A Note on a Parallel-Tuned Transformer Design

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An analysis of the parallel-tuned transformer used in radio-frequency amplifiers has been made for a slightly over-coupled case. The resulting design formulas are simple and practical.

Two cases are discussed: (a) the so-called matched transformer, with resistance loading on each side; (b) a transformer with loading on one side only, which has the same pass-band and phase characteristics as the matched transformer, but gives 3 db more gain when used as an interstage.

A special arrangement of (a) where the matched transformer design is used with one resistor removed, giving a transformer with a considerably double-humped pass-band characteristic and about 6 db more gain, is also discussed.

Introduction

THE parallel-tuned transformer has been used in radio-frequency amplifiers for many years.[1] In an excellent paper,[2] Christopher based design formulas on the principles of the broad band filter. In other publications, simple circuit analyses have been used and design formulas have been based on the assumption of a small ratio of bandwidth to mid-frequency which often fails to be adequate when the wide bands required for modern television and multiplexing services are encountered.

A transitionally flat transformer design may be obtained by setting the first three derivatives of the absolute value of the transfer impedance, with respect to frequency, equal to zero. The resulting design formulas are somewhat unwieldy.

The transformer design to be described here is based upon two simple circuit conditions[3] applied to the fundamental case of a parallel-tuned transformer with resistance loading on each side:

I. Both sides of the transformer are tuned to the same frequency.

II. The transmission loss is zero at the tune frequency. (This condition is responsible for the term "matched transformer" used to describe this case).

The resulting transformer has a slightly double-humped characteristic with less than 0.005 db dip for a coupling coefficient of 0.5. Because of the slight overcoupling this transformer design gives a little more gain and bandwidth than the critically coupled case. Its main advantage lies in the fact that simpler design formulas can be used.

[1] H. T. Friis and A. G. Jensen, High Frequency Amplifiers, *B. S. T. J.*, Vol. III, April 1924, pp. 181–205.

[2] A. J. Christopher, Transformer Coupling Circuits for High-Frequency Amplifiers, *B. S. T. J.*, Vol. XI, Oct. 1932, pp. 608–621.

[3] This analysis is based on an old unpublished report by H. T. Friis.

Derivation of Design Formulas

(a) *Matched, or Symmetrically Loaded Transformer.*

In the circuit of Fig. 1:

$$\left.\begin{aligned}
E_1 &= I_1(R_1 + 1/j\omega C_1) + I_2(-\,1/i\omega C_1)\\
0 &= I_1(-\,1/i\omega C_1) + I_2(j\omega L_1 + 1/j\omega C_1) + I_3(-j\omega M)\\
0 &= I_2(-\,j\omega M) + I_3(j\omega L_2 + 1/j\omega C_2) + I_4(-\,1/j\omega C_2)\\
0 &= I_3(-\,1/j\omega C_2) + I_4(R_2 + 1/j\omega C_2)
\end{aligned}\right\} \quad (1)$$

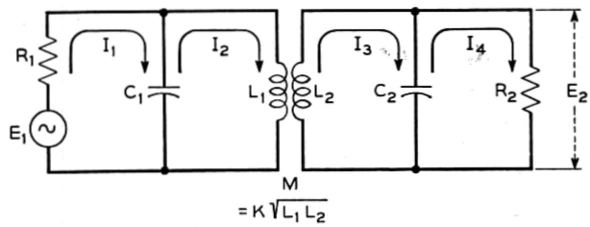

Fig. 1.—Parallel tuned, matched, or symmetrically loaded transformer.

Eliminating I_1, I_2 and I_3 among these equations gives:

$$E_1/I_4 = \frac{L_1R_2 + L_2R_1}{M} + \omega^2\left[\left(M - \frac{L_1L_2}{M}\right)(C_1R_1 + C_2R_2)\right]$$
$$+ j\left(\omega^3\left[\left(M - \frac{L_1L_2}{M}\right)C_1C_2R_1R_2\right] + \omega\left[\frac{R_1R_2}{M}(L_1C_1 + L_2C_2) - \left(M - \frac{L_1L_2}{M}\right)\right] - \frac{1}{\omega}\cdot\frac{R_1R_2}{M}\right) \quad (2)$$

If the coefficient of mutual coupling is k, and the first and second inductance and capacitance are resonant at ω_1 and ω_2 radians per second, respectively, we can substitute in (2) the expressions,

$$M = k\sqrt{L_1L_2},\; L_1 = 1/\omega_1^2C_1,\; L_2 = 1/\omega_2^2C_2,\; I_4 = E_2/R_2 \quad (3)$$

This gives the general expression,

$$E_1/E_2 = \frac{R_1\sqrt{C_1C_2}}{k}\left\{\left[\frac{\omega_1}{\omega_2}\cdot\frac{1}{R_2C_2} + \frac{\omega_2}{\omega_1}\cdot\frac{1}{R_1C_1}\right] - \omega^2\left[\frac{1-k^2}{\omega_1\omega_2}\left(\frac{1}{R_2C_2} + \frac{1}{R_1C_1}\right)\right]\right.$$
$$\left. + j\left(\frac{-\omega^3}{\omega_1\omega_2}(1-k^2) + \omega\left[\frac{\omega_2}{\omega_1} + \frac{\omega_1}{\omega_2} + \frac{1-k^2}{\omega_1C_1R_1\,\omega_2C_2R_2}\right] - \frac{\omega_1\omega_2}{\omega}\right)\right\} \quad (4)$$

The application of circuit condition I, $(\omega_1 = \omega_2 = \omega_0)$ to (4) gives,

$$E_1/E_2 = \frac{R_1\omega_0\sqrt{C_1C_2}}{k}\left\{\left(\frac{1}{\omega_0 C_1 R_1} + \frac{1}{\omega_0 C_2 R_2}\right) - \left(\frac{\omega}{\omega_0}\right)^2 (1 - k^2)\left(\frac{1}{\omega_0 C_1 R_1} + \frac{1}{\omega_0 C_2 R_2}\right) - j\left[\left(\frac{\omega}{\omega_0}\right)^3 (1 - k^2) - \frac{\omega}{\omega_0}\left(2 + \frac{1 - k^2}{\omega_0^2 C_1 C_2 R_1 R_2}\right) + \frac{\omega_0}{\omega}\right]\right\} \tag{5}$$

Circuit condition II (zero transmission loss at $\omega = \omega_0$) is satisfied if $|E_1/E_2| = 2\sqrt{R_1/R_2}$. Substituting this condition in (5) and solving for k gives:

$$k^2 = \frac{\omega_0^2 C_1 C_2 R_1 R_2 + 1 \pm j(\omega_0 C_1 R_1 - \omega_0 C_2 R_2)}{(\omega_0 C_1 R_1)^2 + (\omega_0 C_2 R_2)^2 + (\omega_0^2 C_1 C_2 R_1 R_2)^2 + 1} \tag{6}$$

Since k must be real,

$$\omega_0 C_1 R_1 = \omega_0 C_2 R_2 \tag{7}$$

and from (6) and (7)

$$\omega_0 C_1 R_1 = \omega_0 C_2 R_2 = \sqrt{1 - k^2}/k \tag{8}$$

From (5) and (8) the transmission formula for this transformer is,

$$\frac{E_1}{E_2} = \frac{\sqrt{1 - k^2}}{k^2}\sqrt{\frac{C_2}{C_1}}\left[\frac{2k}{\sqrt{1 - k^2}}\left(1 - \left(\frac{\omega}{\omega_0}\right)^2 (1 - k^2)\right) - j\left(\left(\frac{\omega}{\omega_0}\right)^3 (1 - k^2) - \frac{\omega}{\omega_0}(2 + k^2) + \frac{\omega_0}{\omega}\right)\right] \tag{9}$$

or, $\dfrac{E_1}{E_2} = \left|\dfrac{E_1}{E_2}\right| e^{j\phi}$, where,

$$\left|\frac{E_1}{E_2}\right| = \frac{\sqrt{1 - k^2}}{k^2}\left(\frac{C_2}{C_1}\right)^{\frac{1}{2}}\left\{\frac{4k^2}{1 - k^2}\left[1 - \left(\frac{\omega}{\omega_0}\right)^2 (1 - k^2)\right]^2 + \left[\left(\frac{\omega}{\omega_0}\right)^3 (1 - k^2) - \frac{\omega}{\omega_0}(2 + k^2) + \frac{\omega_0}{\omega}\right]^2\right\}^{\frac{1}{2}} \tag{10}$$

$$\phi = \arctan \frac{\sqrt{1 - k^2}}{2k} \cdot \frac{(\omega/\omega_0)^4(1 - k^2) - (\omega/\omega_0)^2(2 + k^2) + 1}{(\omega/\omega_0)^3(1 - k^2) - \omega/\omega_0} \tag{11}$$

The transmission loss, $\mathfrak{L}$, defined as the ratio of the maximum available power from the generator to the output power may be obtained from (10),

$$\mathfrak{L} = \frac{R_2}{4R_1}\left|\frac{E_1}{E_2}\right|^2 = \left\{\frac{1}{k^2}\left[1 - \left(\frac{\omega}{\omega_0}\right)^2 (1 - k^2)\right]^2 + \frac{1 - k^2}{4k^4}\left[\left(\frac{\omega}{\omega_0}\right)^3 (1 - k^2) - \frac{\omega}{\omega_0}(2 + k^2) + \frac{\omega_0}{\omega}\right]^2\right\} \tag{12}$$

or the loss in decibels equals 10 log $\mathfrak{L}$.

The transmission delay, δ, may be obtained from (11).

$$\delta = \frac{d\phi}{d\omega} = \frac{\left(\frac{\omega}{\omega_0}\right)^4 (1 - k^2)^2 - \left(\frac{\omega}{\omega_0}\right)^2 (1 - k^2)^2 + 4k^2 - 1 + \left(\frac{\omega_0}{\omega}\right)^2}{\mathfrak{L}} \cdot \frac{\sqrt{1 - k^2}}{2k^3\omega_0} \tag{13}$$

seconds.

The input impedance seen from the generator terminals is,

$$Z_{in} = R_1\left\{\frac{\left(\frac{\omega}{\omega_0}\right) k\sqrt{1 - k^2} - j\left[1 - \left(\frac{\omega}{\omega_0}\right)^2 (1 - k^2)\right]}{\left[\frac{\sqrt{1 - k^2}}{k}\left(\frac{\omega_0}{\omega}\right)\right]\left[-1 + 2\left(\frac{\omega}{\omega_0}\right)^2 - \left(\frac{\omega}{\omega_0}\right)^4 (1 - k^2)\right] - j\left[1 - \left(\frac{\omega}{\omega_0}\right)^2 (1 - k^2)\right]}\right\} \tag{14}$$

The formulas (12), (13) and (14) give the important characteristics of this transformer. Table I gives expressions for these characteristics in the general case, and for $k = 0.5$, for several picked frequencies. Of these frequencies ω_0 and $\omega_0/\sqrt{1 - k^2}$ are the frequences of zero transmission loss, $\omega_0/\sqrt[4]{1 - k^2}$ is the geometric midband, and $\omega_0/\sqrt{1 + k}$ and $\omega_0/\sqrt{1 - k}$ are the cut-off frequencies of an infinite filter made of identical transformers of this type.

The input impedance, transmission loss, and the transmission delay are plotted against ω/ω_0 for a matched transformer with $k = 0.5$ in Fig. 2. From these curves, and from Table I the following important characteristics of this transformer will be noted:

(1) The pass-band is approximately symmetrical with frequency, rather than with the logarithm of frequency, as in circuits which have a low-pass analog.

TABLE I

Relative Frequency ω/ω_0	Transmission Loss (db)		Delay $\times\ \omega_0$ (radians)		Input Impedance	Input Standing Wave Ratio in db. $k = 0.5$
	General Case	$k = 0.5$	General Case	$k = 0.5$		
1	0	0	$\frac{2\sqrt{1-k^2}}{k}$	3.464	R_1	0
$1/\sqrt{1-k^2}$	0	0	$\frac{2\sqrt{1-k^2}}{k}$	3.464	R_1	0
$1/\sqrt[4]{1-k^2}$	$\frac{0.136k^4}{2-k^2}$ approx.	0.0048	$\frac{3+\sqrt{1-k^2}}{4+k^4/8(2-k^2)} \cdot \frac{2\sqrt{1-k^2}}{k}$ approx.	3.344	$R_1\left(\dfrac{\dfrac{k\sqrt[4]{1-k^2}}{1-\sqrt{1-k^2}} - j}{\dfrac{2-\sqrt{1-k^2}}{k} - j}\right)$	0.585
$1/\sqrt{1+k}$	$10 \log \frac{5-k}{4}$	0.52	$\frac{6-k}{5-k} \cdot \frac{2\sqrt{1-k^2}}{k}$	4.234	$R_1(1 + j\sqrt{1-k})$	6.02
$1/\sqrt{1-k}$	$10 \log \frac{5+k}{4}$	1.39	$\frac{6+k}{5+k} \cdot \frac{2\sqrt{1-k^2}}{k}$	4.094	$R_1(1 - j\sqrt{1+k})$	10.06

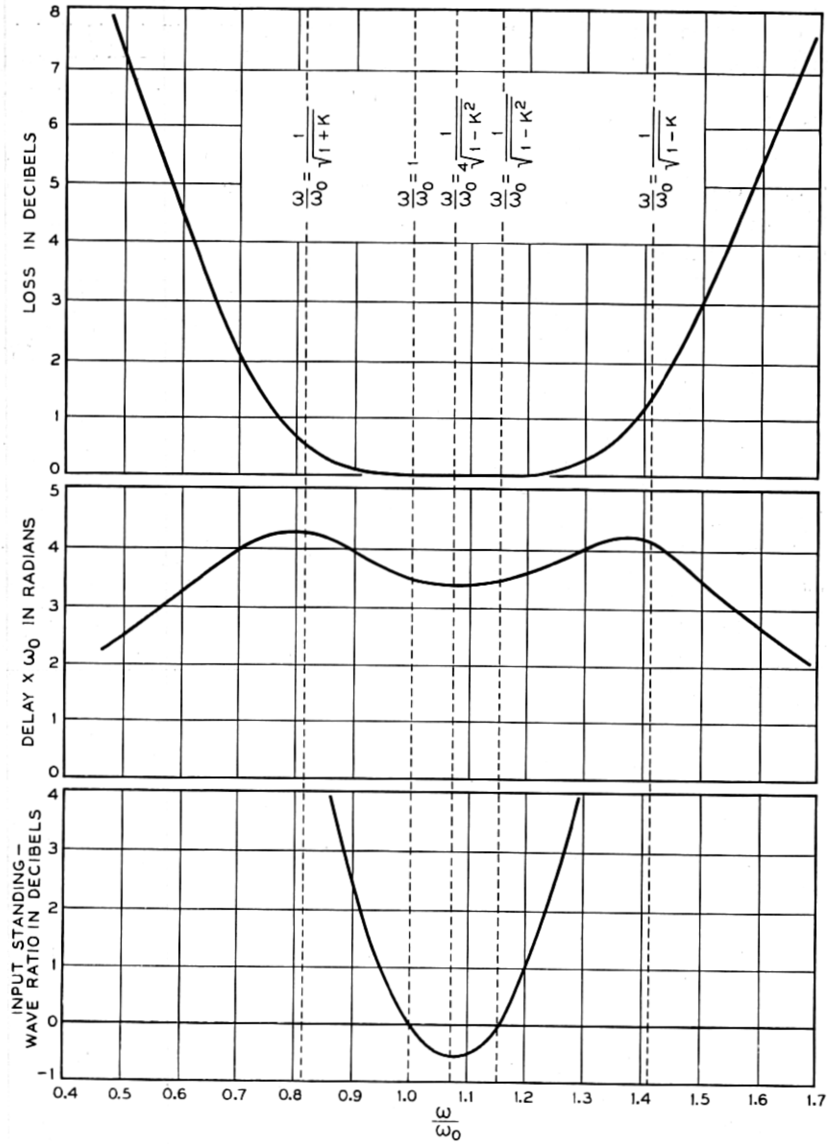

Fig. 2.—Loss, delay, and input standing-wave ratio versus relative frequency for the matched transformer for k = 0.5. Loss and delay curves also apply to the case of the mis-matched transformer for k = 0.632.

(2) The delay curve is also quite symmetrical. Circuits which can be derived from low-pass forms have more delay on the low-frequency side, where loss increases faster. The symmetry of the delay curve makes phase compensation easier.

(3) The input standing-wave ratio passes through zero on either side of mid-band, and although it has a 0.6 db dip at mid-band for the case of $k = 0.5$, it is under one db over a larger frequency range than in the transitionally coupled case.

(4) The bandwidth between points one db down on the loss curve is very nearly equal to the bandwidth between the cut-off frequencies. This bandwidth is simply related to the tune frequency f_0 and the geometric midband frequency f_m as shown below.

$$\left.\begin{aligned} \Delta f &= f_0/\sqrt{1-k} - f_0/\sqrt{1+k} \\ &\cong f_0 k/\sqrt{1-k^2} \\ &\cong f_m k/\sqrt[4]{1-k^2} \end{aligned}\right\} \tag{15}$$

Thus, given required values of f_m, Δf, C_1 (or R_1), and C_2 (or R_2), the matched transformer can be designed by the use of formulas (3), (8) and (15). These formulas are summarized at the end of this paper.

A transformer of different phase and loss characteristics results if the loading resistance is removed from one side of a matched transformer. Equation (5), if $R_2 = \infty$, becomes,

$$\begin{aligned} \frac{E_1}{E_2''} = \frac{\sqrt{1-k^2}}{k^2}\sqrt{\frac{C_2}{C_1}}\Bigg[&\frac{k}{\sqrt{1-k^2}}\left(1-\left(\frac{\omega}{\omega_0}\right)^2(1-k^2)\right) \\ &-j\left(\left(\frac{\omega}{\omega_0}\right)^3(1-k^2)-2\frac{\omega}{\omega_0}+\frac{\omega_0}{\omega}\right)\Bigg] \end{aligned} \tag{16}$$

At $\omega = \omega_0$,

$$|E_1/E_2''|^2 = C_2/C_1 \tag{17}$$

while from (10) the corresponding ratio for the matched transformer is,

$$|E_1/E_2|^2 = 4C_2/C_1 \tag{18}$$

Thus this mis-matched transformer has 6 db more gain at the tune frequency than the corresponding matched transformer, whether used as an interstage, an output or an input transformer.

The transmission loss referred to the loss at $\omega = \omega_0$ is,

$$\begin{aligned} \mathfrak{L}'' = \frac{C_1}{C_2}\left|\frac{E_1}{E_2''}\right|^2 = &\frac{1}{k^2}\left[1-\left(\frac{\omega}{\omega_0}\right)^2(1-k^2)\right]^2 \\ &+\frac{1-k^2}{k^4}\left[\left(\frac{\omega}{\omega_0}\right)^3(1-k^2)-2\frac{\omega}{\omega_0}+\frac{\omega_0}{\omega}\right]^2 \end{aligned} \tag{19}$$

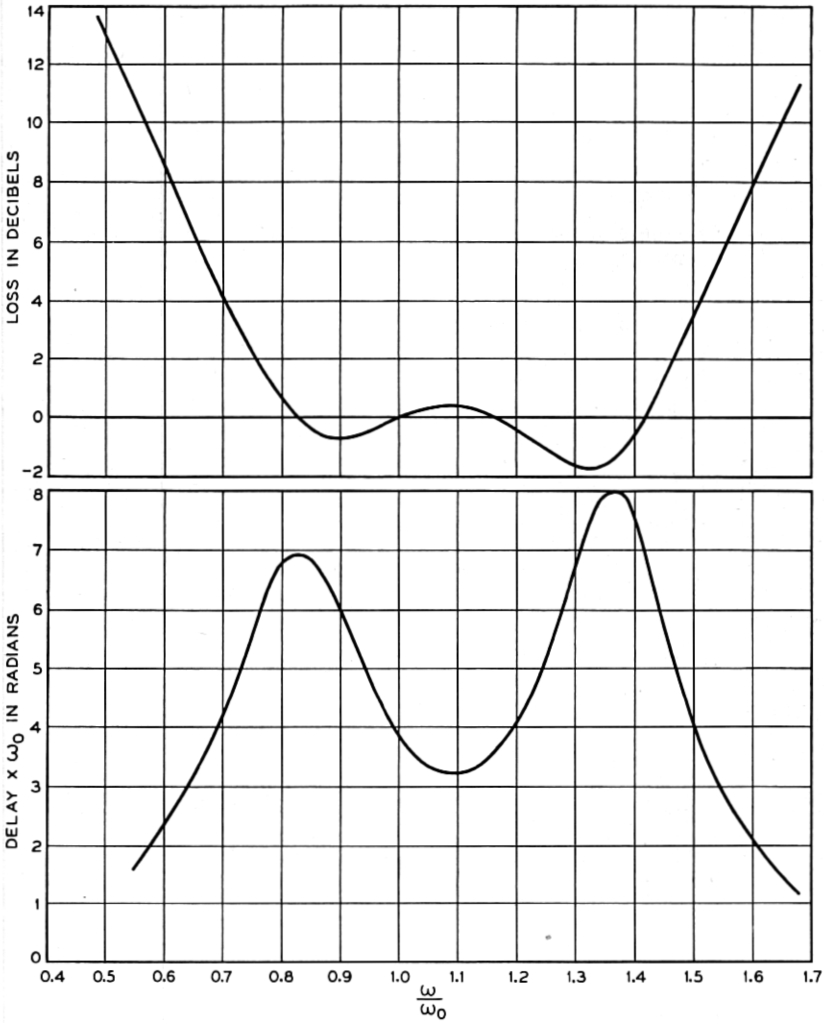

Fig. 3.—Loss and delay versus relative frequency for the mis-matched transformer obtained by making R_2 in Fig. 1 infinite, for k = 0.5.

The transmission delay δ' in this case is,

$$\delta' = \frac{\sqrt{1 - k^2}}{\omega_0 k^3} \cdot \frac{\left(\frac{\omega}{\omega_0}\right)^4 (1 - k^2)^2 - \left(\frac{\omega}{\omega_0}\right)^2 (1 - k^2) + (3k^2 - 1) + \left(\frac{\omega_0}{\omega}\right)^2}{\mathfrak{L}''} \text{ seconds.} \tag{20}$$

The loss is equal at four points,

$$\omega = \omega_0, \quad \omega = \omega_0/\sqrt{1 - k^2}, \quad \omega = \omega_0/\sqrt{1 - k}, \quad \omega = \omega_0/\sqrt{1 + k}.$$

The curves of Fig. 3 show the loss and delay for this mis-matched transformer for the case where $k = 0.5$. The double-humped band-pass characteristic of the mis-matched transformer can be compensated for in an amplifier by de-tuning slightly to equalize the humps and using a single-tuned circuit in another interstage in which,

$$Q = \omega_m CR \cong \sqrt{1/k^2 - 3/4 - 3k^2/32} \tag{21}$$

The above combination of a mis-matched transformer and a single-tuned transformer, in successive interstages, will give approximately 6 db more gain than if two matched transformers were used. The actual gain obtained will depend upon the ratio of the input and output capacitances. This gain advantage carries with it the penalty of increased sensitivity to tube capacitance variations. However, the mis-matched transformer may be used to advantage in the first interstage of an amplifier to minimize the effect of the second tube on the signal-to-noise ratio. It may also be used in the last interstage to offset any lack of power-handling capacity in the second-last tube.

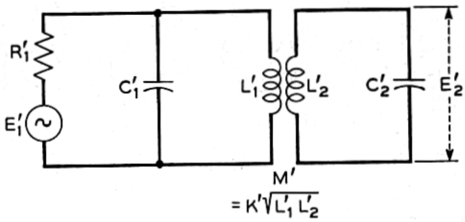

Fig. 4.—Mismatched or unsymmetrically loaded transformer.

(b) *Mismatched or Unsymmetrically Loaded Transformer*

A transformer with only one loading resistor, as in Fig. 4, can always be designed to have the same bandpass and phase characteristics as the matched transformer[4] shown in Fig. 1. No power transfer will occur, but the ratio of output voltage to the square root of the available input power can be determined, and set equal to this ratio for the matched transformer, except

[4] This important principle has been described by S. Darlington in a paper: Synthesis of Reactance 4-Poles, *Journal of Mathematics and Physics*, Vol. XVIII, Sept. 1939, pp. 257–353. Independently, this principle was demonstrated to the author by W. J. Albersheim in an unpublished memorandum dealing with the transitionally flat transformer.

for a constant factor. For the matched case this ratio is, from equation (9),

$$\sqrt{\frac{E_1^2/4R_1}{E_2^2}} = \left| \frac{1}{k}\sqrt{\frac{C_2}{C_1R_1}} - \left(\frac{\omega}{\omega_0}\right)^2 \frac{1-k^2}{k}\sqrt{\frac{C_2}{C_1R_1}} - j\left(\frac{\omega}{\omega_0}\right)^3 \frac{(1-k^2)\sqrt{1-k^2}}{2k^2}\sqrt{\frac{C_2}{C_1R_1}} + j\left(\frac{\omega}{\omega_0}\right)\frac{(2+k^2)\sqrt{1-k^2}}{2k^2}\sqrt{\frac{C_2}{C_1R_1}} - j\left(\frac{\omega_0}{\omega}\right)\frac{\sqrt{1-k^2}}{2k^2}\sqrt{\frac{C_2}{C_1R_1}} \right| \tag{22}$$

For the mismatched transformer the corresponding ratio can be obtained from equation (4) by letting R_2 approach infinity, giving,

$$\sqrt{\frac{E_1'^2/4R_1'}{E_2'^2}} = \left| \frac{1}{2k'}\frac{\omega_2'}{\omega_1'}\sqrt{\frac{C_2'}{C_1'R_1'}} - \frac{\omega^2}{\omega_1'\omega_2'} \cdot \frac{1-k'^2}{2k'}\sqrt{\frac{C_2'}{C_1'R_1'}} - j\frac{\omega^3}{\omega_1'\omega_2'}\frac{1-k'^2}{2k'}\sqrt{R_1'C_1'C_2'} + j\omega\left(\frac{\omega_2'}{\omega_1'}+\frac{\omega_1'}{\omega_2'}\right)\frac{\sqrt{R_1'C_1'C_2'}}{2k'} - j\frac{\omega_1'\omega_2'}{\omega}\frac{\sqrt{R_1'C_1'C_2'}}{2k'} \right| \tag{23}$$

The two transformers are to have the same phase characteristics, and the same band-pass characteristics except for a constant factor N where,

$$N = \sqrt{\frac{E_1^2/4R_1}{E_2^2}} \div \sqrt{\frac{E_1'^2/4R_1'}{E_2'^2}} \tag{24}$$

The terms involving voltages in (22) and (23) can be eliminated by combining with (24). Equating the coefficients of like powers of ω gives (25), below.

$$\left.\begin{aligned}
\frac{1}{k}\sqrt{\frac{C_2}{C_1R_1}} &= \frac{1}{2k'}\sqrt{\frac{C_2'}{R_1'C_1'}}\,N\frac{\omega_2'}{\omega_1'} \\
\frac{1-k^2}{k}\sqrt{\frac{C_2}{C_1R_1}}\frac{1}{\omega_0^2} &= \frac{1-k'^2}{2k'}\sqrt{\frac{C_2'}{C_1'R_1'}}\left(\frac{N}{\omega_1'\omega_2'}\right) \\
\frac{1-k^2}{k}\sqrt{R_1C_1C_2}\cdot\frac{1}{\omega_0^2} &= \frac{1-k'^2}{k'}\sqrt{R_1'C_1'C_2'}\left(\frac{N}{\omega_1'\omega_2'}\right) \\
\frac{2+k^2}{k}\sqrt{R_1C_1C_2} &= \frac{\sqrt{R_1'C_1'C_2'}}{k'}\left(\frac{\omega_2'}{\omega_1'}+\frac{\omega_1'}{\omega_2'}\right)N \\
\frac{\sqrt{R_1C_1C_2}}{k}\,\omega_0^2 &= \frac{\sqrt{R_1'C_1'C_2'}}{k'}\,(\omega_1'\omega_2'N)
\end{aligned}\right\} \tag{25}$$

The last three equations were simplified by the use of equation (8).

Equations (25) yield the five relations below:

$$\left.\begin{aligned} R_1'C_1' &= R_1C_1/2 \\ \omega_1 &= \omega_0 \\ \omega_2 &= \omega_0/\sqrt{1+k^2} \\ k' &= \frac{\sqrt{2}\,k}{\sqrt{1+k^2}} \\ N &= 2\sqrt{\frac{C_2}{C_2'}} \end{aligned}\right\} \tag{26}$$

Summary of Design Formulas

A transformer must usually meet certain requirements as to bandwidth and mid-frequency. Loss curves for various values of k, plotted against relative frequency may be used. To a very close approximation the geometric mean frequency $f_m = \omega_m/2\pi$, may be used for the mid-band frequency, and Δf, the bandwidth between cut-off points, may be used for the bandwidth between one db points. Then,

$$\omega_m = \omega_0/\sqrt[4]{1-k^2} \tag{27}$$

And, from (15),

$$k \cong \frac{\Delta f/f_0}{\sqrt{1+(\Delta f/f_0)^2}} \tag{28}$$

The other necessary relations for the matched transformer and for the mismatched transformer with the same transmission characteristics, as derived from equations (3), (8) and (26) are given below.

(a) Matched Transformer.

$$\left.\begin{aligned} \frac{1}{\sqrt{L_1C_1}} &= \omega_0 \\ \frac{1}{\sqrt{L_2C_2}} &= \omega_0 \\ \frac{M}{\sqrt{L_1L_2}} &= k \\ R_1C_1 = R_2C_2 &= \frac{\sqrt{1-k^2}}{\omega_0 k} \end{aligned}\right\} \tag{29}$$

(b) Mismatched Transformer:

$$\left.\begin{aligned} \frac{1}{\sqrt{L_1' C_1'}} &= \omega_0 \\ \frac{1}{\sqrt{L_2' C_2'}} &= \frac{\omega_0}{\sqrt{1+k^2}} \\ \frac{M'}{\sqrt{L_1' L_2'}} &= k' = \frac{\sqrt{2}\,k}{\sqrt{1+k^2}} \\ R_1' C_1' &= \frac{\sqrt{1-k^2}}{2\omega_0 k} \end{aligned}\right\} \quad (30)$$

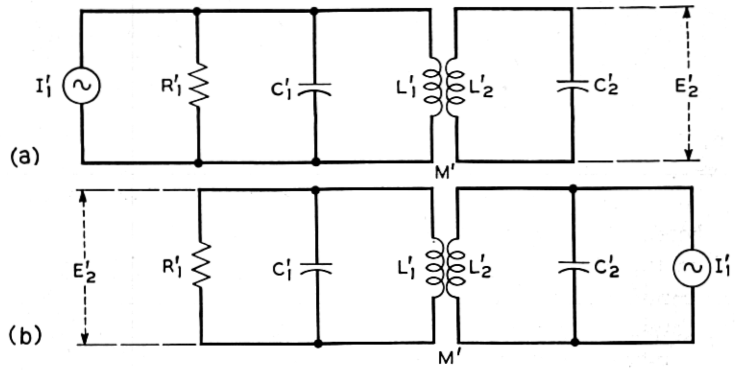

Fig. 5.—Mismatched transformer (a) fed by constant current generator, (b) with input and output reversed.

The ratio of the output voltages in the two cases is given by:

$$\left|\frac{E_2'}{E_2}\right|^2 = N^2 \left(\frac{E_1'^{\,2}/4R_1'}{E_1^{\,2}/4R_1}\right) = 4\,\frac{R_1 C_2}{R_1' C_2'}\left|\frac{E_1'}{E_1}\right|^2 \quad (31)$$

The relative gains of the two transformers, as given by equation (31), will be examined for three important cases:

(1) *Input Transformer*—In this case $R_1 = R_1'$, and $C_2 = C_2'$, so that from (31), the mismatched transformer gives four times, or 6 db more gain.

(2) *Output Transformer*—The voltage source in Fig. 4 may be replaced by the constant current source $I_1' = E_1'/R_1'$, by Norton's Theorem, as shown in Fig. 5(a). By the Reciprocity Theorem we may exchange the positions of the current source and the output voltage as shown in Fig. 5(b), without changing the value of the latter. This may also be done for the

matched transformer. The resultant circuit is that for a pentode output transformer in each case, where the plate resistance of the tube is so high compared to circuit impedance that we can consider it to be a constant current generator of $g_m E_g$ amperes, where g_m is the grid-plate transconductance and E_g the grid signal voltage. Thus in this case the mis-matched transformer also gives 6 db more gain than the matched transformer.

(3) *Interstage Transformer*—If the voltage generators of Figs. 1 and 4 are replaced by their equivalent current generators $g_m E_g = E_1/R_1$ and $g_m' E_g' = E_1'/R_1'$, then (31) becomes,

$$\left| \frac{E_2'}{E_2} \right|^2 = 4 \frac{R_1' C_2}{R_1 C_2'} \left| \frac{g_m' E_g'}{g_m E_g} \right|^2 \tag{32}$$

The first relation in (26) may be substituted in (32) giving,

$$\left| \frac{E_2'}{E_2} \right|^2 = 2 \left| \frac{C_1 C_2 g_m' E_g'}{C_1' C_2' g_m E_g} \right|^2 \tag{33}$$

Thus, in the interstage case, this mismatched transformer has only 3 db more gain than the matched transformer.

Here, as in the case of the first mismatched transformer discussed in this paper, there is an increased sensitivity to capacity variations. In practice it appears that this is not always serious in wide-band intermediate-frequency amplifiers. Some improvement in noise figure and power handling may be obtained, as before, without encountering difficulties with double-peaked pass-band characteristics.

The design formulas given in this paper have been used successfully in the design of experimental intermediate frequency amplifiers in the 65 mc region having band widths of the order of 10 to 20 mc.