Targets for Microwave Radar Navigation

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The effective echoing areas of certain radar targets can be calculated by the methods of geometrical optics. Other more complicated structures have been investigated experimentally. This paper considers a number of targets of practical interest with particular emphasis on trihedral and biconical corner reflectors. The possibility is indicated of using especially designed targets of high efficiency as aids to radar navigation.

Introduction

IT NOW seems likely that radar, developed during the war, will find increasing application as a navigational aid for aircraft and surface vessels. In fact there are good reasons for expecting that peace-time radar can be made even more efficient than its war-time prototype.

There are two ways of improving radar performance. One may concentrate on the radar set proper with the object of increasing either the power of the transmitter or the sensitivity of the receiver. Or, one may take steps to improve the echoing efficiency of the targets. The latter was, of course, not possible during the war since most of the targets of interest were controlled by the enemy. It is a purpose of the present paper to consider the design of various targets of high echoing efficiency and wide angular response which may be placed at strategic points as aids to radar navigation. The ideal reflector to serve as a "beacon" or "buoy" for guiding radar-equipped aircraft or ships would present a highly effective area to incident radiation over a full 360° in azimuth, and would also be operative over a fairly broad vertical angle. The value of a particular target for navigational purposes may therefore be considered in terms of two factors: effective area, and angular response.

The echo received by a radar from a particular target can be calculated by the formula:[1]

$$W_R = W_T \frac{A_R^2 A_{eff}^2}{\lambda^4 d^4} \tag{1}$$

where W_R = echo power available at the terminals of the radar antenna.
W_T = power launched by radar.
A_R = effective area of radar antenna assuming that the same antenna is used both for transmission and reception.

[1] This equation follows directly from Equation (1) of a paper by H. T. Friis, "A Note on a Simple Transmission Formula," *Proc. I.R.E.*, Vol. 34, pp. 254–256, May 1946. The radar transmission formula is obtained by applying Friis' formula twice; first to the transmission from the radar to the target, then to the transmission from the target to the radar.

A_{eff} = effective area of target.[2]
λ = wavelength.
d = distance between radar and target.

The above formula applies to the case where free-space propagation prevails; that is, where multiple path or anomalous transmission effects are absent.

It is apparent from the formula that, at a given wavelength and range, the received echo power can be increased by increasing the transmitted power, the size of the antenna, or the effective area of the target. The present paper will consider only the latter.

In some cases the effective area of a target can be calculated from simple geometrical optics. For the more complicated structures it is always possible to measure the effective area by comparing the signal reflected by the object in question to the signal reflected from a simple target of known effective area.

Flat Plates

The simplest target for which the effective area can be calculated is a flat metal plate oriented so as to be perpendicular to the incident radiation. It can be demonstrated that a flat plate with all linear dimensions large in proportion to the wavelength of the incident radiation has an effective area which is substantially equal to its geometrical area. Diffraction effects at the edges of such a plate are small in comparison with the energy reflected from the central portion of the plate.

Flat plates, however, have the serious disadvantage that, in order to create strong echoes, they must be maintained accurately perpendicular to the incident rays. At other angles of incidence the echoes fall off rapidly. For this reason flat plates are of limited value as targets for use in navigation.

Dihedral Corner Reflectors

A dihedral corner reflector consists of two perpendicular, plane conducting surfaces which are usually arranged so that they intersect along a common line. Figure 1 shows a typical dihedral reflector. The dihedral reflector has the important property that a ray which enters the corner will experience a reflection from each of the surfaces and will return in the direction from which it came, provided of course that the entering ray lies in a plane which is perpendicular to the line of intersection of the planes which form the

[2] The term "effective area" as used in this paper refers to the equivalent flat plate area of a target. The echoing effectiveness of a target may alternatively be expressed in terms of the cross section of an equivalent isotropic reflector as described by Schneider, "Radar," *Proc. I.R.E.*, Vol. 34, p. 529, August 1946. The alternative unit is called the "scattering cross section" and is frequently denoted by the symbol σ, although Schneider uses S. The two quantities are related by the equation $\sigma = 4\pi A^2_{eff}/\lambda^2$. Both units are useful. For most of the targets considered in the present paper, A_{eff} does not vary with λ and is therefore preferable.

corner. The latter restriction constitutes the principal objection to the practical use of dihedrals. The path of a typical ray is shown in Fig. 1.

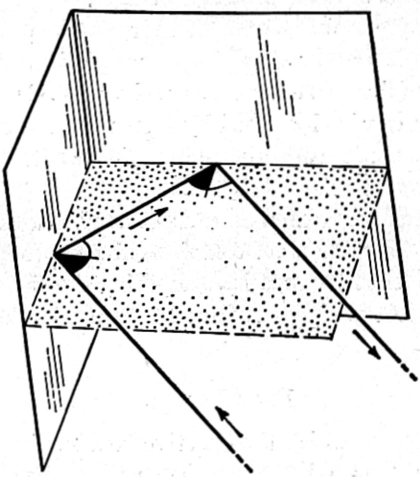

Fig. 1—Dihedral corner reflector.

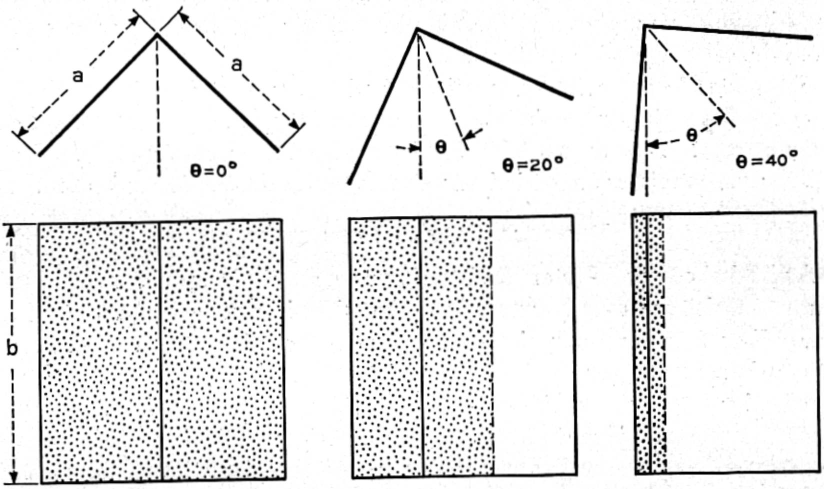

Fig. 2—Variation of effective area of a dihedral with aspect angle.

The effective area of a dihedral reflector depends upon both the size of the reflector and the orientation of the reflector with respect to the incident rays. Figure 2 shows how the effective area varies as the dihedral is rotated about the line of intersection of the two planes. The effective areas for the different orientations are shown by the shaded regions in the lower part of the figure. For a reflector having the dimensions shown in the figure the effective area for different angles of incidence θ can be calculated by the formula.

$$A_{eff} = 2\,a\,b \sin\,(45° - \theta)$$

where θ is always considered positive and less than 45°.

Figure 3 shows the polarization of the reflected ray for differently polarized incident rays. For our purpose, the incident rays may be assumed to enter the left side of the reflector shown in the figure and the reflected rays may be assumed to emerge from the right. It is apparent that if the incident ray is polarized either parallel or perpendicular to the line of intersection of the two surfaces the reflected ray will be polarized in the same plane as the inci-

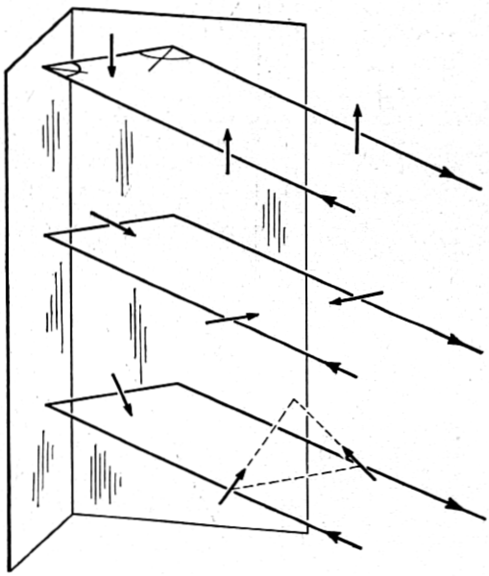

Fig. 3—Polarization effect in a dihedral reflector.

dent ray. If the incident ray is polarized at an angle of 45° to the line of intersection, the reflected ray will be polarized perpendicularly to the incident ray. In the latter case the signal received back at the radar will not ordinarily be accepted by the same antenna which launched the incident radiation.

Trihedral Corner Reflector

Assume that three reflecting surfaces AOB, AOC, and BOC are placed so as to form the right-angled corner illustrated in Fig. 4. In general, electromagnetic waves, upon striking an interior surface of the device, will undergo a reflection from each of the three planes and return in a direction parallel to and with the same polarization as the incident ray. The path of a typical ray is shown by line 1, 2, 3, the particular ray chosen having entered the reflector along a line perpendicular to the plane of the paper. Points 1 and 3 represent the initial and final points of reflection, respectively, whereas point 2 represents the intermediate reflection point.

Two important conclusions can be drawn from a careful inspection of the path 1, 2, 3; namely, the projections of points 1 and 3 are diametrically opposite on a circle drawn about point O as a center, and points 1 and 3 appear to be images of point 2; i.e., the ingoing ray at 1 appears to be directed toward

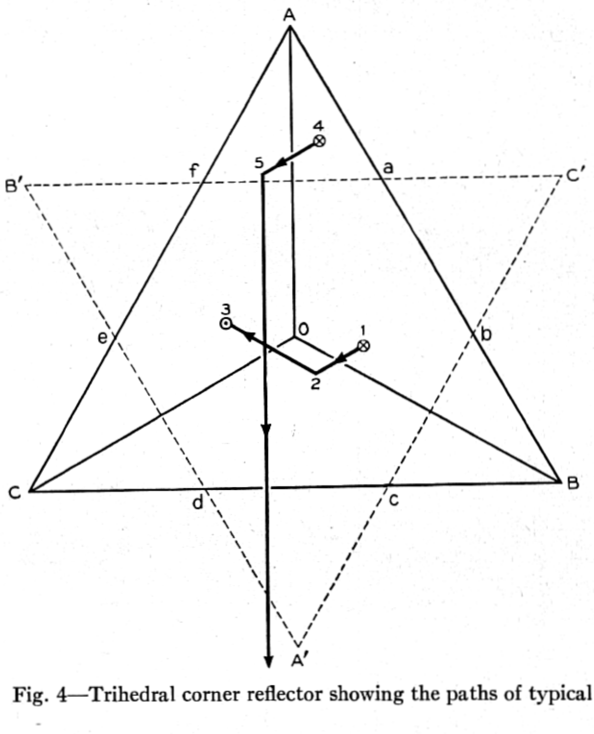

Fig. 4—Trihedral corner reflector showing the paths of typical rays.

the image of point 2 in plane AOB, and the outcoming ray at 3 appears to come from the image of point 2 in plane AOC.

Not all rays falling upon a corner reflector of finite dimensions will be reflected in the direction of the source. For example, a ray striking point 4 in Fig. 4 may be reflected successively at points 4 and 5, but if the plane BOC is not sufficiently extended it will not undergo the necessary third reflection required to return the ray in the incident direction.

The portion of the projected cross-section of a corner reflector which is able to return incident radiation to the source is called the "effective area." It is, of course, a function of the aspect, that is to say, the angle at which the reflector is being viewed, as well as the geometrical configuration of the reflector. For some of the simpler configurations the effective area can be readily determined by the following procedure.

Project the aperture of the reflector through the apex O to form the image (A′ B′ C′ of Fig. 4); then project the aperture and its image upon a plane perpendicular to the incident rays. The area common to the projections of the aperture and its image is equal to the effective area. The effective area of the triangular reflector of Fig. 4 is, therefore, represented by the hexagon a b c d e f. Only those rays, perpendicular to the plane of the paper, which

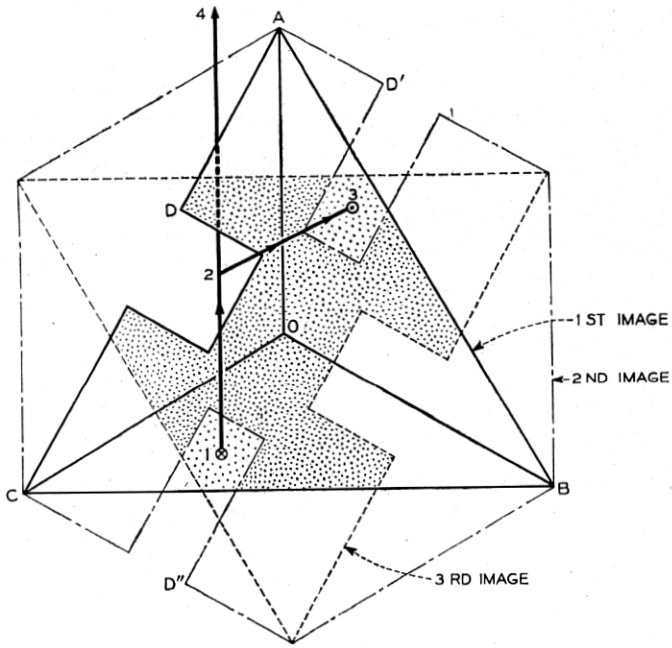

Fig. 5—Determination of effective area of trihedral corner reflector.

fall inside the hexagon will be returned. Exactly the same procedure is used in determining the effective area for other aspect angles.

The above rule must, however, be applied with caution. Situations arise in which rays falling upon the area determined by this method do not return to the source. Figure 5 shows a reflector in which this difficulty is encountered. This reflector differs from the previous reflector in that it has a notch cut in one of the reflecting surfaces. The projection of the aperture upon the plane of the paper is indicated by the solid line; that of its image by the dotted line. According to the rule of the preceding paragraph, one would expect the effective area to be defined by the total shaded area of the figure.

Such, however, is not the case. It was stated earlier that the ingoing rays appear to be directed toward the images of the intermediate reflecting points. This requires that the images of the intermediate reflecting points fall inside of the effective area. In Fig. 5, the images of the notch fall inside of what would otherwise be the effective area. Since the notch is incapable of serving as an intermediate reflector, the more lightly shaded areas are excluded from the effective area. In the absence of the notch, a ray entering at 1 would be reflected at 2 and emerge at 3. In the presence of the notch, however, it passes through plane AOC and escapes in the direction of 4.

Therefore, in order to determine the effective area of a corner reflector of arbitrary shape and aspect, one must take account of three loci of points defined by the aperture as follows:

1) The aperture itself

2) The locus of points determined by taking the direct mirror image of each point of the aperture with respect to each of the two surfaces of the trihedral not containing the point. For example, point D of Fig. 5 will have the images D′ and D″ with reference to planes AOB and BOC, respectively. The complete locus of points determined in this way is represented by the dot-dash line of Fig. 5.

3) Locus of points on aperture after each has been assumed to have been projected through the vertex. This image is pictured by the dotted lines of Fig. 5.

These three images of the aperture can, for simplicity, be referred to as the first, second, and third images, respectively. The effective area is the area common to the projections of the first, second, and third images of the aperture upon a plane passing through the apex of the reflector and perpendicular to the incident rays. For a given aperture and aspect, a corner reflector can theoretically be replaced by a flat plate located at the apex. The size and shape of the flat plate will vary with the aspect as well as with the configuration of the aperture. The above procedure has been of considerable aid in studying reflectors having apertures of arbitrary shape.

Although the graphical analysis just given is sufficient to enable one to compute the effective area of a reflector for any aspect angle, it is frequently more conveneint to determine the complete response pattern of a reflector experimentally. Most of the experimental results reported in this paper were obtained with a 1.25 centimeter radar arranged as shown in Fig. 6. Echo levels were measured on the screen of a type-A indicator using a calibrated intermediate frequency attenuator to restore the signal to an arbitrary reference level. It is believed that the levels measured in this way are accurate to within $\pm \frac{1}{2}$ decibel. The coordinate system used in recording and presenting the data is given in Fig. 7. The reflector was mounted on a turntable which could be rotated about horizontal and vertical axes.

Curves for the response patterns of a corner reflector of triangular aperture are shown in Fig. 8. These curves were obtained with a reflector constructed of silver-painted plywood whose aperture was in the form of a 24-inch equilateral triangle. It had been previously determined that, with suitable paints, reflectors of this construction behaved exactly as though they were made of sheet metal.

Depending upon its angle of arrival, a ray may be reflected by a corner reflector in one of four ways. If the angle is too oblique, the ray may not be returned in the direction of the source at all. If the incoming ray is

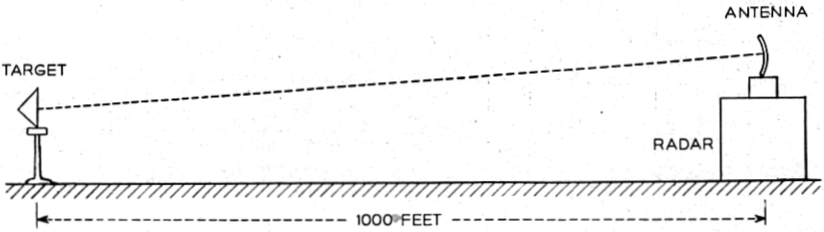

Fig. 6—Arrangement of apparatus for measuring effective areas of targets.

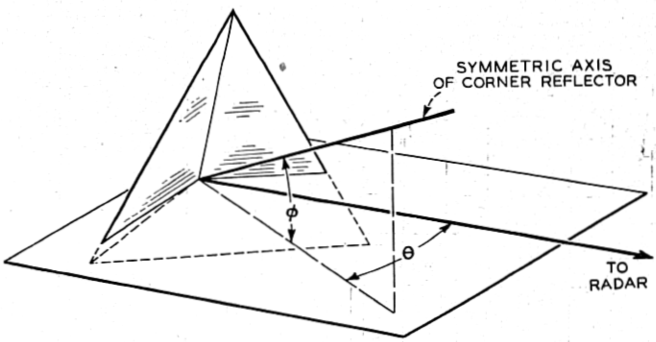

Fig. 7—Coordinate system used in presenting data.

exactly perpendicular to one of the three reflecting planes, it will be returned to its source after only one reflection. Should the ray arrive in a direction exactly parallel to one of the three planes, it will again be returned in the direction of its source but in this case it is reflected twice as in a dihedral. This particular mode of reflection is illustrated by the sharp peaks at the extremities of the curves in Fig. 8. For all remaining angles of approach the ray will be returned after three reflections in the manner already described. The central regions of the curves represent this type of reflection which is of principal interest in practice.

The effective area of the triangular trihedral reflector along the symmetric axis ($\theta = O, \phi = O$) can be computed from the geometry of Fig. 4.

$$A_{eff} = 0.289\, \ell^2 \tag{3}$$

where ℓ is the length of one side of the aperture such as CB. The effective area at other aspect angles can be computed by relating the echo level at the aspect in question with that along the symmetric axis.

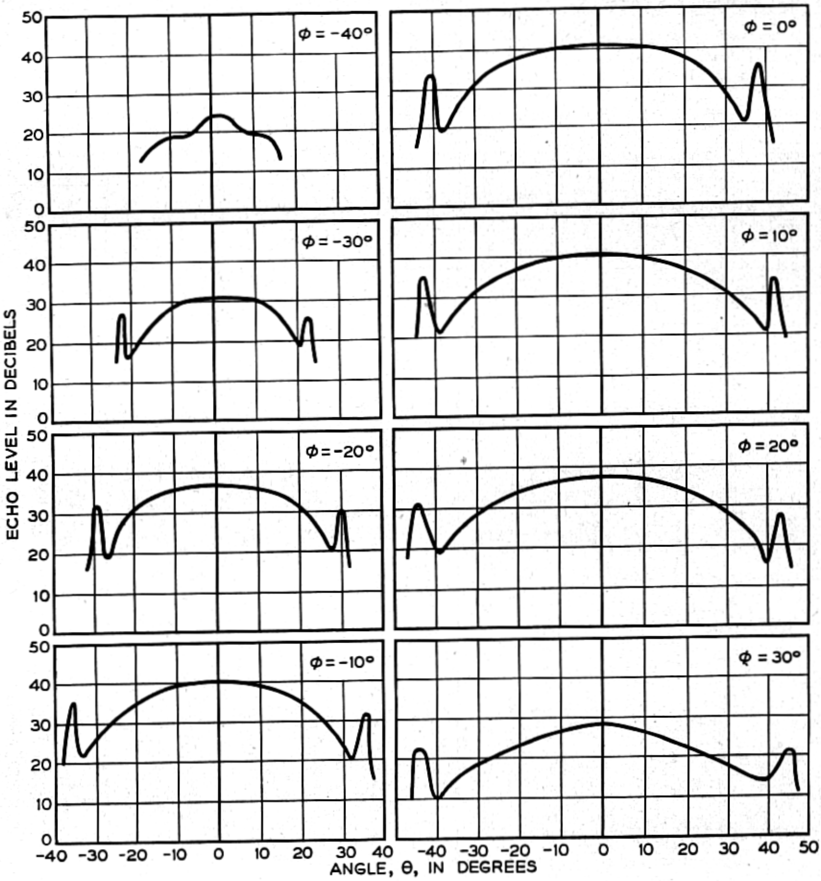

Fig. 8—Echo-response patterns of a triangular trihedral reflector.

It should be pointed out that the effective area for trihedral reflections is independent of wavelength where the reflector is large enough so that geometrical optics prevail. If the wavelength is increased, however, the sharp dihedral peaks at the edges of the pattern will be broader.

In the case of the triangular corner reflector the response levels for aspect angles of 30°, as measured from the symmetric axis, are down by 10 decibels. For many applications a flatter response pattern is desirable.

The present investigation led to the discovery that the response pattern of a corner reflector can be modified by a suitable alteration of the geometrical configuration of the aperture. There is even the suggestion that the response can, to a certain degree, be made to conform to a somewhat arbitrary pattern within a region extending to approximately 30° from the principal axis. The procedure for accomplishing this has, so far, been one of trial and error since the difficulties of a general mathematical solution appear to be

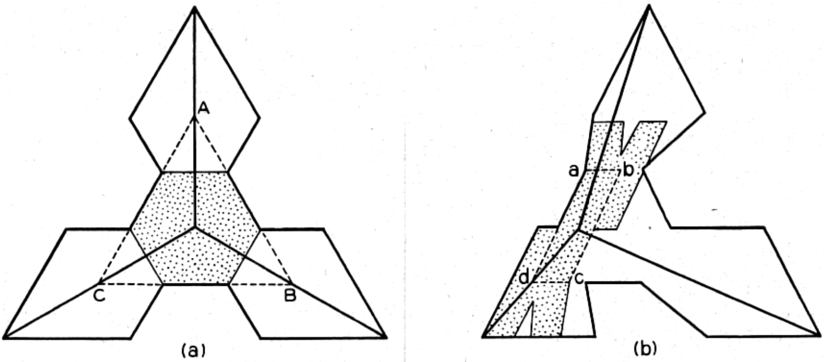

Fig. 9—Compensated corner reflector.

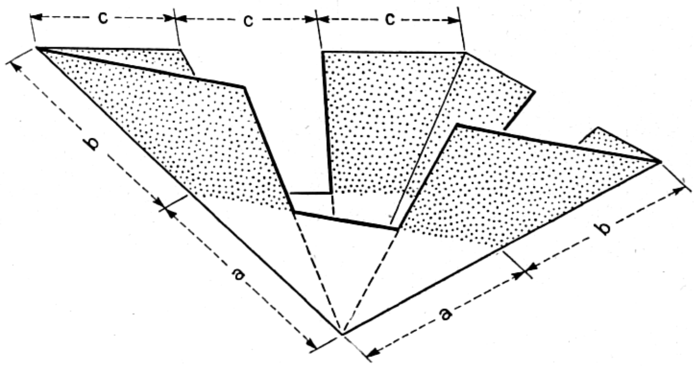

Fig. 10—Dimensions of face of compensated reflector.

insurmountable, at least in a practical sense. For practical purposes, however, it is comparatively easy to conduct a few graphical experiments in order to design a reflector having the desired response pattern.

Figures 9a and 9b show two views of a modified corner reflector which was designed to have a relatively flat response characteristic out to angles of 30° from the central axis. Each of the three sides of the reflector, instead of being triangular as formerly, has the contour shown in Fig. 10. The shaded regions of Fig. 10 represent the surface which has been added.

In Fig. 9a, one is assumed to be looking into the reflector along the symmetric axis. The effective area is represented by the shaded hexagon. Evidently, the effective area of the modified reflector is identical to the effective area of the original triangular reflector A B C. Therefore, for this particular aspect, the effective area has not been changed by the addition of the material at the corners.

Figure 9b is a view of the reflector at $\theta = 30°, \phi = 0°$. Again, the shaded region represents the effective area, and the parallelogram abcd is the effective area of the reflector before modification. The modification has evidently introduced a substantial gain in effective area for this aspect. A graphical comparison of the effective areas of Figs. 9a and 9b shows them to be of comparable magnitude.

With the dimensions defined as in Fig. 10, a corner reflector was constructed with a= b= 17″. The response curves of this reflector are plotted in Fig. 11, along with the curves of the ordinary triangular reflector. A substantial improvement in response is exhibited by the compensated reflector. In the region extending out to 30° from the axis, the response level varies by no more than a couple of decibels. The response appears to rise slightly in the vicinity of 20°. This could, perhaps, be reduced by a more appropriate shaping of the sides of the reflector.

The variation of the response curve with the ratio $\frac{b}{a}$ has been studied briefly. It appears that a value of $\frac{b}{a} = 1$ is about right, for the 30° contour to equal the axial response. If $\frac{b}{a} < 1$, the reflector will only be partially corrected; if $\frac{b}{a} > 1$, it will be overly corrected. In the uncorrected reflector with triangular aperture, $\frac{b}{a} = 0$.

If $b/a = \infty$, that is, if $a = 0$, one would expect to obtain a response curve having a minimum value on the axis and rising to a maximum on either side. A reflector having these properties is illustrated in Figs. 12a and 12b. Again Fig. 12a is the axial aspect, whereas Fig. 12b is the 30° aspect. In the former, the effective area should be zero; in the latter, it has the value represented by the shaded portion. A reflector of this kind, in which b = 34″ and a = 0, was constructed and tested. The experimental results are shown in Fig. 13. The minimum is, perhaps, not as low in value as expected because of residual reflections from the support upon which the reflector was mounted. As expected, however, the curve passes from a minimum on the axis to maxima on either side.

The above examples serve to illustrate some of the results which can be realized with trihedral reflectors. We have seen that the response characteristic can be controlled by appropriate modifications of the geometrical configuration of the aperture.

Experiments were performed in order to determine the reduction in echo caused by errors in the internal angles at the corner.

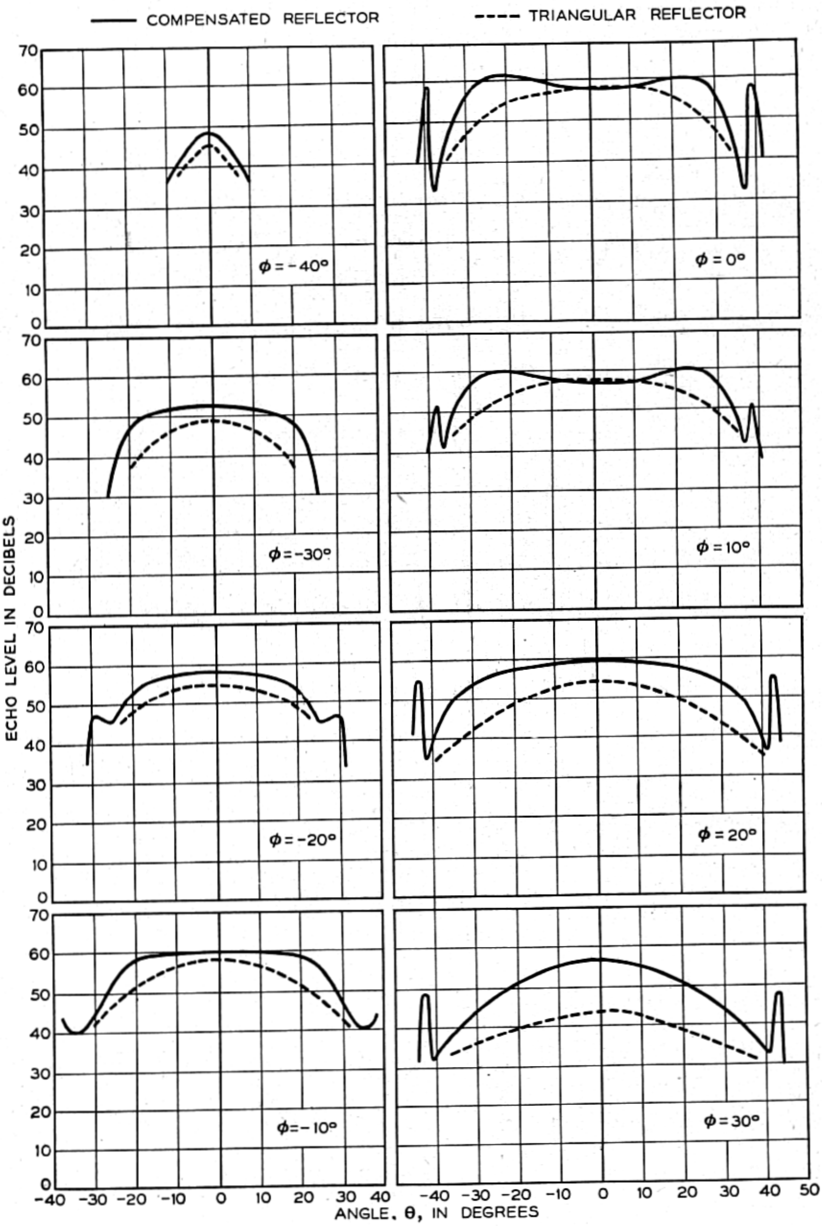

Fig. 11—Response of compensated reflector compared with that of triangular reflector.

In the first set of experiments only one of the internal angles was altered from its nominal value of 90°. Figure 14 shows the apparatus used in the

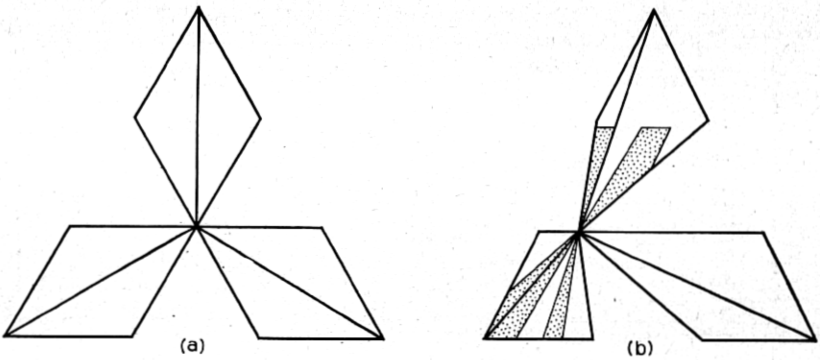

Fig. 12—Modified reflector having minimum response on axis.

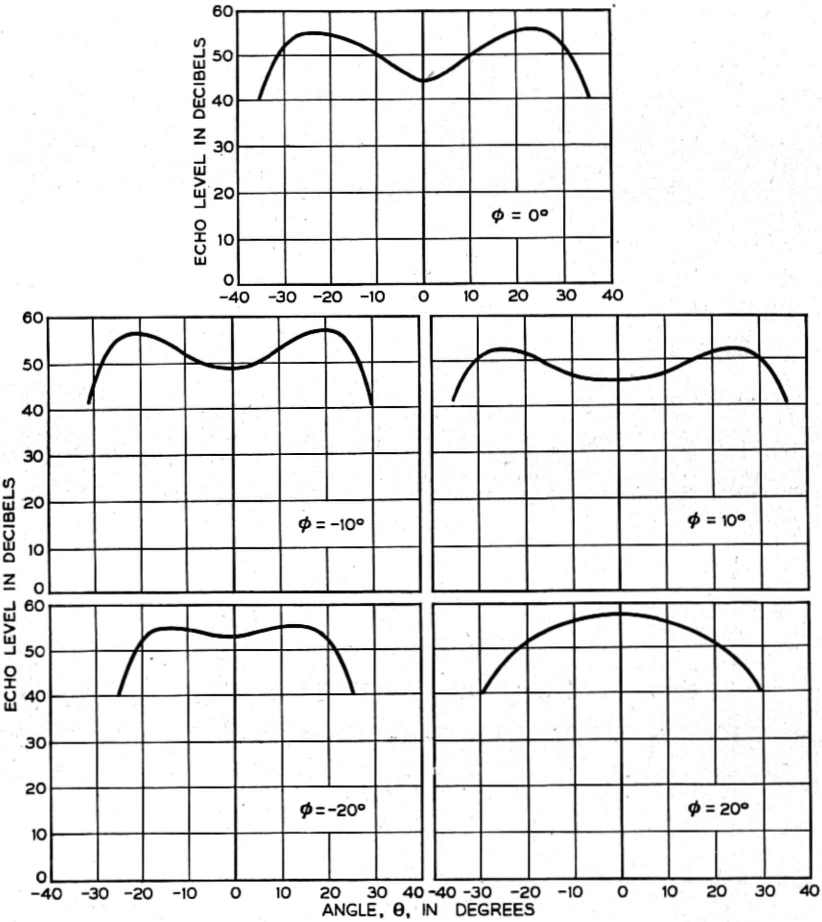

Fig. 13—Response patterns of reflector of Fig. 12.

experiment. The 24-inch reflector was constructed of silver-painted plywood and hinged along the intersection of the two upper surfaces so that the angle α could be varied at will. A series of response patterns were taken for various values of α. These are shown in the lower part of Fig. 14. It will be observed that one effect of changing α is to lower the echo level. This appears, however, to be accompanied by a somewhat flatter response curve. The radar used in this experiment had a wavelength of 1.25 centimeters.

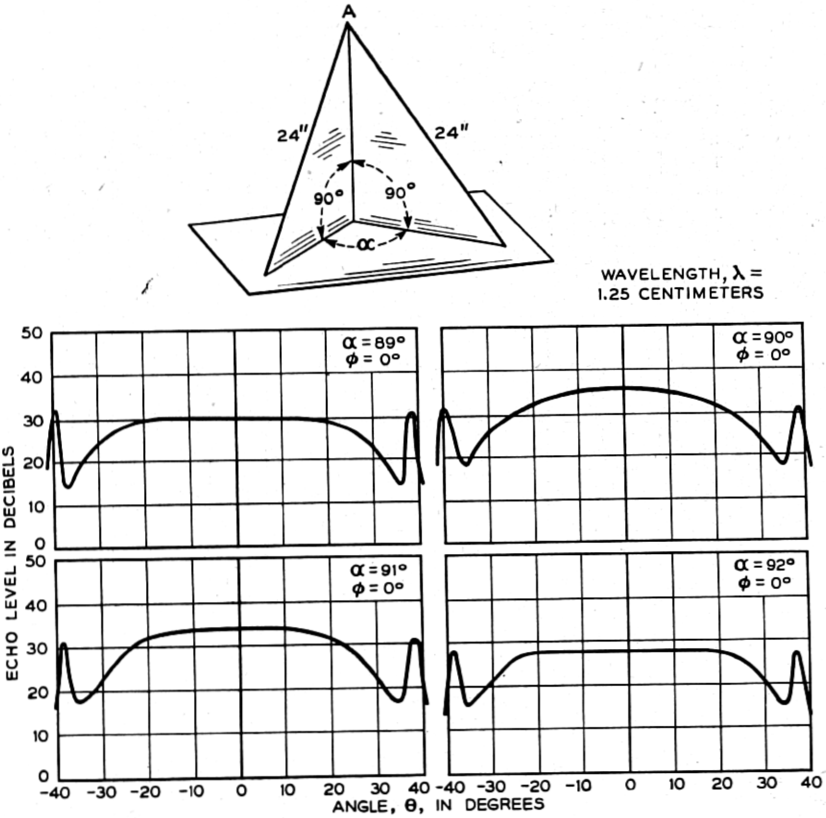

Fig. 14—Effect of an error in one of the corner angles of a trihedral upon its performance.

A second series of experiments was conducted in which all three internal angles were varied simultaneously. The curves shown in the upper half of Fig. 15 show the axial echo level for various sizes of reflector as a function of the internal angles α. It will be noted that for the 24-inch reflector an angular error of $\frac{1}{2}$ degree results in an echo-level reduction of two decibels, while the same angular error in the $9\frac{3}{8}$ inch reflector produces only a negligible reduction. The lower part of the figure shows some response patterns taken with a 24-inch triangular reflector for several values of α. A wavelength of 1.25 centimeters was used in obtaining both sets of curves. Later, similar measurements were made at a wavelength of 3.2 centimeters. It was found that the loss of signal is a function of the linear error of the aperture in wavelengths rather than the angular error in degrees. Thus a given angular error in a $9\frac{3}{8}$-inch reflector at a wavelength of 1.25 centimeters will produce the same loss in signal as the same angular error in a 24-inch reflector operating at a wavelength of 3.2 centimeters.

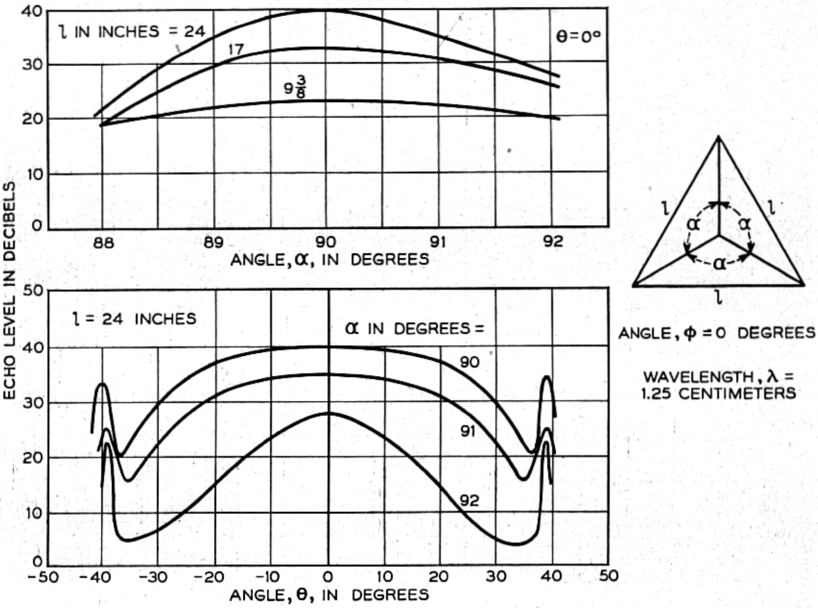

Fig. 15—Effect of an error in all three corner angles upon the performance of a trihedral.

Spheres and Cylinders

Formulas for the effective areas of spheres and cylinders which have dimensions large in comparison with a wavelength have been supplied by J. F. Carlson and S. A. Goudsmit of the Radiation Laboratory.[3] The effective area of a sphere of radius R is given by

$$A = \frac{R\lambda}{2} \tag{4}$$

where λ is the wavelength.

For a cylinder of radius R and height L, both large with respect to a wavelength, the effective area for rays perpendicular to the axis of the cylinder is

$$A = L\sqrt{\frac{R\lambda}{2}} \tag{5}$$

[3]Unpublished Report

It will be of interest to compare the sphere and the cylinder with corner reflectors and flat plates. The response pattern of a sphere is ideal in that it is uniform in all directions. Unfortunately its effective area is small in comparison with that of corner reflectors or flat plates having the same cross-sectional area. The cylinder has a symmetrical response pattern in the plane

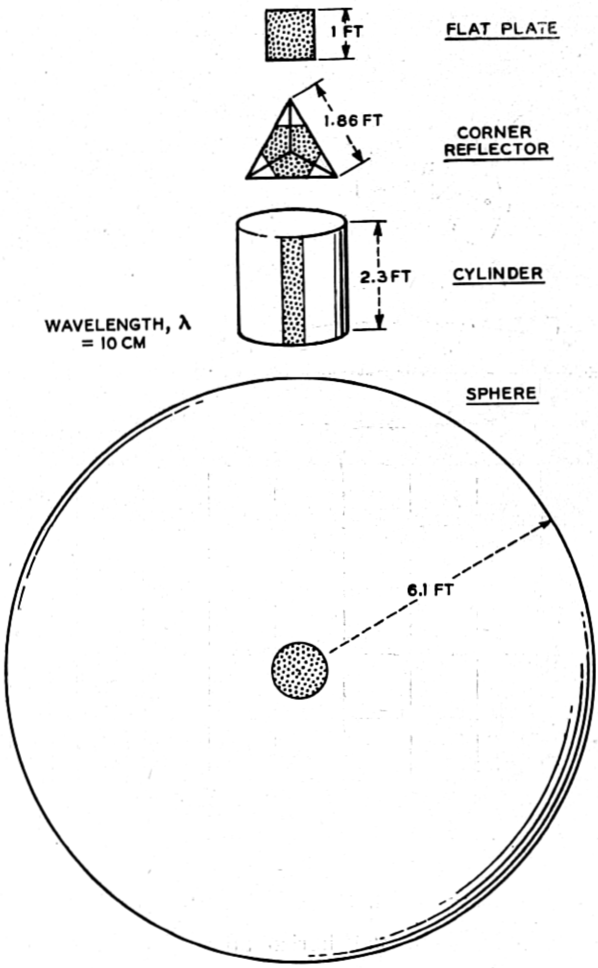

Fig. 16—A comparison of several representative targets having equal effective areas.

perpendicular to the axis but is very sharp in the plane of the axis. The effective area of a cylinder is intermediate between that of a corner reflector and a sphere. Figure 16 is a scale view of a flat plate, a corner reflector, a cylinder, and a sphere all having an effective area of one square foot at a wavelength of 10 centimeters. For shorter wavelengths the flat plate and the corner reflector would remain the same size, whereas the cylinder and the sphere would have to be larger in order to maintain the same effective areas. At a wavelength of 1 centimeter the sphere would have to have a radius of about 60 feet.

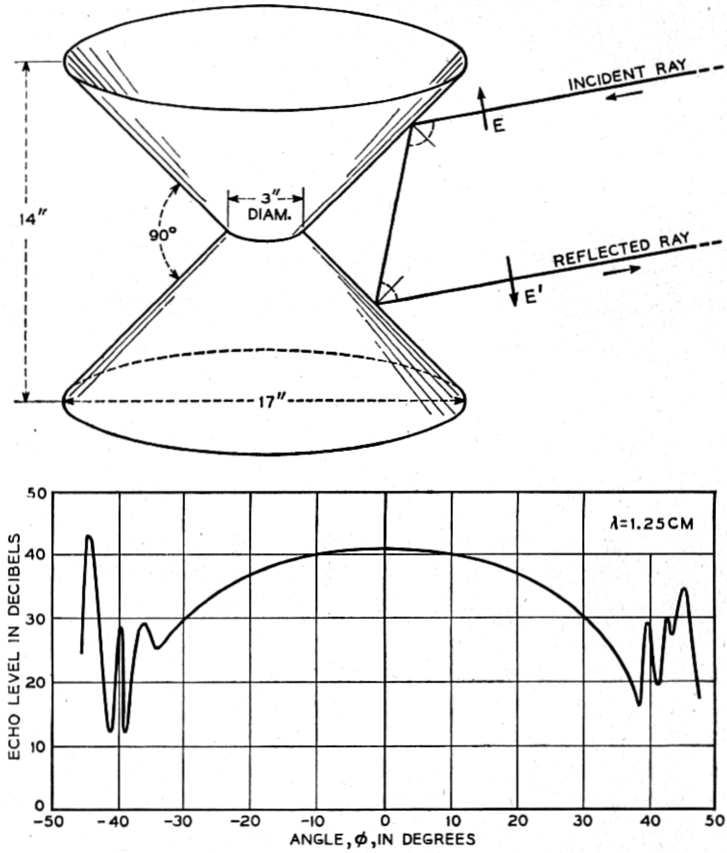

Fig. 17—Properties of the biconical corner reflector.

Biconical Corner Reflector

A reflector, combining the 360° horizontal response characteristic of the cylinder with a vertical response like that of the corner reflector, was evolved and is illustrated in Fig. 17. As shown here, the device consists of two conical surfaces placed in juxtaposition such that the generatrices of one cone intersect those of the other at right angles. The operation of the reflector is somewhat like that of a dihedral corner reflector in that a ray, upon striking one of the cones, is reflected to the other and then returns in the direction of the source. The "Biconical" reflector may perhaps be likened to a cylinder which automatically orients itself so that the impinging rays are always perpendicular to the axis.

A biconical reflector was constructed of sheet metal having the dimensions indicated in Fig. 17. The vertical response pattern was measured and is plotted in the lower portion of the figure. Because of the circular symmetry of the reflector, the vertical response curve shown will be equally valid for all angles of azimuth. At $\phi = 0°$ the reflector exhibited an effective area of 0.16 square feet. The measurements were made at a wavelength of 1.25 centimeters.

In the above experiment the incident radiation was polarized in the plane of the axis of the cones. In another test with the polarization perpendicular to the axis the received echo was reduced by four decibels. This effect is not as yet entirely explained. It probably results from a depolarizing effect similar to that encountered in the dihedral corner reflector, complicated however by the curvature of the cones.

Only a limited amount of data is available for predicting the effective area of a biconical reflector over a wide range of sizes and wavelengths. The available data indicate that to a rough approximation and for a given polarization the effective area varies directly as the square root of the wavelength and as the three-halves power of the diameter of the cones, assuming that the height of the reflector is approximately equal to the diameter.