Evaluating the Relative Bending Strength of Crossarms

By RICHARD C. EGGLESTON

OVER a million crossarms are produced annually in the United States. In the open wire lines of the Bell System alone there are now about 20 million arms in use. It is natural, therefore, that public utility engineers should have an interest in the strength of such an important item of outside plant material; and, consequently, an interest in any tool or means of evaluating the strength of such material. It is believed that the moment diagram is a convenient and reasonably reliable tool for estimating the loads an arm will support, for measuring the effect of knots of various sizes and of pinhole locations on arm strength, and for answering similar questions relating to the bending strength of crossarms under vertical loads.

Two moment diagrams are shown in Fig. 1 for Bell System Type A crossarms; and in the pages that follow are presented the method used in constructing the diagrams and a discussion of their use. While the calculation results apply particularly to the type and quality of arm referred to, they would also be of value as a time saving reference in future studies that may be proposed relating to the strength of the same or other types of arms involving different knot allowances.

The resisting moment of a beam is the product of its section modulus by the unit stress on the remotest fiber of the beam. The section modulus of a beam of uniform cross-section is constant and readily determinable. The section modulus, however, of a beam of nonuniform cross-section, such as a crossarm, varies because of the different cross-sectional shapes and dimensions involved.

In this study the following five different shapes were recognized:

(1) Roofed section between pinholes
(2) Roofed pinhole section
(3) Roofed brace bolt hole section
(4) Rectangular pole bolt hole section
(5) Rectangular section without bolt holes

The dimensions of the sections investigated were as follows:

Section of Arm	Dimensions	
	Minimum	Nominal
	(Inches)	(Inches)
Roofed section, except at end of arm	$3\frac{3}{16}$ x $4\frac{3}{32}$	$3\frac{1}{4}$ x $4\frac{3}{16}$
Roofed section at end of arm	$3\frac{3}{16}$ x 4	$3\frac{1}{4}$ x $4\frac{3}{16}$
Unroofed sections	$3\frac{3}{16}$ x $4\frac{3}{16}$	$3\frac{1}{4}$ x $4\frac{1}{4}$

Since there is little, if any, engineering interest in the strength of structural members of maximum size, no investigations were made of sections of maximum dimensions.

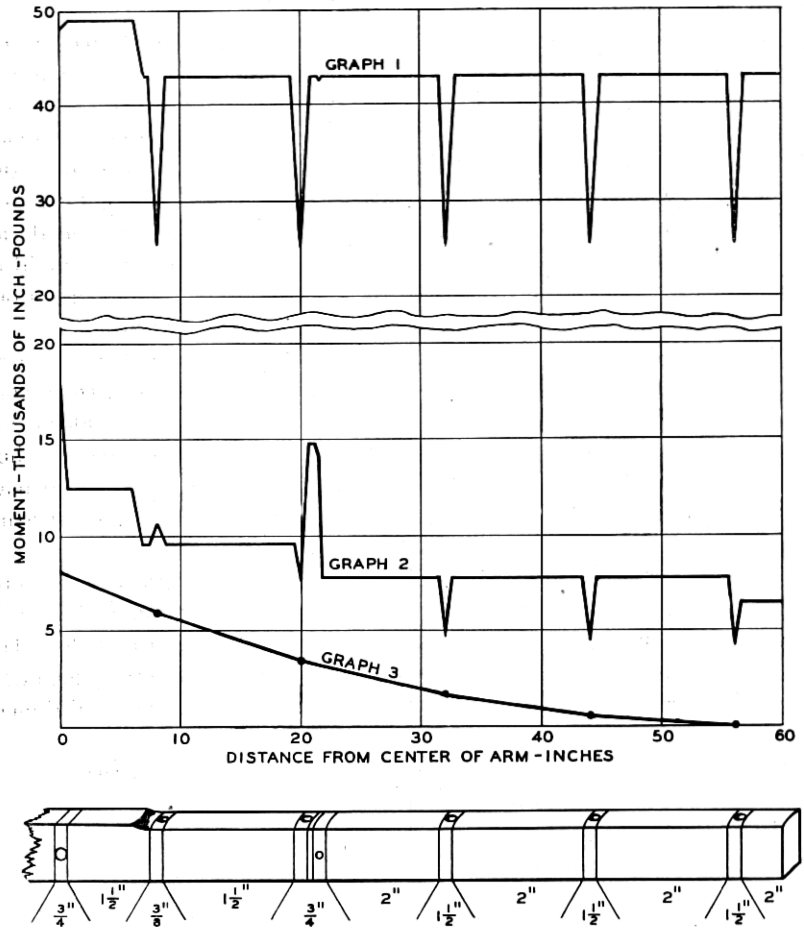

Fig. 1—Moment diagram for Type A southern pine and Douglas fir crossarms per Specification AT-7075:

Graph 1—Resisting moments of arms of nominal dimensions, straight grained and free from knots. (Fiber stress 5000 psi).

Graph 2—Resisting moments of arms of minimum dimensions, having maximum slant grain (1″ in 8″), and containing knots of the maximum sizes permitted (viz., sizes shown at bottom of arm sketch). (Fiber stress 3250 psi)

Graph 3—Bending moments from a load of 50 pounds at each pin position.

Section modulus calculations were made of each shape of minimum and nominal size, both with and without knots. Tests have shown that, because of the distortion of the grain that occurs around them, knots are fully as injurious to the strength of structural timbers as knot holes.[1] Therefore, in dealing with sections containing knots, it was assumed for the purposes of this study that the knot extended across the section in the same manner as a hole having a diameter equal to the diameter of the knot. It was also assumed that the knot was located in, or reasonably close to, the most damaging position in the arm section.

In the calculations of the section modulus of all roofed arm sections, it was necessary first to compute the moments of inertia of the whole or parts of the top segments of such sections (viz. nominal and minimum sections

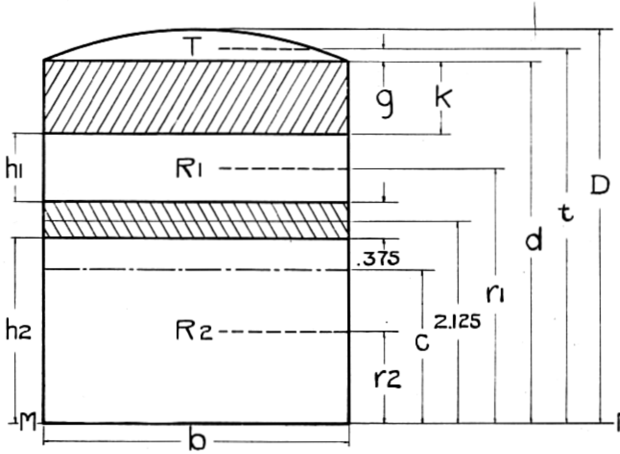

Fig. 2—Brace bolt hole section containing a $\frac{3}{4}$ inch knot located immediately below the top segment (knot and bolt hole shaded).

between pinholes, and nominal and minimum pinhole sections). Accordingly, four such computations were made and the results used in calculating the section moduli of all the roofed sections investigated. The details of the four computations are shown in the Appendix. To insure uniformity in the results, the degree of precision used in these computations was considerably greater than is ordinarily employed in dealing with timber products. All of the work, however, was done on a computing machine, and it was just about as easy to carry the operations to eight decimal places (which was the capacity of the machine used) as to a lesser number. As a matter of interest in this connection, it was found by actual trial in Computation I that absurd results would occur if fewer than five decimal places were used.

For convenience, all of the section modulus calculations were made in tabular form. In such form the procedure employed would not be readily

[1] Pg. 6 Dept. Circular 295, U. S. Dept. of Agriculture, "Basic Grading Rules and Working Stresses for Structural Timbers," by J. A. Newlin and R. P. A. Johnson.

apparent. Therefore, a sample calculation follows showing the method of finding the section modulus of the brace bolt hole section containing a $\frac{3}{4}$ inch knot.

Sample Calculation

Referring to Fig. 2, it will be noted that the knot and bolt hole divide the section into three parts: the top segment (T) and two rectangular portions ($R1$ and $R2$). The moment of inertia (I) of such a compound section about its neutral axis (at a distance c from M-M) is equal to the sum of the moments of inertia (IT, $IR1$ and $IR2$) of the component parts T, $R1$ and $R2$ about axes through their own centers of gravity, plus the areas of the component parts multiplied by the squares of the distances of their own centers of gravity from the neutral axis of the compound section. The section modulus (S) of this section is found, of course, by dividing its moment of inertia by the distance (y) from the neutral axis of the section to the most remote fiber.

Dimensions:

$b = 3.1875''$ (Width of Section)
$k = 0.7500''$ (Diameter of Knot)
$d = 3.7625''$ (See Computation I in Appendix)
$h1 = 0.7000''$ $(d - 2.125'' - 0.1875'' - k)$
$h2 = 1.9375''$ $(2.125'' - 0.1875'')$
$g = 0.1330''$ (See Computation I)
$t = 3.8955''$ $(d + g)$
$r1 = 2.6625''$ $(\frac{1}{2}\, h1 + 2.3125'')$
$r2 = 0.96875''$ $(\frac{1}{2}\, h2)$
$D = 4.09375''$ (Depth of Section)

Areas:

$T = 0.7099$ sq. ins. (See Computation I)
$R1 = 2.2313$ " $(bh1)$
$R2 = 6.1758$ " $(bh2)$

9.1170 sq. ins.

Moments about M — M:

$Tt = 2.7654$
$R1r1 = 5.9408$
$R2r2 = 5.9828$

$14.6890 = 9.1170\, c$; and hence
$c = 1.6112$

Moments of Inertia:

$IT = 0.0053$ (See Computation I)
$IR1 = 0.0911$ $(bh1^3 \div 12)$
$IR2 = 1.9319$ $(bh2^3 \div 12)$
$T(t - c)^2 = 3.7043$
$R1(r1 - c)^2 = 2.4661$
$R2(c - r2)^2 = 2.5490$

$I = 10.7477$
$y = 2.48255$ (D − c)

Section Modulus:

$$S = \frac{I}{y} = 4.3293$$

The same general procedure shown in this sample calculation was followed in dealing with the other cross-sectional shapes. For this reason, only the final results of the several calculations are presented; although, for

TABLE 1.—*Section Modulus of Roofed Sections between Pinholes*

	Knot Diameter—Inches								
	No Knot	½	¾	1	1¼	1½	2	2½	3
Calculation 1: (Knots located at top of section) Section Size*:									
Minimum		6.86		5.08		3.57	2.33	1.35	0.64
End. Min.				4.78			2.13		0.53
Nominal		7.37		5.50		3.91	2.59	1.54	0.76
Calculation 2: (Knots located at bottom of section) Section Size*:									
Minimum		6.11	5.24	4.42	3.71	3.05	1.92	1.05	0.47
Calculation 3: (Knots located immediately below top segment) Section Size*:									
Minimum	8.03	5.45	4.56	3.86	3.34	2.95	2.50	2.34	2.37
End. Min.	7.65			3.65					
Nominal	8.60			4.16					

TABLE 2.—*Section Modulus of Roofed Pinhole Sections*

	Knot Diameter—Inches						
	No Knot	¼	½	¾	1	1½	2
Calculation 4: (Knots vertical) Section Size*:							
Minimum	4.50	3.84	3.21	2.63	2.25		
End. Min.	4.29			2.50			
Nominal	5.11			2.88			
Calculation 5: (Knots horizontal) Section Size*:							
Minimum		3.63	2.96	2.40	1.97	1.41	1.11
End. Min.				2.26		1.33	
Nominal				2.76		1.64	

* Section Sizes:
Minimum = $3\frac{3}{16}''$ x $4\frac{3}{32}''$
End. Min. = $3\frac{3}{16}''$ x $4''$ (viz. minimum at end of arm)
Nominal = $3\frac{1}{4}''$ x $4\frac{3}{16}''$

convenience, reference is made to the calculations by number in the pages that follow. These results are shown in Tables 1, 2, 3 and 4, and a brief discussion of the scope and use made of them follows.

TABLE 3.—*Section Modulus of Bolt Hole Sections*

	Knot Diameter—Inches								
	No Knot	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	
Calculation 6: *Brace bolt hole section*									
Section Size*:									
Minimum	7.97	6.47		5.28	4.33	3.58		2.62	
Nominal	8.55				4.71			2.78	
Calculation 7: *Pole bolt hole section*								$2\frac{1}{2}''$ *Knot*	$3''$ *Knot*
Section Size*:									
Minimum	9.25		7.42		5.63		3.24	1.51	.75
Nominal	9.74				6.05		3.61	1.66	.85

* Section Sizes:
Minimum = $3\frac{3}{16}''$ x $4\frac{3}{32}''$
End. Min. = $3\frac{3}{16}''$ x $4''$ (viz. minimum at end of arm)
Nominal = $3\frac{1}{4}''$ x $4\frac{3}{16}''$

TABLE 4.—*Section Modulus of Rectangular Section without Bolt Holes* (*Calculation 8*)

Section Size	Knot Diameter	Section Modulus
Minimum ($3\frac{3}{16}''$ x $4\frac{3}{16}''$)	(No Knot)	9.32
	$\frac{1}{4}$	8.24
	$\frac{1}{2}$	7.22
	$\frac{3}{4}$	6.28
	1	5.40
	$1\frac{1}{2}$	3.84
	2	2.54
	$2\frac{1}{2}$	1.51
	3	.75
Nominal ($3\frac{1}{4}''$ x $4\frac{1}{4}''$)	(No Knot)	9.78
	$\frac{1}{4}$	8.67
	$\frac{1}{2}$	7.62
	$\frac{3}{4}$	6.64
	1	5.72
	$1\frac{1}{2}$	4.10
	2	2.74
	$2\frac{1}{2}$	1.66
	3	.85

ROOFED SECTIONS BETWEEN PINHOLES

As indicated in Table 1, three tabular calculations were made for roofed sections between pinholes. In Calculations 1, 2 and 3 it was assumed that the knots present were located (1) at the top, (2) at the bottom, and (3) immediately below the top segment of the section, respectively. The results relating to the $3\frac{3}{16}''$ x $4\frac{3}{32}''$ section are plotted as Curves 1, 2 and 3, respectively, in Fig. 3.

With respect to the knot positions considered, it is apparent from an examination of the three curves (Fig. 3) that knots up to approximately $1\frac{1}{2}''$ in diameter are most damaging when located immediately below the roofed portion of the arm; and that the worst position for knots over $1\frac{1}{2}''$ in diameter is at the bottom of the arm. However, since under usual loading

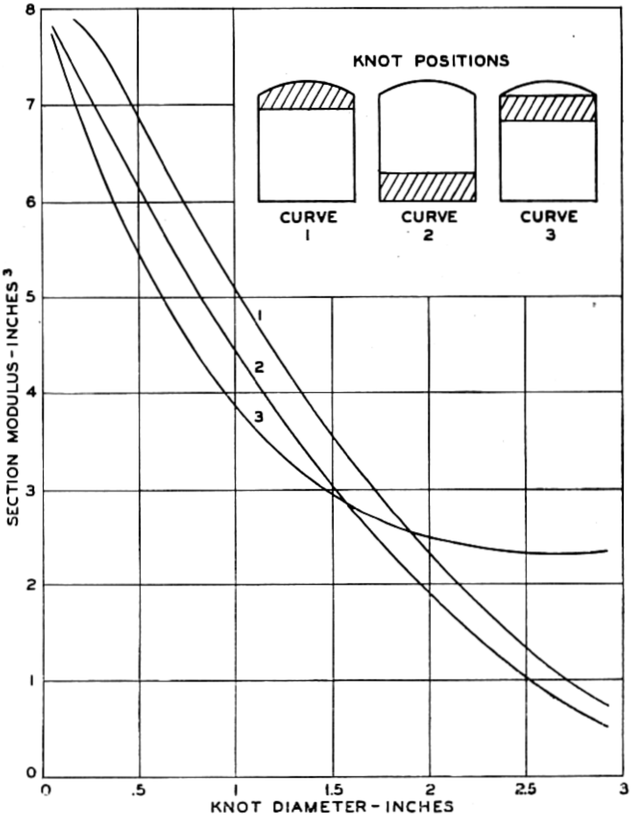

Fig. 3—Sections between pinholes. Section modulus of crossarm sections containing knots of the sizes shown on the base line and located in the positions indicated. The data apply to sections of minimum size ($3\frac{3}{16}''$ x $4\frac{3}{32}''$).

conditions knots at the bottom of an arm section are in compression, and thus would have less influence on strength than they would have on the tension side,[2] it was felt that the strength value shown by Curve 2 may be ignored; and that the values shown by a smooth curve, combining the values

[2] On Page 69 of U. S. Dept. of Agriculture Tech. Bul. 479, "Strength and Related Properties of Woods Grown in the United States" by L. J. Markwardt and T. R. C. Wilson, is the following statement: "Knots have approximately one-half as much effect on compressive as on tensile strength."

of Curve 3 up to the $1\frac{1}{2}''$ knot point with those of Curve 1 for 2″ and larger knots, would be the practical minimum section moduli for roofed sections between pinholes. Accordingly, such a smooth curve was constructed and is shown as Curve 2 in Fig. 4. The results of Calculations 1 and 3 for nom-

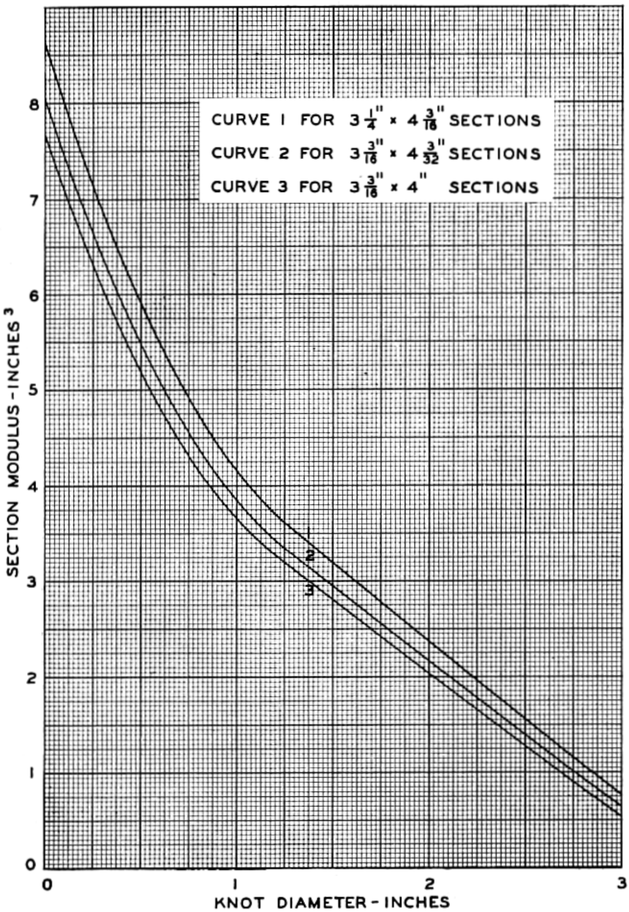

Fig. 4—Sections between pinholes. Section modulus of crossarm sections containing knots of the sizes shown on the base line and located in damaging positions.

inal and arm-end minimum sections were also plotted, and Curves 1 and 3 drawn for those sections.

Roofed Pinhole Sections

Two calculations were made for the pinhole sections: Calculation 4, in which the knots were assumed to be located adjacent to the pinhole in a *vertical* position; and Calculation 5, in which the knots were assumed to be immediately below the top segment in a *horizontal* position. The results of these two calculations are shown in Table 2. It has heretofore been generally assumed that in pinhole sections knots less than 1″ in diameter were more damaging in a vertical position than in a horizontal position. The results of Calculations 4 and 5, however, show that the horizontal knots immediately below the top segment are the more damaging. In order to compare the effect of knots so located with the effect of knots at the extreme

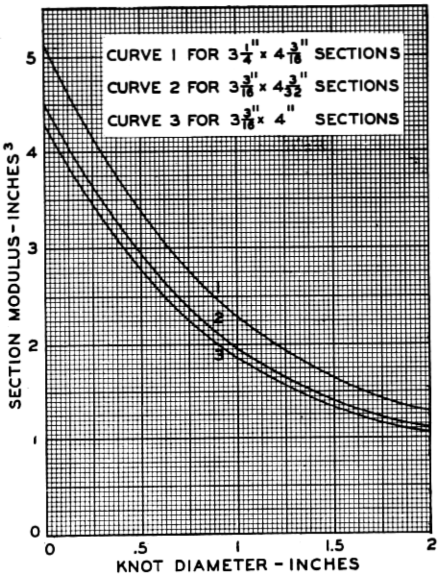

Fig. 5—Pinhole sections. Section modulus of crossarm sections containing knots of the sizes shown on the base line and located in damaging positions.

top of the section, the following two computations assumed 1″ and 2″ horizontal knots at the latter location:

1″ Knot at Section Top:

$$\tfrac{1}{2}S = \frac{.02875\ (3.09375)^2}{6} = 1.48156$$

$$S = 2.9631$$

2″ Knot at Section Top:

$$\tfrac{1}{2}S = \frac{.92875\ (2.09375)^2}{6} = .67857$$

$$S = 1.3571$$

As the section modulus (S) values for sections containing 1″ and 2″ horizontal knots located immediately below the top segment are 1.97 and 1.11, respectively, (Calculation 5, Table 2) it is apparent that in pinhole sections horizontal knots immediately below the top segment are the more damaging. The results of Calculation 5 were accordingly plotted in Fig. 5 and smooth curves drawn to show the section modulus for each of the three sections containing knots of any size.

Roofed Brace Bolt Hole Section

The worst position for knots in the brace bolt hole section was assumed to be substantially the same as in the roofed sections between pinholes; and in Calculation 6, the results of which are shown in Table 3, knots up to $1\frac{1}{2}''$ in diameter were assumed to be so located, viz. immediately below the top segment.

To check this assumption with respect to worst position, the following analysis was made of the minimum sections:

Distance from top of section:

To top of bolt hole	1.78″
To bottom of bolt hole	2.16″

Distance from bottom of top segment:

To top of bolt hole	1.45″
To bottom of bolt hole	1.83″

It is apparent that any knot ranging in diameter from 1.78″ to 2.16″, when located at the top of the section, would enter the bolt hole. The section modulus of any section containing a knot within that size range would be the section modulus of the remaining portion of the section, or $\frac{bd^2}{6}$, where b is the width of the section and d the depth below the bolt hole. Thus

$$S \text{ (minimum arm)} = \frac{3.1875\ (1.9375)^2}{6} = 1.9943$$

It is also evident that any knot from 1.45″ to 1.83″ in diameter, when located immediately below the top segment, would likewise enter the bolt hole; and that the section modulus, on this basis of knot location, would be the same for any section containing a knot within the size range mentioned. Continuing the analysis the following tests were made:

2″ Knot:

The distance between the top segment and the bottom of the bolt hole of a minimum section is 1.83″. Therefore, a 2″ knot located immediately below the top segment would extend beyond the hole; and its effect would be the same as in Calculation 3 (Table 1), where the section modulus of a section containing a 2″ knot similarly located was found to be 2.50. On the other hand, since a 2″ knot is within the limits 1.78″ and 2.16″, the section modulus of a section containing such a knot located at its top would be 1.99.

1.78″ Knot:

A knot of this size immediately below the top segment would enter the bolt hole since it is within the 1.45″ and 1.83″ limits, and the section modulus value associated with it would be the same as shown in the Calculation 6 results (Table 3) for a section containing a $1\frac{1}{2}$″ knot, or S = 2.62. But, as evident from previous discussion, the section modulus associated with this knot, if located at the top of the section, would be 1.99.

1.5″ Knot:

It can be shown that the section modulus of a section containing a knot of this size located at the top of the section would be 2.55; and that the section modulus associated with a similarly located *1″ knot* would be 4.55.

The foregoing analysis for minimum sections may be summarized as follows:

Knot Size	Section Modulus	
	Knot at Top	Knot below Top Segment
(Inches)	(Inches^3)	(Inches^3)
2.0	1.99	2.50
1.78	1.99	2.62
1.5	2.55	2.62
1.0	4.55	3.58

A study of this summary shows that knots $1\frac{1}{2}$″ and over are more damaging when located at the section top; and that knots under $1\frac{1}{2}$″ are more damaging when located immediately below the top segment. The section modulus values associated with $2\frac{1}{2}$″ and 3″ knots would be the same as shown in the Calculation 1 results (Table 1).

By a similar analysis for arms of nominal size it can be shown:

(1) That the more damaging position for knots $1\frac{1}{2}$″ and under is immediately below the top segment;

(2) That the more damaging position for any knot within the diameter range from 1.875″ to 2.25″ and all the larger knots is at the top of the section;

(3) That the section modulus associated with 1.875″ to 2.25″ knots would be $\frac{3.25(1.9375)^2}{6} = 2.0334$; and

(4) That the section modulus values associated with $2\frac{1}{2}$″ and 3″ knots would be the same as shown in the Calculation 1 results (Table 1).

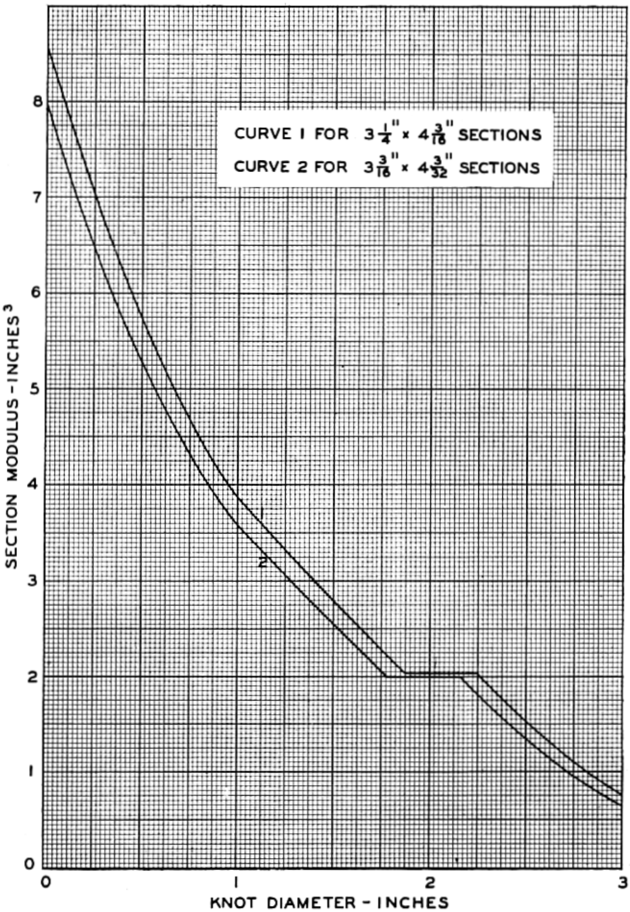

Fig. 6—Brace bolt hole sections. Section modulus of crossarm sections containing knots of the sizes shown on the base line and located in damaging positions.

The results of Calculation 6 (Table 3), and of the foregoing analyses, together with the Calculation 1 results for $2\frac{1}{2}$″ and 3″ knots, were plotted in Fig. 6 for both minimum and nominal sections.

Rectangular Pole Bolt Hole Section

The most damaging position for knots in the pole bolt hole section was assumed to be at the top of the section. They were so figured in Calcula-

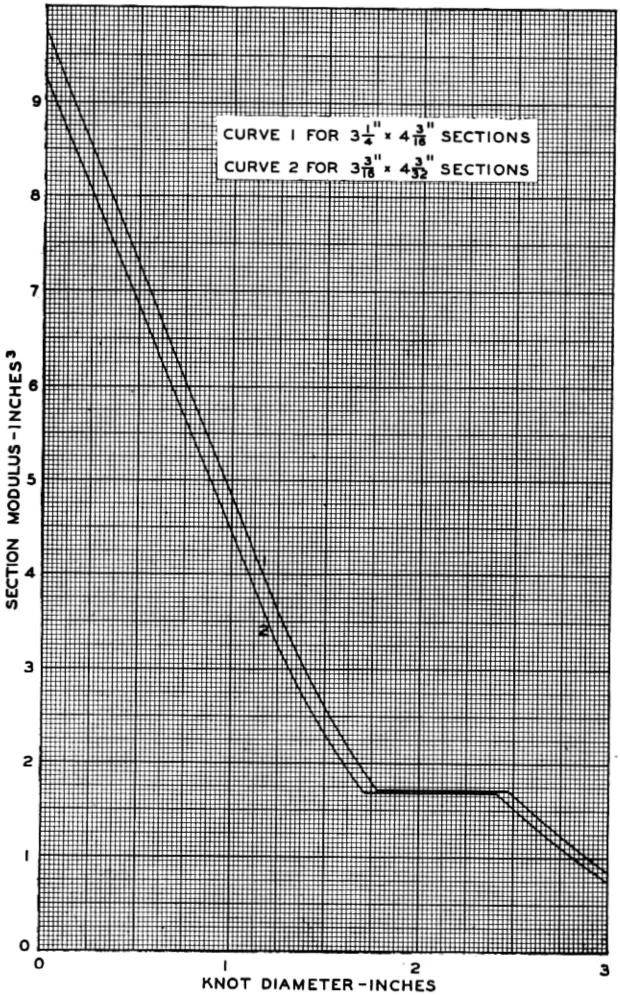

Fig. 7—Pole bolt hole section. Section modulus of crossarm section containing knots of the sizes shown on the base line and located in damaging positions.

tion 7, the results of which are shown in Table 3 and plotted in Fig. 7 for both minimum and nominal arms.

Rectangular Sections without Bolt Holes

Here too the most damaging position for knots was assumed to be at the top of the section. In Calculation 8 the section moduli of sections containing knots from ¼″ to 3″ in diameter were determined for both minimum and nominal sections. The results are shown in Table 4. As section modulus values for sections containing knots of other sizes than those shown may be found so simply by the formula for rectangular sections, $S = \frac{bd^2}{6}$, no curves of the results of this calculation were drawn.

Moment Diagrams

From the results of this study as shown in Table 4 and in Figs. 4, 5, 6 and 7, section modulus values for clear arms and for arms containing knots of various sizes may be read and multiplied by appropriate fiber stresses to determine the resisting moments throughout the length of such arms. For example, the section moduli of clear arms of nominal dimensions, and of arms of minimum dimensions with the maximum knots permitted under the current Bell System crossarm specification (AT-7075) are as follows:

Section of Arm	Arms of nominal size and free from knots	Arms of minimum size with maximum knots	
	Section Modulus	Section Modulus	Diameter of Max. Knots
Pole bolt hole	9.74	5.63	¾″
Brace bolt holes	8.55	4.33	¾″
Pole pinholes	5.11	3.28	⅜″
Other pinholes in middle section[3]	5.11	2.38	¾″
End pinholes	5.11	1.33	1½″
Other pinholes in end sections[3]	5.11	1.41	1½″
Unroofed part of middle section	9.78	3.84	1½″
Roofed part of middle section	8.60	2.95	1½″
Solid part of brace bolt hole zones[4]	8.60	4.56	¾″
Between pinholes in end sections	8.60	2.17	2″
Extreme ends	8.60	2.03	2″

These section modulus values were used in preparing the moment diagrams shown in Fig. 1. The clear arm of nominal dimensions was also assumed to be straight grained. The fiber stress factor used for it was 5000 psi, which is the ultimate fiber stress value that has been employed in the Bell System for many years for sawn southern pine and Douglas fir. The

[3] For the purposes of specifying knot limitations, crossarms under Specification AT-7075 are divided into a middle section (between brace bolt holes) and end sections (beyond brace bolt holes).

[4] Where a brace bolt hole zone is less than four (4) inches from a pinhole zone, these zones and the portion of the arm between them are considered as a single zone.

fiber stress factor used in computing the resisting moments for the arm of minimum size with maximum slant grain and maximum knots was 3250 psi, which is simply 5000 psi discounted 35% to allow for slant grain of 1″ in 8″, which is the maximum permitted by Specification AT-7075. A discount is, of course, unnecessary for the presence of knots, since allowance for their effect on strength was made in the section modulus values used.

Since the 5000 psi value is an ultimate fiber stress and not a working stress, and since the arms were assumed to be made of clear, straight grained material, Graph 1 (Fig. 1) represents an idealized condition. The resisting moments shown are probably the maximum that may be expected from any commercial lots of southern pine or Douglas fir crossarms,[5] notwithstanding the fact that the dimensions of some of the arms may exceed the nominal specified. With respect to Graph 2 (Fig. 1), the objection may be raised that 35% is not a sufficient discount for a 1″ to 8″ slant of grain and that the 3250 psi value makes no allowance for the effect of long continued loading. On the other hand, the graph assumes the simultaneous occurrence of the maximum knot in a most damaging position in every section of an arm of minimum dimensions and having the maximum slant of grain allowed. Since the probability of such simultaneous occurrence of these defects and conditions is extremely small, it is felt that the resisting moments of Graph 2 represent the minimum strength of any arm of the two species concerned that may be furnished under Specification AT-7075.

Under the assumptions made, Graphs 1 and 2 (Fig. 1) may be regarded as the upper and lower limits of the bending strength of specification crossarms. On the same diagram may be plotted the graph or graphs of the moments resulting from any given load at each pin position, or any single load concentrated at any point on the arm. As an illustration, Graph 3, showing the bending moments from a load of 50 pounds per pin, is shown in the diagram (Fig. 1). A load of 50 pounds per pin is calculated to be the load of size 165 wire coated with ice having a radial thickness of $\frac{1}{4}$ inch in span lengths of 235 feet, or of wire of the same size in 100 foot spans where the radial thickness of the ice coating is $\frac{1}{2}$ inch. Since Graph 3 is wholly below Graph 2, even an arm of lowest specification quality would support the assumed loads with some margin of strength to spare. This margin or factor of safety, would, of course, be increased greatly if the quality of the arm under consideration approached the quality assumed in Graph 1. As previously indicated, the probability is extremely remote that any single arm will ever be furnished of a quality as low as assumed in Graph 2. It

[5] Graphs 1 and 2 (Fig. 1) are for southern pine and Douglas fir crossarms. It is estimated that the resisting moments of comparable graphs for the other woods included in Specification AT-7075 should be about 20% lower.

follows, therefore, that the average strength of any lots of southern pine or Douglas fir arms produced under Specification AT-7075 may be expected to lie well above the Graph 2 limit.

Graph 2 and a bending moment graph for vertical loads at each pin position are of considerable value to the material design engineer, since the degree of parallelism between the two will show whether a consistent strength relationship exists throughout the length of the crossarm. As a matter of interest in this connection, moment diagrams were used as a guide in setting the knot limitations shown in Specification AT-7075.

Resisting and bending moment graphs may also be used to determine the location of the critical section of a crossarm by noting the point of coincidence between a *maximum* bending moment graph and the resisting moment graph for a clear arm. It can be shown by such graphs that this point in all types of Bell System crossarms, designed for vertical loads, is located at the pole pinholes. If the comparison were made between a maximum bending moment graph and the resisting moment graph of an arm containing all of the maximum defects permitted, the location of the point of coincidence between the graphs might or might not fall at the pole pinholes, depending on the magnitude and location of the defects allowed. It should be noted, however, that for such arms the critical section locations so determined apply only when the arms are actually of the assumed minimum quality; and, since the probability of such being the case is so extremely remote, it is concluded that the maximum stress or critical section locations in arms of that quality are of academic interest only, and that for all practical purposes the critical section of any $3\frac{1}{4}''$ x $4\frac{1}{4}''$ x $10'$ crossarm is located at the pole pinhole.

This conclusion does not mean that every arm broken in service or under test will break at the pole pinhole; for, obviously, if some other section is relatively weaker because of some hidden defect which reduces its section modulus or its fiber strength, it will break at such section regardless of any mathematical determination of the break location. But the conclusion does mean that, generally speaking, when a crossarm breaks the break will occur at, or be closely related to, the pole pinholes. To check the accuracy of this conclusion, an examination was made of all available crossarm strength test data in which the break locations were recorded. The examination revealed that, out of 258 arms tested, the breaks in 219, or 85 per cent, were either at, or directly related to, the pole pinholes. Six per cent of the breaks were located between the two pole pinholes, and 9 per cent at points outside the pole pinholes.

As an illustration of another use to which such a moment diagram may be put, the following specific example is cited. Before the present standard Bell System specification for crossarms was drafted, it was decided to include a new type ("W6") with 16 pin positions. It was felt that, if the additional pin holes in the type W6 did not unduly weaken the arm, it could not only replace the old type "JW" arm with 8 pin positions but also be used in installations where greater flexibility in wire spacings might be required.

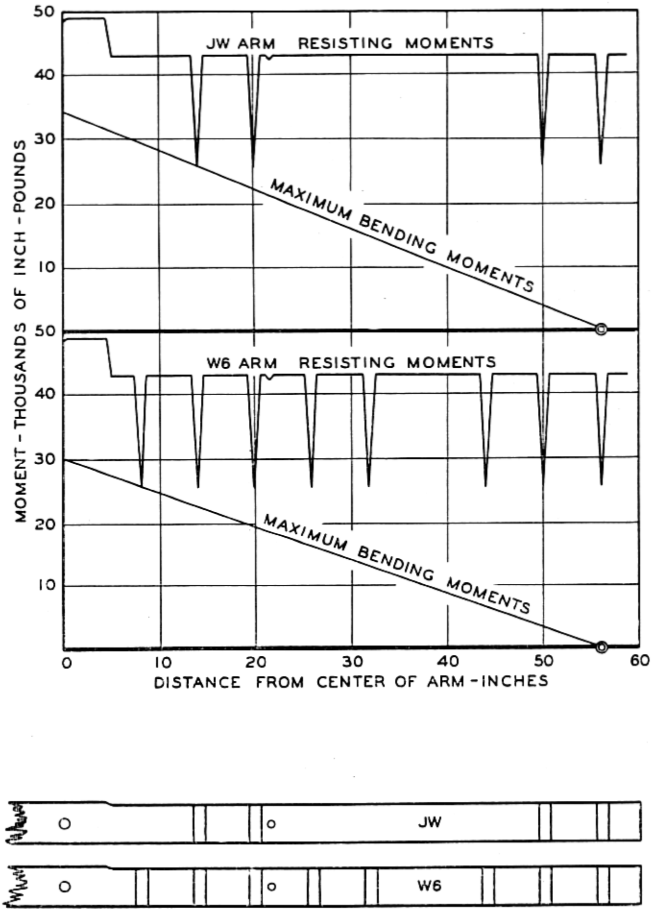

Fig. 8—Resisting moments and maximum bending moments for clear JW and W6 crossarms.

In order to obtain an estimate of the strength relationship between the two types, strength tests were made of 10 matched arms of each type. The test arms were made of air-seasoned, clear Douglas fir. The dimensions of the crossarm blanks were $3\frac{1}{4}'' \times 4\frac{1}{4}'' \times 20'$. In selecting the 10 blanks from which the test arms were made, only straight grained pieces free from evidence of manufacturing and other defects were chosen. Each blank chosen was cut into two 10′ lengths, one of which was made into a JW arm and the other into a W6 arm, making 10 matched arms of each type. The tests were made on an Amsler testing machine. The average breaking load at the end pinholes was 1159 pounds for the JW arms and 1002 pounds for W6 arms.

At the same time an estimate was made of the theoretical *strength relationship* between the two types by means of the moment diagrams shown in Fig. 8. In this figure are shown the graphs of the resisting moments (fiber stress factor—5000 psi) of clear JW and clear W6 arms, together with the graphs of the bending moments due to the maximum loads these arms would withstand when the loads are concentrated at the end pinholes. These maximum loads were determined by dividing the moments at the points of coincidence between the graphs (critical pole pinhole sections) by the distances to the end pinholes. The maximum loads, so determined, are 608 pounds for the JW arm and 532 pounds for the W6 arm. The fact that these loads are low as compared with the actual breaking loads shows, of course, that the average ultimate fiber stress developed by these selected arms was considerably greater than 5000 psi, which is not surprising in view of their exceptionally high quality. However, so far as the information sought is concerned—namely, to determine not the actual strength but the strength relationship between the two types—the result would be the same regardless of the fiber stress factor used in the moment diagram.

The ratio of the strength of the W6 arm to that of the JW arm as shown both by the actual strength tests and by the moment diagrams was as follows:

	Strength Ratio W6 to JW
	(Per cent)
Actual strength tests — $\frac{1002}{1159} \times 100 =$	86.5
Moment diagrams — $\frac{532}{608} \times 100 =$	87.5

These ratios show a remarkably close agreement between theory and actuality and justify the belief that the crossarm moment diagram may be employed to obtain reasonably accurate estimates of relative bending strength.

Summary

The results of this study may be summarized as follows:

1. The moment diagram is a useful guide in setting specification limitations on defects.

2. It is shown that the critical section of a crossarm is located at the pole pinholes. The practical value of this observation is that it emphasizes the need for keeping the pole pinhole sections and the portion of the arm between them reasonably free from strength reducing defects.

3. Only by breaking tests can the *actual* bending strength of crossarms be determined. The *relative* bending strengths, however, of two or more arms of different types or quality may be estimated with sufficient accuracy by means of the moment diagram, regardless of the fiber stress used in its construction.

4. If the fiber stress factor employed is dependable, the moment diagram may be used to estimate the minimum factor of safety that would obtain for an arm of any type or any assumed quality. In this connection, it is believed that the strength of Bell System crossarms is well above the minimum required to support the loads ordinarily carried.

5. The section modulus curves of Figs. 4, 5, 6 and 7 will simplify the construction of moment diagrams for arms of the same sizes shown in the figures but differing with respect to type and quality.

The uses listed lead to the general conclusion that the crossarm moment diagram is a convenient and reasonably reliable engineering tool.

APPENDIX

Computation I. Moment of Inertia of Top Segment of Minimum ($3\frac{3}{16}''$ x $4\frac{3}{32}''$) Section between Pinholes:

The moment of inertia (IT) of a segment (T) with respect to an axis through its center of gravity and parallel to its base may be found by the formula

$$IT = I_{BB} - Ax^2$$

where I_{BB} is the moment of inertia of the segment about the axis BB, A the area of the segment and x the distance between the two axes. The values I_{BB}, A and x are given by:

$$I_{BB} = \tfrac{1}{4}Ar^2\left[1 + \frac{2\sin^3 a \cos a}{a - \sin a \cos a}\right] \qquad (1)$$

$$A = \tfrac{1}{2}r^2\,(2a - \sin 2a) \qquad (2)$$

$$x = \tfrac{2}{3}\,\frac{r^3 \sin^3 a}{A} \qquad (3)$$

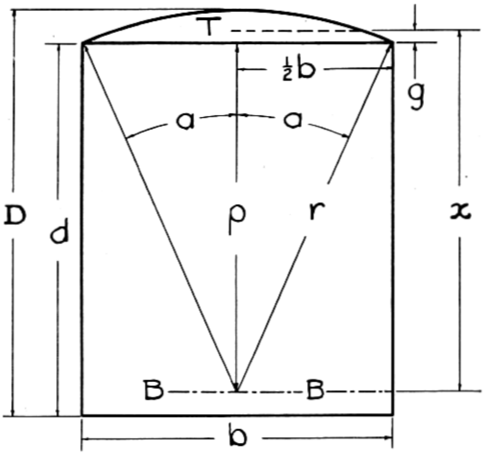

Fig. 9—Crossarm section between pinholes.

The significance of r and a in these formulae, and of the other symbols used in the computations that follow will be clear from a glance at Fig. 9.

$D = 4.09375''$

$b = 3.1875''$

$\frac{1}{2} b = 1.59375''$

$r = 4''$

$r^2 = 16.000000$

$(1/2\ b)^2 = 2.540039$

$p^2 = 13.459961$

$p = 3.668782''$

$\text{Sin } a = \dfrac{1/2\ b}{r} = 0.39843750$

$a = 23° 28' 49.93''$

$a = 0.40981266$ radians

$2\ a = 46° 57' 39.86''$

$\text{Sin}^3\ a = 0.063252925$

$\text{Sin } 2\ a = 0.73089017$

$\text{Cos } a = 0.91719548$

$\text{Sin } a \text{ Cos } a = 0.36544507$

$d = p + (D - r) = 3.7625''$

$A = 0.7099$ sq. ins. [Area of T by Formula (2)]

$x = 3.8018''$ [By Formula (3)]

$g = x - p = 0.1330''$

$I_{BB} = 10.2654$ [By Formula (1)]

$Ax^2 = 10.2601$

$IT = 0.0053$

(Note: While the results of this and the following computations are shown to four decimal places, the actual work was done by machine and carried to eight decimal places as mentioned in the text.)

Since the width of the section in this computation and the radius of its roof is the same as for the minimum $3\frac{3}{16}''$ x $4''$ section at the end of the arm, the top segments of the two are identical, and the only value that will differ will be the depth (d) of the rectangular portion of the section, which for the smaller will be $p + (D - r)$, or

$$3.6688 + (4 - 4) = 3.6688''$$

Computation II. Moment of Inertia of Top Segment of Nominal ($3\frac{1}{4}''$ x $4\frac{3}{16}''$) Section between Pinholes:

As this computation was made in exactly the same manner as Computation I, only the results are here shown:

$$d = 3.8593''$$
$$g = 0.1317''$$
$$A = 0.7168 \text{ sq. ins}$$
$$IT = 0.0053$$

Computation III. Moment of Inertia of Top Segment of Minimum ($3\frac{3}{16}''$ x $4\frac{3}{32}''$) Pinhole Section:

It will be noted in Fig. 10 that the top segment is divided into four parts: the small segment (T_1) at the top of the pinhole, the rectangular portion

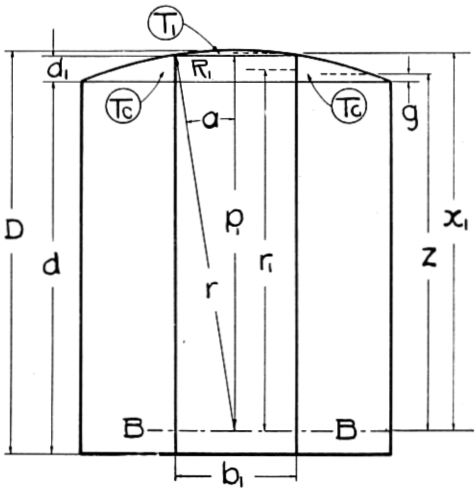

Fig. 10—Crossarm pinhole section.

R_1, with a width of b_1 and a depth of d_1, and two portions designated Tc. The purpose of this computation is to determine the moment of inertia of one of the Tc portions with respect to its gravity axis parallel to its base. The moment of inertia of the two Tc portions about the axis BB may be found by deducting the moments of inertia of T_1 and R_1 about this axis from the moment of inertia of the entire top segment about the same axis.

$D = 4.09375''$ $\qquad$ $\text{Sin } a = \dfrac{1/2\, b_1}{r} = 0.16625$

$b_1 = 1.33''$ $\qquad$ $a = 9^\circ\, 34'\, 11.49''$

$\frac{1}{2} b_1 = 0.665''$ $\qquad$ $a = 0.16702554$ radians

$r = 4.00''$ $\qquad$ $2\, a = 19^\circ\, 8'\, 22.98''$

$r^2 = 16.000000$ $\qquad$ $\text{Sin}^3\, a = 0.0045949941$

$(1/2\, b_1)^2 = 0.442225$ $\qquad$ $\text{Sin } 2\, a = 0.32787285$

$p_1^2 = 15.557775$ $\qquad$ $\text{Cos } a = 0.98608364$

$p_1 = 3.9443345''$ $\qquad$ $\text{Sin } a \text{ Cos } a = 0.16393640$

d (Computation I) $= 3.7625''$

$d_1 = p_1 + (D - r) - d = 0.2756''$

$r_1 = p_1 - 1/2d_1 = 3.8065''$

Area $R_1 = b_1 d_1 = 0.3665$ sq. ins.

$A_1 = 0.0494$ sq. ins. [Area of T_1 by Formula (2)]

$x_1 = 3.9666''$ [By Formula (3)]

By Computation I, $IT_{BB} = 10.2654$

IT_{1BB} [Formula (1)] $= 0.7777$

$$IR_{1BB} = \frac{b_1 d_1^3}{12} + R_1 r_1^2 = 5.3126$$

6.0903

$2ITc_{BB} = 4.1751$

The moment of inertia of the 2 Tc areas with respect to the axis through their own centers of gravity is given by

$$2ITc = 2ITc_{BB} - 2Tc\, z^2$$

where

2 Tc is the area of the two Tc portions of the top segment and is given by

$$2Tc = A - (A_1 + R_1)$$

in which A is the area of the entire top segment as shown in Computation I; and

where, by the principle of moments,

$$z = \frac{Tx - T_1 x_1 - R_1 r_1}{2Tc}$$

in which Tx, $T_1 x_1$ and $R_1 r_1$ are the moments of the areas of T, T_1 and R_1, respectively, about the axis BB. ($Tx = Ax$ of Computation I.)

Thus

$$2Tc = 0.2940 \text{ sq. ins.}$$
$$z = 3.7680''$$

As previously shown, $2ITc_{BB} = 4.1751$

$$
\begin{aligned}
2Tc\, z^2 &= 4.1738 \\
\hline
2ITc &= 0.0013 \\
ITc &= 0.0007 \\
D - r &= 0.09375'' \\
z &= 3.7680'' \\
\hline
 & \ 3.8618'' \\
d &= 3.7625'' \\
\hline
g \text{ for } Tc &= 0.0993''
\end{aligned}
$$

The results of this computation apply also to the minimum $3\frac{3}{16}''$ x $4''$ pinhole section at the ends of the arm. The depth (d) of the rectangular portion of the end pinhole sections will be the same as at the extreme ends of the arm, viz. 3.6688″.

Computation IV. Moment of Inertia of Top Segment of Nominal ($3\frac{1}{4}''$ x $4\frac{3}{16}''$) Pinhole Section:

Since this computation was made in the same manner as Computation III, only the results are here shown:

$$
\begin{aligned}
d &= 3.8593'' \\
g &= 0.1019'' \\
Tc &= 0.1630 \text{ sq. ins.} \\
ITc &= 0.0008
\end{aligned}
$$