.

In theory, the indicial response forms a somewhat different approach to the problem of obtaining the optimum characteristics of telephone receivers at the upper end of the frequency range. The greatest value of the square wave analysis lies in the fact that it gives us an entirely different conception of the behavior of an ideal sound system in terms of the unit function. The frequency response characteristic is ordinarily interpreted on the theory that any transient, such as an interval of conversation, may be represented by a Fourier series of sinusoidal frequencies of constant intensity lasting over the entire interval. If these equivalent component frequencies are to be reproduced in their true proportions, the ideal sound system must have mathematically uniform response for all single frequencies. On the other hand, the indicial response characteristic is judged from the Carson extension theorem, which shows that the more closely this characteristic approaches the unit function, the more perfect will be the reproduction of any given transient. Thus, the unit function and the sinusoid may be used as mutually complementary tools of analysis to show different aspects of the same type of problem.

In sound systems which are not ideal, due to inherent physical limitations, we tend to apply the Fourier Theorem out to a certain frequency, just as if it were an ideal system out to this frequency, and then beyond this frequency we do not attempt to sustain the higher frequencies. For most faithful reproduction of transients, it would seem that such practices might be altered somewhat to advantage by allowing the frequency response to drop off more gradually wherever it seems feasible to do so. The exact shape of the ideal curve under these circumstances is a matter of compromise between excessive delay on the one hand and excessive oscillations on the other. In practice, however, a fairly good picture is soon formed when curves such as the last in Figs. 6, 8, and 9 are found to approach the ideal more closely than those of other forms. Such listening tests as have been made tend to confirm these views, but cannot be regarded as being more than an indication.

Square wave analysis is somewhat limited in its practical applications to cases which may be interpreted by inspection. Systems having only a single cutoff frequency, or in the case of an additional low-end cutoff, ratios of the upper and lower cutoff frequencies f_2/f_1 of 100 or more, seem necessary to interpret the results by inspection.

The use of indicial response is not necessarily limited to any particular coupler or method of response measurement, since frequency response and indicial response are so closely related that one is a function of the other. The choice of a closed coupler measurement does, however, permit some interpretation of the results to be made by inspection, whereas other types of measurement may require laborious mathematical means to obtain an interpretation. Other types of vibration instruments, such as recorders, vibration pickups, crystal phonograph reproducers and carbon transmitters, which sustain their response down to zero frequency, should lend themselves to such methods of analysis.

In conclusion, the writer wishes to acknowledge the assistance of Mr. T. J. Pope in connection with the oscillographic work of this paper, and to express his sincere appreciation.

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