A Precise Direct Reading Phase and Transmission Measuring System for Video Frequencies

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THE evolution of transmission networks for communications systems progresses through three fairly well-defined phases—design, synthesis and final adjustment. The design phase ordinarily involves no problem of measurement. In the synthesis stage, during which the physical model is constructed from the paper design, precise equipment is often needed for measuring the magnitude of the various components comprising the network. The adjustment stage, in which the network is actually tested as an element in a transmission circuit, generally requires the most complex instrumentation. In the latter category we may include insertion loss, gain, and phase measurement systems.

Television and broad-band carrier facilities, such as the New York-Midwest video cable link, employ vast numbers of transmission networks. These include, for example, filters, equalizers, and repeaters. The final adjustment of these networks requires a large number of precise insertion phase and transmission measurements during both development and manufacturing stages. Consequently, the measurement equipment must combine laboratory accuracy with speed of measurement suitable for use in production testing.

The quantities measured are defined in Fig. 1. Conforming with current usage, the term *Transmission* is used herein to designate insertion loss and gain.

The performance of the system with respect to frequency range, measurement range and accuracy is as follows:

Frequency Range: 50–3600 kilocycles

Generator and Network Termination Impedance: 75 Ω

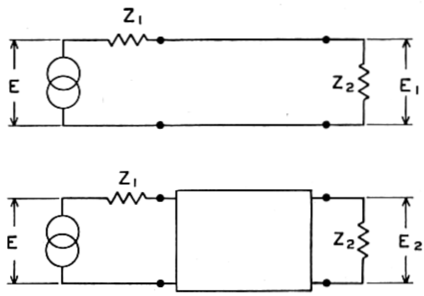
Transmission Range: +40 db to −40 db; Accuracy ±0.05 db
−40 db to −60 db; reduced accuracy.

Insertion Phase Shift Range: 0–360°; Accuracy ±0.25 degree (+40 db to −40 db)

The measuring circuit is based on the heterodyne principle whereby the phase and transmission of the *unknown* are translated from the variable frequency to a constant intermediate frequency at which the phase and transmission standards operate. Accurate phase-shifters and variable attenuators with negligible phase shifts are constructed readily for fixed frequency operation. This advantage more than offsets resulting problems of modulator design and automatic frequency control.

Conforming with the definitions of insertion phase and transmission, the measurement system compares, with respect to phase and amplitude, the outputs of two transmission channels energized from the measurement frequency source, one of which serves as a standard or reference channel, while the other contains the apparatus under test. This is illustrated by the block drawing in Fig. 2.

For loss measurements the range attenuator I_1 (Fig. 2) is set at 0 db. Measurement frequency F from the master oscillator is applied to both standard "S" and unknown "X" channels through splitting pad I. The voltages at "S" and "X" modulator inputs, points A and B respectively



$$\left\{\begin{matrix}\text{INSERTION LOSS}\\ \text{OR}\\ \text{INSERTION GAIN}\end{matrix}\right\} = 20 \text{ LOG } \frac{|E_1|}{|E_2|}$$

$$\text{INSERTION PHASESHIFT} = \angle E_2 - \angle E_1$$

Fig. 1—Definition of quantities measured.

in Fig. 2, differ with respect to phase and amplitude because of the transmission differential introduced between the two channels by the apparatus under test. By frequency conversion in the "S" and "X" modulators these amplitude and phase differences at frequency F are translated at points C and D to a constant intermediate frequency, 31 kc. The second input to the "S" and "X" modulators, of frequency $F + 31$ kc, is supplied by the slave oscillator which automatically tracks at constant 31 kc difference with respect to the master oscillator. By selective filtering, only the difference frequency appears at the modulator outputs C and D. 31 kc has been chosen as the intermediate frequency, primarily on the basis of filtering requirements in the modulators. The detector (Fig. 2) compares the voltages of the "X" and "S" channels at K and L as to magnitude

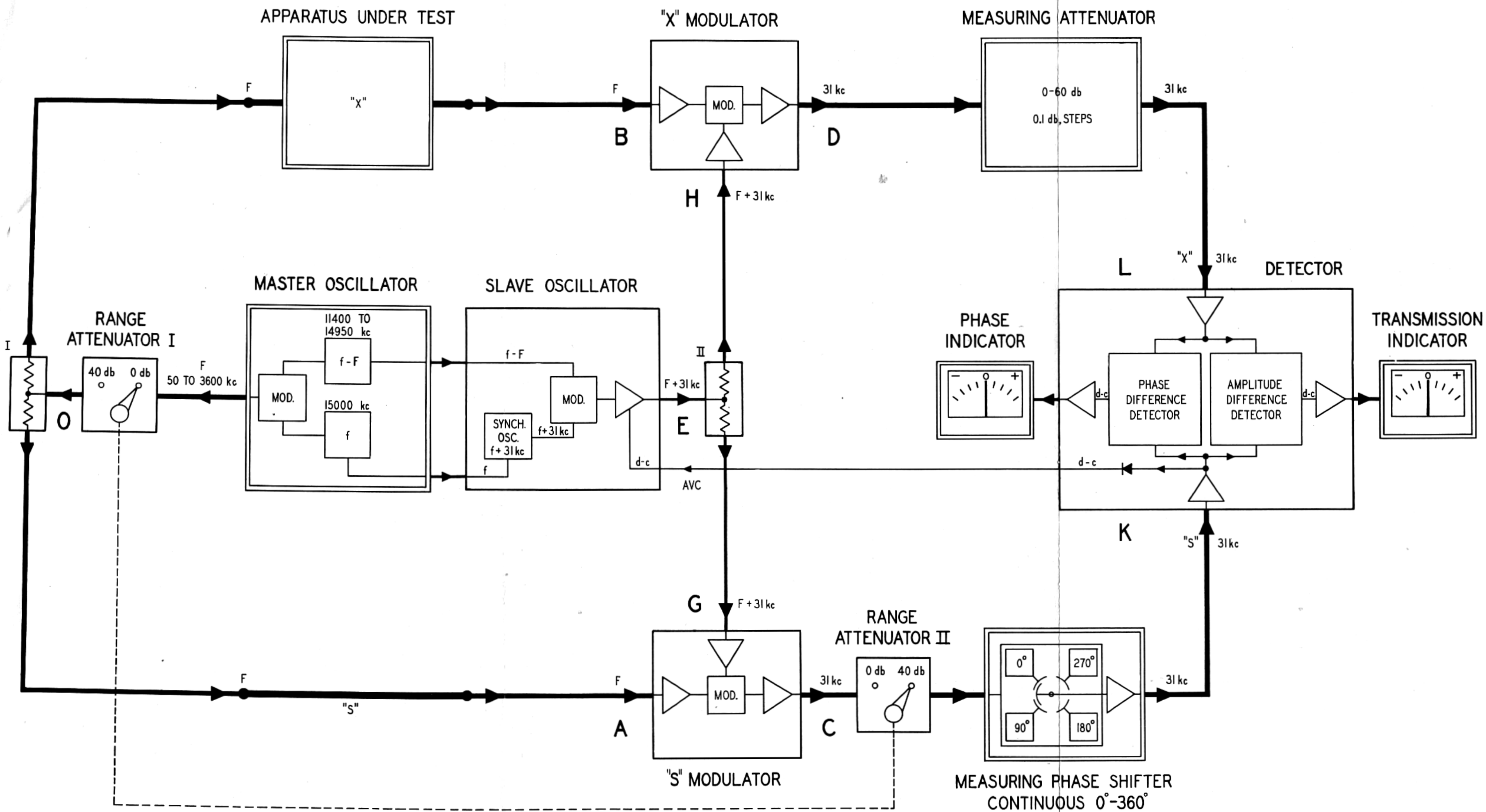


Fig. 2—Block schematic of the phase and transmission measurement system.

and phase, and indicates their difference on the direct reading scales of the indicator meters.

If the measuring attenuator is set at 60 db loss, and the range attenuator *II* at 40 db loss, "*S*" and "*X*" channels are in balance when the *apparatus under test* is replaced by a *zero loss* strap. The phase-shifter has, by design, 20 db loss; so that under these conditions "*S*" and "*X*" channels are nominally in balance, except for small residual phase and transmission differentials which may be *zeroed-out* by initial adjustment of the phase-shifter and of the relative gain between "*S*" and "*X*" channel amplifiers within the detector. Null readings on the phase and transmission difference indicating meters tell when exact phase and transmission balance between the two channels has been established. The phase-shifter and attenuator dials are arranged to read zero after this initial balance has been made. To measure apparatus transmission and phase, the strap is replaced by the *apparatus under test* and the balance restored by adjustment of the phase-shifter and the measuring attenuator. The insertion phase and transmission of the apparatus under test are then read directly from the calibrated dials of the phase shifter and attenuator.

When measuring loss, attenuation in the measuring attenuator is reduced by the amount of attenuation introduced in the high-frequency portion of "*X*" channel by the "apparatus under test." In measuring gain, the attenuation through the measuring attenuator must be increased by the amount of apparatus gain. To insure that "*S*" and "*X*" channel modulators are not overloaded by excessive input, range attenuator I is set to 40 db loss during gain measurements. This attenuator is common to both channels and therefore introduces no phase differential. Simultaneously and automatically, the range attenuator *II* immediately following the "*S*" modulator, is operated, removing 40 db loss from the 31 kc standard channel.

The measuring attenuator is self-computing and indicates directly in illuminated figures the gain or loss of the apparatus under test. A simple switching arrangement automatically controls the dial-lighting circuit of the measuring attenuator. When measuring gain the dial indications increase in one direction, and when measuring loss the indications increase in the opposite direction (Fig. 3).

In addition to the null-balance method, a deflection method of measurement using direct reading scales of the phase and transmission difference indicating meters is also possible. An automatic volume control circuit assures invariance of the indicator scale factors with either the modulator frequency-transmission characteristic, or input voltage variation at the "*S*" modulator caused by reflections from apparatus under test. The automatic volume control circuit regulates the output voltage of the slave oscillator to maintain the amplitude of the "*S*" channel input to the dif-

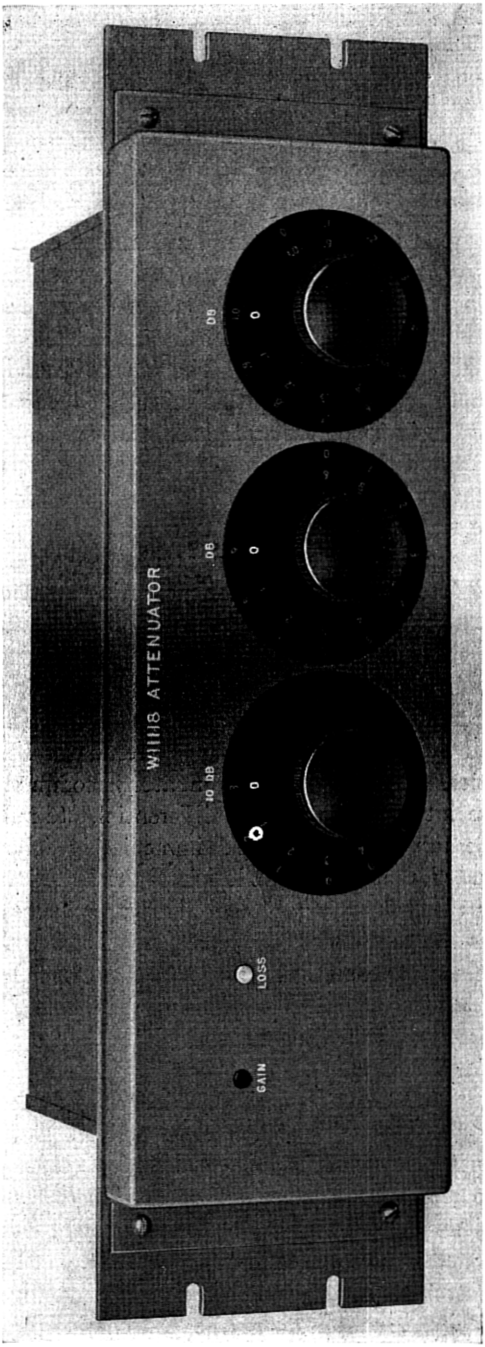


Fig. 3—Measuring attenuator with computing dials.

ferential detector constant. As the control action simultaneously affects both "*S*" and "*X*" modulators uniformly, the system zero is undisturbed.

Careful attention has been given to the problem of obtaining an electrical match between "*S*" and "*X*" modulators and coaxial cable lengths in the high-frequency channels. (RG $6/U$ cable contributes a phase shift of 0.2°/inch at 3600 kc.) Consequently, with the apparatus under test replaced by a *coaxial strap*, a balance indication on the phase and transmission difference indicators may be obtained which shifts less than 0.1 degree in phase and 0.02 db in transmission when the master oscillator frequency is varied over its entire band.

Because of the frequency independence of the system zero and the automatic frequency control of the slave oscillator, the master oscillator may be swept through the entire frequency band for rapid appraisal of the network performance by observation of the phase and transmission difference indicators.

The component chassis of the set are mounted in a specially designed console, shown in Fig. 4, which places all controls within easy reach of the operator. This console houses as much apparatus as three 6-foot relay racks within a floor space equal to that occupied by a 5-foot laboratory bench. Though not visible, a full bay of apparatus is mounted behind the central meter panel. Easily movable partitions and covers permit accessibility to all units, thus expediting maintenance.

Some of the significant design considerations are discussed separately under the following headings:

(1) Master Oscillator
(2) Slave Oscillator
(3) Modulators
(4) Phase and Transmission Detector
(5) Phase-shifter

Master Oscillator

As indicated in Fig. 2 and Fig. 5C, the master oscillator is of the heterodyne type. It employs 15,000 and 11,400–14,950 kc local oscillators. A high degree of frequency stability has been achieved through special oscillator circuit design. A motion picture film type scale, 300 inches in length, calibrated every 10 kc, and further subdivided every 2 kc, covers the entire frequency range 50–3600 kc without band-switching. A 0–10 kc interpolation dial with 100-cycle divisions, which operates on the fixed local oscillator frequency, is used to interpolate between adjacent 2 kc graduations on the main film scale. By oscilloscopic comparison with a 10 kc standard of frequency, the oscillator can be set within 50 cycles of any desired frequency in its band. An A.V.C. circuit maintains the output power at six db above one milliwatt.

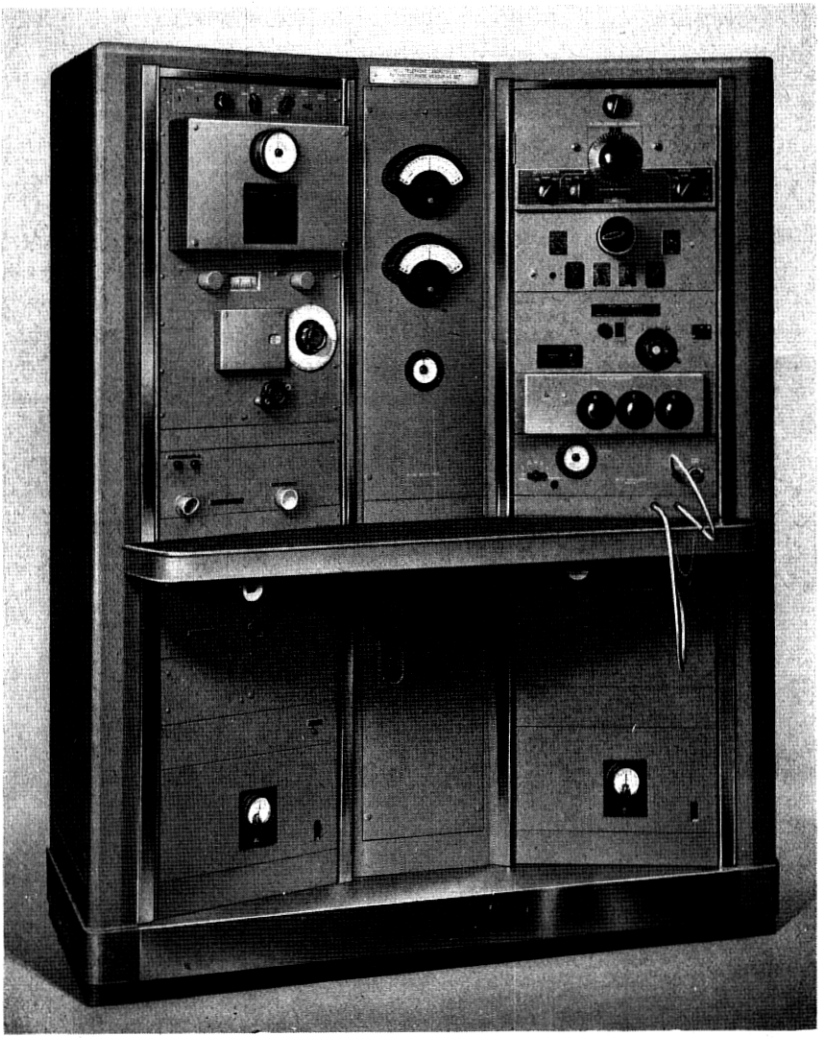


Fig. 4—The assembled phase and transmission measuring system.

Slave Oscillator

To make possible the operation of the measuring attenuator, phase-shifter, and phase and transmission difference detectors at constant frequency, the inputs to the "S" and "X" channel modulators from the master and slave

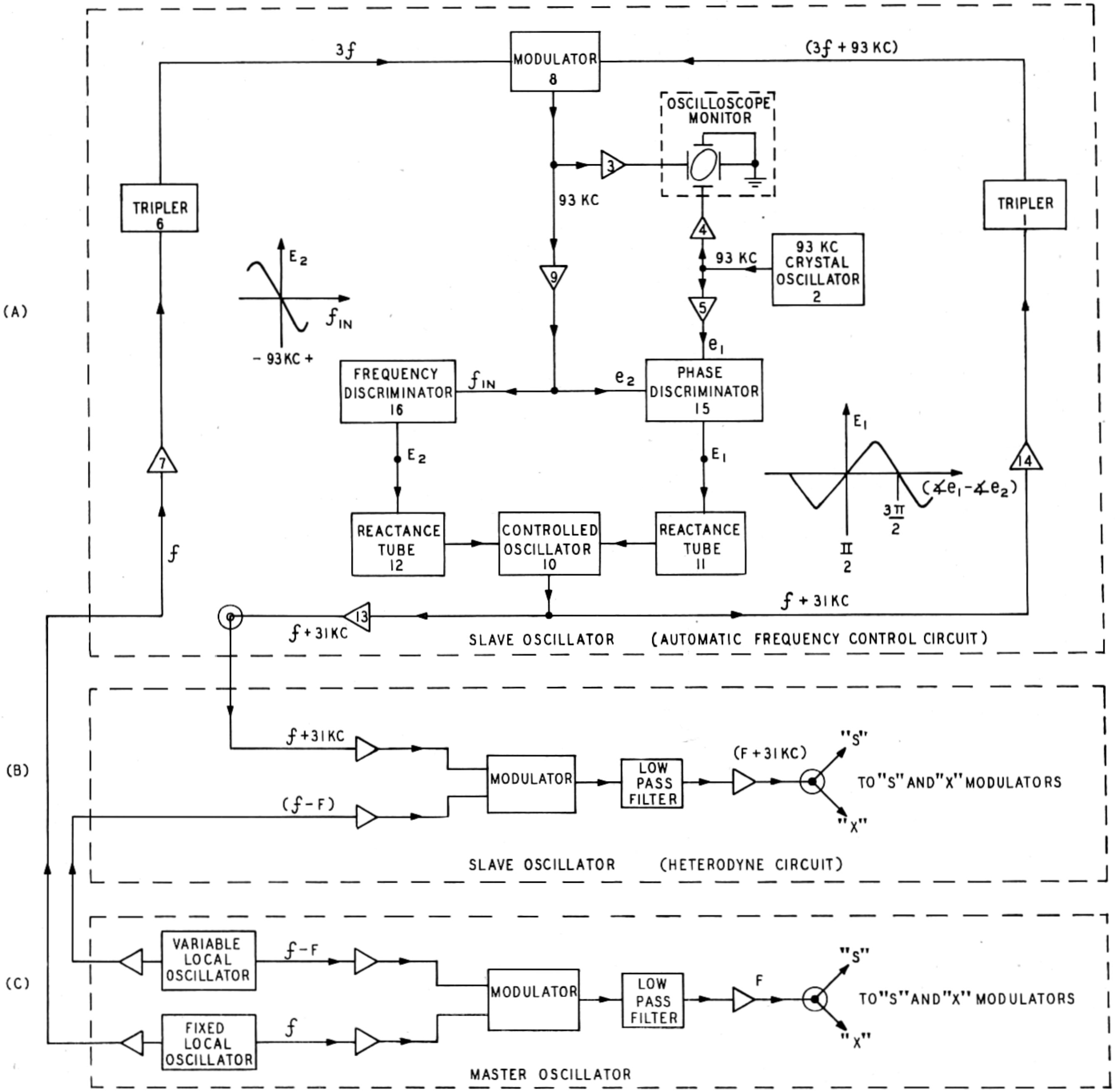


Fig. 5—Master and slave oscillators.

oscillators must always differ in frequency by a constant amount. This difference is maintained at 31 kc by the control of the master oscillator over the slave oscillator frequency.

Very briefly, the scheme consists in applying the fixed local oscillator frequency, f, of the master oscillator, to an automatic frequency control circuit which produces an output frequency $f + 31$ kc. $f + 31$ kc is then modulated with variable local oscillator frequency, $f - F$, of the master oscillator, resulting in an output of frequency $F + 31$ kc. Frequency F, formed by modulation of f and $f - F$, is the master oscillator frequency.

In the automatic control circuit, frequency f is compared with that of a controlled oscillator, by detecting their difference in a modulator. The nature of the control is such, that any deviation of this difference from 31 kc causes the frequency of the controlled oscillator to change in the direction which eliminates the deviation. While it is simpler to compare f and the controlled oscillator frequency directly, in the slave oscillator the comparison is made between the outputs of tripler circuits energized from the latter frequencies. In this way more complete isolation is realized between f and the controlled frequency than would be afforded with only buffer amplifiers. Because of the tripling, it follows that the oscillator must be controlled according to the departure of the difference between the tripler circuit frequencies from 93 kc. This, however, has the advantage of avoiding the generation of 31 kc anywhere in the automatic frequency control circuit, which could, by spurious modulation, cause the $f + 31$ kc output to be contaminated with small traces of frequency f. The necessity for exceptional purity of $f + 31$ kc output arises in the measurement of high losses where minute amounts of F at the $F + 31$ kc input to "*S*" and "*X*" modulators may produce appreciable error.

Owing to phase tracking requirements between "*S*" and "*X*" intermediate frequency channels, and to the frequency dependence of the phase-shifter calibration, it is necessary to maintain the intermediate frequency as closely as possible to the precise value, 31,000 cycles. The permissible deviation from the correct value has been limited to ± 1 cycle. This precise control is maintained in the presence of 10 kc changes in f, which may occur when the setting of the 0–10 kc interpolation dial of the master oscillator is varied in the course of measurement.

Figures 5A and 5B illustrate the automatic frequency control and heterodyne circuits of the slave oscillator. The frequency of oscillator 10 in Fig. 5A is controlled by the reactance tubes 11 and 12. Reactance tube 12 is actuated by direct voltage from frequency discriminator 16, so that it controls oscillator 10 according to frequency error. Frequency error is the difference between the input frequency to discriminator 16 from amplifier 9, and 93 kc, the frequency of zero voltage output from the discriminator. The voltage from phase discriminator 15 controls oscillator 10 according to the difference of phase between the input from stage 9, and an input of reference phase from amplifier 5. This difference of phase is proportional to the time integral of the frequency error. The gross effect, therefore, is to control the oscillator 10 according to the controller law, *proportional to frequency error + time integral of frequency error*, or, in the terminology of feedback regulators, *proportional + integral control*[1]. When in equilibrium, the system operates with a static phase difference between the phase discriminator inputs, a condition which can exist only when these inputs are of equal frequency. The system is thus endowed with the property *zero frequency error*, and the frequency at the output of modulator 8 is maintained in exact equality with crystal oscillator 2 frequency. Consequently the intermediate frequency difference between input, f, and controlled oscillator 10 is held precisely at the value 31,000 cycles.

Automatic frequency control circuits of the phase sensitive type have been previously described[2,3,4].

The system of combined phase and frequency sensitive control in the slave oscillator is superior to those which use only phase or frequency sensitive control. In a control circuit which uses only a phase discriminator and associated reactance tube, the controlled oscillator may lock-in at either of two sideband frequencies. These are $f + 31$ kc, and $f - 31$ kc. Operation is at upper sideband when control stabilizes on the positive slope of the phase discriminator output voltage curve in Fig. 5A, and at lower sideband if control is along the negative slope. Thus an ambiguity of sideband exists, though the attribute of *zero frequency error* is retained. When only a frequency discriminator and reactance tube are used, lock-in is possible at only one of the two sideband frequencies, determined by the poling of the frequency discriminator output voltage. A frequency error, however, is present.

The combination in Fig. 5 of the two systems operating jointly utilizes the phase sensitive discriminator to insure close control of oscillator frequency, and the polarizing property of the coarser frequency discriminator to eliminate the possibility of synchronization at the undesired sideband.

The joint system of phase and frequency sensitive automatic control has the further virtue of possessing a far greater degree of stability than is obtainable with the phase discriminator loop acting alone.

In the heterodyne circuit of Fig. 5B, $f + 31$ kc from the automatic frequency control circuit is modulated with $f - F$, the variable local oscillator frequency of the master oscillator. The frequency at the output of the heterodyne circuit is $F + 31$ kc, and this is modulated with frequency F in the "S" and "X" modulators to produce the constant intermediate frequency, 31 kc, in the measurement portion of the set.

The Modulators

The difficulties of precise measurement over a wide frequency band essentially are concentrated in the modulator. With the precision to which measurement must be made, effects ordinarily of small concern assume importance. The following discussion is valid for any modulator, though the specific example of the vacuum tube is used.

It is the function of the modulator to convert linearly changes in amplitude and phase from the input frequency F to the output frequency 31 kc. The linear range of conversion is limited by overload at the high-level limit and by noise at the low-level limit.

Let the input x to a modulator consist of two frequencies F_1 and F_2. In the ideal square law modulator[5] perfect linearity results between changes in the input signal F_1 and the output signal $F_2 - F_1$. The output filter rejects all frequencies but $F_2 - F_1$.

In actual tubes the plate current is

$$I_p = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots. \tag{1}$$

The effect of the term $a_4 x^4$ and higher even-order terms is to contribute output currents of frequency $F_2 - F_1$ which do not vary linearly with the input.[5] In addition to this the effect of remodulation in plate, screen and suppressor circuits is that the coefficients a_2, a_4 etc. are not independent of the input x and so contribute to the distortion. Further, in presence of modulation of higher than second order, the d-c. term in even-order modulation will cause distortion if cathode bias is used. Removal of d-c. degeneration using fixed bias eliminates this effect.

The high-level limit may be defined as the signal value for which the total error due to overload equals the desired limits of modulator performance.

The lowest input level into the modulator which may be tolerated, and hence the lower limit of loss which can be measured, is determined by the effective signal-to-noise ratio at the modulator output. If no amplification exists preceding the modulator the input grid noise is usually limiting. The signal-to-noise ratio of the signal F_1 and a noise band centered on F_1 is unaffected by the modulation process as only the modulated portion of the noise band passes through the output filter. Yet for a noise band centered on the intermediate frequency $F_2 - F_1$ for which the output filter is transparent the modulator acts as a straight amplifier; hence the effective signal-to-noise ratio is degraded approximately by the ratio of amplifier gain to conversion gain of the modulator.

The low-level limit may be defined as the signal value for which the error due to noise equals the desired modulator performance limit. For example for a noise error of 0.01 db, a signal-to-noise ratio of 1000 to 1 or 60 db is required.

To obtain maximum signal-to-noise ratio, a tube must be chosen to have the lowest product of *noise multiplied by the ratio of amplifier to conversion gain*, the latter requirement being in conflict to overload requirements. When inputs below the low-level limit are to be utilized a preamplifier ahead of the modulator tube is required. This amplifier also contains a noiseband centered on $F_2 - F_1$. If the amplifier is selective and rejects this noise band or if an $F_2 - F_1$ rejection filter is inserted ahead of the modulator tube the resultant new low-level limit is determined by the signal-to-noise ratio of the preamplifier at the signal frequency F_1 only.

Dynamic range is defined as the useful range of a modulator limited by the high-level limit on one end and by the low-level limit on the other. The dynamic range of a number of pentodes was determined. It was found experimentally that differences in dynamic range between pentodes of different power ratings, such as 6AK5, 6AC7, 6AG7, 6L6, 829B, are small. A dynamic range of 30–36 db can be realized with a 6AK5 for a .01 db linearity requirement. The 6AK5 was the most suitable tube of those investigated considering all other requirements of the circuit such as band width, available signal levels, etc.

Buffer amplifiers are required ahead of the modulator tube to prevent crosstalk between measuring and reference modulator through common paths. These buffer amplifiers are of conventional video amplifier design, with phase and gain characteristics closely controlled to the order of 0.01 db and 0.1 degree.

The Phase and Transmission Detector

In the null type of phase measurement an initial *circuit zero* is made. When the circuit is rebalanced with the *apparatus under test* inserted, the phase detector must be able to verify that the same phase relationship has been reestablished as existed when the initial circuit zero balance was made. Bridge circuits yield high sensitivity and a high degree of independence of input voltage amplitudes. In Fig. 6 a four-arm resistance phase bridge is shown, which has two inputs E_1 and E_2, and two outputs E_S and E_D corresponding to the vectorial sum and difference of the input voltages E_1 and E_2.

As derived in the appendix, for the equal arm bridge, the amplitudes of the voltages E_S and E_D are equal for phase angles of $\varphi = \pi/2 + n\pi$, where n is any integer, regardless of the amplitudes of E_1 and E_2. Thus equality of $|E_S|$ and $|E_D|$ is convenient to define the circuit phase zero. Equality of $|E_S|$ and $|E_D|$ by itself does not distinguish between 90° and 270° phase shifts. This ambiguity can be resolved with a detection circuit which responds to both the amount and the sign of the difference $|E_S| - |E_D|$ and by making provision for the introduction of a small increase $\Delta\varphi$

in phase angle φ of a known direction. From equation (11) in the appendix for equal amplitudes E_1 and E_2

$$(2) \qquad |E_S| - |E_D| = |E_1| [\cos(\varphi/2) - \sin(\varphi/2)]$$

Substituting (a) $\varphi = \pi/2 + \Delta\varphi$ and (b) $\varphi = 3\pi/2 + \Delta\varphi$ into equation (2) it is evident that the sign of $|E_S| - |E_D|$ in substitution (b) is the reverse of substitution (a).

$|E_S|$ and $|E_D|$ are equal every 180° only if all arms of the phase bridge are exactly equal. If the arms are unequal, balance exists for all angles $\varphi = \pi/2 + 2n\pi + \Delta\theta_1$ and $\varphi = \pi/2 + (2n - 1)\pi + \Delta\theta_2$ where $\Delta\theta$ may be called departure angle.

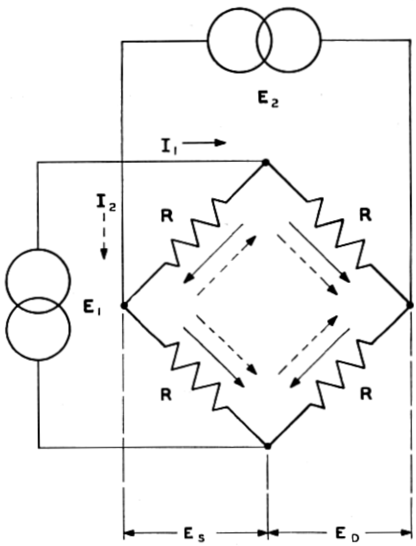


Fig. 6—The phase bridge.

The phase detector can also be used as a deflection bridge. If the phase indicating meter is calibrated according to equation (2), phase angle departures from $\pi/2 + n\pi$ may be read directly on the indicator when $|E_1| = |E_2|$ and the scale factor is adjusted for the amplitude of E_2 which is maintained constant by the overall automatic volume control circuit. Equation (2) is almost a linear function and is plotted in Fig. 7.

In using the deflection on the indicator to measure phase shift, an error $\Delta\psi$ is incurred if $|E_1| \neq |E_2|$. The maximum permissible ratio of $|E_2|/|E_1|$ for a given error $\Delta\psi$ is given by

$$(3) \qquad |E_2|/|E_1| = \cos\varphi/\cos(\varphi + \Delta\psi) + \sqrt{[\cos\varphi/\cos(\varphi + \Delta\psi)]^2 - 1}$$

Equation (3) is derived in the appendix and plotted for several values $\Delta\psi$ in Fig. 8.

The phase bridge essentially converts the measurement of phase into the measurement of voltage difference. Vacuum tube diodes are used as dif-

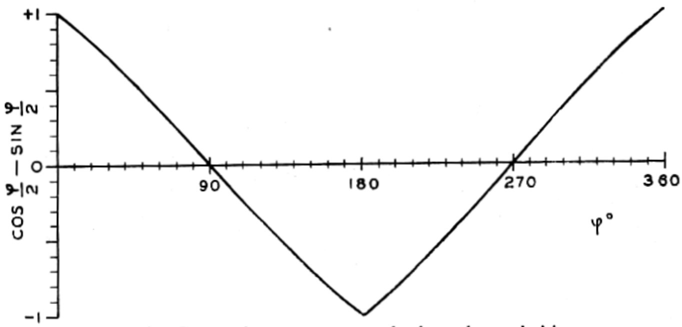


Fig. 7—Deflection response of the phase bridge.

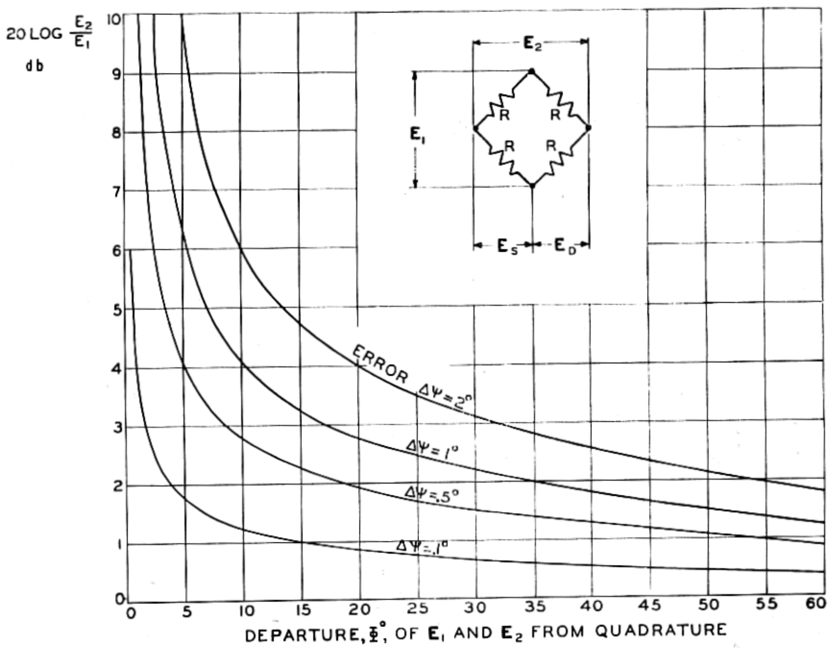


Fig. 8—Phase error $\Delta\psi$ for unequal inputs.

ferential rectifiers with a high resistance load consisting of hermetically sealed carbon deposited resistors closely matched for value and temperature coefficient and specially mounted to minimize temperature differentials. The differential output of the rectifiers is amplified in a feedback stabilized d-c. amplifier which has adjustable gain to adjust scale factors on the indicator. The phase detection circuit is energized from the output of the phase bridge and the almost identical transmission detection circuit is energized from the inputs to the phase bridge. Thus both phase and transmission are measured simultaneously.

Each indicator (Fig. 9) has three scales, *fine* ($-5°$ to $+5°$; -1 db to $+1$ db), *coarse* ($-90°$ to $+90°$; -10 db to $+3$ db), and *null balance.* The *fine* and *coarse* scales are linear while the *null balance* scale has maximum

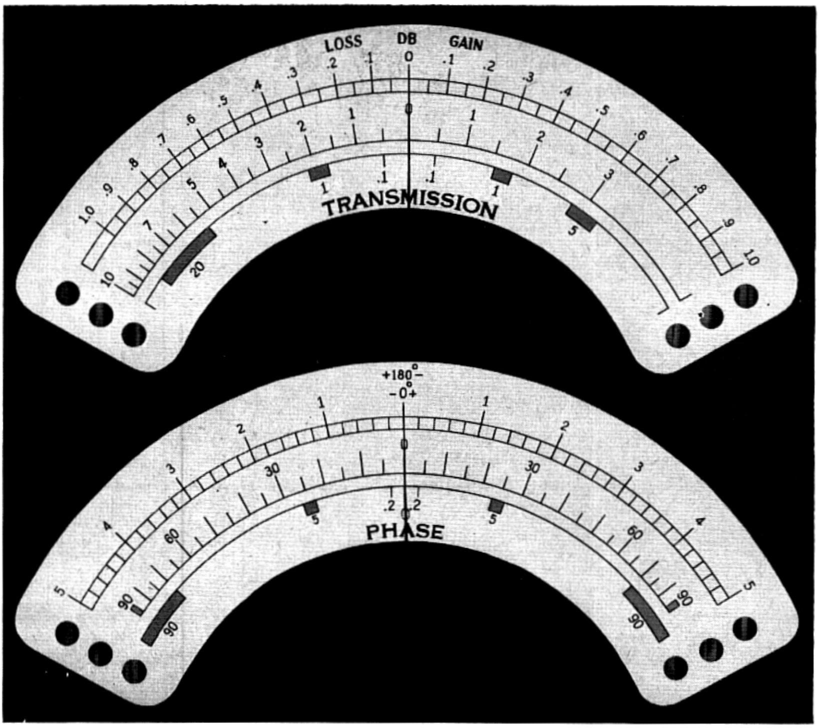


Fig. 9—Phase and transmission indicators.

sensitivity in the neighborhood of the center zero and greatly reduced sensitivity at each end. Varistor shunts across the indicators compress the null scale for large deflections. Colored pilot lights at the ends of the indicator scales, operated by the scale switch, indicate the scale in use directly.

The Phase Shifter

The phase-shifter employs a four-quadrant variable sine condenser. It has two linearly subdivided scales—*coarse* 0–360° on a cylinder and *fine*

0–10° on a dial. The fine dial is connected through reduction gearing to the shaft of the sine condenser. The construction of a phase-shifter which has a sufficiently linear correspondence of electrical phase-shift and mechanical displacement of a shaft is not practical. Instead a movable index for the fine scale permits correction of the deviation from linearity. The index position is controlled by a corrector which is fastened to the condenser shaft. As the corrector is rigidly associated with the sine condenser position and not with the scales this permits shifting the linear scales inde-

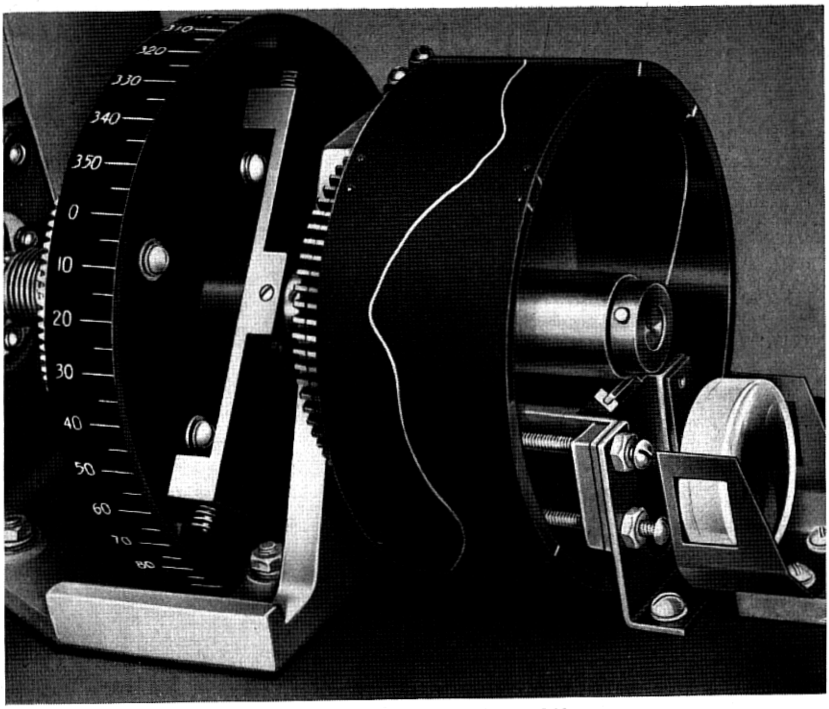


Fig. 10—Optical cam of phase shifter.

pendently without affecting the correction. The correction curve (Fig. 10) is printed on a photographic negative which is placed on a transparent lucite drum and projected optically as an index (Fig. 11) adjacent to the fine dial of the phase-shifter. The calibration curve is obtained by marking the correction at each calibrating point on a piece of cellulose acetate placed on the lucite drum. The correction point is projected on the screen adjacent to the fine scale during the calibration and problems arising from divergence or misalignment of the light beam are thus avoided. Since the index is projected upon a surface coplanar with the dial, no parallax exists.

The phase-shifter's deviation from linearity is sufficiently small that no correction is needed on the coarse scale.

Both dials can be moved with respect to their shafts by releasing friction clutches. Thus the measuring system phase zero can be established by an initial balance of the phase-shifter and restoration of the coarse and fine scales to zero. This is only possible because the scales are linear. Thus no zero readings have to be subtracted from the measurement readings and the need for a separate zero setting phase-shifter is avoided.

The phase-shifter is calibrated by a method of substitution. As discussed previously the phase-indicator indicates balance uniquely in mul-

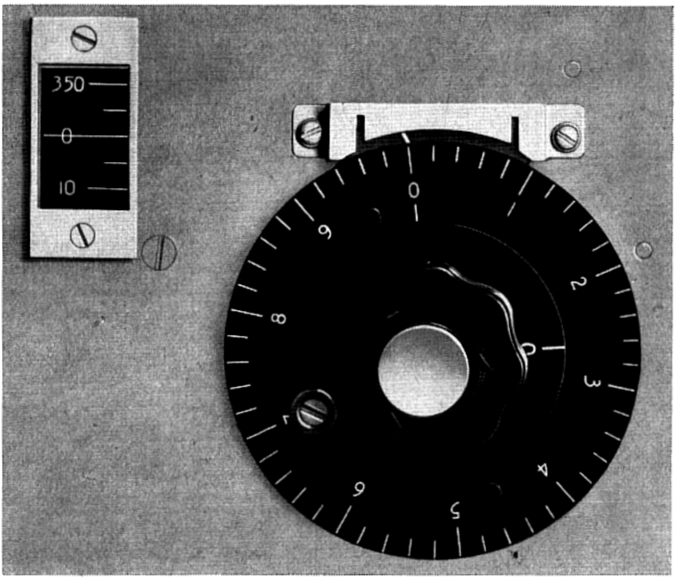


Fig. 11—Phase shifter scales and projected index.

tiples of 360° phase-shift. Exact sub-multiples of 360° can be generated and used to calibrate the phase-shifter.

For example (Fig. 12), to establish an exact 180° phase-shift the standard phase-shifter is set to an arbitrary starting point. With the switches in the position shown a null is obtained on the indicator by adjusting the auxiliary phase-shifter. The network of nominally 180° phase-shift is inserted and a null obtained on the indicator by adjusting the standard phase-shifter. Now the network is removed and the null reestablished by adjustment of the auxiliary phase-shifter. The 180° network again is inserted and a null obtained by adjustment of the standard phase-shifter, which now has been moved through twice the actual phase-shift of the nominal 180° network. The amount the standard phase-shifter failed to return to the original starting point indicates the residual error of the 180° network. The 180° network is adjusted accordingly and the procedure repeated until an error is no longer discernible. Thus the 180° point on the phase-shifter scale can be determined. In similar fashion, by combination of the 180°, 90° and 60° networks, calibration points in multiples of 30° are obtained. The equivalent of a 10° network is obtained by use of the ±5° scale on the indicator and scale factor adjustment. Interpolation to 1° is then made using the scale divisions on the indicator. Calibration to an absolute accuracy of ±0.1° was found adequate for use in the measuring system. Much higher accuracy could be obtained if the need arose. There appears to be no inherent frequency limitation in this calibration method.

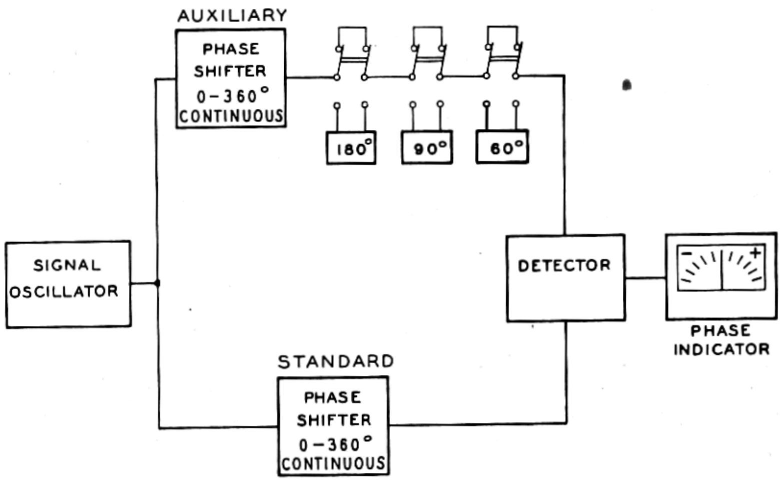


Fig. 12—Phase shifter calibration circuit.

Conclusion

The design effort has been directed toward achieving laboratory precision in measurement and at the same time maintaining the speed necessary for production testing of transmission networks.

The measurement of phase-shift is unambiguous with respect to quadrants and the measurements of insertion phase-shift and loss or gain are independent of each other. The entire frequency range is covered without band switching by use of a heterodyne signal oscillator and the system zero is independent of measurement frequency. Detector tuning is eliminated through the use of frequency conversion, employing a beating oscillator automatically controlled in frequency by the signal oscillator. Phase-shift and transmission may be read directly, without auxiliary computations, from the dials of the phase-shifter and attenuator or from the scales of the indicators.

Acknowledgment

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APPENDIX

The Phase Discriminator Bridge

The general phase relationship of the discriminator is shown in Fig. 13a.

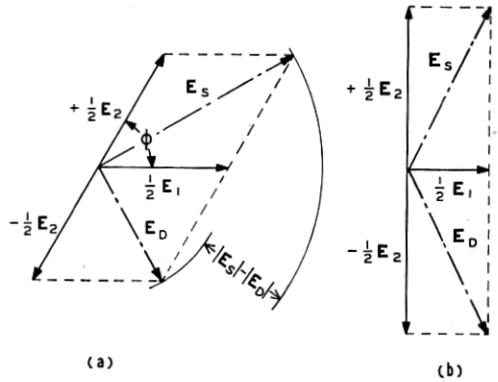

Fig. 13—Phase bridge vector relationships.

Using complex vectorial notation,

$$E_S = (1/2)(E_1 e^{j\varphi_1} + E_2 e^{j\varphi_2}) \tag{4}$$

$$E_D = (1/2)(E_1 e^{j\varphi_1} - E_2 e^{j\varphi_2}) \tag{5}$$

Hence for $\varphi = \pi/2 + n\pi$ where $\varphi = \varphi_1 - \varphi_2$ and n is an integer,

$$| E_S | = | E_D | \tag{6}$$

Stated in words: The amplitudes of E_S and E_D are equal if the relative phase angle φ is equal to 90°, 270°, 450°, etc., independently of the amplitudes of E_1 and E_2 (Fig. 13b).

Inequality of $| E_S |$ and $| E_D |$ can be utilized to measure phase departure from $\varphi = \pi/2 + n\pi$

From (4) and (5),

$$| E_S | = (1/2) \sqrt{| E_1 |^2 + 2 | E_1 E_2 | \cos\varphi + | E_2 |^2} \tag{7}$$

$$| E_D | = (1/2) \sqrt{| E_1 |^2 - 2 | E_1 E_2 | \cos\varphi + | E_2 |^2} \tag{8}$$

If the amplitudes $|E_1|$ and $|E_2|$ are equal, then

$$(9) \qquad |E_S| = (|E_1|/2)\sqrt{2(1 + \cos\varphi)} = |E_1|\cos(\varphi/2)$$

$$(10) \qquad |E_D| = (|E_1|/2)\sqrt{2(1 - \cos\varphi)} = |E_1|\sin(\varphi/2)$$

$$(11) \qquad |E_S| - |E_D| = |E_1|[\cos(\varphi/2) - \sin(\varphi/2)]$$

$$(12) \qquad |E_S|/|E_D| = \operatorname{cotan}(\varphi/2)$$

When $|E_1| \neq |E_2|$ determination of φ by (11) or (12) is in error by $\Delta\psi$. From (7) and (8)

$$(13) \qquad \operatorname{cotan}\frac{\varphi + \Delta\psi}{2} = \sqrt{\frac{|E_1|^2 + 2|E_1E_2|\cos\varphi + |E_2|^2}{|E_1|^2 - 2|E_1E_2|\cos\varphi + |E_2|^2}}$$

Hence

$$(14) \qquad \left|\frac{E_2}{E_1}\right|^2 + 2\,\frac{1 + \operatorname{cotan}^2[(\varphi + \Delta\psi)/2]}{1 - \operatorname{cotan}^2[(\varphi + \Delta\psi)/2]}\left|\frac{E_2}{E_1}\right|\cos\varphi + 1 = 0$$

From trigonometry

$$(15) \qquad \frac{1 + \operatorname{cotan}^2[(\varphi + \Delta\psi)/2]}{1 - \operatorname{cotan}^2[(\varphi + \Delta\psi)/2]} = -\frac{1}{\cos(\varphi + \Delta\psi)}$$

$$(16) \qquad |E_2|/|E_1| = \cos\varphi/\cos(\varphi + \Delta\psi) + \sqrt{[\cos\varphi/\cos(\varphi + \Delta\psi)]^2 - 1}$$

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