A Gun for Starting Electrons Straight in a Magnetic Field

By J. R. PIERCE

In a simple electron gun consisting of a cathode and two apertured planes held at different potentials the apertures act as electron lenses. When the gun is immersed in a uniform axial magnetic field the aperture spacings and potentials can be chosen so that the emerging electrons have no radial velocities.

IN 1931 Davisson and Calbick showed[1] that a circular aperture in a conducting plate which separates regions with different electric gradients normal to the surfaces acts as an electron lens of focal length F given by

$$F = \frac{4V}{V_2' - V_1'} \tag{1}$$

Here V is the potential of the plate with respect to the cathode which supplies the electrons and V_2' and V_1' are the electric gradients on the far and near sides of the aperture respectively.

When an electron beam is produced by means of a plane cathode and an opposed plane positive apertured anode, the fields about the anode aperture form a diverging lens and cause the emerging beam to spread. Sometimes this is very undesirable. A strong uniform magnetic field parallel to the direction of electron flow may be used to reduce such spreading of the beam, as well as the spreading caused by space charge and by thermal velocities.

The magnetic field does not completely overcome the widening of the beam caused by the lens action of the anode aperture, for the radial velocities which the electrons have on emerging from the aperture cause them to spiral in the magnetic field, and the beam produced is alternately narrow and broad along its length.

This paper describes an electron gun consisting of a cathode and two apertured plates together with a uniform axial magnetic field. The gun is designed so that the net lens action is zero and the electrons emerge traveling parallel to the magnetic field.

The electrode system is shown in Fig. 1. The electrons travel from the plane cathode to the aperture in plane electrode A_1 in parallel lines. At A_1 they receive a radial velocity approximately v_{r1}, given by

$$v_{r1} = -\frac{r}{F_1} v_1 \tag{2}$$

[1] C. J. Davisson and C. J. Calbick, "Electron Lenses," *Phys. Rev.*, vol. 38, p. 585, Aug. 1931; vol. 42, p. 580, Nov. 1932.

Here r is the radial position of the electron, F_1 is the focal length of the lens at A_1, and v_1 is the longitudinal velocity at A_1.

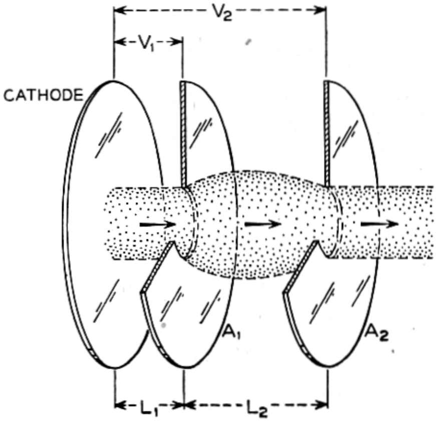

Fig. 1—The gun consists of a planar cathode and two apertured plane electrodes A_1 and A_2, with the spacings and the voltages with respect to cathode which are shown above.

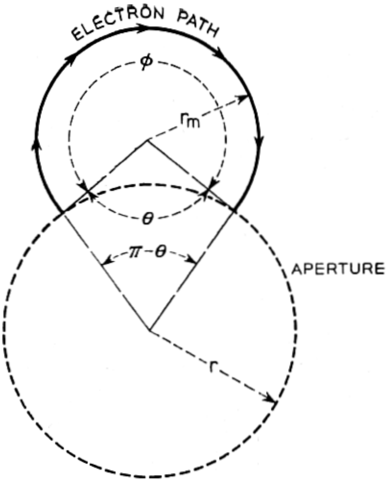

Fig. 2—Between electrodes A_1 and A_2 of Fig. 1, an electron path as seen looking parallel to the axis is a sector of a circle of angular extent Φ.

The magnetic field strength is so adjusted as to return the electrons to the radius r at A_2. Figure 2 shows the motion of an outer electron between A_1 and A_2, seen looking along the axis. Since there is no radial electric field between A_1 and A_2, the electron will move in a circular arc of some radius r_m, and at A_2 the radial velocity will be equal and opposite to that at r_1; that is, it will be $-v_{r1}$.

The change in radial velocity of the electron in passing through the aperture in A_2, v_{r2}, is

$$v_{r2} = -\frac{r}{F_2} v_2 \tag{3}$$

where F_2 is the focal length of the lens at A_2 and v_2 is the longitudinal electron velocity at A_2. F_2 is made such that

$$v_{r2} = v_{r1} \tag{4}$$

Hence, the radial velocity $-v_{r1}$ of the approaching electrons is overcome in passing through the aperture in A_2 and the electrons move parallel to the axis to the right of A_2.

For temperature-limited emission and small space charge, we may assume a uniform gradient between the cathode and A_1, and between A_1 and A_2. Further, we may use the relation

$$v_2/v_1 = \sqrt{V_2/V_1} \tag{5}$$

From (1)–(5) we easily find that the required relation between L_1, the spacing from cathode to A_1, L_2, the spacing between A_1 and A_2, and V_1 and V_2, the potentials of A_1 and A_2 with respect to the cathode, is

$$L_2/L_1 = (\sqrt{V_1/V_2} + 1)(V_2/V_1 - 1) \tag{6}$$

In case of space-charge-limited emission, the space charge will cause the gradient to the left of A_1 to be $\frac{4}{3}$ times as great as in the absence of space charge. If space charge is taken into account in this region only, L_2/L_1 as obtained from (6) should be multiplied by $\frac{3}{4}$.

We have still to determine the magnetic field required to return the electrons leaving A_1 at a radius r to the radius r at A_2.

From Fig. 2 we see that the electrons turn through an angle Φ. Since the angular velocity of electrons in a magnetic field is $(e/m)B$,

$$\Phi = (e/m)B\,\tau \tag{7}$$

where τ is the transit time between A_1 and A_2.

As the electron moves between A_1 and A_2 with a constant acceleration

$$\tau = \frac{2L_2}{v_1 + v_2}$$

$$\tau = \frac{2L_2}{\sqrt{2(e/m)V_2}\,(1 + \sqrt{V_1/V_2})} \tag{8}$$

Now, from Fig. 2 we see also that

$$r_m \sin(\theta/2) = r \sin(\pi/2 - \theta/2) = r \cos(\theta/2)$$
$$\tan(\theta/2) = r/r_m$$

Now

$$\theta = 2\pi - \Phi$$

so

$$\tan(\pi - (\Phi/2)) = r/r_m$$
$$\tan(\Phi/2) = -r/r_m \tag{9}$$

For circular motion with an angular velocity $(e/m)B$ and a circumferential speed $v = v_{r1} = v_{r2}$, the radius of motion r_m is

$$r_m = v_{r2}/(e/m)B \tag{10}$$

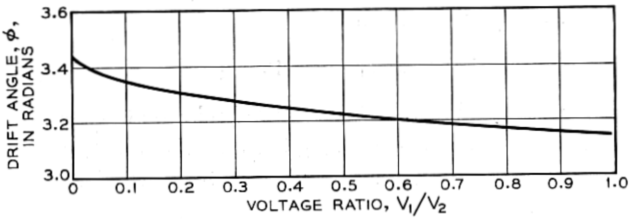

Fig. 3—The ratio L_2/L_1 of the electrode spacings shown in Fig. 1 should satisfy equation (6). When this is so, the angle Φ, measured in radians, is a function of the voltage ratio V_1/V_2, and this function is shown above.

From (1), (3), (9) and (10) we obtain

$$\tan(\Phi/2) = -\left[\frac{(e/m)BL_2}{\sqrt{2\frac{e}{m}V_2(1+\sqrt{V_1/V_2})}}\right]\frac{4}{(1-\sqrt{V_1/V_2})}$$

From (7) and (8) we see that this may be written

$$\tan(\Phi/2) = -(\Phi/2)\frac{4}{(1-\sqrt{V_1/V_2})}$$

$$V_1/V_2 = (1 + 4(\Phi/2)/\tan(\Phi/2))^2 \tag{11}$$

We note that Φ must lie in the third or fourth quadrant. In Fig. 3, Φ is plotted vs. V_1/V_2.

We now have both L_2/L_1 and Φ expressed in terms V_1/V_2, by (6) and (11). From Φ and L_2 we can obtain the proper value of B from (7) and (8)

$$B = \Phi/(e/m)T$$

$$B = (\Phi\sqrt{V_2}/L_2\sqrt{2e/m})(1 + \sqrt{V_1/V_2}) \qquad (12)$$

We see from Fig. 3 that there will be little error in assuming that $\Phi = \pi$.

If we assume complete space charge between the cathode and A_1 and neglect space charge between A_1 and A_2, nothing is altered save the ratio L_2/L_1; as was explained previously, this becomes $\frac{3}{4}$ times the value given by (6).

In the case of slits L_2/L_1 is the same function of V_2/V_1 as for apertures; the correction for space charge is the same, and (12) will give the correct magnetic field with $\Phi = \pi$.