On the Theory of the A-C. Impedance of a Contact Rectifier

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THE a-c. impedance of the rectifying contact between a metal and a semiconductor is measured by superimposing a small a-c. current on a d-c. bias current. It is generally recognized[1] that an equivalent circuit consists of a parallel resistance and capacitance in series with a resistance as shown in Fig. 1. The parallel components represent the impedance of the barrier layer itself and depend on the d-c. bias current flowing. The series resistance is that of the body of the semiconductor. It has been shown theoretically by Spenke[2] that under quite general conditions the parallel capacitance and resistance are independent of frequency. Unfortunately Spenke's proof is highly mathematical and is also not readily available. The derivation of the impedance relations which is presented here is in some ways more general and gives more physical insight into the problem.

The method of analysis which is used is similar to that employed by Miss C. C. Dilworth[3] for the d-c. case. Except for some obvious differences in sign, the theory is the same for *n*- and *p*-type semiconductors.[4] We give the theory for the latter because the signs are a little simpler for positively charged holes than for negatively charged conduction electrons. Before the discusssion of the theory of the a-c. impedance, a brief outline of Schottky's theory of the barrier layer will be given.

A rough schematic energy level diagram, based on Schottky's theory of the barrier layer at a contact between a metal and a *p*-type semiconductor, is illustrated in Fig. 2. The diagram is plotted upside down from the usual one in order to show the energy of holes increasing upward. The energy of electrons increases downward. In a defect or *p*-type semiconductor, such as Cu_2O, electrons are thermally excited to acceptor levels, charging the acceptors negatively, and leaving missing electrons or holes in the filled band. The holes are mobile and provide the conductivity. Electron states with energies lying above the Fermi level in the diagram, corresponding to lower energies for electrons, have a probability of more than one-half of

[1] For an outline of the theory of contact rectifiers together with references to the earlier literature, see H. C. Torrey and C. A. Whitmer, "Crystal Rectifiers," McGraw-Hill Book Company, Inc., New York, New York (1948).
[2] Eberhard Spenke, *Wiss. Veroff.* Siemen's Konzern, **20,** 40 (1941).
[3] C. C. Dilworth, *Proc. Phys. Soc.* London, **60,** 315 (1947). A similar method was used earlier by H. A. Kramers, Physica 1, 284 (1940), in a discussion of the diffusion of particles over potential barriers.
[4] We suppose that only one type of carrier takes part in conduction.

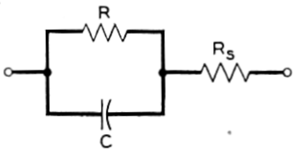

Fig. 1—Equivalent circuit for contact rectifier. The parallel components R and C represent the barrier layer itself and R_s represents the resistance of the body of the semiconductor.

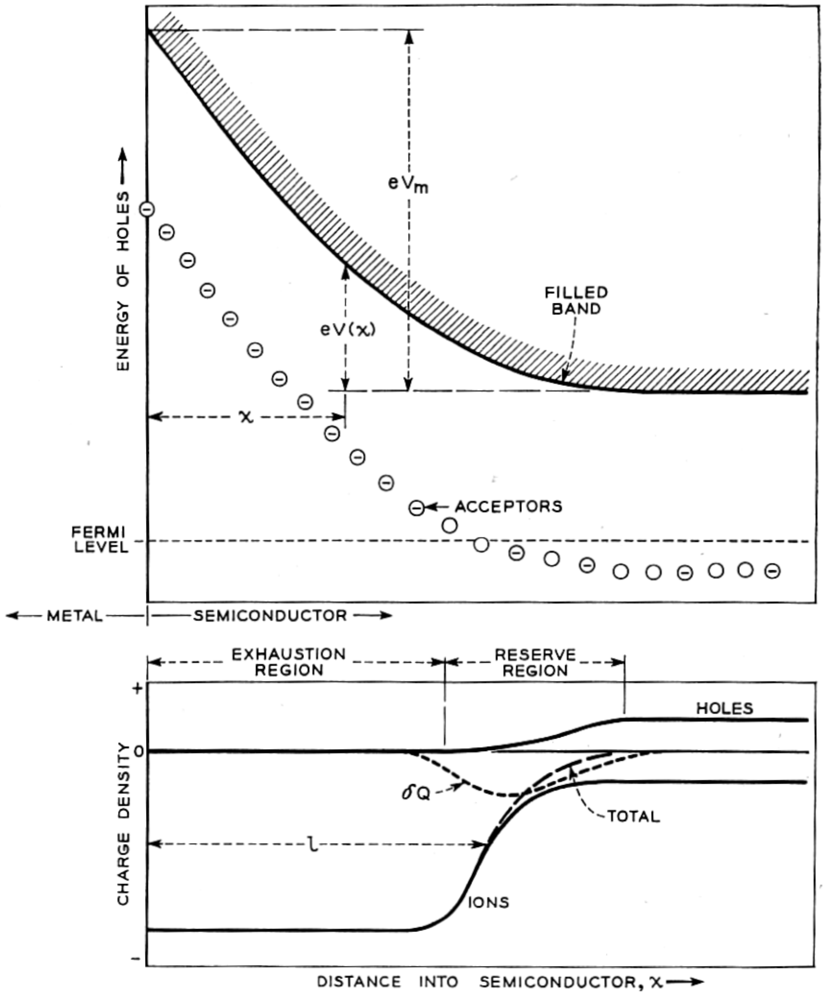

Fig. 2—Schematic energy level diagram of p-type semiconductor in contact with a metal. The diagram is plotted upside down from the usual way in order to show the energy of holes increasing upward. The energy of electrons increases downward. The lower diagram gives the density of charge in the barrier layer. In the body of the semiconductor the space charge of the holes is compensated by the space charge of the negatively charged acceptor ions. Holes are drained out of the barrier layer by the electric field, leaving the negative space charge of the acceptors. The rise in electrostatic potential in the barrier region results from this negative space charge together with the compensating positive charge on the metal. The capacitance of the barrier layer is approximately that of a parallel plate condenser with plate separation l.

being occupied by electrons; those below the Fermi level are most likely unoccupied. Holes are depleted from the barrier layer, leaving the negative space charge of the acceptors. This negative space charge, together with the compensating positive charge on the metal, gives the potential energy barrier which impedes the flow of holes from semiconductor to metal. The thickness of the barrier layer may vary from 10^{-6} to 10^{-4} cm, depending on the materials forming the contact.

In drawing the diagram of Fig. 2 it has been assumed for simplicity that the concentration of acceptors is uniform over the region of interest. In the main body of the semiconductor only a few of the acceptors are charged. Throughout a large part of the barrier layer practically all acceptors are negatively charged and there are very few holes in the filled band. This part of the barrier layer has been called by Schottky the exhaustion region and is in our case a region of uniform space charge, as shown in the lower diagram of Fig. 2. The transition zone in which the concentration of holes is decreasing and the concentration of charged acceptors is increasing is called the reserve region.

In thermal equilibrium, with no applied volatge, the potential drop across the barrier layer, V_m, may be a fraction of a volt. If a voltage is applied in such a direction as to make the semiconductor positive relative to the metal, the effective height of the barrier is reduced and holes flow more easily from the semiconductor to the metal. This is the direction of easy flow. If a voltage is applied in the opposite direction the height of the barrier is increased for holes going from semiconductor to metal and remains unchanged, to a first approximation, for holes going from metal to semiconductor (actually electrons going from the filled band of the semiconductor to the metal). This is the reverse or high resistance direction.

If a voltage is applied in the reverse direction, and equilibrium is established, the thickness of the exhaustion layer increases. The reserve region keeps the same form but moves outward from the metal. A forward voltage decreases the thickness of the space charge layer.

The change in charge density corresponding to a small reverse voltage is shown schematically by the curve marked δQ in the lower diagram of Fig. 2. The maximum of δQ occurs where the total charge density is changing most rapidly with distance. If l is the distance from the metal to this maximum, the effective capacitance C, is approximately that of a parallel plate condenser with plate separation l and with the dielectric constant of the medium equal to that of the semiconductor. The capacitance decreases as l increases with a d-c. bias applied in the reverse direction and the capacitance increases with forward bias. Schottky[5] has shown that information

[5] Walter Schottky, *Zeits f. Phys.* **118**, 539 (1942).

about the concentrations of donors and acceptors can be obtained from the variation of capacitance with bias.

In the equivalent circuit of Fig. 1 the capacitance C is in parallel with the differential resistance, R, of the contact, and the parallel components are in series with the resistance R_s of the body of the semiconductor. Spenke showed that R and C are independent of frequency if the frequency is low enough so that the charge density is in equilibrium during the course of a cycle.

If the applied voltage is suddenly changed, it will take time for the charges to adjust to new equilibrium values. The time constant for the readjustment of charge of the carriers (holes in this case) is $\kappa\rho/4\pi$, where ρ is the resistivity (in e.s.u.) of the body of the semiconductor and κ is the dielectric constant, and is $\sim 10^{-10}$ sec. for a resistivity of 100 ohm cm. Even if a larger value of ρ is used, corresponding to a point in the reserve layer, the relaxation time for the carriers is very short.[6] A much longer time may be required for readjustment of charge on the donor or acceptor ions, giving a variation of R and C at lower frequencies. If the barrier is nonuniform over the contact area, so that much of the current flows through low-resistance patches, the equivalent circuit may consist of a number of circuits like those of Fig. 1 in parallel. In this case, if an attempt is made to represent the contact by a single circuit of this form, it will be found that R and C vary with frequency.

The derivation of the current voltage characteristic for the general case of a time dependent applied voltage follows. The total current per unit area is the sum of contributions from conduction, diffusion, and displacement currents:

$$I(t) = \sigma E - eD(\partial n/\partial x) + (\kappa/4\pi)(\partial E/\partial t), \tag{1}$$

where

$n(x,t)$ = concentration of holes;
$\sigma = n(x,t)e\mu$ is the conductivity;
e = magnitude of electronic charge;
μ = mobility of holes;
$D = \mu kT/e$ = diffusion coefficient;
$V(x,t)$ = electrical potential;
$E(x,t) = -\,\partial V(x,t)/\partial x$ = electric field strength.

The coordinate x extends into the semiconductor from the junction. Equation (1) may be written in the form

$$I(t) = ne\mu(-\partial V/\partial x) - \mu kT(\partial n/\partial x) - (\kappa/4\pi)(\partial^2 V/\partial x\partial t) \tag{1'}$$

[6] Another limit is the transit time of carriers through the barrier layer. This time is generally shorter than the relaxation time of the semiconductor.

The potential V is determined from the charge density, q, by Poisson's equation

$$\partial^2 V/\partial x^2 = -4\pi q/\kappa. \tag{2}$$

Since the charge density may be expressed in terms of $n(x,t)$ and the density of fixed charge, these two equations may be used to determine n and V when $I(t)$ is specified. Spenke eliminates the potential V between (1) and (2) and gets a rather complicated equation for n. We prefer to deal with Eq. (1) directly, to treat the potential $V(x,t)$ as a known function, and to solve for the concentration, $n(x,t)$.

The plane $x = 0$ is taken at the interface between metal and semiconductor and the plane $x = x_1$ just beyond the barrier layer in the semiconductor. It is assumed that $V = 0$ at $x = x_1$. Under thermal equilibrium conditions, with no current flowing, the hole concentration in the barrier layer varies as $exp\ (-eV/kT)$, taking the values:

$$n = n_o \ at\ x = x_1 \tag{3a}$$

$$n = n_m = n_o\ exp\ (-eV_m/kT)\ at\ x = 0, \tag{3b}$$

where n_o is the equilibrium concentration in the body of the semiconductor and V_m is the height of the potential barrier. We suppose that the boundary conditions (3a) and (3b) also hold when a current is flowing and when there is an additional voltage, V_a, across the barrier layer. Our procedure is to solve Eq. (1) for $n(x,t)$, with $V(x,t)$ assumed known, and then to determine $I(t)$ in such a way that the boundary conditions are satisfied. The solution of Eq. (1) which satisfies (3b) is:

$$n(x,t) = n_0 \exp[-e(V - V_a)/kT] - \frac{1}{\mu kT}\int_0^x \left(I + \frac{\kappa}{4\mu}\frac{\partial^2 V'}{\partial x'\,\partial t}\right) \cdot \exp[e(V' - V)/kT]\,dx' \tag{4}$$

The prime indicates that the variable is x' rather than x. At $x = 0$, V is the sum of V_m and the applied potential, V_a:

$$V = V_a + V_m\ at\ x = 0 \tag{5}$$

The current $I(t)$ is determined in such a way that (3a) is satisfied. Setting $x = x_1$, using (3a), and solving the resulting equation for $I(t)$, we get:

$$I(t) = \frac{\mu kT[n_0 \exp(eV_a/kT) - n_0 \exp(eV/kT)] - \int_0^{x_1} \frac{\kappa}{4\pi}\frac{\partial^2 V'}{\partial x'\,\partial t}\exp(eV'/kT)\,dx'}{\int_0^{x_1} \exp(eV'/kT)\,dx'} \tag{6}$$

Provided that the barrier height, $V_m + V_a$, is as much as several times kT/e,[7] the integrand in both integrals is largest near $x = 0$ and drops rapidly with increase in x. Where the integrand is large we may write to a sufficient approximation:

$$V = V_a + V_m - Fx, \tag{7}$$

where F is the field in the semiconductor at the interface. The approximation (7) may be used if kT/eF is small compared with the thickness of the barrier layer. The value of $\partial^2 V/\partial x \partial t$ is nearly constant over the important part of the integration and may be replaced by its value at $x = O$ and taken out of the integral. The upper limit x_1 may be replaced by ∞ without appreciable error, so that we get finally:

$$I(t) = I_m(Q)(1 - exp\,[-eV_a/kT]) + \partial Q/\partial t, \tag{8}$$

where

$$I_m(Q) = (4\pi e\, \mu\, Q\, n_e/\kappa)\; exp\,[-eV_m/kT] \tag{9}$$

and

$$Q = \kappa F/4\pi \tag{10}$$

is the surface charge density at the metal interface.

The current $I_m(Q)$ has a simple interpretation; it is just the conduction current in the semiconductor at the interface resulting from the field F. In equilibrium, this conduction current is balanced by a diffusion current of equal magnitude and opposite sign. A voltage V_a applied in the reverse direction reduces the diffusion current at the interface as compared with the conduction current by the factor $exp\,[-eV_a/kT]$. The current $\partial Q/\partial t$ is the displacement current at the interface.

Actually, the diffusion theory as given above is not complete. The Schottky effect, the lowering of the barrier by the image force, has been neglected. There may be appreciable tunneling through the barrier. There may be a patch field resulting from nonuniformity of the barrier. If the variations in the patch fields are not too large, the modification of current resulting from these factors depends only on the field at the metal and not on the form of the barrier at some distance from the metal. Thus we may expect the form (8) to be generally valid if $I_m\,(Q)$ is considered to be a general function of Q. Equation (10) is also of the form to be expected from the diode theory.[1] In the latter case, $I_m(Q)$ is the thermionic emission current from metal to semiconductor.

If the current is varying in time it is the instantaneous value of Q at

[7] The value of kT/e at room temperature is .025 volts.

time t which is to be used in Eq. (10). At high frequencies, the charge at the interface need not be in phase with the applied voltage. If the frequency is low enough so that the charges maintain their equilibrium values during the course of a cycle, Q will be in phase with V and the parallel capacitance for unit area is simply:

$$C = dQ/dV. \tag{12}$$

The barrier layer may be represented by this capacitance in parallel with the d-c. differential resistance, R.

Both R and C may depend on the d-c. bias current flowing. Variations of R and C with frequency at moderate frequencies may result from large scale nonuniformities of the barrier such that the patch fields extend over a large fraction of the thickness of the barrier layer or from charge relaxation times associated with acceptors, donors or trapped carriers. At low frequencies, drift of ions may be involved.

Attempts which have been made to determine the variation of resistivity in the barrier layer from impedance data are invalid. It is not correct to take the impedance of an element of thickness dx to be

$$dx/[\sigma(x) + (j\omega\kappa/4\pi)]$$

and integrate over dx to obtain the impedance of the layer. This procedure omits terms arising from diffusion and changes of concentration in time. It is possible to obtain an integral of Eq. (1′) if both sides are divided by $ne\mu$. Integrating over x from $x = 0$ to $x = x_1$, and using the boundary conditions (3a), (3b) and (5), we get

$$V_a = \int_0^{x_1} \frac{I(t) + (\kappa/4\pi)(\partial^2 V/\partial x\, \partial t)}{ne\mu}\, dx, \tag{13}$$

which means that the integral of the conduction current over the conductivity gives the applied voltage. This is consistent with the representation of the barrier by a resistance and capacitance in parallel.

Acknowledgments

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