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A Study of the Delays Encountered by Toll Operators in Obtaining an Idle Trunk

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THE aim of the Bell System is to give the fastest possible toll service consistent with costs. The aim of the Intertoll Trunk Engineer is to provide the proper number of trunks in each group to obtain that objective. His problem is to gauge the effect of his work on the overall speed of service.

Overall speed of toll service is the elapsed interval from the filing of a call until conversation starts or until there is a definite report about the called party. This overall speed includes the operating time or interval required for the operators to establish the connection; the subscriber time or interval required for the calling party to give the details of the call, for the called party to answer his telephone, etc.; and the circuit delay time or interval of waiting for a trunk to become idle. This last factor may be termed the *trunk speed* interval.

The proportion of this trunk speed to the overall speed is an important factor in determining the number of trunks to be provided. If it is a large proportion of the total, a marked improvement may be expected as a result of providing more trunks. Conversely, if the trunk speed is a small proportion of the total, the improvement to be expected as a result of providing more trunks will be small also, with a diminishing rate of improvement until the trunk speed ceases to be a factor. The trunk speed in turn depends upon three factors:

Group size—number of trunks to the called city or in the direction of the called city.

The per cent. usage—the degree to which the trunks are kept busy in carrying the load offered.

Holding time—the length of time that a trunk is in use each time it is used.

Since the trunk speed depends in part on the per cent. usage, it follows that this interval will be longer in the busiest hour when the usage is greatest and will be shorter in the hours which are less busy. Consequently, the trunk speed interval over the total day will be much less than in the busiest hour.

Purpose of Study

Earlier information, based on data assembled in Cleveland in 1929 and 1930, was formulated as a series of relationships between varying degrees of loading (in terms of busy hour per cent. use) on trunk groups of different sizes and the *overall* speed of service. These relationships were set forth in a table which was to be used as a guide to the trunk provision needed to accomplish a desired overall service result.

The table also furnished the *percent* calls encountering an NC (no circuit) condition but made no specific reference to the average *duration* of NC although from the data shown it could be inferred and demonstrated that other factors, such as operating method, operating and party delays, normally have a more pronounced influence on the total day overall speed of service than the busy hour trunk provision. That being so, as changing conditions since 1930 have affected these other factors, either in the direction of faster or slower service, the relationships in terms of overall speed of service shown in that table have become less valuable as engineering guides.

The purpose of the current study, therefore, was to improve the engineering and management tools used in determining the number and arrangement of trunks required to attain faster toll service so that the investment in facilities may be used as effectively as possible.

Study Procedure

The study was based on the premise that if the size of group, per cent. usage and holding time are known, the trunk speed can be determined and will remain constant under that particular set of conditions. With this constant known, it would then become possible to construct from analyses of overall speed of service data for groups, offices, areas or networks the going relationship between the trunk speed of service and the overall speed of service and to predict with reasonable assurance the effect on the overall speed which would be brought about by changes in the group sizes or traffic characteristics. The effect of foreseen changes in operating method, force conditions or the character of the toll traffic on the overall speed can be estimated separately and taken into consideration in determining the basis of trunk provision. With such information available, trunks can be provided where they will be most effective. This is especially important during periods of major change such as the transition from war to peacetime conditions or from the ringdown to the dial method of toll operation.

The problem was therefore to determine the average delay in securing a trunk with various sizes of groups at various levels of usage with a view to:

Stating that portion of the overall speed of service which results from inability to secure a circuit, and

Constructing engineering tables based on a preselected constant circuit delay or trunk speed of service.

Arrangements were made with several Associated Companies to furnish data for this purpose which would show:

1. The average overall speed of service on different sizes of groups under various conditions of loading.
2. The minimum average overall speed interval on these same groups at times when circuit provision was not a factor, i.e., when NC conditions were not encountered.

The speeds obtained in Item 2 were subtracted from those obtained in Item 1, the difference representing that portion of the overall speed which can be attributed to circuit delay, or the trunk speed.

In order to determine these trunk speeds it was necessary to obtain from several sources as much data as possible of the following nature:

Per cent. circuit usage, by hours, as derived from group busy timing registers on selected groups of various sizes. Hours during which the traffic over a group was handled subject to posted delay were disregarded.

The number of originating terminal calls handled over the groups during the hours corresponding to the usage data and the average speed of service on these calls. The call and speed of service data were summarized first to include all calls and then separately for calls not encountering NC. Correction was made for transfer of tickets to point-to-point positions by subtracting from the speed shown on each such ticket an interval representing the average length of time required to send a ticket to point-to-point positions in the office in which the data were obtained, provided the transfer time was included in the overall speed interval. This interval of transfer time is not properly chargeable as part of the trunk speed.

These data were obtained for trunk groups of various sizes ranging from one up to eighteen trunks. To secure a comparable amount of data for the smaller groups which handle fewer calls, it was necessary to include more of the smaller groups or to continue the record for a longer period of time on such groups.

The data for all hours of the day or evening were useful because as the volume of traffic recedes from the busy hour the data are typical of the busy hour condition of other groups engineered on a more liberal basis. The very light hours also show the minimum speed interval which can be obtained when lack of an available circuit is not a factor.

Five Associated Companies obtained data at eleven toll offices on 112 intertoll groups having 561 trunks. Approximately 17,000 calls (occurring during hours when the groups were at least 40% busy) were included.

The data were summarized by size of group and by circuit usage. Separate counts were maintained for groups with and without alternate routes and for person and station traffic. The following tabulation shows the type of data available for each point, i.e., each size of group at each level of usage.

ONE TRUNK—WITH ALTERNATE ROUTE—61–70% USE

% Use	Type	All Calls			Calls Not Encountering NC		
		Minutes	Calls	Speed	Minutes	Calls	Speed
61–65	Station	111	23		39	17	
	Person	109	38		68	32	
66–70	Station	103	25		33	21	
	Person	117	28		54	25	
Total		440	114	3.86	194	95	2.04

Average Speed—All Calls	3.86 Mins.
Average Speed—NC not Encountered	2.04 Mins.
Average Delay Due to NC (Trunk Speed)	1.82 Mins.

The results were plotted for each level of usage by steps of 10% as shown in Fig. 1, using a 3-point moving average to smooth out the deviations and to establish a more definite trend. Each of these curves was then redrawn in relation to the others and combined results are shown on Fig. 2.

The delay intervals indicated in Fig. 2 represent the total delay which resulted from the fact that there was no circuit available when the operator was first ready to make use of one. It includes not only the time spent in waiting for a circuit to become idle but also the time required for the operator herself to return to that call if she had engaged in some other work in the meantime. If the operator is not free to utilize the circuit as soon as it becomes available some other operator may use it for another, later call. The subsequent call is then delayed less than the average, or not at all, but the original call is delayed longer than the average. While the delays experienced by individual calls may vary considerably from the average, the data have been treated in terms of averages for engineering purposes.

MATHEMATICAL FORMULAE

The summarized data were referred to the different mathematical expressions frequently applied to trunking problems, such as the Poisson, Erlang "B" and Erlang "C" formulae. It was found that the observed average NC delays were considerably shorter than the theoretical average delays in those formulae which make allowance for variable holding times, such as would be encountered in local trunking where the average trunk use is short and the deviations from average on a percentage basis are apt to be appreciable.

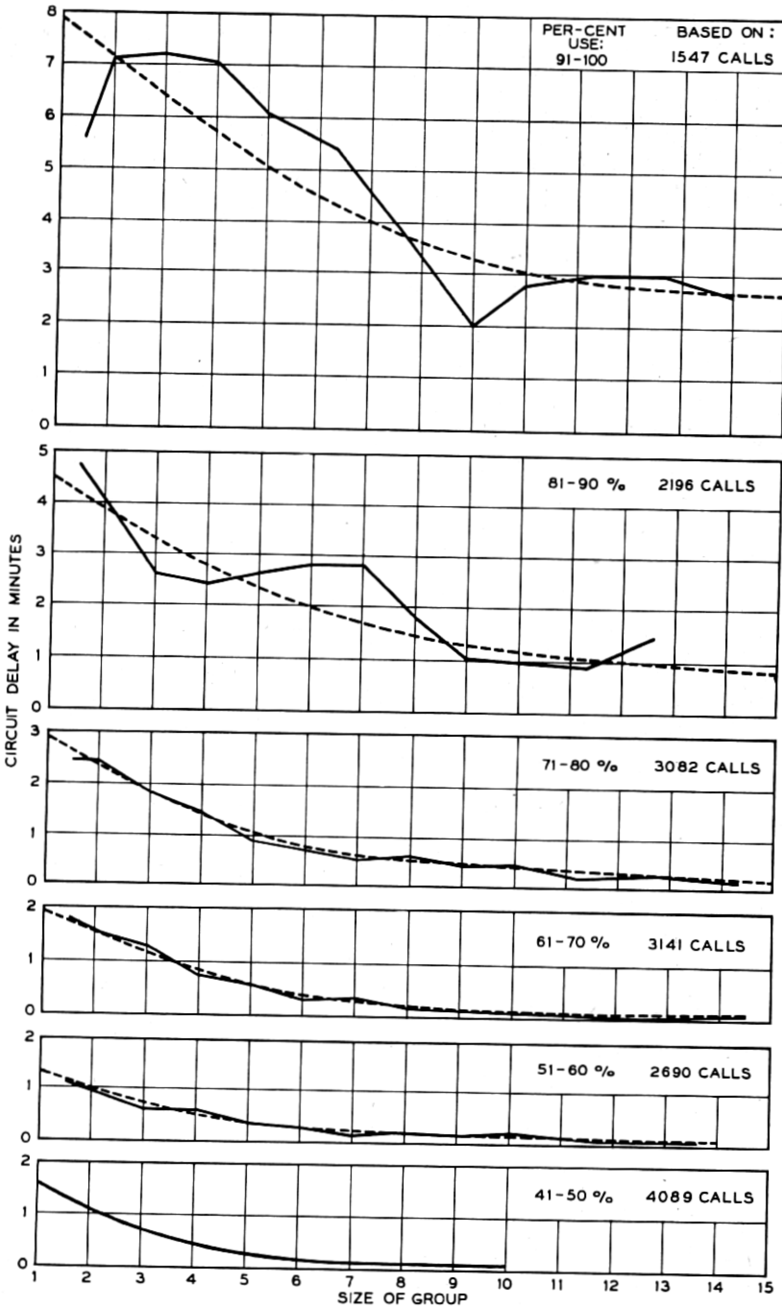

Fig. 1—Average circuit delay on all calls (with alternate routes where authorized). Circuit delay = average speed on all calls minus average speed on calls which did not encounter NC. Based on 3-point moving average.

However, further reference to mathematical studies of telephone traffic indicated that a delay theory based on constant holding times, first developed

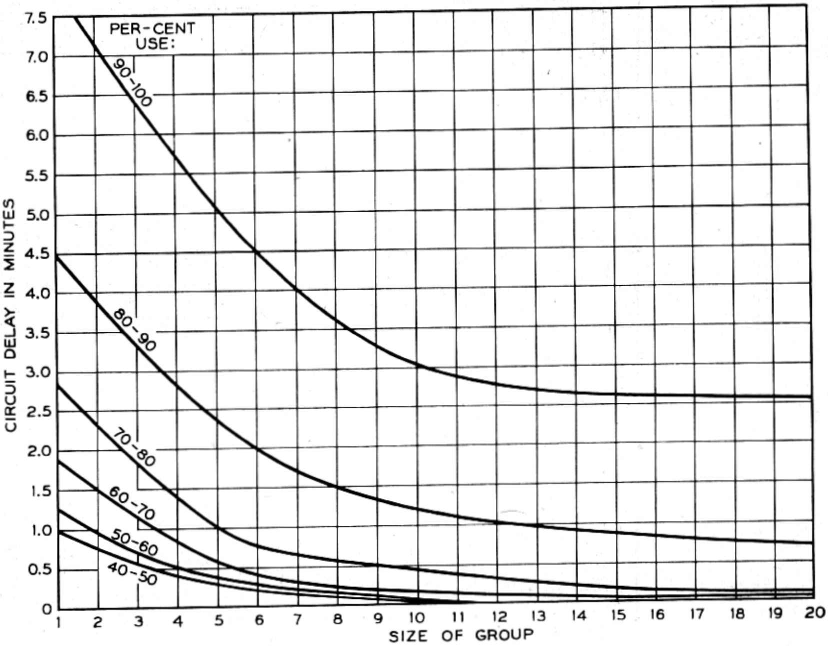

Fig. 2—Average circuit delay on all calls (with alternate routes where authorized). Circuit delay = average speed on all calls minus average speed on calls which did not encounter NC. Combined curves based on 16,745 calls.

by Felix Pollaczek in Germany and amplified by C. D. Crommelin in England, closely approximated the empirical data. This formula[1] is:

$$d = \sum_{w=1}^{\infty} e^{-aw} \left[\sum_{u=wc}^{\infty} \frac{(aw)^u}{\lfloor u} - \frac{c}{a} \sum_{u=wc+1}^{\infty} \frac{(aw)^u}{\lfloor u} \right]$$

In which

d = average delay on all calls

a = average simultaneous calls submitted to a group of c trunks (trunk hours)

c = number of trunks in group

It may be quite reasonable that the Pollaczek constant holding time formula should better represent toll delays than an exponential holding time formula since the toll charge and perhaps other factors ordinarily cause these calls

[1] C. D. Crommelin, "Delay Probability Formulae," *P.O.E.E. Journal*, Jan. 1934, p. 266

to exhibit considerably less percentage deviation from their average than is found in the exponential distribution.

In order to compare the empirical data with the Pollaczek formula it was necessary to assume a holding time per attempt since the formula expresses the delays in terms of the average interval of use whenever the circuit is in use. The average holding time as reported by the companies for the groups included in the study was 8.3 minutes per message. Recent data show 1.42 attempts per call disposed of. Relating this figure to the 8.3 minutes results in an average holding time per circuit use of 5.85 minutes. Six

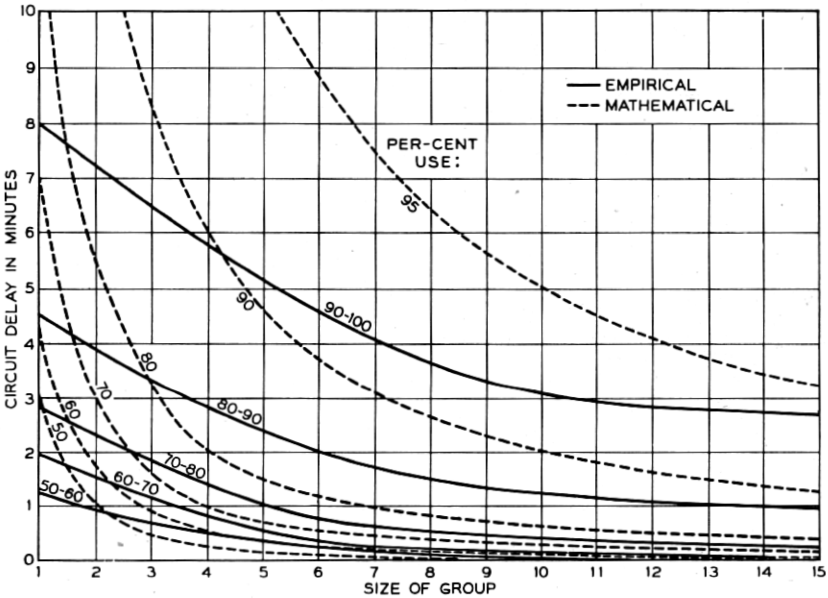

Fig. 3—Average circuit delay on all calls.
Comparison of empirical data (with alternate routes) and Pollaczek formula (no alternate routes) using a 6 minute holding time.

minutes is therefore well within the limits of accuracy required for this purpose.

Curves were prepared from the Pollaczek formula for various levels of usage at a 6-minute holding time per attempt. The corresponding curves derived from the empirical data were then superimposed for comparative purposes as shown in Fig. 3. It will be seen that the shape and levels of the curves are very similar *except* for the smaller groups on which the effect of alternate routes tends to reduce the average length of delay.

As a further check on the validity of the Pollaczek formula, the delay data from the Cleveland (1929–1930) study were expressed in terms of holding times for different group sizes at different levels of usage and compared with delay intervals developed from the formula. This comparison is

COMPARISON OF 1945 STUDY WITH CLEVELAND STUDY OF 1929–30
Based on 3.5 Min. HT per Circuit Attempt
or 5.25 Min. HT per Message

No. of Trunks	% Use	Minutes Delay		% of H.T.	
		Cleveland	1945	Cleveland	1945
1	55–60	.8	.8	21	22
1	65–70	1.1	1.2	30	35
1	75–80	1.5	1.9	43	53
1	85–90	2.7	3.0	76	86
3	55–60	.3	.5	09	13
3	65–70	.5	.8	14	22
3	75–80	.9	1.3	24	36
3	85–90	1.8	2.4	51	68
6	55–60	.1	.1	03	04
6	65–70	.3	.3	09	08
6	75–80	.6	.6	16	16
6	85–90	1.3	1.7	36	48
10	55–60	.1	—	02	01
10	65–70	.3	.1	07	03
10	75–80	.4	.3	12	08
10	85–90	1.0	1.0	28	28
14	55–60	.1		01	
14	65–70	.2	.1	05	02
14	75–80	.3	.2	09	05
14	85–90	.8	.7	23	19

The minutes of circuit delay shown above for the Cleveland study are derived by subtracting the minimum speed of 1.65 minutes from the actual overall speed for the various sizes of groups and levels of usage. The comparable 1945 figures are taken from Fig. 5.

It will be noted that the principal differences occur on the smaller groups at the higher levels of usage. This is undoubtedly due to the fact that the alternate routes are more heavily loaded today than they were in 1929–30 and therefore are less helpful in absorbing the overflow from the first route.

The holding time used in this comparison as probably typical of 1929–30 is derived as follows:

Conversation time	3.00 minutes
Operating time	2.25 minutes
	5.25 minutes
No. of Circuit Uses per Message	1.50
HT per Circuit Use (5.25 ÷ 1.50)	3.50

Fig. 4

shown in Fig. 4. It will be seen that there is substantial agreement between the two sets of delay factors, such differences as there are being explained in the notes on that figure.

The delay in securing a circuit varies directly with the holding time, i.e., a call waiting for a circuit will be delayed twice as long if the group is handling 10-minute calls as would be the case with 5-minute calls. This is best illustrated by one call awaiting access to a single circuit group. The new call may appear at any time during the progress of the existing call, but the average delay for many such delayed calls will be one-half the holding time of the existing call. If the existing call uses the circuit for five minutes, the new call will wait $5 \div 2 = 2.5$ minutes. If the existing call uses the circuit for ten minutes, the new call will wait five minutes. It should be noted that the average delay depends upon the average length of time that the circuit is in use each time that it is used, in other words, the holding time per circuit attempt.

The Cleveland study did not go into this phase in detail, the statement being made that the effect of holding time on speed of service "is slight." This is so when considering the *overall* speed of service, with which that study was primarily concerned, because of the weight of operating and subscriber time intervals. Reference to that study shows that the circuit delay increased about in proportion to the holding time when a minimum operating and subscriber time interval is subtracted from the overall speed, as follows:

	Holding Time		
	5′	7.5′	10′
Total Overall Speed	2.2	2.6	3.0
Minimum Operating and Subscriber Time Interval	1.6	1.6	1.6
Average NC Delay-Trunk Speed	.6	1.0	1.4

Combination of Mathematical and Empirical Methods

Because of the apparent close agreement between the Pollaczek delay formula and two representative large samples of actual NC delay data taken at different periods and under widely different conditions, 1930 and 1945, the Pollaczek formula can be used for deriving expressions of the average duration of NC with intertoll trunk operation without alternate routes. The effect of alternate routes in reducing the duration of NC can be shown with sufficient accuracy for practical needs from the empirical data. The curves shown in Fig. 5 were constructed on this basis. For large groups the formula has been extended in Fig. 6.

It is advantageous to have available an acceptable mathematical formula for expressing the relationship between the loads carried by the trunk groups and the length of time that the average call will be delayed because of NC conditions, i.e., the part played by trunk provision in the overall speed of service. With such a formula results can be predicted for any given set of assumptions without going through the burdensome process of accumulating actual data in reliable volume and with useful frequency. The formula

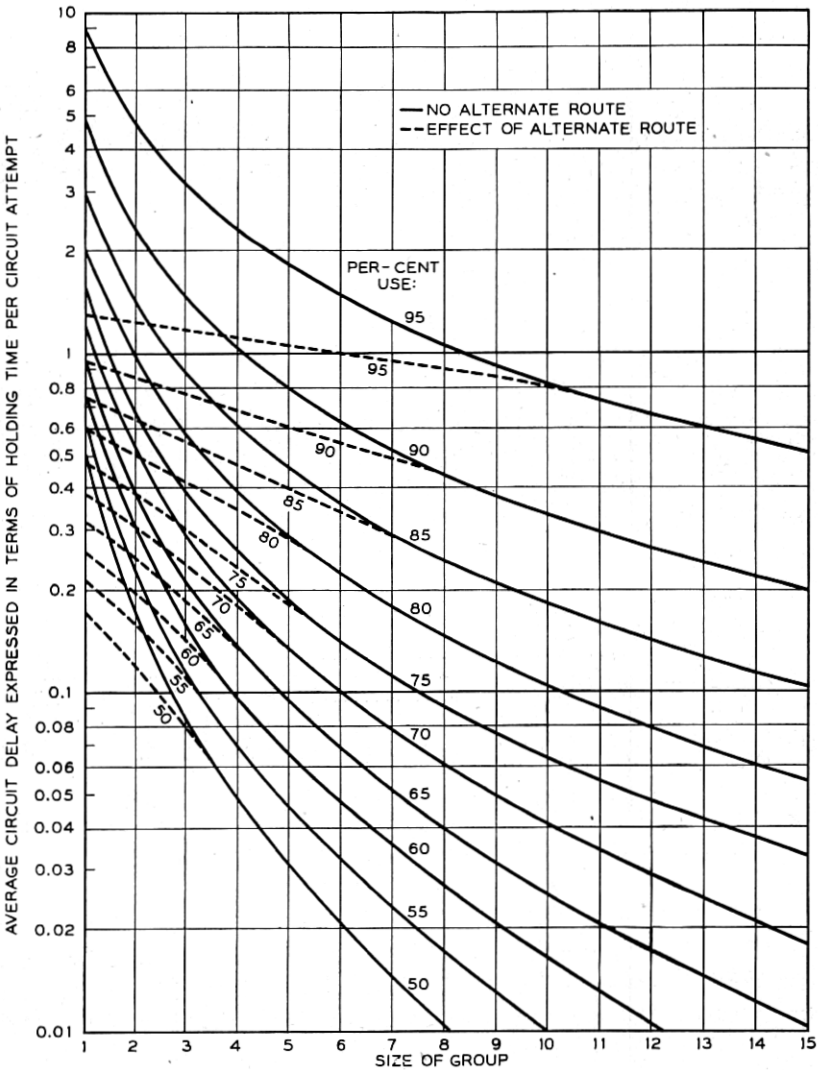

Fig. 5—Average circuit delay on all calls expressed in terms of the holding time per circuit attempt.

also provides a convenient means of checking the adequacy of the trunk facilities in any unusual situations which may be observed from time to time.

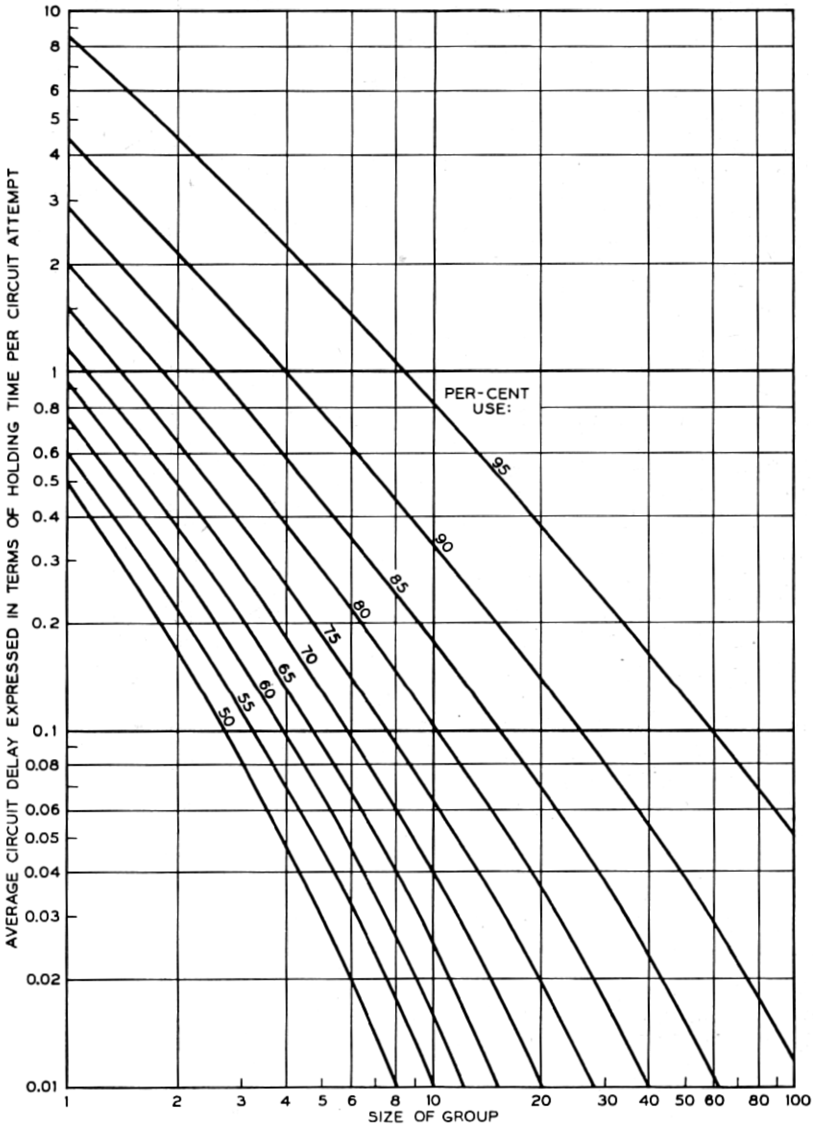

Fig. 6—Average circuit delay on all calls expressed in terms of the holding time per circuit attempt.
Based on Pollaczek formula.

In the case of the alternate route effect, where a variable is introduced which the formula does not encompass, it may be necessary later to recheck this factor by observed data should the conditions governing the use of alternate routes change substantially or should the actual results at some future time on groups provided with alternates be found to differ from those currently predicted.

Delays are expressed in Figs. 5 and 6 as a percentage of the holding time per *average use* of the trunk. The holding time factors used in intertoll trunk engineering generally are expressed in terms of the holding time *per message.* Therefore, in order to use the curve conveniently it is necessary to reduce the holding time per message to the holding time per trunk

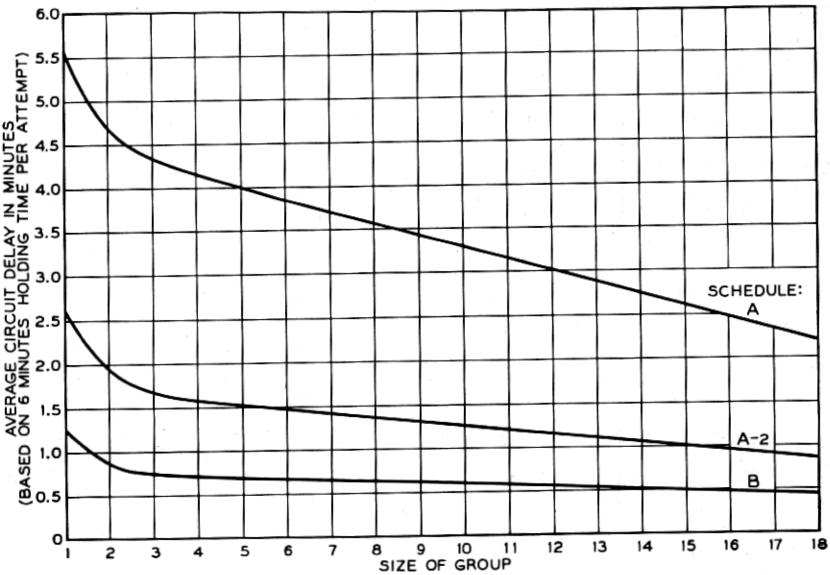

Fig. 7—Average circuit delay on all calls resulting from present intertoll trunk capacity schedules (without alternate routes)—Using Pollaczek formula.

use or attempt. This can be done with sufficient accuracy for this purpose by a ratio of 1.5 outward attempts per call disposed of.

Description of Engineering Tables

Having determined the average circuit delay, as previously described, the delays which result from the present Intertoll Trunk Capacity Tables A, A2, and B can be determined by referring the percentages of use on these tables to the curves in Figs. 5 and 6. Figure 7 indicates that each of these existing tables results in variable delays depending upon the size of the group.

The curves in Figs. 5 and 6 can also be used to construct new capacity tables which should produce a relatively uniform delay for any size of group. If we select an average delay of three-tenths of a holding time as an example

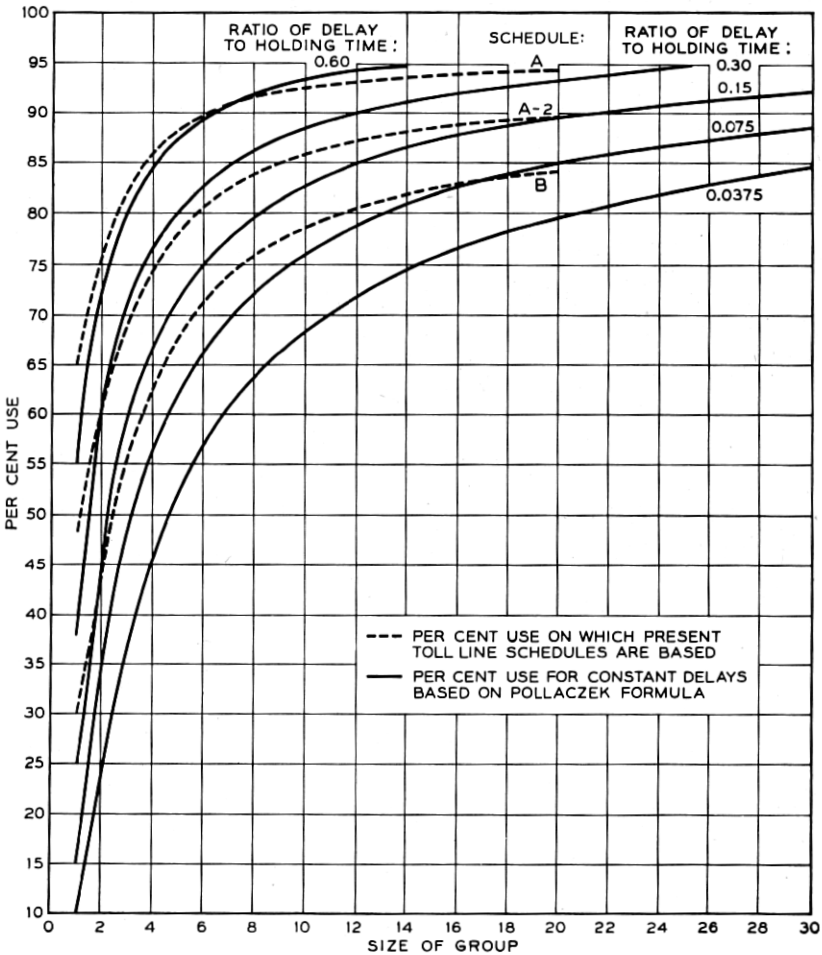

Fig. 8—Per cent. use with present intertoll trunk schedules and for constant delays based on Pollaczek formula.

and follow the .3 line across these curves, we see that a two-trunk group can be kept busy 60.8 per cent. of the time; a three-trunk group can be in use about 71.1 per cent. of the time; about 76.7 per cent. for four trunks, etc.

Figure 8 shows the per cent. usage for groups of from one to thirty trunks which result in average delays of .0375, .0750, .15, .30, and .60 of a holding time. The usage obtained from present capacity tables is also shown for comparison. From this figure it will be seen that five new tables based on these average delays will adequately cover the field encompassed by those now in use. The usage curves for these five selected delay intervals were

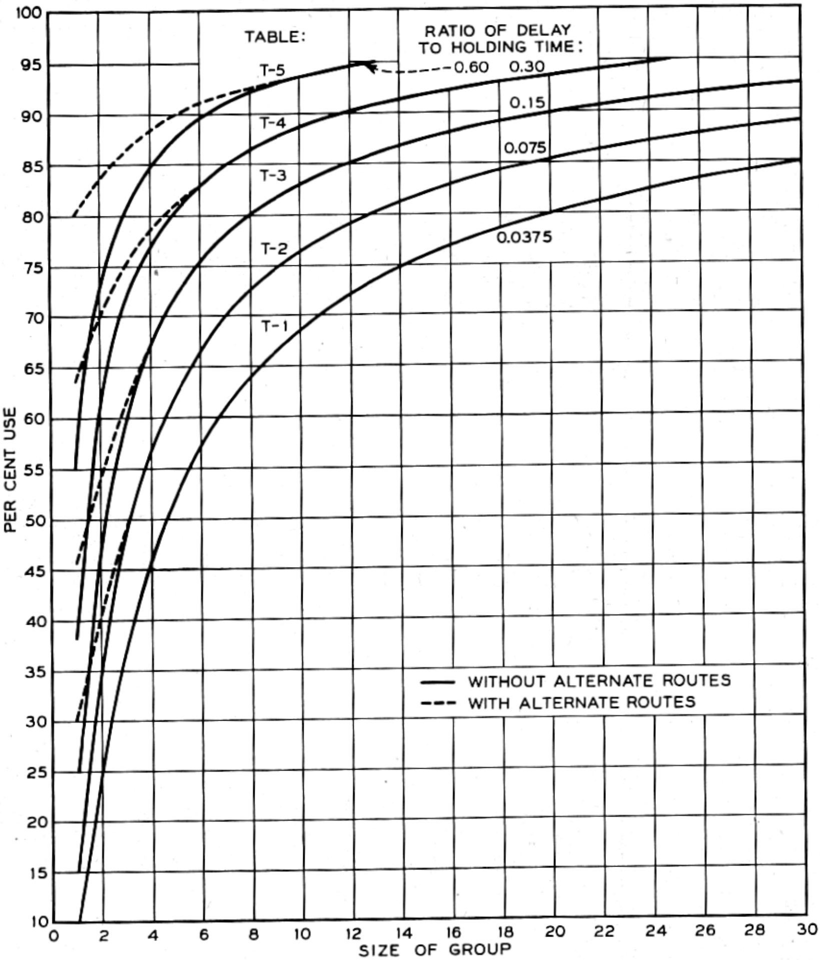

Fig. 9—Per cent. use for constant delays based on Pollaczek formula.

redrawn in Fig. 9, on which the effect of alternate routes is also shown. Capacity Tables T-1 to T-5 (Fig. 10) were constructed from this curve, Table T-1 representing the shortest (.0375) delay and Table T-5 the longest (.60) delay.

For situations where it may be necessary for service reasons to provide trunks on a basis more liberal than Table T-1, the interlocal trunk capacity tables should be used. These latter tables, constructed from the Poisson formula, express a service relationship in terms of the percentage of calls *encountering NC* rather than the average *duration of NC*. When trunks are provided on a basis as liberal as that implied by the interlocal tables the frequency of NC is deemed to be the more important service consideration because the duration of NC is so short as to be of lesser consequence.

Since the Pollaczek formula expresses the delay as a percentage of the holding time per *circuit attempt* but the intertoll trunk engineer is accustomed to dealing with holding times per *message* the delays indicated for each of Tables T-1 to T-5 have been expressed in the latter terms, using a 1.5 ratio of attempts per message. This is consistent with previous computations. With Table T-4 and a five-minute message holding time, for example, we have:

5-minute HT per message $\div$ 1.5 = 3.33 minutes HT per attempt
3.33 minutes x .30 delay = 1.0 minute delay

This one minute delay shown in the heading of Table T-4 is, therefore, actually the delay per attempt when the holding time per message is five minutes. The delay per attempt is the important criterion because the "speed of toll service" as quoted from service observations is generally the speed of the first attempt and the two are, therefore, comparable.[2]

As the usage approaches 100 per cent. there may be an indeterminate backed-up potential demand and the normal relationship between service and loading no longer holds true. From other data assembled for this purpose, it appears that about 96–97 per cent. represent the practical upper limit of usage, beyond which trunk speeds can not be accurately predicted. For practical reasons, therefore, group capacities have not been computed for percentages of use higher than 97 even though, from a theoretical viewpoint, the curves derived from the Pollaczek formula would permit extending the usage virtually to 100 per cent. This would also apply to Tables T-1 and T-2 if they were extended above 75 trunks.

Relation of Circuit Delays in Busy Hour to Total Day

Up to this point the trunk speed of service has been discussed in terms of the busiest hour. However, the overall speed of service is generally quoted in terms of the total day. Therefore, one additional step is necessary, namely to determine the relationship of the trunk speed in the busy hour to that of the total day.

[2] There is one exception to the statement that the "speed of toll service" is the speed on the first attempt. That is the case of a built-up connection where an NC condition is encountered at an intermediate office which persists so long that the first circuit is released. When the connection is established it is at least the second attempt. The full time interval during which the ticket is held awaiting completion is included in the speed quoted on that call. Similar intervals were also included in the empirical data for this study and the results are, therefore, comparable in this case also.

Fig. 10—Intertoll trunk capacity tables

Trunk Speed when average H.T. per Msg. is:	Table T-1				Table T-2				Table T-3				Table T-4				Table T-5			
4–6 Mins.	.13 Minutes				.25 Minutes				.5 Minutes				1.0 Minutes				2.0 Minutes			
7–9 Mins.	.20 Minutes				.40 Minutes				.8 Minutes				1.6 Minutes				3.2 Minutes			
10–12 Mins.	.28 Minutes				.55 Minutes				1.1 Minutes				2.2 Minutes				4.4 Minutes			
	No Alt. Route		Alt. Route		No Alt. Route		Alt. Route		No Alt. Route		Alt. Route		No Alt. Route		Alt. Route		No Alt. Route		Alt. Route	
No. of Trunks	% Use	Minutes Capacity	% Use	Minutes Capacity	% Use	Minutes Capacity	% Use	Minutes Capacity	% Use	Minutes Capacity	% Use	Minutes Capacity	% Use	Minutes Capacity	% Use	Minutes Capacity	% Use	Minutes Capacity	% Use	Minutes Capacity
1	10.0	6	—	—	15.0	9	30.0	18	25.0	15	50.0	30	38.3	23	63.4	38	55.0	33	80.0	48
2	25.0	30	—	—	35.0	42	40.0	48	48.3	58	53.3	64	60.8	73	70.0	84	73.3	88	83.4	100
3	37.2	67			48.9	88			60.0	108	61.1	110	71.1	128	75.0	135	80.6	145	86.6	156
4	46.3	111			56.7	136			67.5	162			76.7	184	78.3	188	85.0	204	88.4	212
5	52.7	158			62.3	187			72.3	217			80.3	241	80.7	242	87.6	263	90.0	270
6	57.5	207			66.7	240			75.4	272			83.1	299			89.5	322	91.1	328
7	61.2	257			70.0	294			78.1	328			85.0	357			91.0	382	91.9	386
8	64.2	308			72.7	349			80.2	385			86.5	415			92.1	442	92.5	444
9	66.7	360			74.8	404			81.9	442			87.6	473			93.0	502		
10	68.8	413			76.5	459			83.1	499			88.7	532			93.6	561		
11	70.6	466			78.0	515			84.2	556			89.6	591			94.1	621		
12	72.2	520			79.3	571			85.3	614			90.3	650			94.5	680		
13	73.6	574			80.4	627			86.2	672			90.9	709			94.9	740		
14	74.8	628			81.4	684			86.9	730			91.4	768			95.2	800		
15	75.9	683			82.3	741			87.6	788			91.9	827			95.5	860		
16	76.9	738			83.1	798			88.1	846			92.3	886			95.8	926		
17	77.8	793			83.9	855			88.6	904			92.6	945			96.1	980		
18	78.6	848			84.5	912			89.0	962			93.0	1,005			96.3	1,040		
19	79.3	904			85.0	969			89.5	1,021			93.4	1,065			96.5	1,100		
20	80.0	960			85.5	1,026			90.0	1,080			93.7	1,125			96.6	1,160		

21	80.7	1,017			86.0	1,084			90.4	1,139			94.0	1,185			96.8	1,220		
22	81.3	1,074			86.5	1,142			90.8	1,198			94.3	1,245			97.0	1,280		
23	82.0	1,131			87.0	1,200			91.1	1,257			94.5	1,305						
24	82.5	1,188			87.4	1,258			91.4	1,316			94.8	1,365						
25	83.0	1,245			87.7	1,316			91.7	1,375			95.0	1,425						
30	85.0	1,530			89.7	1,606			92.8	1,670			95.9	1,725						
35	86.5	1,816			90.5	1,901			93.7	1,968			96.4	2,025						
40	87.8	2,106			91.5	2,196			94.6	2,268			96.8	2,325						
45	88.8	2,398			92.3	2,491			95.1	2,568										
50	89.7	2,692			92.9	2,786			95.6	2,868										
55	90.5	2,987			93.4	3,081			96.0	3,168										
60	91.2	3,282			93.8	3,376			96.3	3,468										
65	91.7	3,577			94.3	3,675			96.6	3,768										
70	92.2	3,872			94.7	3,975			96.8	4,068										
75	92.7	4,170			95.0	4,275			97.0	4,368										

Note:—The Trunk Speeds of Service indicated at the heading of each Table represent the average busy hour delay in securing a trunk on each attempt, when the holding times per message are as shown.

To develop this relationship it was necessary first to determine a typical distribution of traffic throughout the day, i.e., the ratio of the busy hour to each of the other hours. Actual delays experienced on a particular group may deviate somewhat from those developed herein to the extent that the actual distribution varies from the typical distribution.

The probable total day circuit delays are derived from Figs. 5 and 6 and from a typical distribution of traffic based on a five-day record on each of 20 groups in Ohio and 24 groups in Illinois.

Hours	No. of Calls	% of Total Traffic in 14-hour day (Used as weighting factor)
1 (Busy Hour)	100	12.80
1	90	11.53
1	90	11.53
1	80	10.25
1	80	10.25
1	75	9.62
1	70	8.98
1	65	8.35
1	55	7.07
5	75	9.62
14	780	100.0
10	20	
24	800	

It will be noted that the above distribution shows a busy hour which is 12.5% of the total 24-hour day but that the weighting factors are based on a total day of 14 hours. This corresponds to the normal service observing period so that the results will be comparable with the overall total day speeds obtained from service observations.

The total day delays for Tables T-1 to T-5 for each size of group were computed as illustrated in the following sample calculation:

TABLE T-5—FIVE TRUNKS

Hours	% of BH	% Use in BH (Table T-5)	% Use Each Hour	Circuit Delay (Fig. 6)	Weight Factor	Weighted Delay
1	at 100 ×	87.6 =	87.6	.60	× 12.80% =	.0768
1	90		78.8	.26	11.53	.0300
1	90		78.8	.26	11.53	.0300
1	80		70.2	.13	10.25	.0133
1	80		70.2	.13	10.25	.0133
1	75		65.7	.10	9.62	.0096
1	70		61.3	.07	8.98	.0063
1	65		56.9	.06	8.35	.0050
1	55		48.2	.03	7.07	.0021
5				None	9.62	.0000
14					100.00	.1864

The figure .1864 derived above represents the average delay expressed as a per cent. of the holding time per circuit use (attempt). The last step, therefore, is to relate this figure to the message holding times contemplated in Table T-5, as follows:

Holding Time Per Message		Ratio of Attempts Per Message		Holding Time Per Attempt		% of H. T. Delayed		Average Delay
5 min.	÷	1.50	=	3.33	×	.1864	=	.62 min.
8 min.	÷	1.50	=	5.33	×	.1864	=	1.00 min.
11 min.	÷	1.50	=	7.33	×	.1864	=	1.37 min.

The results of similar calculations are summarized in Fig. 11.

As pointed out previously, actual delays experienced will deviate from those shown in Fig. 11 to the extent that the actual hourly distribution varies from that which has been used. If a particular group has a higher per cent busy hour the total day delays should be less than indicated. Conversely, if the group has a lower per cent busy hour the delays should be greater. However, the variations in distribution which are most likely to be encountered in practice will not have any marked effect on the total day delays except possibly for groups of about five trunks or less which are loaded as heavily as indicated in Tables T-4 and T-5.

Per Cent. NC Encountered

The per cent. of calls delayed by NC as noted by the operators on the tickets analyzed for this study was plotted for each level of usage in steps of 10% as shown in Fig. 12, using a 3 point moving average. Each of these curves was then redrawn in relation to the others and the combined results are in Fig. 13. The results are very similar to those obtained in the Cleveland study of 1929–30, as shown in the same figure.

It should be noted that there is a difference between the per cent. calls encountering NC and the per cent. NC existing. In the present study no data were obtained to indicate NC existing. However, the Cleveland study included such data which showed that the NC existing follows the Erlang "B" formula (Fig. 14) in this respect. The individual points in Fig. 14 were derived by selecting from the Erlang "B" table of overflows the point at which the call-seconds carried (offered minus overflow) gave the desired level of usage.

The difference between NC existing and NC encountered may be due to several factors, some of which are suggested below:

1. Effect of alternate routes.

No. of Trunks	Table T-1			Table T-2			Table T-3			Table T-4			Table T-5		
	Holding Time—In Minutes														
	5	8	11	5	8	11	5	8	11	5	8	11	5	8	11
	Trunk Speed—Busy Hour—In Minutes														
	.13	.20	.28	.25	.40	.55	.5	.8	1.1	1.0	1.6	2.2	2.0	3.2	4.4
	Trunk Speed—Total Day—In Minutes														
1	.09	.14	.20	.19	.31	.43	.38	.60	.83	.67	1.07	1.47	1.20	1.92	2.64
2	.08	.13	.18	.16	.25	.34	.29	.47	.65	.52	.83	1.14	.92	1.48	2.03
3	.07	.11	.16	.14	.22	.30	.25	.39	.54	.43	.69	.95	.77	1.24	1.70
4	.06	.10	.14	.12	.19	.26	.22	.35	.48	.39	.62	.85	.68	1.09	1.50
5	.06	.09	.13	.11	.18	.24	.20	.32	.44	.35	.56	.77	.62	1.00	1.37
6	.05	.09	.12	.10	.17	.23	.18	.29	.40	.33	.52	.72	.57	.91	1.25
7	.05	.08	.11	.10	.15	.21	.17	.28	.38	.31	.49	.67	.53	.85	1.17
8	.05	.08	.11	.09	.14	.20	.16	.26	.36	.29	.46	.63	.50	.80	1.10
9	.05	.07	.10	.09	.14	.19	.16	.25	.34	.27	.44	.60	.48	.76	1.05
10	.04	.07	.10	.08	.13	.18	.15	.24	.33	.26	.42	.58	.46	.73	1.00
11	.04	.07	.09	.08	.13	.18	.14	.23	.32	.25	.41	.56	.44	.70	.96
12	.04	.07	.09	.08	.12	.17	.14	.22	.31	.24	.39	.54	.43	.68	.94
13	.04	.06	.09	.08	.12	.17	.13	.21	.29	.23	.37	.52	.41	.66	.91
14	.04	.06	.09	.07	.12	.16	.13	.21	.29	.23	.36	.50	.40	.64	.88
15	.04	.06	.08	.07	.11	.15	.13	.20	.28	.22	.35	.48	.39	.63	.86
16	.04	.06	.08	.07	.11	.15	.12	.20	.27	.22	.35	.48	.38	.61	.84
17	.04	.06	.08	.07	.11	.15	.12	.19	.26	.21	.34	.47	.38	.60	.83
18	.04	.06	.08	.07	.11	.15	.12	.19	.26	.21	.33	.45	.37	.59	.81
19	.03	.05	.07	.06	.10	.14	.11	.18	.25	.20	.33	.45	.37	.59	.81
20	.03	.05	.07	.06	.10	.14	.11	.18	.24	.20	.32	.44	.36	.58	.79
21	.03	.05	.07	.06	.10	.14	.11	.18	.24	.20	.32	.43	.35	.57	.78
22	.03	.05	.07	.06	.10	.13	.11	.17	.24	.19	.31	.43	.35	.56	.77
23	.03	.05	.07	.06	.09	.13	.11	.17	.23	.19	.31	.42	.35	.55	.76
24	.03	.05	.07	.06	.09	.13	.10	.17	.23	.19	.31	.42	.34	.55	.76
25	.03	.05	.07	.06	.09	.13	.10	.17	.23	.19	.30	.41	.34	.54	.75
30	.03	.05	.06	.05	.09	.12	.10	.15	.21	.18	.28	.39	.33	.52	.72
35	.03	.04	.06	.05	.08	.11	.09	.15	.21	.17	.27	.37	.32	.51	.70
40	.03	.04	.06	.05	.08	.10	.09	.14	.20	.16	.26	.36	.31	.50	.68
45	.03	.04	.06	.05	.07	.10	.09	.14	.19	.16	.26	.35	.30	.49	.67
50	.02	.04	.05	.05	.07	.10	.09	.14	.19	.16	.25	.34	.30	.48	.66
55	.02	.04	.05	.04	.07	.10	.08	.14	.19						
60	.02	.04	.05	.04	.07	.10	.08	.13	.18						
65	.02	.04	.05	.04	.07	.09	.08	.13	.18						
70	.02	.04	.05	.04	.07	.09	.08	.13	.18						
75	.02	.04	.05	.04	.06	.09	.08	.13	.18						

Fig. 11—Relation of trunks speeds in busy hour to total day

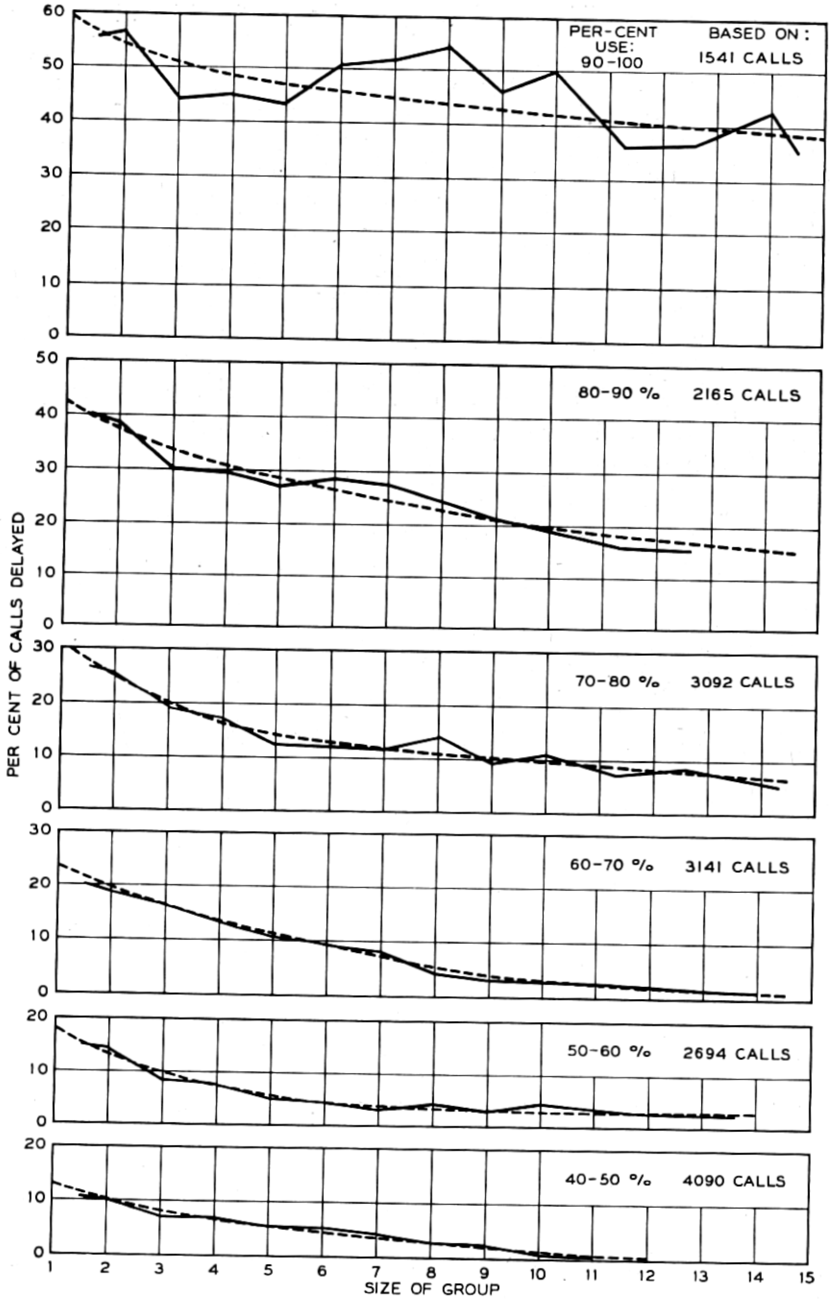

Fig. 12—Per cent. of calls delayed by NC as noted by the operators (with alternate routes where authorized).

2. The operators did not make NC notations on the tickets in a certain proportion of the cases where NC was actually encountered.
3. Because the operator does not test for an idle trunk with machine-like finality there are probably many cases where she does not consider that an NC condition existed if it was of such short duration that it did not materially affect her ability to secure a trunk.
4. The possibility of some "limited sources" effect in the case of small groups. The number of people in Newark who have occasion to call York, Pa. must be relatively small since only one trunk is provided. Therefore, while the trunk is in use on one call there is less likelihood that a second call will be originated than would be the case if there were a greater community of interest between the two places. Although the NC condition exists during the period of one call, no NC is encountered unless there is a second and overlapping call.

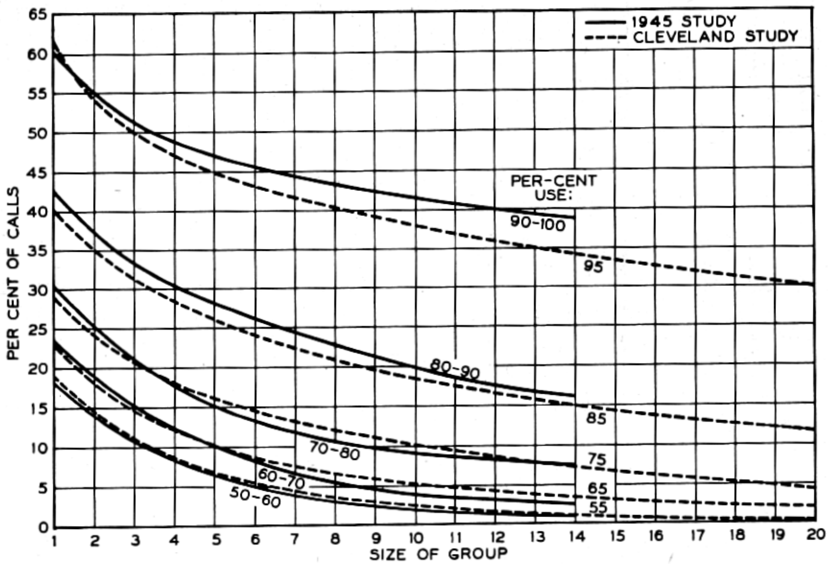

Fig. 13—Per cent. of calls delayed by NC (with alternate routes where authorized).

Because of the importance of Items 2 and 3 above, both of which involve the testing of trunks by operators, the empirical data should be used as representative of per cent. NC encountered with ringdown operation (operator testing) and the Erlang "B" per cent. NC existing should be used as representative of intertoll dialing conditions (mechanical testing).

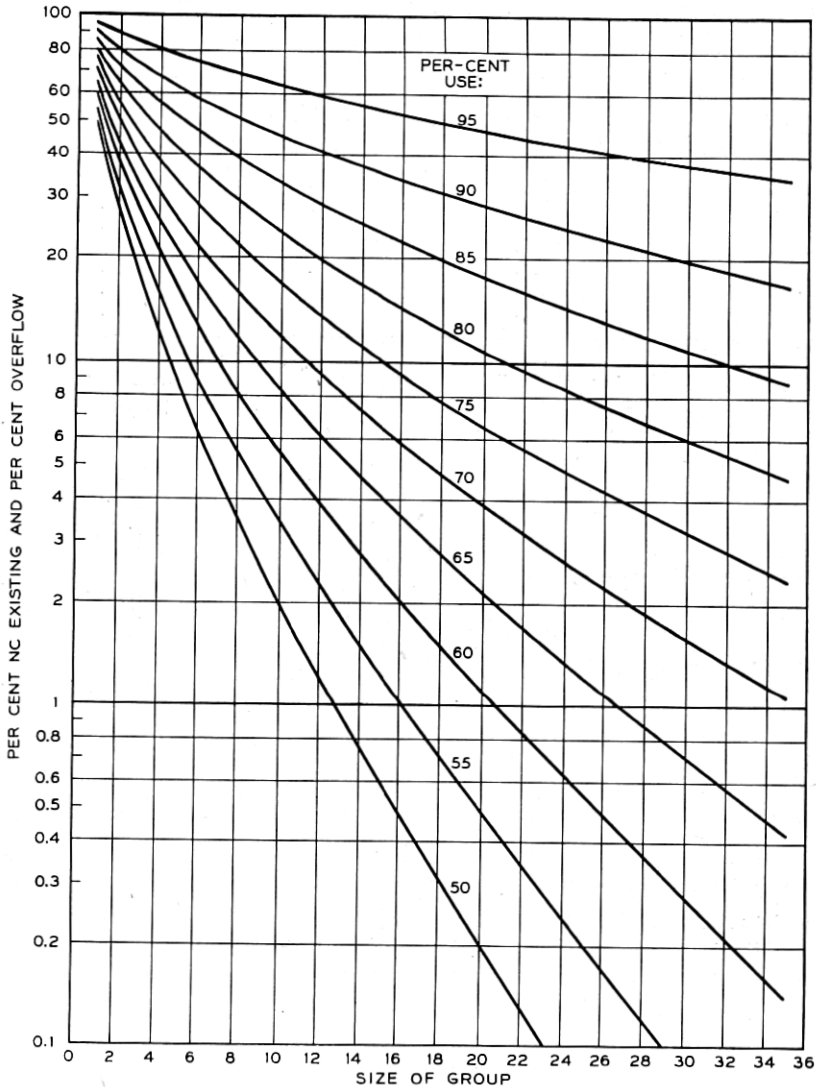

Fig. 14—Per cent. NC existing and per cent. overflow at various levels of usage. Based on Erlang "B" formula.

It is interesting to note that each of the proposed Capacity Tables T-1 to T-5 results in a fairly uniform percentage of NC encountered by the operators as determined from the empirical data (Fig. 13) and also a fairly uniform percentage of NC existing as determined from the Erlang "B" formula (Fig. 14). Using Table T-1 as an example, the NC conditions are as follows:

% Use	No. of Trunks Table T-1 from Fig. 9	%NC Encountered from Fig. 13	% NC Existing from Fig. 14
50	4.6	6.5	10.0
55	5.5	6.0	10.0
60	6.7	6.0	10.7
65	8.4	6.5	10.9
70	10.7	7.0	11.0
75	14.1	7.5	11.0
80	20.0	7.0	10.7
85	30.0		10.8

Similar comparisons made with the other capacity tables indicate similar uniformity in the most frequently used portion of the tables, i.e., up to 20 or 30 trunks. The results are as follows:

Capacity Table	% NC Encountered by Operators (from empirical data) With Alternate Routes for the Small Groups	% NC Existing (from Erlang "B") Without Alternate Routes
T-1	6–7	10–11
T-2	9–11	18–19
T-3	14–18	27–29
T-4	21–26	39–42
T-5	33–35	55–57

Conclusion

Since the primary function of an intertoll trunk capacity table is to translate a desired speed of toll service into the number of trunks required for that level of service, the table used should be indicative, within reasonable limits, of the probable effect of trunk provision on the overall speed. For this reason, tables which reflect a uniform service situation will be more useful in intertoll trunk engineering and administration than the present tables which have inherent service variations. Capacity tables such as Tables T-1 to T-5 will therefore be substituted for present Schedules A, A2 and B.

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