Intermodulation Interference in Radio Systems

Frequency of Occurrence and Control by Channel Selection

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Intermodulation interference becomes a serious factor in frequency usage when a block of consecutive channels is provided for a given type of radio service in a confined area. Formulas are presented which show the number of potentially interfering 3rd and 5th order intermodulation products that can be formed in a band of n consecutive channels. The probability of encountering interference when a number of operating channels are picked at random from this band of n channels is developed and the number of interference free operating channels that can be obtained by careful selection in this same band is also derived.

When a block of consecutive radio channels is used in a confined area to provide a given type of service, interference becomes a serious problem. The situation is aggravated by the fact that whenever energy at two or more radio frequencies combines in a nonlinear circuit, as in transmitter output stages or in receiver input stages, products at other than the original frequencies are created. These are called intermodulation products, and they are capable of causing serious interference within the block of channels assigned to a given type of service as well as in other bands assigned to other types of service.

It is important in engineering a service to know something about the nature of these products in order to evaluate their interference potentialities and to study means of controlling or minimizing that interference. The numbers and locations of various types of intermodulation products are susceptible to mathematical computation. Whether or not all of these products would produce actual interference depends on the geographical locations of transmitters and receivers, and on their specific electrical characteristics. This paper discusses the number of potential interferences, and in effect, envisages a situation where potential interferences are strong enough to be actual interferences.*

GENERAL

Intermodulation products are commonly referred to as 2nd, 3rd, 4th ··· nth order products depending on the order of nonlinearity which gives rise to the products. Interference within a system is not generally experienced from the even order products because the frequency separation between the channels involved and the product formed by them is so great that the selectivity of the transmitter and receiver radio frequency circuits is sufficient to reduce it to a negligible amount. Some of the odd order products can be discounted also for the same reason. There are odd-order products, however, involving both sums and differences of operating frequencies in such fashion that the frequencies of the products formed are very close to those which generated them. These products are those referred to throughout the remainder of this paper since they are the most likely to cause interference. The most general form of 3rd order interference occurs when three frequencies, A, B and C, intermodulate in such fashion as to produce interference on a channel operating at frequency D. In this case

$$A + B - C = D$$

Another form of 3rd order interference occurs when the second harmonic of A intermodulates with B to produce interference on a channel operating at frequency C. In this case

$$2A - B = C$$

In like fashion the following forms of 5th order interference may occur.

$$A + B + C - D - E = F$$

$$2A + B - C - D = E$$

$$A + B + C - 2D = E$$

$$2A + B - 2C = D$$

$$3A - B - C = D$$

$$3A - 2B = C$$

* Bullington, K., Frequency Economy in Mobile Radio Bands. Page 42 of this issue.

NUMBER OF INTERMODULATION PRODUCTS AND FREQUENCY BAND AFFECTED

It is of interest to know how many intermodulation products can be produced by a block of n uniformly spaced channels and where they will fall with respect to the frequency band occupied by the n channels.

Third Order Products

When all possible combinations of n uniformly spaced operating channels are activated three at a time, $n^2(n - 1)/2$ products will be formed and they will lie between $A - (n - 1)$ and $A + 2(n - 1)$ where the operating band lies between channels A and $A + (n - 1)$. This means that the bandwidth of the intermodulation products is very nearly three times that of the operating channels. The products are symmetrically distributed with respect to the midpoint of the operating band. There will be

$$\frac{2n^3 + 3n^2 - 2n - 6}{24}$$

third order products that will fall in the $n - 1$ channels immediately below the operating band and a like number of products that will fall in the $n - 1$ channels immediately above the operating band. The remaining products,

$$\frac{4n^3 - 9n^2 + 2n + 6}{12}$$

in number, will fall in the n channels that constitute the operating band. Not all of these products, however, are capable of producing interference since some products of the $A + B - C$ type can fall on the very transmitting channels that combine to produce them. These products are harmless since they do not fall on receiving channels and are generally of much lower level than the carrier on the channels in which they do fall. If such products are not counted, there remain

$$\frac{n}{3}(n - 1)(n - 2)$$

products which fall within the operating band. The formulas presented here and in Table I are empirical and were derived for values of n up to 10. However, it is believed that they are reasonably accurate for much larger values of n.

Fifth Order Products

When all possible combinations of n operating channels are activated five at a time

$$\frac{n^2(n-1)(n^2-n+4)}{12}$$

fifth order products will be formed and they will lie between $A - 2(n - 1)$ and $A + 3(n - 1)$ where the operating band lies between channels A and $A + (n - 1)$. This means that the bandwidth of the intermodulation products is very nearly five times that of the operating channels. The products are symmetrically distributed with respect to the midpoint of the operating band. It has been found empirically that if all products known to fall on transmitting channels are excluded, there can still be formed

$$\tfrac{1}{6}[6n^4 - 87n^3 + 575n^2 - 2214n + 3922]$$

potentially interfering products which fall within the operating band. This is strictly true only for values of n that exceed 8 and are not multiples of 3. There will remain

$$\tfrac{1}{12}[n^5 - 14n^4 + 179n^3 - 1154n^2 + 4428n - 7844]$$

products that will fall either outside the band or on transmitting channels within the band. Since relatively few of these products fall on transmitting channels within the band the above expression gives to a high degree of approximation the number of products that will fall outside the operating band.

Numbers of Potentially Interfering 3rd Order and 5th Order Products

The formulas given in the two preceding sections were developed by considering individually the various types of 3rd order and 5th order products. Table I shows the general formulas which apply to these individual types of 3rd and 5th order products from which the summation formulas given in the preceding sections were obtained. The number of potentially interfering 3rd and 5th order products is shown in Table II for specific transmission bands containing 10 and 20 consecutive channels.

PROBABILITY OF INTERMODULATION INTERFERENCE WHEN OPERATING CHANNELS ARE PICKED AT RANDOM

In an uncoordinated radio communication system it may be assumed that p operating channels are assigned on a random basis in a band con-

TABLE I — NUMBER OF POTENTIALLY INTERFERING INTERMODULATION PRODUCTS

n = Number of consecutive channels in available band

Type of Product	Number of Products	
$2A - B$	$\frac{(n-1)^2}{2}$ if n is odd	$\frac{n(n-2)}{6}$ if n is even
$A + B - C$	$\frac{(n-1)(n-3)(2n-1)}{6}$ if n is odd	$\frac{n(n-2)(2n-5)}{6}$ if n is even
$3A - 2B$	$\frac{n}{3}(n-3)$ if n is a multiple of 3	$\frac{(n-1)(n-2)}{3}$ other n's
$3A - B - C$	$3(n^2 - 11n + 32)$	
$2A + B - 2C$	$\frac{1}{2}(n^3 - 9n^2 + 34n - 56)$	
$2A + B - C - D$	$3(n-3)(n-5)^2$ for $n > 6$	
$A + B + C - 2D$	$4(n-3)(n-5)^2$	
$A + B + C - D - E$	$(n-5)(n-6)(n^2 - 11n + 37)$ for $n \neq 8$	

taining n consecutive channels. Let us suppose further that the average busy time of the channels is such that T represents the portion of time that an average channel is activated by a transmitter and R represents the portion of time that an average channel is connected to a receiver. The probability of interference, I, may be defined as the probability that one or more of the intermodulation products that are formed when pT channels are transmitting will fall on a specific channel in the operating band. The method used to determine I is based on the assumption that the distribution of intermodulation products is uniform over the operating band. It is further assumed that the magnitudes of the intermodulation products as encountered at the receiver input, are always strong enough to cause interference. Table III shows formulas for I that have been developed for each type of third order and fifth order product. Fig. 1 shows plots of p versus I when only 3rd order products are considered and Fig. 2 shows plots of p versus I when both 3rd and 5th order products are considered with n and T as independent variables.

1. Fig. 2a shows that a band in excess of 500 adjacent channels is required to limit the probability of 3rd and 5th order interference to 10 per cent ($I = 0.1$) when 10 operating channels are picked at random from that band if traffic is such as to fully use these 10 channels, 5 for

TABLE II — TYPE AND NUMBER OF INTERMODULATION PRODUCTS

Type of Product	Number of Products	
	10 Channels	20 Channels
$2A - B$	40	180
$A + B - C$	200	2,100
$3A - 2B$	24	114
$3A - B - C$	66	636
$2A + B - 2C$	192	2,512
$2A + B - C - D$	525	11,475
$A + B + C - 2D$	700	15,300
$A + B + C - D - E$	540	45,570

TABLE III — FORMULAS FOR I, PROBABILITY OF INTERFERENCE

n = Number of consecutive channels in available band.
p = Number of operating channels picked at random from those in band.
T = Portion of time average channel is activated by transmitter.
m = Number of intermodulation products of a specific type expected to fall within band of n channels when pT channels are activated.

Type of Intermodulation	Formula for m
$2A - B$	$m_1 = \frac{pT(pT-1)(n-2)}{2(n-1)}$
$A + B - C$	$m_2 = \frac{pT(pT-1)(pT-2)(2n-5)}{6(n-1)}$
$3A - 2B$	$m_3 = \frac{pT(pT-1)(n-2)}{3n}$
$3A - B - C$	$m_4 = \frac{3pT(pT-1)(pT-2)(n^2-11n+32)}{n(n-1)(n-2)}$
$2A + B - 2C$	$m_5 = \frac{pT(pT-1)(pT-2)(n^3-9n^2+34n-56)}{2n(n-1)(n-2)}$
$2A + B - C - D$	$m_6 = \frac{3pT(pT-1)(pT-2)(pT-3)(n-5)^2}{n(n-1)(n-2)}$
$A + B + C - 2D$	$m_7 = \frac{4pT(pT-1)(pT-2)(pT-3)(n-5)^2}{n(n-1)(n-2)}$
$A + B + C - D - E$	$m_8 = \frac{pT(pT-1)(pT-2)(pT-3)(pT-4)(n-5)(n-6)(n^2-11n+37)}{n(n-1)(n-2)(n-3)(n-4)}$

$I = 1 - \left(1 - \frac{1}{n}\right)^{m_1 + m_2}$ when only third order products are considered.

$I = 1 - \left(1 - \frac{1}{n}\right)^{m_1 + m_2 + --- + m_8}$ when both third and fifth order products are considered.

transmitting and 5 for receiving. ($T = 0.50 = R$) In this case it is the $2A + B - 2C$ type of product that requires the use of such a wide band.

2. Fig. 2a shows that there is practical certainty of interference in a band of 500 adjacent channels when 30 operating channels, picked at random from that band, are fully used. In this case it is the $A + B + C - D - E$ type of product that requires the use of such a wide band.

3. If the same total traffic as was assumed in (1) is handled by a greater number of operating channels, the number of available consecutive channels required for the same probability of interference remains the same. Thus, Fig. 2b shows that a band in excess of 500 consecutive channels is still required to limit the chance of interference to ten per cent when the traffic is distributed among 20 randomly selected operating channels; ($T = 0.25 = R$) similarly Fig. 2c shows that the required number of available consecutive channels remains the same when the traffic is distributed among 40 operating channels. ($T = 0.125 = R$).

CHANNEL SELECTION FOR THE ELIMINATION OF INTERMODULATION INTERFERENCE

Discounting the effect of selectivity in the radio equipment, it was shown in the preceding section that only a very limited number of channels can operate together without some degree of mutual interference when these channels are picked at random from a very considerable number of available channels. This is of course extravagant of frequency space. In this section, it is proposed to determine whether frequency space can be conserved by carefully selecting the operating channels in such fashion that the various types of intermodulation products that are formed will all fall on other than operating channels. This is readily accomplished in the case of 3rd order products by selecting the operating channels in such fashion that the frequency difference between any pair of these channels is unlike that between any other pair of channels. Many other rules inherently more complicated and more cumbersome to apply than the one stated above must be obeyed if 5th order as well as 3rd order products are to be controlled in this way. Table IV presents p operating channels selected from a band of n adjacent channels (numbered sequentially in order of ascending frequency) in such fashion as to avoid 3rd order interference within the system. Considerable effort has been spent in selecting these channels to insure that the number n associated with each value of p is the lowest

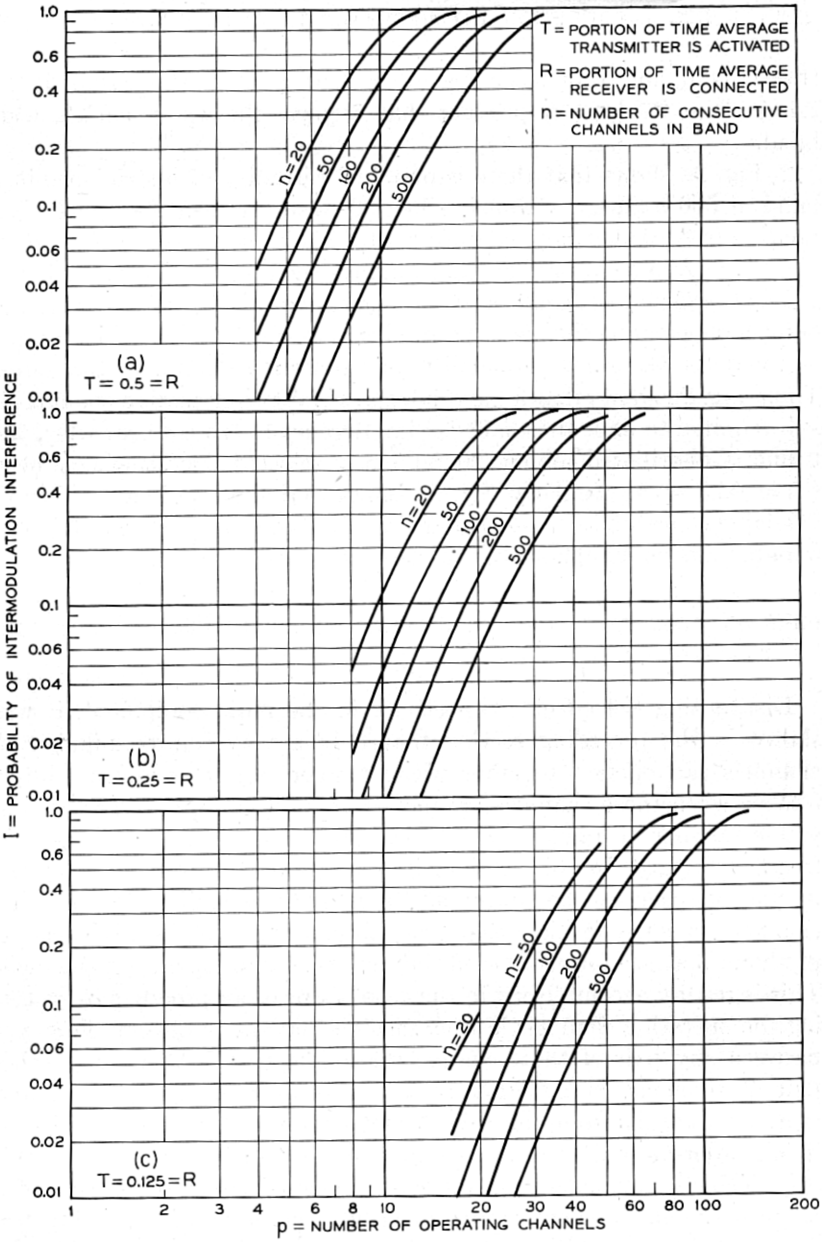

Fig. 1—Probability of intermodulation interference I versus number of operating channels p when only 3rd order products are considered. (a) T = Portion of time average transmitter is activated = 0.50; R = Portion of time average receiver is connected = 0.50. (b) Same as above except $T = 0.25 = R$. (c) Same as above except $T = 0.125 = R$.

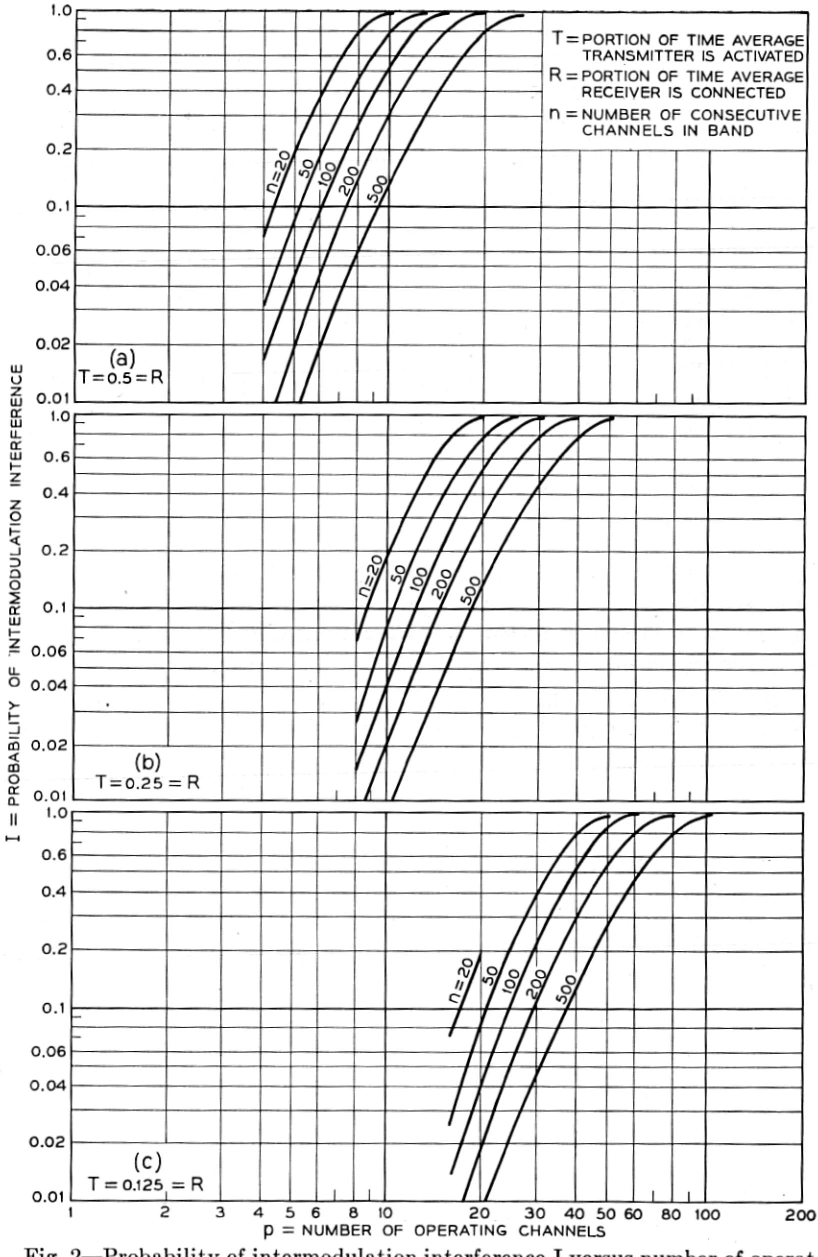

Fig. 2—Probability of intermodulation interference I versus number of operating channels p when both 3rd and 5th order products are considered. (a) T = Portion of time average transmitter is activated = 0.50; R = Portion of time average receiver is connected = 0.50. (b) Same as above except $T = 0.25 = R$. (c) Same as above except $T = 0.125 = R$.

TABLE IV — SPECIFIC OPERATING CHANNELS HAVING NO THIRD ORDER INTERMODULATION INTERFERENCE

p is the number of interference free operating channels which can be obtained from *n* consecutive channels.

p	*n*	Operating Channels Having No 3rd Order Interference
3	4	1, 2, 4
4	7	1, 2, 5, 7
5	12	1, 2, 5, 10, 12
6	18	1, 2, 5, 11, 13, 18
7	26	1, 2, 5, 11, 19, 24, 26
8	35	1, 2, 5, 10, 16, 23, 33, 35
9	46	1, 2, 5, 14, 25, 31, 39, 41, 46
10	62	1, 2, 8, 12, 27, 40, 48, 57, 60, 62
8*	137	1, 2, 8, 12, 27, 50, 78, 137

* Neither 3rd nor 5th order interference exists with this selection of eight operating channels.

possible number from which *p* channels having no 3rd order interference can be selected.

A plot of *p* versus *n* based on the above table is shown in Fig. 3. The curve which includes both 3rd and 5th order intermodulation products shows that 300 consecutive channels must be made available to provide for the careful selection of 10 operating channels which are interference-free at all times, regardless of the traffic load. For comparison, it was shown earlier (Fig. 2) that more than 500 consecutive channels must be available to permit picking at random 10 operating channels

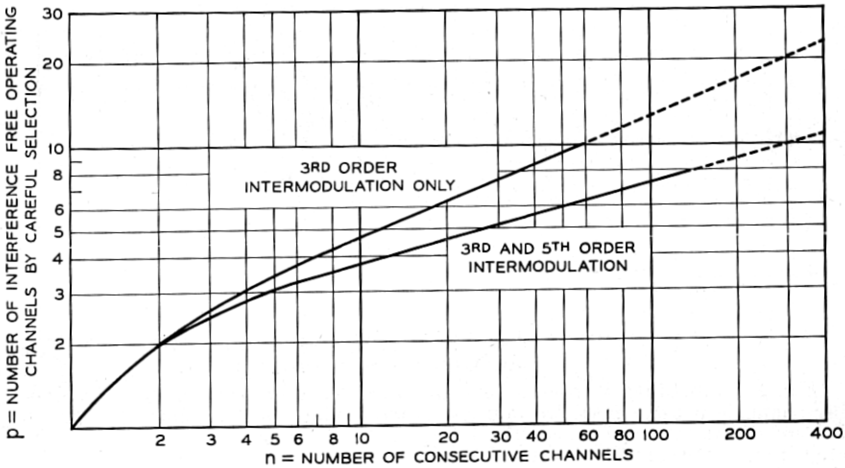

Fig. 3—Number of consecutive channels *n* required to provide a number of interference free operating channels *p*.

which are subject to interference 10 per cent of the time when fully loaded with traffic.

Careful channel selection is therefore a step toward better frequency usage. It is, however, only a small step, and a more effective solution must be found if efficient frequency usage is to be achieved in areas where sizeable numbers of operating channels are required. Such a solution for the mobile radio service is a "coordinated system," proposed in a companion paper previously referred to, wherein additional measures are described for reducing the probability of interference by proper geographical system layout and the use of certain practicable operational features.