A Broad-Band Microwave Noise Source

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Measurements of the microwave noise power available from gaseous discharges, such as in an ordinary fluorescent lamp, show remarkable uniformity and stability. Such tubes are therefore suitable for a new type of standard noise source.

Introduction

A STANDARD noise source, such as a hot resistance or a temperature limited diode, has been used advantageously for making measurements of the noise figure of radio receivers in the short-wave and the ultra-short wave region. The use of such a tool eliminates the possible errors which are practically inescapable when using the large amounts of attenuation which are needed for the determination of the ratio of power levels encountered in measuring noise figures with a standard signal generator. For example, the power from a standard signal generator might be measurable and known accurately at a level of 40 db below a watt, whereas the noise power available from a resistance might be 141 db below one watt.[1] It is difficult to ascertain accurately power ratios of this magnitude, 10^{10}.

Another advantage of using a standard noise source arises from the fact that ordinarily the bandwidth of the receiver need not be considered, thereby eliminating a time consuming measurement. This assumes, of course, that the bandwidth of the noise source is much greater than that of the amplifier under test.

In the microwave region it is possible to match a resistive element to the waveguide over a wide enough band, but ordinary resistive materials will not stand the high temperatures (5000 degrees or more) needed to measure the noise figures encountered in practice. The noise diode is capable of furnishing adequate noise power, but one with wide bandwidth has yet to be developed. A good, stable, broadband microwave noise generator is needed.

Another possible source of noise power consists of a gaseous discharge.[2] Before we examine the data which have led us to conclude that the gaseous discharge is a good, broad-band, stable microwave noise generator and possibly a calculable noise standard, we review our definitions of noise figure

[1] This figure, 141 db below one watt, assumes that the effective bandwidth is 2 mc. The resistance noise power available from a generator at 290° Kelvin is 204 db below one watt per cycle.

[2] G. C. Southworth, *Journal of the Franklin Institute*, Vol. 239, #14, pp. 285-298, April 1945.

and gain,[3] and discuss the factors involved in making noise figure measurements by means of a noise source.

Notes on Noise Figure

Definition: The Noise Figure of a network, with a generator connected to its input terminals, is the ratio of the available signal-to-noise power ratio at the signal generator terminals (weighted by the network bandwidth) to the available signal-to-noise power ratio at its output terminals.

Definition: The Gain of a network is the ratio of the available signal power at the output terminals of the network to the available signal power at the output terminals of the signal generator.

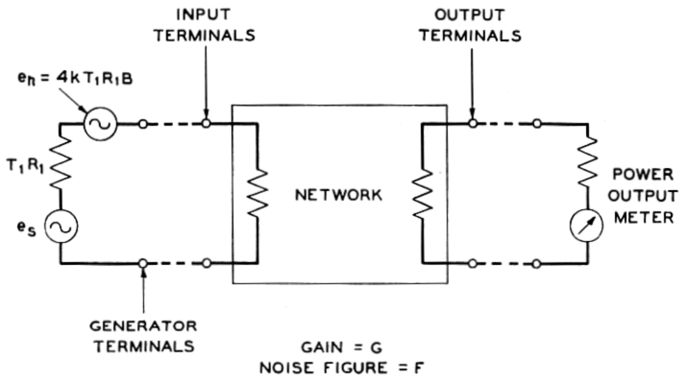

Fig. 1—Schematic diagram of generator, network and output power meter.

These definitions apply to a circuit consisting of a generator, a network and an output power meter as shown schematically in Fig. 1. The signal power available from the generator, having an open circuit voltage e and an internal resistance R_1, is:

$$P_{SA} = \frac{e^2}{4R_1} \tag{1}$$

The noise power available from the signal generator resistance, R_1, at absolute temperature T_1, is

$$P_{NA} = \frac{4KT_1R_1B}{4R_1} = kT_1B \tag{2}$$

where B is the effective bandwidth of the network, by which the generator noise is weighted in this case.

[3] H. T. Friis, *Proc. I. R. E.*, Vol. 32, #17, pp. 419–422, July, 1944.

The weighted available signal-to-noise ratio at the generator terminals is:

$$\frac{P_{SA}}{P_{NA}} = \frac{\frac{e^2}{4R_1}}{KT_1B} \tag{3}$$

The network amplifies (or attenuates) the generator's signal power by the factor G, the gain of the network, so that the available signal power at the output terminals of the network is:

$$P_{SO} = G\,\frac{e^2}{4R_1} \tag{4}$$

The network amplifies (or attenuates) the generator noise power by the same factor G, and also delivers noise power which originates within itself, N_N, so that the total available noise power at the output terminals of the network is:

$$P_{NO} = GkT_1B + N_N \tag{5}$$

The available signal-to-noise ratio at the output terminals of the network is then:

$$\frac{P_{SO}}{P_{NO}} = \frac{G\frac{e^2}{4R_1}}{GkT_1B + N_N} \tag{6}$$

We now express the noise figure of the network, F, which by definition is the ratio of equation (3) to equation (6), thus,

$$F = \frac{GkT_1B + N_N}{GkT_1B} \tag{7}$$

We should pause at this point to consider this equation further, for it leads us to a simpler definition of noise figure.

Definition: The noise figure of a network is the ratio of the noise power output of that network to the noise power output which would exist if the network were noiseless. The temperature of the signal generator resistance is 290 degrees Kelvin.

The choice of generator temperature of 290 degrees is an arbitrary one, which makes $kT_1 = 4(10)^{-21}$ watts per cycle bandwidth; $-10 \log kT_1 =$ 204 db below one watt per cycle. Putting $T_1 = 290$ in equation (7) gives:

$$F = \frac{Gk\,290\,B + N_N}{Gk\,290\,B} \tag{8}$$

Rearranging (8) we have:

$$N_N = (F - 1)Gk\,290\,B \tag{9}$$

Equation (9) will now be used to illustrate one method of measuring noise figures. In this method, the network output noise power is measured for two known values of the temperature of the generator resistance, T_2 and T_1. When the generator is hot, the output noise power is, by equation (5):

$$P_{NOH} = GkT_2B + N_N \tag{10}$$

When the generator is cool, the output noise power is:

$$P_{NOC} = GkT_1B + N_N \tag{11}$$

Calling the ratio of these two noise powers Y:

$$Y = \frac{P_{NOH}}{P_{NOC}} = \frac{GkT_2B + N_N}{GkT_1B + N_N} \tag{12}$$

Substituting for N_N the value given in equation (9), we have for the noise figure:

$$F = \frac{\left(\frac{T_2}{290} - 1\right) - Y\left(\frac{T_1}{290} - 1\right)}{Y - 1} \tag{13}$$

In practice T_1 is often near enough to 290 degrees so that the second term in the numerator of equation (13) is negligible. Setting T_1 equal to 290 degrees, equation (13) becomes:

$$F = \frac{\frac{T_2}{290} - 1}{Y - 1} \tag{14}$$

The determination of noise figure by this method is independent of the gain of the network, the degree of mismatch and the bandwidth, provided that the band of the noise source is broad compared with the overall RF band of the network and the output power meter.

The Noise Source

The limitations at microwaves of a noise source such as a heated wire will now be discussed. In particular we are interested in measuring amplifiers which have noise figures between 10 and 100 (10 db to 20 db) and bandwidths up to 200 mc. If a hot wire could be matched to the impedance of a waveguide over a wide enough band, and raised to a temperature of 10×290 degrees our Y factor would be (rearranging eq. 14):

$$Y = \frac{\frac{T_2}{290} - 1}{F} + 1 \tag{15}$$

and setting $T_2 = 2900$ degrees Kelvin

$$Y = 1.9 \text{ for } F = 10$$

$$Y = 1.09 \text{ for } F = 100$$

Assuming that Y can be read to within $\pm 1\%$ our accuracy in determining F would be within about $\pm 1\%$ for $F = 10$ but only within about $\pm 10\%$ for $F = 100$. If the noise source had a temperature of 40×290 degrees, our experimental errors would be reduced accordingly to about $\pm 1/4\%$ for $F = 10$ and $\pm 2.5\%$ for $F = 100$. Since metal wires will not stand such temperatures, we must look to something different for the noise source if these accuracies are to be achieved.

In view of the foregoing considerations, the nonoscillating reflex klystron presented one possibility of a suitable microwave noise source. This, however, was not exploited because the bandwidth was not wide enough.

Another possibility was found to be an electrical gas discharge. This type of source was determined to generate noise at microwave lengths when the open end of the input-waveguide of a sensitive microwave receiver was directed toward various gaseous discharge tubes, including a 721A TR tube containing water vapor and hydrogen, a neon light in a stroboscope, a mercury vapor rectifier and an ordinary fluorescent desk lamp. Of these, the commercial fluorescent lamp appeared to lend itself most readily to mounting in a waveguide without the complication of the effects of the internal metal electrodes, so further tests were performed on it.

Microwave Measurements

A T-5, 6-watt, daylight fluorescent lamp,[4] lighted from a d-c. source, was mounted with its axis parallel to the magnetic vector in a waveguide as illustrated in Fig. 2. The lamp itself was 9″ long, with cathodes at each end. These could be isolated from the field in the 1″ x 2″ waveguide by enclosing the portion of the lamp which extended beyond the walls of the waveguide in cylindrical metal shields which formed waveguides beyond cutoff. Thus, energy was kept from reaching the cathodes, and the noise source was effectively confined to that part of the discharge which appeared inside the main waveguide. A piston in back of the gaseous discharge tube served to tune out the susceptance and a trimming screw provided an additional adjustment. The conductance could be adjusted by varying the direct current.

The admittance of the combination could be adjusted for an impedance

[4] A commercial fluorescent lamp contains about two mm. of argon and six to ten microns of mercury gas. The argon merely facilitates the initiation of the discharge; the mercury furnishes the radiation which excites the fluorescent material.

match at any operating frequency from 3700 mc to 4500 mc. The admittance diagram when the circuit was adjusted for match at 3960 mc is shown in Fig. 3; the standing wave ratio was less than 2.9 db from 3700 to 4240 mc.

At 3960 mc the conductance of the gaseous discharge varied directly with the direct current, while the negative susceptance had a broad maximum of $-j.62\ Y_0$ mhos at a current of 65 to 100 milliamperes, as shown in Fig. 4. These values are for the gaseous discharge; the susceptances of the enclosing glass tubing, the back piston and the holes in the sidewalls have been subtracted from the measured results. It is interesting to note that the discharge appears to be inductive.

The waveguide circuit containing the gaseous discharge tube was connected to the input waveguide of a sensitive microwave receiver which was used as a relative noise power meter. The noise power available from the

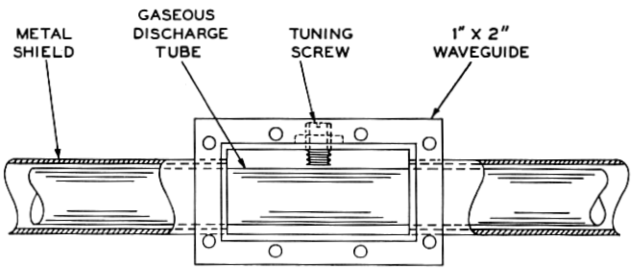

Fig. 2—Waveguide circuit for microwave noise generator using a gaseous discharge tube.

gaseous discharge was substantially independent of the direct current from 40 ma to 140 ma. These data are plotted in Fig. 5, which gives $10 \log\left(\frac{T}{290} - 1\right)$ versus direct current in milliamperes. The ordinate has been chosen so as to conform with absolute measurements made subsequently. The r.m.s. deviation from the straight line which represents a probable coefficient of only $-.003$ db per milliampere was about $\pm.05$ db. We do not claim to be able to achieve even this degree of accuracy with our present measuring equipment and hence do not place much confidence in the numerical value of this coefficient. Actually the decrease in noise with increasing current may have been associated with a change in the ambient temperature rather than with the increased current density. At least it is in the right direction for this to be the case.

The temperature coefficient of the noise from the discharge was found to be negative; when a piece of dry ice was held on the tubular shield of the circuit for a few minutes (long enough for frost to form on the brass) the output noise power of the discharge increased 0.6 db. The circuit was heated on a hot plate and allowed to return to room temperature gradually, then cooled with an air stream and allowed to warm up gradually while the output noise and the temperature of the waveguide were being recorded. This revealed the temperature coefficient of −.055 db per degree centigrade. The data (plotted in Fig. 6) show an r.m.s. deviation of ±.114 db from this coefficient.

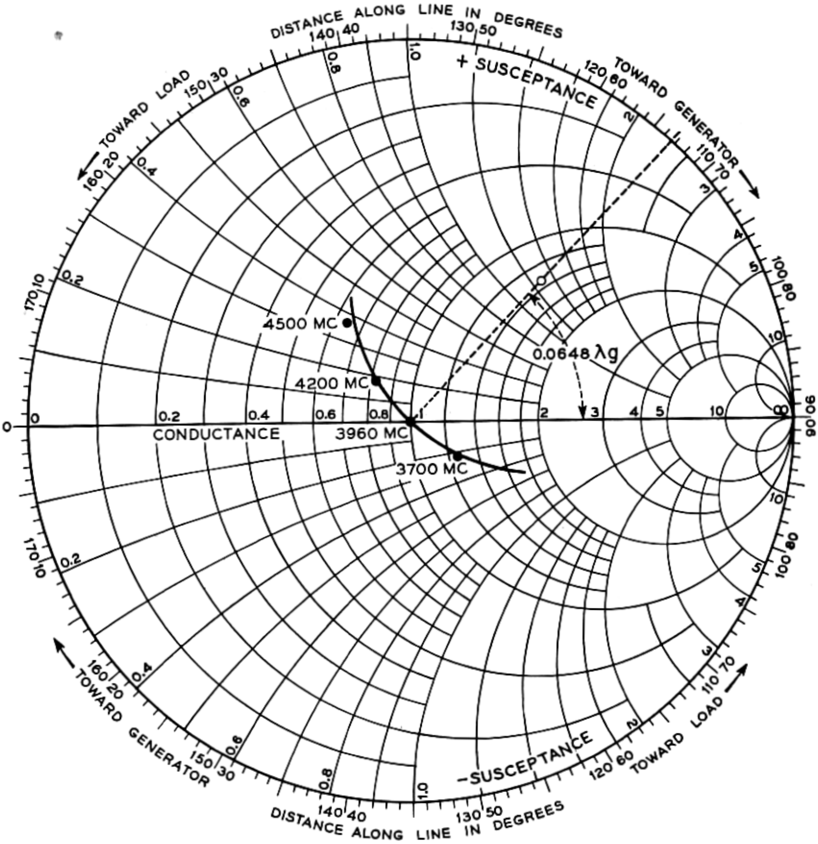

Fig. 3—Admittance diagram of microwave noise generator.

The ambient temperature of the waveguide circuit had very little effect on the admittance of the gaseous discharge.

As a check on variability with respect to time, two of these noise sources were compared, one against the other, at five-minute intervals for 65 minutes. During this time the waveguide temperature of source #1 rose from 34° to 35.2° C and that of source #2 rose from 33.7° to 36.1°. Each comparison was corrected, according to the coefficient of −.055 db per degree C and the observed temperature, to a common temperature of 34° C. Assuming that the noise figure of the microwave receiver was constant, source #1

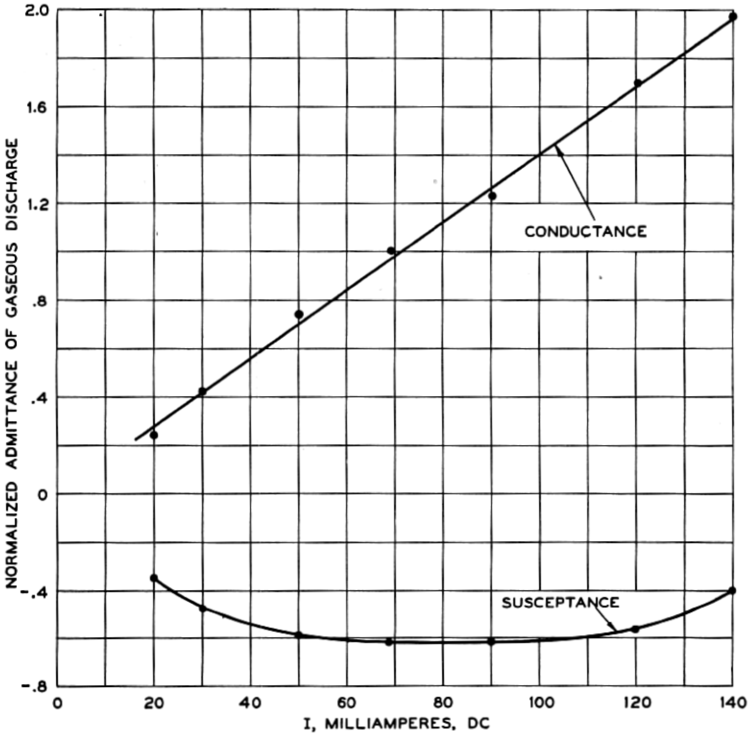

Fig. 4—Admittance of the gaseous discharge at 3960 mc as a function of the direct current in the discharge.

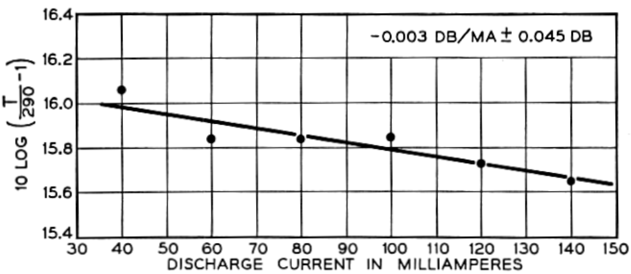

Fig. 5—The microwave noise power is practically independent of the discharge current.

showed variations whose r.m.s. deviation was ± 0.11 db, while source # 2 had similar deviations of ±.092 db. Assuming on the other hand that source #1 held constant and that the microwave measuring set varied with time, source #2 displayed r.m.s. deviations of ±.088 db. These variations are in fact comparable with the probable experimental error, and the proof that they actually exist at all still remains to be demonstrated.

Of thirty-two different lamps, including 10 different types of fluorescent coatings such as used in the pink, red, gold, soft white, daylight, green, white, 4500° white, black light and blue, thirty-one[5] were all within ±0.25 db of each other as was also a germicidal lamp with no fluorescent coating. Thus it appears that the source of the microwave noise energy lies chiefly in the gaseous discharge rather than in the fluorescent coating.

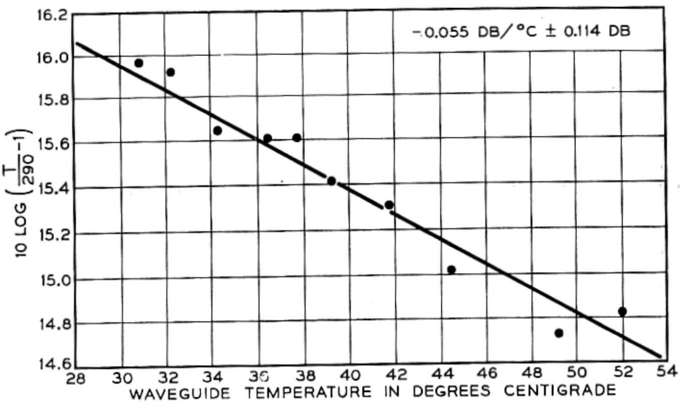

Fig. 6—The microwave noise power depends slightly upon the temperature of the waveguide circuit.

If this noise is tied up with the electron temperature of the discharge, we should expect the noise to be flat, or "white" noise. Corroborative evidence of this was observed when the spectrum of the noise was examined over the band from 3700 to 4500 mc at points 20 mc apart and no irregularities were found. The nature of the experiment was such that frequency bands of excessive noise power would have been observed had they been present. Further tests should indicate whether or not a gradual change in noise with frequency exists. It appears, however, unlikely that such a slope exists at 4000 mc.

Furthermore, since the level of the noise energy is so constant with respect to time, reproducible from tube to tube, practically independent of the current and only slightly affected by the ambient temperature, we might expect that it is being controlled or limited by some invariant physical property of the atoms and ions within the gaseous discharge. If this is the case, an absolute measurement of the noise power might lead us to some

[5] One of the 32 lamps flickered erratically. At times its noise was ½ db higher than the average.

theoretical explanation which, when applied to the case in hand, would explain the observed results qualitatively and quantitatively, thereby establishing a new absolute standard noise source for microwave measurements.

The microwave noise power from such a discharge tube was measured at 3950 mc in cooperation with Mr. C. F. Edwards on his calibrated measuring set on two different occasions, 16 days apart. The values obtained were 15.86 db and 15.80 db respectively for 10 log $\left(\frac{T}{290} - 1\right)$.[6] This places the temperature, T, in the neighborhood of 11,400 degrees Kelvin. It is believed that the absolute measurements are correct to within $\pm.25$ db or better.

Having determined the temperature of this noise source, we might ask, "If we should terminate our waveguide in a black body at 11,400 degrees, how much microwave noise power would we get from it?" The black body radiates with three polarizations, only one of which is propagated along the waveguide, and this available power is given by Nyquist:[7]

$$P_{NA} = \frac{hfB}{e^{hf/kT} - 1} \tag{16}$$

where $h = 6.61\ (10)^{-34}$ joule sec.
$k = 1.381\ (10)^{-23}$ joule/deg.
f = frequency in cycles per sec.
B = bandwidth in cycles per sec.

At 4000 mc, $\frac{hf}{kT}$ is, for $T = 290$ degrees, 6.6 $(10)^{-4}$ which is so small that the denominator of (16) can be replaced by $\frac{hf}{kT}$. This gives us the familiar expression for thermal noise:

$$P_{NA} = kTB \text{ watts} \tag{17}$$

In other words, thermal noise is black body radiation with but one polarization.

Going one step further we might also ask the question, "If we should examine the radiation from this black body with an optical spectroscope, at what wavelength would we find its maximum radiated energy?" The spectroscope detects radiation having three polarizations, and Planck's radiation law applies. From Wien's displacement law, the wavelength of maximum radiation is given by the relation:

$$\lambda_m T = 0.289 \text{ cm deg.} \tag{18}$$

[6] The temperature of the waveguide was 32°C when these values were measured.
[7] H. Nyquist, *Phys. Rev.*, Second Series, Vol. 32, pp. 110–113, July 1928.

Substituting $T = 11{,}400$ degrees,

$$\lambda_m = 2535\ (10)^{-8}\ \text{cm} \tag{19}$$

This is indeed an interesting result, since the mercury vapor discharge in the fluorescent lamp radiates most of its energy at $\lambda = 2536.52\ (10)^{-8}$ cm. The design of the lamp was guided by the effort to accentuate the radiation at this wavelength, and the manufacturers state that this has been achieved so that no other spectral line is excited to radiate more than two percent of the input power.[8] The conversion loss from dc to 2536 $(10)^{-8}$ cm is only 2 or 3 db.

The striking similarity between the black body and the mercury vapor discharge at these two wavelengths, 7.6 cm and 2536 $(10)^{-8}$ cm, suggests the following hypothesis:

Hypothesis: In a gaseous discharge which is radiating light energy substantially monochromatically at a particular wavelength, λ_m, the microwave noise energy is the same as that available from a black body which radiates its maximum energy at that wavelength.

Applying this hypothesis to the case in hand, where λ_m is 2536.52 $(10)^{-8}$ cm, and using Wien's displacement law (eq. 18) we calculate the temperature to be

$$T = \frac{0.289}{2536.52} = 11{,}394^\circ \tag{21}$$

$$\frac{T}{290} = 39.29$$

$$\left(\frac{T}{290} - 1\right) = 38.29 \tag{22}$$

$$10 \log \left(\frac{T}{290-1}\right) = 15.84\ \text{db} \tag{23}$$

Since this calculated value is so close to the measured values of 15.8 db and 15.86 db, it will be assumed to be correct until proved otherwise.

CONCLUSIONS

A commercial fluorescent lamp is a reliable source of microwave noise energy. At 4000 mc its effective temperature is 11,394 degrees Kelvin which is convenient for measuring noise figures of 20 db or less. The noise power is practically independent of the fluorescent coating, the current density and only slightly affected by the room temperature. The lamp lends itself readily to a broad-band impedance match in the waveguide.

[8] G. E. Inman and R. N. Thayer, *A. I. E. E. Transactions*, Vol. 57, pp. 723–726, Dec. 1938.