Solderless Wrapped Connections

PART II — NECESSARY CONDITIONS FOR OBTAINING A PERMANENT CONNECTION

By W. P. MASON and T. F. OSMER

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In order to study the stresses and strains occurring in a solderless wrapped connection, a photoelastic technique using photoelastic bakelite and a photoplastic technique using polyethylene have been used. Polyethylene has a stress strain curve similar to a metal and can be used to investigate strains in the plastic region. Using these techniques, it is shown that the connection is held together by the hoop stress in the wrapping wire. In order to lock this in, a dissymmetry from a circular form has to occur. This may be in the direction of an oval shape or a square or rectangular shape. Sharp corners are preferred since a more definite contact area results. A number of rules are derived for constructing the most satisfactory solderless wrapped connection.

It is shown that the connection between the wire and terminal is intimate enough to permit solid state diffusion, but the strains are not high enough to cause cold welding of the connection. The life of the joint depends on the twin processes of stress relaxation and self diffusion. Stress relaxation occurs at a rate such that half the hoop stress is relaxed in 2500 years at room temperature. This loss of stress is compensated by the diffusion of one part of the joint into the other. Since the activation energies for stress relaxation and self diffusion are approximately equal for most metals, the two effects complement each other and produce a connection which should remain unchanged for times in excess of forty years under any likely ambient conditions.

INTRODUCTION

The solderless wrapped connection described in the paper by R. F. Mallina (see page 525) provides a very satisfactory and economical method for making connections with apparatus terminals when such connections are properly made. Not all methods of wrapping or all types of terminals are equally satisfactory and it is the purpose of this paper to describe investigations that have been made to determine the necessary conditions for the best wrapped terminal. These investigations include a photoelastic investigation of the stresses in the terminal and a photoplastic investigation of the strains in the outside wrapping wire. A new photoplastic material, polyethylene, has been used which has a stress strain curve similar to a metal and a birefringence proportional to the strain. The use of this material makes possible the evaluation of strains in the plastic region and may find applications in other plastic flow problems such as the extrusion of metals.

Even after such terminals have been satisfactorily made, there remains the question of whether they will have sufficient life to satisfy the requirements of the telephone plant. A design objective for most relays and other switching apparatus of the telephone plant is an uninterrupted trouble-free life of forty years. Hence, unless the connections are to be the limiting factor in the maintenance of the equipment, they also should have a minimum life of forty years under the conditions for which the apparatus is designed. In order to investigate the probable length of life of such connections, theoretical and experimental work on stress relaxation in metals has served as the basis for calculations and tests. These have been extended to the materials and conditions of the wrapped solderless connection and the results indicate that the life should be adequate even under very severe ambient conditions.

PHOTOELASTIC ANALYSIS OF STRAINS IN TERMINALS OF THE SOLDERLESS WRAPPED CONNECTION

In studying the conditions necessary to insure a good solderless wrapped connection, it is desirable to know what strains occur in the terminals and in the wrapping wires and how these vary with the terminal shape, the winding force and other variables entering into the construction of the connection. While some of these strains can be surmised from the winding conditions and the shape of the terminals, it is difficult to obtain any quantitative results by calculations on account of the fact that the desirable terminal shapes are rather complicated and because a good many of the strains are in the plastic region.

To remedy this difficulty, use has been made of a photoelastic and a photoplastic technique. For the inside terminal, all the strains, except at the corners where the wires make contact with the terminals, are elastic and can be approximated with an ordinary photoelastic tech-

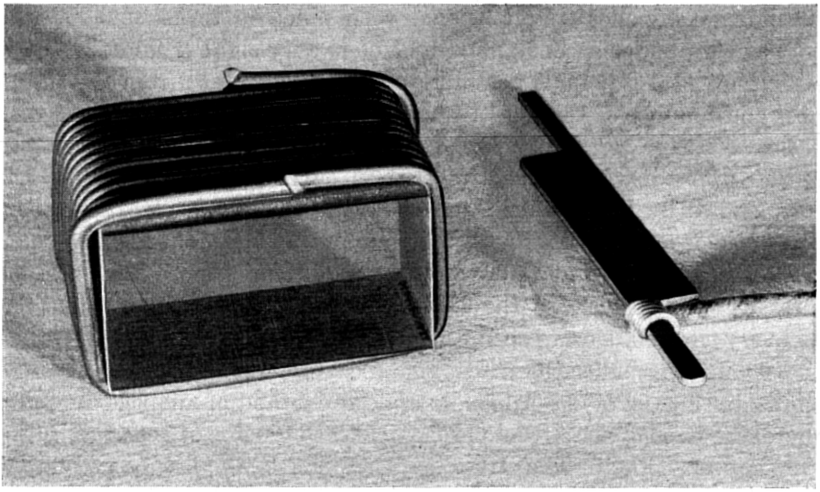

Fig. 1 — Photograph of photoelastic model and solderless wrapped connection.

nique using photoelastic bakelite. Fig. 1 shows one of the photoelastic models as compared with the metal solderless wrapped connection that it simulates. While the ratios of the wire diameter to the terminal dimensions are different in the model from those in the connection, considerable information can be obtained about strains in the terminals and wires from the photoelastic model.

The wrapping gun described in the previous paper puts a tension on the wire. To simulate the tension, the photoelastic model is placed in a chuck, one end of the wire is anchored to the chuck and an appropriate weight is applied to the other end of the wire. The specimen is then rotated by the chuck and a definite number of turns of copper wire are wound around the specimen. The extra wire is then clipped off and it is found that the wire tightly adheres to the terminal in the manner of a metal solderless wrapped connection. The specimen is then polished on the two ends up to the end wires and is put into the polariscope of a photoelastic analyzer. For obtaining the isochromatic lines, i.e., the lines occurring when the ordinary and extraordinary rays differ in path length by a half wavelength or some multiple of a half wavelength, the elements of the polariscope, as shown by Fig. 2, contain a quarter-wave plate before and after the specimen. These have the effect of making the plane wave from the polarizer circularly polarized and eliminate the isoclinic lines which mark the directions of the slow and fast axes of the material. Fig. 3 shows the isochromatic lines for a square specimen 0.4 inches square wound with nineteen turns of 0.050-inch copper wire under a constant weight of twenty-eight pounds, which is a stress of 14,300 pounds per square inch. It is evident from the sharpness and number of the lines that can be seen that we are dealing with a case of plane stress that can be analyzed by the method discussed in the Appendix. Since the stress strain curve of copper wire has the form shown by Fig. 4 with a yield stress* of about 26,000 pounds per square inch and a breaking stress of about 34,000 pounds per square inch, the applied winding stress is about 42 per cent of the breaking stress. Fig. 4 shows also the recovery measured for copper wire. The recovery curves are quite accurately parallel to each other, but the larger the strain the smaller the percentage recovery. Using the isoclinic lines shown by Fig. 24 of the Appendix and the method of analysis discussed there, the stresses across and perpendicular to the line of "eyes" of Fig. 3 are shown by Fig. 5. The stress perpendicular to the line of eyes measures the total compressive stress put on the terminal by the hoop stress in the wire and from this measurement the average hoop stress remaining in the wire can be calculated as follows. The cross section of which

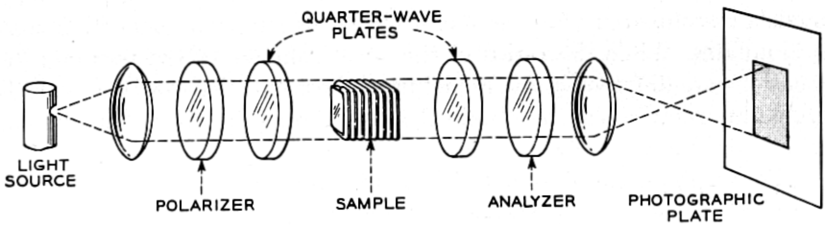

Fig. 2 — Elements of polariscope.

this force is applied is the width 0.4 inches by the length of the specimen 0.95 inches and hence the force applied by all the turns is

$$F = 0.95 \times 0.40 \times 2000 = 760 \text{ pounds.} \tag{1}$$

Since there were nineteen turns of wire wound around the specimen and each turn has two sides exerting a tension on the bakelite, the average tension remaining in the wire, required to balance the compressive stress, is

$$T = \frac{760}{2 \times 19} = 20 \text{ pounds,} \tag{2}$$

* In this paper the yield stress is taken as the point of greatest curvature of the stress-strain curve.

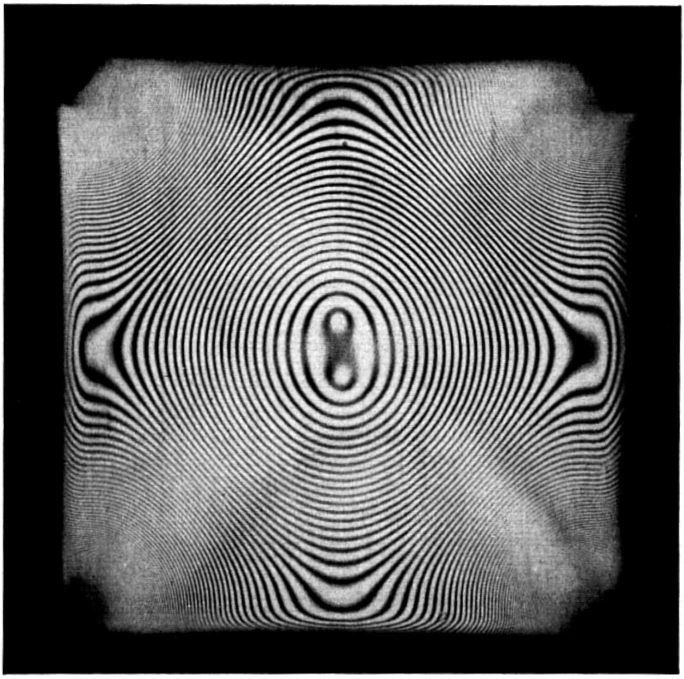

Fig. 3 — Isochromatic lines for a square model 0.4 inches on a side wrapped with nineteen turns of 0.050 inch copper wire with a constant load of 28 pounds.

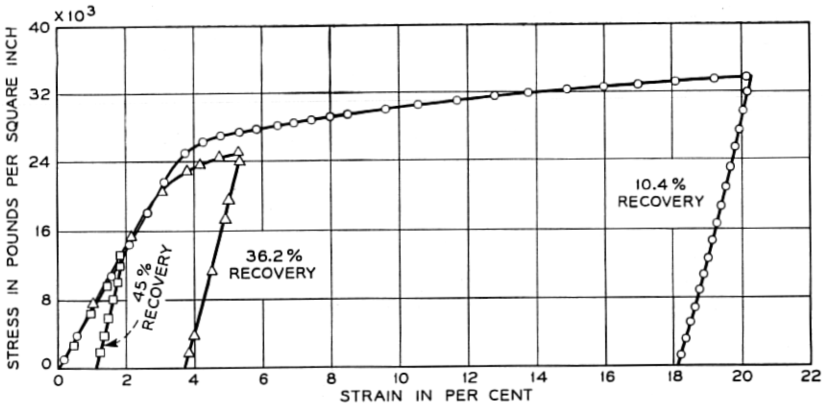

Fig. 4 — Stress-strain recovery curves for copper wire.

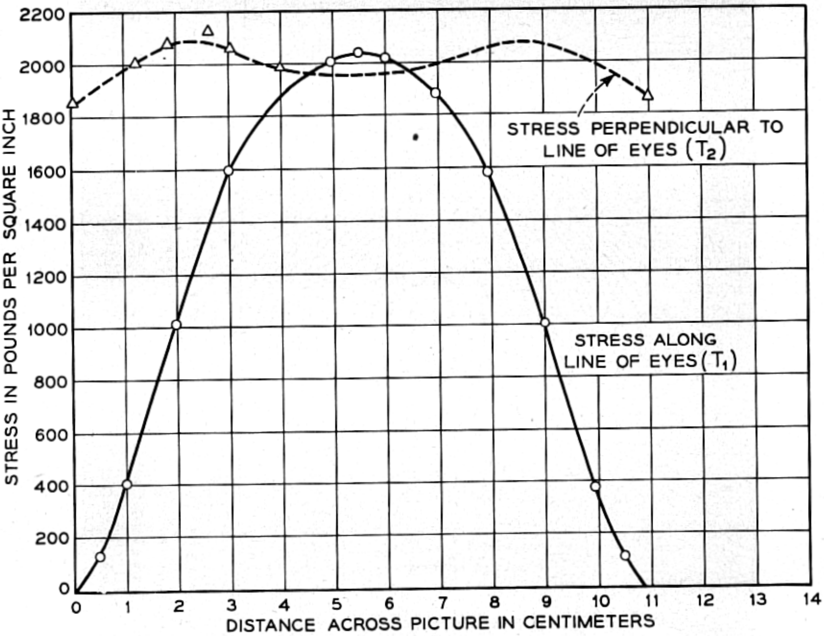

Fig. 5 — Stresses along X and Y directions for a square terminal.

or 71 per cent of the winding stress applied to the wire. The rest of the stress is lost in the contraction that occurs when the wire causes plastic flow to occur at the corners of the specimen. This plastic flow causes the terminal (both bakelite and metal) to flow around the wire and provides an air tight joint which is an essential requirement for a good solderless wrapped connection. That the stress in the terminal is high enough to do this can be seen directly from the photoelastic picture, Fig. 3. Counting the lines as far as they can be distinguished by a microscope, there are 72 lines from the eyes of the picture (which are isotropic points) up to 0.0525 inches from the geometrical corner lines. Using the stress constant of photoelastic bakelite which is 88 pounds per square inch per fringe per inch length along the optic path, this corresponds to a stress per unit area of

$$T = \frac{72 \times 88}{.95} = 6700 \text{ pounds per sq in.} \tag{3}$$

Since the total force put on by the wire is supported by successively smaller cross sections, as one approaches the corners the yield stress of bakelite of 15,000 pounds per square inch will be attained at a radius of

$$\left(\frac{0.0525}{x}\right) \times 6700 = 15{,}000 \text{ pounds/sq in. or } x = 0.0235 \text{ in.} \quad (4)$$

Hence, plastic flow should occur for about 23 mil inches into the plastic. Unwrapping the wires from the terminal, it is found that depressions of this order are cut in the terminals. Since one of the requirements of a better connection is that an air tight bond shall be formed between the wire and terminal, it is obvious that the terminal should have a low enough yield stress so that a sizable groove can be cut in it by the hoop stress of the wire. This rules out such terminals as hardened steel in the most satisfactory connections. It has been found that copper, brass, aluminum, soft iron and nickel silver are soft enough to meet this requirement. Any material with a plastic flow limit in compression, much lower than photoelastic bakelite, would probably have such a deep groove that it would be difficult to maintain the desired hoop stress.

Using the photoelastic technique as a tool, considerable data has been obtained on desirable shapes for the terminal and limiting winding stresses that can be used. One of the most used terminals is the rectangular terminal and Fig. 6 shows a photoelastic picture of a terminal 0.8 inches by 0.4 inches wound with nineteen turns of 0.050-inch copper wire with a winding load of 28 pounds. This figure is particularly easy to analyze for stress across a line half way down the long edge since the stress along this in, the direction of the line varies only a little. Hence,

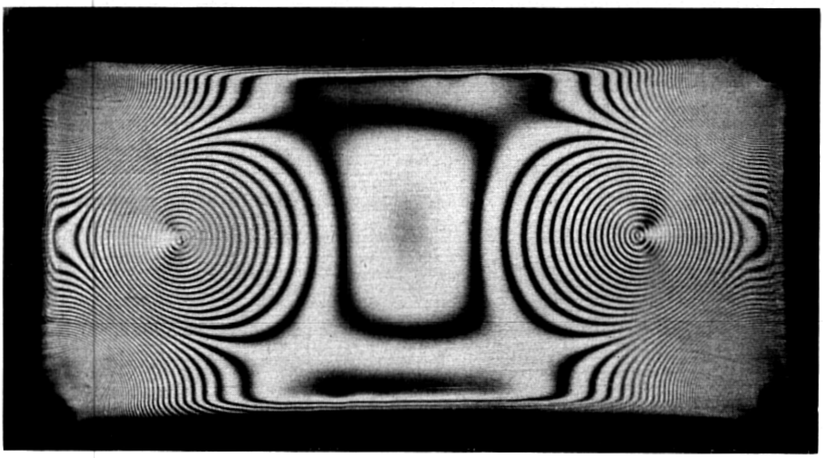

Fig. 6 — Isochromatic lines for a rectangular model 0.4 inches by 0.8 wrapped with nineteen turns of 0.050 inch copper wire with a constant load of 28 pounds.

one can count the number of fringes from the "eye" (which is an isotropic point) to the center of the specimen which in this case is eighteen fringes. Using the stress constant, 88 pounds per square inch per fringe per inch along the optic path, the compressive stress normal to the mid line is

$$\frac{18 \times 88}{0.95} = 1670 \text{ pounds/sq in.} \tag{5}$$

and the hoop residual stress per wire is

$$\frac{1670 \times 0.4 \times 0.95}{2 \times 19} = 16.6 \text{ pounds} \tag{6}$$

which is 60 per cent of the winding stress. Hence, the change in shape has not made any appreciable difference in the residual hoop stress. A specimen 0.4″ x 1.6″ was also tried and this had a residual stress of 56 per cent of the winding stress.

In order to determine the number of turns required to make a satisfactory joint, measurements were made of the residual stresses as a function of the number of turns with the results shown by Fig. 7. All of these experiments were made with the same weight, 28 pounds, which results in a stress of 14,300 pounds/square inch. Down to five turns, about 50 per cent of the winding stress is maintained. The results are consistent with assuming that the wire unwinds to the extent of two corners on each end while 60 per cent of the winding stress is maintained in all the other turns. As seen from Fig. 8, this is what one might expect, for when the constant tension is released, recovery will cause the

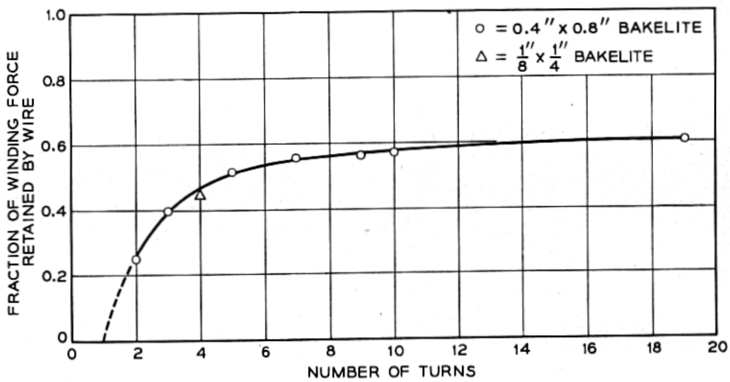

Fig. 7 — Residual hoop stress as a function of number of turns.

first corner to bend out and lose contact with the terminal. When the second corner attempts to unwind, it pulls the first corner up against the sample and no unwinding beyond the second corner can occur. In order to obtain the most satisfactory connections, at least five or six turns should be used.

The fact that the first two corners on each end do not make close contact to the inside terminal produces a very beneficial result when the connection is subject to vibration due to the handling and operation of

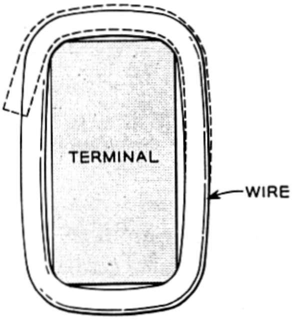

Fig. 8 — Locking in effect in a rectangular terminal which allows unwrapping at only two corners on each end.

a relay. In a soldered connection, large bending strains are caused by vibrations at the point of contact between the wire and the solder and in standard vibration tests when large 60-cycle vibrations are impressed upon the wires, the wires fatigue and break off at the point where the wire enters the solder in times in the order of fifty hours. Similar tests have been carried out for solderless wrapped connections and up to times of 2000 hours and longer no breaks have occurred. This is due to the fact that a bending strain is not enhanced by a sharp discontinuity as it is in the soldered connection and strains for a given vibration amplitude should be less than half as large as those for a soldered connection. Since the relation between fatigue and strain is such that a reduction of strain of two to one or more caused an increase in the number of cycles before breakage of factors of 1000 or more, the increased life under vibration for the solderless wrapped connection is not surprising.

Next a series of measurements were made on the value of the winding stress necessary to preserve a hoop stress in the wire. This was measured by winding wires with different weights on photoelastic samples and measuring the residual hoop stress by the technique described above. This was done for both copper and aluminum wires with the re-

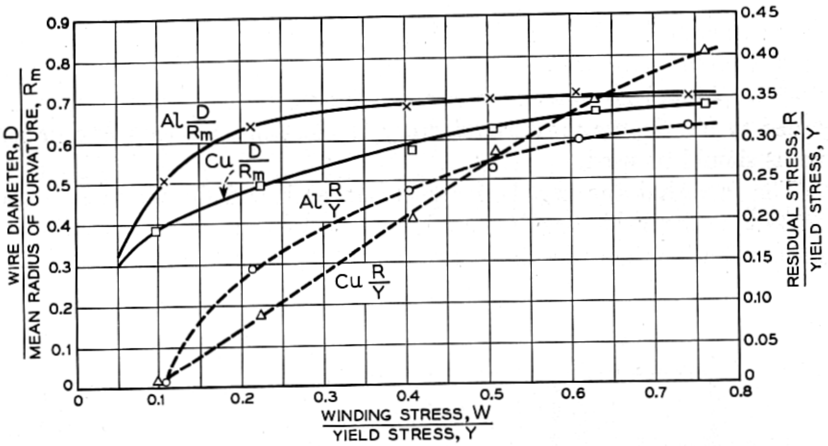

Fig. 9 — Relation between remanent hoop stress, ratio of wire curvature to wire diameter and the applied winding stress for copper and aluminum wire.

sults shown by the dashed lines of Fig. 9. To obtain a hoop stress greater than zero, the ratio of winding stress to yield stress must be greater than 0.1. To obtain a hoop stress that is 0.2 of the yield stress, a winding stress of 35 per cent of the yield stress has to be employed. This is the value recommended to give the most stable terminal. The winding stresses were carried up to 0.8 of the yield stress. At this stress the copper wire at the corners tends to draw down to a low value and may break when it unwinds and hence this is probably the upper limit for winding stresses. The radius of curvature of the middle of the wire, as it is bent around a corner was also measured from photographs similar to that shown on Fig. 6 and the ratio of the wire diameter to the mean radius of curvature is shown by the solid lines of Fig. 9. This is an alternate way of specifying the necessary winding force which may be useful for other shapes of terminals.

While square and rectangular terminals are very satisfactory shapes for the inside terminal, they are not the only ones that can be used. A number of coined and U shaped terminals are in general use as discussed by Mallina. In order to investigate the necessary requirements for such shapes, a number of experiments have been made on circular and elliptical terminals. When a wire under tension is wound around a rod of circular cross section, there are two sets of opposing stresses, one of which tends to make the helix smaller and the other to make it larger. As shown by Fig. 10, the tension in the wire tends to make the helix hug the cylinder while the bending strains introduced by the wrapping of the wire around the cylinder tend to make the helix open up when the constant stress is released. A number of experiments were made on wrapping copper wire 20 mil inches in diameter on a steel cylinder having a diameter of 0.124 inches. In all cases even up to stresses of 80 per cent of the yield stress, the helix failed to grip the cylinder. Some further measurements were made with smaller sized inner cylinders down to 20 mil inches in diameter. At this small radius the wire barely gripped the cylinder and it took about 70 grams stripping force to pull the wrapped wire off the cylinder. As seen from Fig. 11, the normal force per unit length against the cylinder is balanced by the hoop force in the wire according to the equation

$$\theta F_N r = 2F_H \sin \frac{\theta}{2} \qquad \text{or} \qquad rF_N \doteq F_H. \tag{7}$$

The stripping force SF, when the wire does not dig into the terminal, should be equal to

$$SF = 2\pi rnF_N f = 2\pi nfF_H , \tag{8}$$

where n is the number of turns, f the coefficient of friction which is about 0.15 to 0.2 between metals. For a stripping force of 70 grams for 6 turns, the remaining hoop stress is equal to

$$F_H = 9.3 \text{ grams}. \tag{9}$$

Since the wire was wound with a 700-gram force, it is evident that only about 1 per cent is maintained in the wire, which is entirely inadequate.

In order to obtain a good wrapped connection with high hoop stress in the wires, some means has to be employed to eliminate the unwrapping effect of the strain due to bending. This can be accomplished by changing the shape of the terminal from a circular cylinder to a dissymmetrical shape. For then, as shown by Fig. 8, the tendency to un-

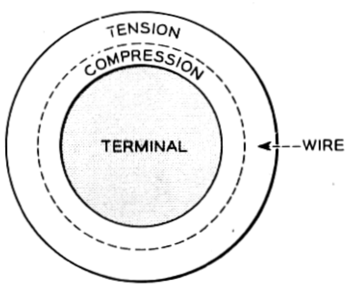

Fig. 10 — Strains in a wire wrapped around a circular terminal.

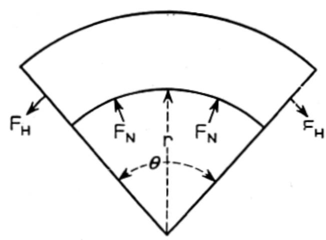

Fig. 11 — Relation between hoop stress and stripping force for terminal not indented.

wind is opposed by the locking in effect of the dissymmetry which in the most preferred types of terminals, as in Fig. 8, comprises abrupt changes in direction around the periphery of the terminal cross section. Some experiments were made with a cylinder whose cross section, as shown by Fig. 12, consisted of parallel sides with a semi-circle on each end. When the length A was twice the width, the top curve of Fig. 12 shows the stripping force as a function of the winding force. Assuming that most of the grip occurs on the two semi-circular surfaces, the hoop

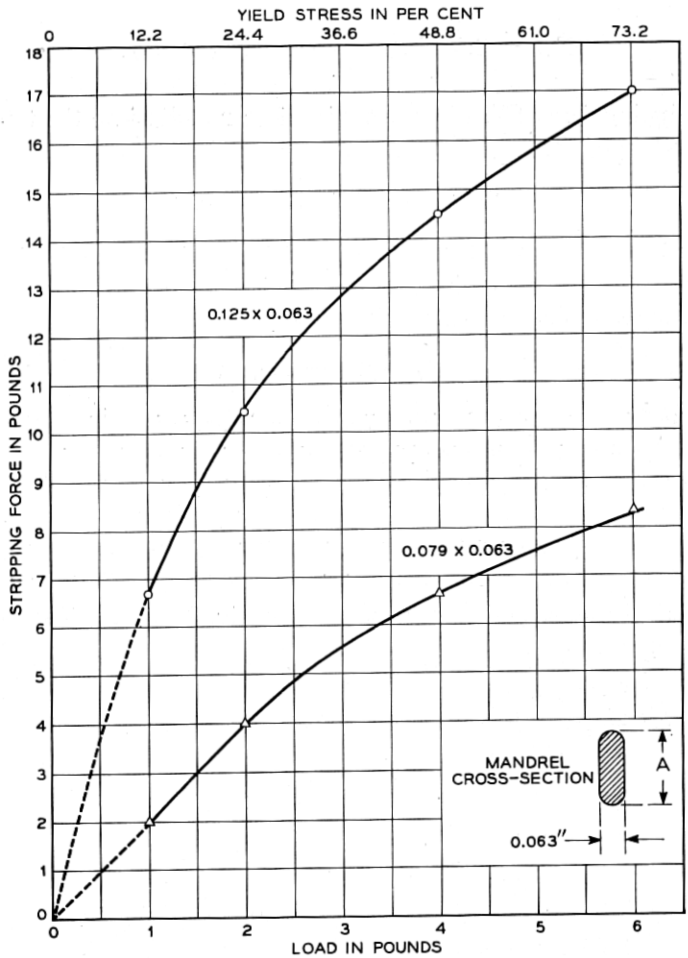

Fig. 12 — Stripping force as a function of winding load for an elliptical type terminal.

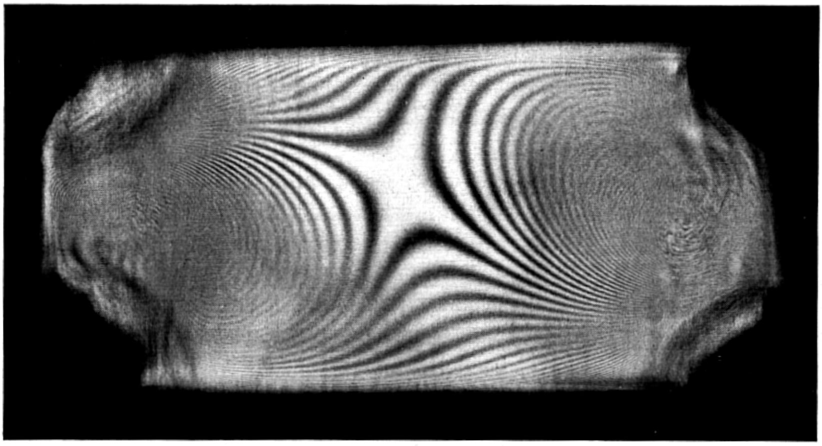

Fig. 13 — Photoelastic picture of a terminal when the outside wire diameter approaches the terminal diameter.

stresses indicated are in the order of 50 to 60 per cent of the winding stresses in agreement with the photoelastic experiments. Ratios of 5 to 1 on the length-width ratio were also tried with substantially the same result. To determine how much dissymmetry is necessary to lock in the bending stresses, a ratio of 1.25 to 1 of the length to width was tried with the result shown by the lower curve of Fig. 12. The stripping force for this ratio is about half that for the larger ratios. The conclusion is that the stripping force decreases as the symmetry increases but that a small deviation from circular symmetry is sufficient to lock in the bending stress. However, for the more satisfactory connections, other factors such as adequate intimate gas tight contact areas indicate that the required dissymmetry involves abrupt surface changes in the nature of edges having appreciable penetrating power with respect to the wire. Therefore, while a terminal like that of Fig. 12 may have the ability to lock in the bending stresses, it may not, for other reasons, be the better terminal to use.

All of the photoelastic and other types of stress measurements were made for terminals that have a large stiffness in torsion and as seen by the photoelastic pictures, the stresses are nearly plane stresses, i.e., they are all tensions, compressions or shears in a plane perpendicular to the axis of the connection. If, however, the torsional stiffness becomes low, another type of deformation can take place, namely, a twist of the whole terminal due to the torque put on by the helical form of the winding. Fig. 13 shows a photoelastic picture of the strain in the terminal when the size of the outside wire is comparable to the size of the terminal and it is obvious that a twist in the terminal is occurring. This is a case of three dimensional stress rather than plane stress and cannot easily be analyzed from the photograph. The photograph does, however, show a twist of the terminal.

The twisting strain can be most easily analyzed by taking a long section of terminal of low torsional stiffness, winding 100 or more turns on the terminal and measuring the angle of twist as discussed in the paper by Mallina. Calculations by Love[1] show that a twist in an elliptical section with its length along the Z axis introduces shearing strains in the X, Z and Y, Z planes, i.e., $e_{ZX} = S_5$ and $e_{YZ} = S_4$ shearing strains equal to

$$S_5 = -\tau\left(\frac{2a^2}{a^2 + b^2}\right)y; \qquad S_4 = \tau\left(\frac{2b^2}{a^2 + b^2}\right)x, \tag{10}$$

where $2a$ is the diameter of the ellipse along the X direction and $2b$ the diameter of the ellipse along the Y direction and τ the angle of twist in radians per centimeter. For the terminal with 100 turns of 0.020 mil copper wire discussed by Mallina whose data are given by Fig. 13, 45° angle of twist occurs in 2 inches giving a value of $\tau = 0.157$. This causes a shearing strain of about 2 per cent in the worst case which is enough to cause a considerable permanent set. While this twist is useful in studying stress relaxation in the wire, it is undesirable for a solderless wrapped connection to have too much twist since it may cause the terminal to twist off in the winding process. According to the data of Fig. 13 of Mallina's paper, no terminal set occurs for nickel silver if the twist in radians per centimeter is less than 0.09 which corresponds to a maximum shearing strain in the X, Z plane of 1.1 per cent. Hence in order to avoid excessive permanent set and twisting off of the inner terminal, the size and shape of the terminal should be controlled so that shearing strains due to twisting should be less than 1 per cent. For standard shapes such as rectangles and ellipses formulae are available to relate the maximum strain to the dimensions of the terminals and the moment due to the winding stress. For a given wrapping tension, this moment can be calculated from Appendix I of Mallina's paper.

Summarizing the results of this section, the necessary conditions that the terminal should meet are:

1. The wrapped connection is held together by the hoop stress in the outside wrapping wire. This can be locked in if the terminal has a dissymmetrical shape in which the length-width ratio is 1.5 or greater,

[1] Love, Theory of Elasticity. Chap. XIV, p. 310, 4th Edition, Cambridge University Press.

or if some regular shape such as a square, rectangle or rhombus is used. Sharp corners are helpful since a sharp bend in the wire occurs around them.

2. The material of the terminal must be strong enough so that the wire will not deform or cut through the terminal but must be plastic enough so that an appreciable groove can be cut in it by the hoop stress of the wire, in order that an air tight connection shall be made. The most satisfactory metals are brass, copper, soft iron, nickel silver and aluminum.

3. In the most satisfactory solderless wrapped connection, the wrapping wire unwinds to the extent of half a turn on each end and about six turns or more are required to make a good connection.

4. To maintain sufficient hoop stress, the constant wrapping stress should be from 0.2 to 0.7 of the breaking stress of the wire.

5. Shearing strains in planes parallel to the axis of the terminal should not exceed 1 per cent in order to eliminate terminal set.

PHOTOPLASTIC ANALYSIS OF STRAINS IN WIRES OF A WRAPPED SOLDERLESS CONNECTION

All of the strains in the inner block or terminal are elastic except at the corners. Hence, for the interior terminal, photoelastic bakelite is a satisfactory material for strain investigations. However, the outer wire is necessarily stressed beyond its elastic limit and ordinary photoelastic techniques cannot be applied. In order to see if the wire strains could be studied with photoelastic bakelite, some time was spent in heating rods to a temperature for which they become elastic, winding them under a stress and cooling under the applied stress. Although the winding process was carried out successfully several times, the bakelite rod always broke on cooling. This appeared to be due to the fact that bakelite is nearly linear up to the breaking point, i.e., it suffers from brittle fracture and does not simulate a metal in this respect.

Some measurements had previously been made at the Bell Laboratories and in England[2] on polyethylene which indicated that it had properties similar to a metal in the plastic range. Stress-strain curves up to 15 per cent strain are shown by Fig. 14, and it is evident that on the ascending part, the curve is very similar to that for copper or soft iron. On the relief from stress, however, a considerably larger recovery is obtained than for a metal. This material is fairly transparent and the lower curve marked R/t shows the relative retardation for a 5461 A°

[2] Miss S. M. Crawford and Dr. H. Kolsky, Stress Birefringence in Polyethylene. Proc. Phys. Soc., Section B, London, **6**, Part 2, pp. 119–125, Feb. 1, 1951.

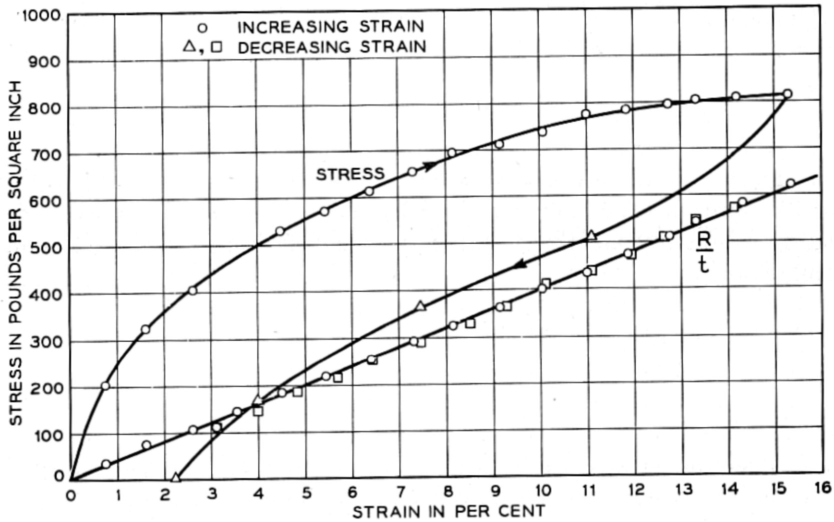

Fig. 14 — Stress and birefringence strain characteristic of quenched polyethylene.

mercury line. This relative retardation R/t is related to the number of fringes N and the wavelength λ by the equation

$$\frac{R}{t} = \frac{N\lambda}{t} \tag{11}$$

and hence for a 1 per cent strain there are 7.3 fringes for a 1-cm path length in the optic direction. It will be observed that for both increasing and decreasing strains, the birefringence is directly proportional to the strain. Hence, by using polyethylene it appeared possible to measure the strain even in the plastic region.

The first experiment tried was to wrap a square metal rod with a polyethylene "wire" one-sixteenth inch in diameter with a wrapping stress about half the yield stress. It was found, however, that the wire sprang off the metal rod when the constant stress was released. This is due to the fact that the polyethylene has considerably more recovery than the metal wire and the dissymmetry is not sufficient to lock in the bending stress. This shows that one of the requirements of the wire is that the recovery shall not be too large.

If we are to use polyethylene as a photoplastic material, it is necessary to simulate the unloading curve as well as the loading curve. This can be done by heating up the polyethylene when it is wound under a load, cooling under a load, and then removing the load at room temperature. Fig. 15 shows the loading and unloading curves for polyethylene as a function of temperature. The stress required to produce a given strain decreases very rapidly as the temperature increases, although the recovery remains about the same irrespective of the temperature. Suppose now that we apply a load and cool the polyethylene down to room temperature maintaining the strain. When the weight is taken off, the unloading curve will parallel that of the 20°C curve, and 42 per cent recovery will be obtained at 50°C and 12 per cent for 90°C.

In order to see if a wrapped joint would be simulated by this means, a one-quarter inch rod of polyethylene was heated up to 97°C, was wound with a one pound winding weight and was cooled to room temperature with the weight attached. This was sufficient to prevent the "wire" from unwrapping and when the weight was removed, the polyethylene "wire" gripped the metal closely and formed a bond similar to the wrapped connection. In order to analyze the strain, one has to cut a section through the center of the wire and put the section in the polariscope. First, a number of unstrained polyethylene samples were cut by various techniques and it was found that if they were cut with a

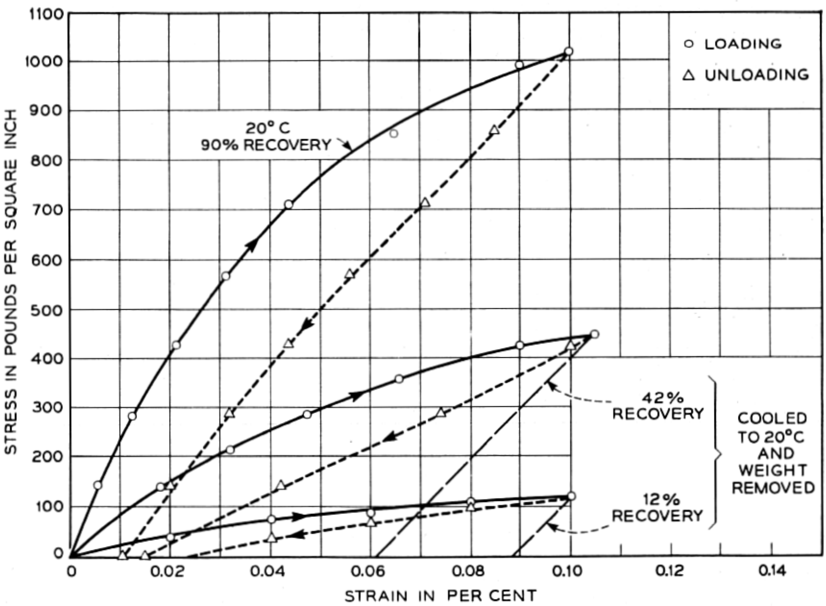

Fig. 15 — Stress strain recovery curves for quenched polyethylene.

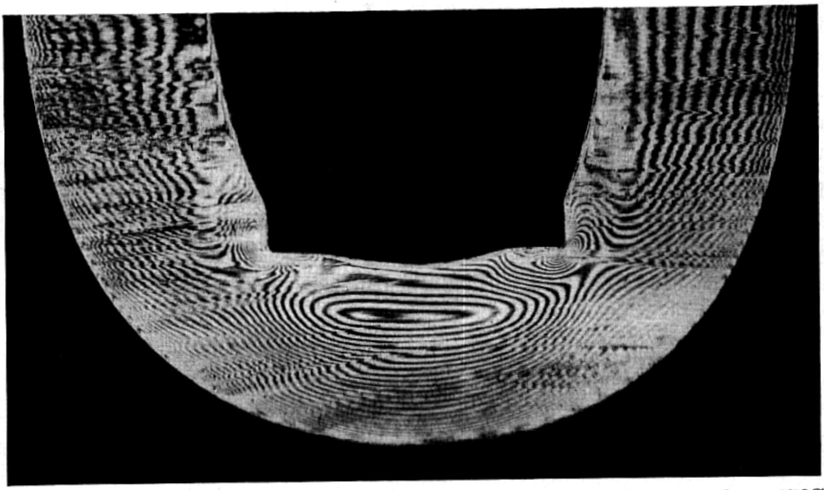

Fig. 16 — Photoelastic picture of a polyethylene "wire" wrapped at 97°C around a rectangular terminal and cooled off under the applied load.

jewelers saw with a good deal of set to the teeth, no strains were introduced in the sawing process. Using a jig with two parallel guides a sample 0.040 inches thick was cut through the polyethylene wire. Taking one in the center of the wrap, a photoelastic picture was taken with the result shown by Fig. 16 with an enlargement of one corner shown by Fig. 17.

The easiest parts to analyze are the strains in the two legs of the sample since the strains are similar to those for a bent section. The long leg which was twice as long as the short leg has its zero order fringe along the inside edge. There are twelve lines to the outer edge which corresponds to a tensile strain of 16 per cent with an average tensile strain of about 8 per cent which is about the strain caused by the loading weight. In the short leg the zero order fringe is the inside oval and the strain is about 11 per cent compressive at the inner edge of the segment and about 38 per cent tensile at the outer edge. These values are consistent with the radius of curvature that the wire is bent around for it can be shown that the strain in a wire of diameter d bent around a cylinder of diameter D without tension is equal to

$$S = \frac{2\rho}{D + d}, \tag{12}$$

where ρ is the radial distance measured from the center of the wire outward, D the diameter of the cylinder and d the diameter of the wire. If ρ is positive or the point is outside the center line, the strain is positive or tensile, while if ρ is inside the center line the strain is negative or compressive. From measurement of Fig. 16, it appears that the equivalent inner cylinder that corresponds to the radius of the center of the wire is about 3.0 times the wire diameter and hence the strain should be 25 per cent tensile at the outer edge and 25 per cent compressive at the inner edge. The addition of a tension of 13 per cent due to the winding force makes the outer strain 38 per cent and the inner one 12 per cent compressive which are close to the values found. This indicates that the tensile strain is somewhat higher in the short leg than in the long one. The same tensile strain occurs around the outside periphery of the wire opposite the corners of the terminal, but a very high

Fig. 17 — Enlargement of one corner of the photograph of Fig. 16.

point of compression develops just below the point of contact between the wire and terminal. The distribution of compressive and tensile strains in the wire is shown pictorially by Fig. 18. The distribution of strains in a metal wire can be considered to be quite similar except that the tensile strain due to winding should be only 2 to 3 per cent as seen from Fig. 4, while the strains due to bending may be even higher, for as seen from Fig. 9 the ratio of D/d may be in the order of 1 and the bending strains may be as high as 50 per cent.

In order to specify the properties that the wire must have in making a good solderless connection, experiments have been made on how much recovery can be tolerated in the wire. Since it is difficult to get a series of metal wires having different amounts of recovery, the technique was resorted to of heating polyethylene to a definite temperature, winding under a load equal to half the yield stress at the winding temperature, and cooling to room temperature under the load. As shown by Fig. 15, known recoveries can be obtained in this way. It was found that the largest amount of recovery that could be tolerated to make a joint at all was 20 per cent while a reasonable hoop stress was not obtained until the recovery was less than 10 per cent.

In summary, the necessary conditions that the wire should fulfill are:

1. Since strains of 50 per cent may be encountered in bending wires around sharp corners, wires should be used which have a large difference between the yield strain and the breaking strain. Copper, aluminum and soft iron are materials of this class while phosphor bronze and music wire are not as satisfactory.

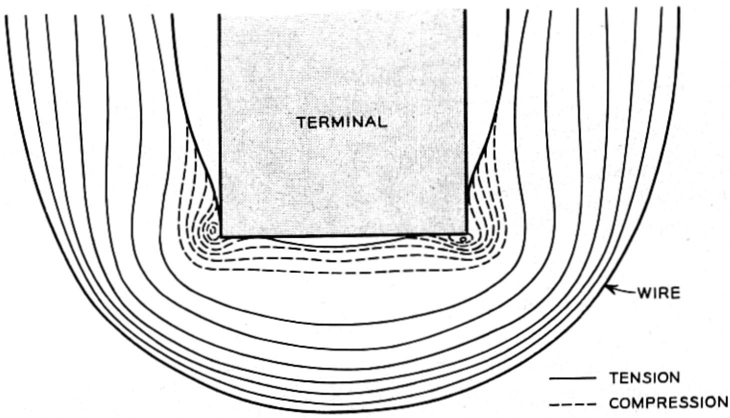

Fig. 18 — Distribution of compressional and tensile stress in the wire of a wrapped solderless connection.

2. The amount of recovery from strains of 20 to 50 per cent should not be greater than 20 per cent.

PERMANENCE OF WRAPPED SOLDERLESS CONNECTIONS

By the photoelastic, photoplastic and strain analysis of the previous two sections, it has been demonstrated that the wrapped solderless connection is held together by the hoop stress in the outside wire whose value is determined by the winding stress and the locking in effect dependent on a dissymmetry of the terminal. The high stresses cause plastic flow in the wire and terminal in such a manner that the two materials flow together and produce an intimate air tight joint. The intimate nature[3] of this contact has been demonstrated by dip coating nickel silver terminals with pure tin and wrapping them with cleaned bare copper wire. The wrapped terminals were then placed in a glass tube, evacuated, sealed off and heated for 400 hours to 180°C (37°C below the melting point of tin). The samples were then removed, mounted vertically, polished, etched and examined microscopically for distinguishing constituents. It is believed that if such a constituent appeared on the originally bare copper wire after such treatment, that the contact was sufficiently intimate to permit solid state diffusion. A section of the wire in contact with the corner is shown by Fig. 19. The copper is seen

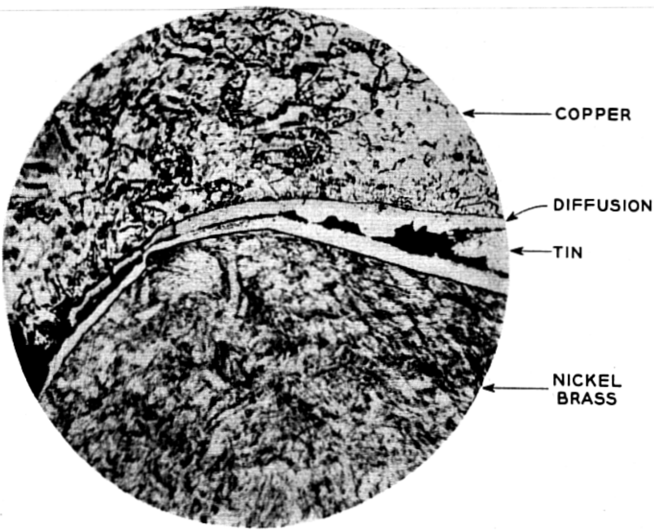

Fig. 19 — Solid state diffusion of tin into copper in a wrapped solderless connection.

[3] This experiment was conducted by G. S. Phipps.

to have a heavy layer of tin constituent at the contact surface. From the test, it can be concluded that wrapped connections are sufficiently tight to allow solid state diffusion and are therefore good electrical contacts.

On the other hand the contacts are not welded contacts such as occur when two pieces of aluminum are cold pressed together with strains in excess of 75 per cent. This is shown experimentally by the simple process of unwinding the wire from the terminal which takes place with no excess force when the wire is removed from a terminal corner. Since the strains at the points of contact do not exceed 30 to 40 per cent, one would not expect cold welding. It is possible, however, that in tinned terminals, some long time diffusion takes place at room temperature in the manner demonstrated at higher temperatures by the data of Fig. 19. This would occur very slowly and cannot be relied upon solely to maintain the contact.

Hence, it appears that long life in the connection depends on maintaining sufficient hoop stress in the wrapping wire to keep the elements of the connection sufficiently tightly pressed together so that no corrosion can occur in the connection in such a manner as to interrupt the electrical continuity. This is a problem in stress relaxation rather than creep. Stress relaxation is intrinsically a simpler phenomenon since the major fraction of stress that can be relaxed will be relieved through viscous flow in previously formed slip bands or along grain boundaries, and no generation of new slip bands is required. However, under ordinary creep conditions, an increase in stress is presumably attended by the generation of new slip bands. It appears likely then that stress relaxation phenomena even at quite high stresses should more nearly follow the conditions that have been established for low stresses than would be the case for creep phenomena.

A good deal of work has been done on stress relaxation at low stresses, particularly by Zener[4] and his coworkers, and this will be briefly reviewed. According to these studies, stress relaxation can be caused by several mechanisms including stress induced migration of impurities in the metal, viscous behavior of slip band material and the viscous behavior of grain boundaries. At the common junction between the two metal grains, there is an amorphous layer of material which acts as a viscous medium, i.e., if there is a shearing stress applied across it, the two grains will move with respect to each other with a velocity

[4] Zener, C., Elasticity and Anelasticity of Metals. University of Chicago Press, 1948. T'ing-Su Ké, Experimental Evidence of the Viscous Behavior of Grain Boundaries in Metals. Phys. Rev., **71**, No. 8, pp. 533–546, April 15, 1947. T'ing-Su Ké, Anelastic Properties of Iron. Tech. Publication No. 2370, Metals Technology, June, 1948.

$$V = \eta T/D, \tag{13}$$

where D is the thickness of the layer, η the coefficient of viscosity and T the shearing stress. Hence no matter how small the shearing stress, one grain will move with respect to the other in a finite time. The amount that the grains can move is limited by the necessity of making the grain boundaries fit. According to Zener, the situation is analogous to the case of a jigsaw puzzle in which the overall configuration possesses rigidity in spite of the fact that no shearing stress exists between adjacent pieces. Zener has calculated that the ratio of the relaxed stress to

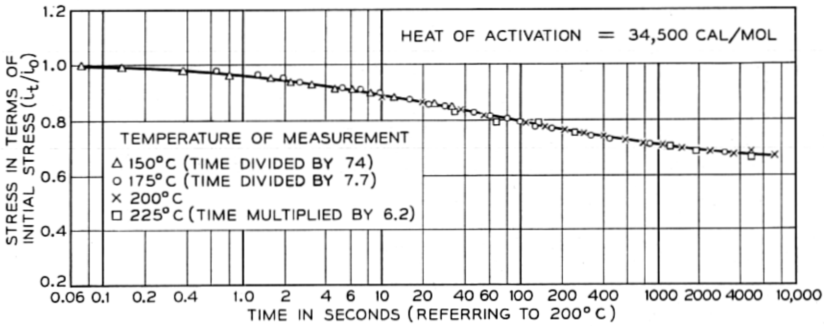

Fig. 20 — Stress relaxation in aluminum at three temperatures for strains less than 10^{-4} (after Ké).

the initial stress is equal to

$$\frac{T_R}{T_I} = \frac{\frac{1}{2}(7 + 5\sigma)}{7 + \sigma - 5\sigma^2}, \tag{14}$$

where σ is the value of Poisson's ratio. For values of Poisson ratio of from 0.25 to 0.5 this ratio lies between 0.595 and 0.76.

Fig. 20 shows measurements of stress relaxation plotted against time for aluminum for three different temperatures. These were obtained[4] by twisting an aluminum wire through a definite angle and observing the force required to hold it at this angle as a function of time and the temperature of the wire. All the curves can be made to coincide by multiplying the times by different factors. If we define the relaxation time τ as the time required to relax half of the variable component of stress, i.e. $\frac{1}{2}(1 - 0.67)$ of the stress, this relaxation time fits an equation of the form

$$\tau = Ke^{H/RT}, \tag{15}$$

where K is a constant, H an activation energy, T the absolute temperature in degrees Kelvin and R the Boltzman constant for one gram mole of the material. R is closely equal to 2 calories per degree K. Hence, if H is expressed in calories per gram mole and the value of K is obtained to fit equation (15), we have

$$\tau = 9.2 \times 10^{-15} \times e^{\frac{34,500}{2T}} .$$

The constant K is close to that given by the Langmuir-Dushman[5] theory

$$K = \frac{hN}{H} = \frac{6.62 \times 10^{-27} \times 6.06 \times 10^{23}}{34,500 \times 4.187 \times 10^{7}} = 2.7 \times 10^{-15}, \qquad (16)$$

where h = Planck's constant equal to 6.62×10^{-27} ergs, N is Avogadro's number equal to 6.06×10^{23} and H is the activation energy expressed in ergs. Su Ké[4] has shown that the activation energy for grain boundary slip is essentially the same as for self diffusion and for creep.

Similar results have been found for α-brass and α-iron. These have activation energies shown by equation (17)

$$\begin{array}{ll} \alpha\text{-brass} & 41 \text{ kilocalories per mole,} \\ \alpha\text{-iron} & 85 \text{ kilocalories per mole.} \end{array} \qquad (17)$$

Although measurements have not been made for copper, the activation energy of self diffusion is about 57.2 kilocalories[6] per mole, but 39.9 kilocalories for the principle impurity silicon.

All of these measurements were made for strains under 10^{-4}, and the question arises as to whether these concepts are valid for the much higher strains experienced in the wrapped solderless joint. From the photoelastic pictures, Figs. 16 and 17, it is obvious that the greatest stress inhomogeneity occurs in the neighborhood of the corners and flow will take place in such a way as to relieve the high stress concentration. This will have the effect of making the terminal and wire mate even closer and may result in a slight transient lowering of the hoop stress. After the initial formation, however, it will be the long time relaxation of the hoop stress in the wire that determines the lasting quality of the joint.

As discussed previously, the twist that the terminal takes is determined by the mean value of the hoop stress in the wire, and any relaxation in this hoop stress can be studied by observing the angle of twist as a function of time and temperature. By using a long terminal wound

[5] S. Dushman and I. Langmuir, Phys. Rev. **20,** (1922) p. 113, 1922.
[6] Zenner, Elasticity and Anelasticity of Metals. Table 12, p. 98, Chicago University Press.

with 100 turns or more of copper wire, a twist of 50° or more can be obtained which is sufficient to measure.

In order to test first the relaxation in the copper wire alone, the inner terminal was made of clock spring steel 0.0124 inches by 0.062 inches. This was wound with 100 turns of 0.020 inch copper wire tensioned at 2.87 pounds (9,300 pounds per square inch). A twist of 25° was obtained which is sufficiently large to measure. If one observed the angle after transient creep has occurred, the angle decreased on the average about 17 per cent in the first month as shown by the circles of Fig. 22. The values agree with the solid curve which was established by relaxation measurements as a function of temperature as discussed in the next paragraph. At room temperature, further decreases in the angle of about 10 per cent were observed out to times in the order of a year.

If, however, the wrapped connection was heated up, a faster relaxation of the hoop stress occurred. Fig. 21 shows the ratio of angle measured to the initial angle as a function of time when the connection is subjected to a temperature of 200°C. As shown by the dashed line, which is a plot of the exponential equation

$$R = e^{-\alpha t} \text{ where } \alpha = 2 \times 10^{-4}, \tag{18}$$

this is not a single relaxation of the type found for grain boundary motion but is a sum of effects occurring with different activation ener-

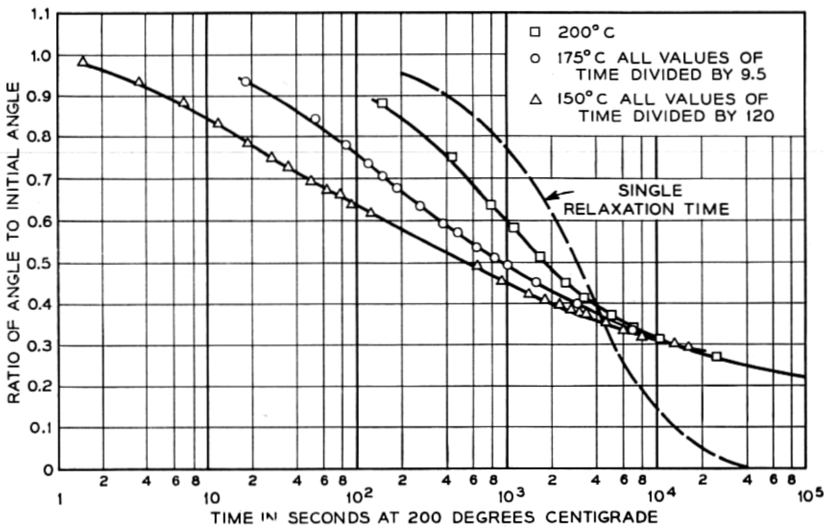

Fig. 21 — Ratio of angles of twist at time shown to initial angles.

gies. Furthermore, the change is not limited to two-thirds as in the case of grain boundary motion, but is much more tending toward a value of 0.2 for very long times. It appears that several processes are involved in addition to grain boundary motion. These are probably connected with slips along slip planes which occur with a lower activation energy when the stress is high. As the slip increases, strain hardening occurs with a resultant increase in activation energy until the activation energy of grain boundary movements is reached. For 0.3 relaxation and lower, the activation energy remains constant and equal approximately to 40 kilocalories per mole — the self-diffusion value — as can be calculated from the 150°C, 175°C and 200°C relaxation curves of Fig. 21. These curves show that the activation energy varies from 12.5 kilocalories at 0.9 relaxation, to 40 kilocalories for long time effects.

Similar measurements have been made on nickel silver terminals which are the terminals actually used and as seen from Fig. 21 of Mallina's paper, these agree quite well with those measured for spring steel terminals. The nickel silver terminals had the dimensions 0.0148 inches by 0.062 inches. A twist of 46° was obtained for 100 turns of 0.020 inch copper wire with 2.87 pounds winding stress applied. At this angle of twist, a permanent set of 19° occurred when the outside wire was unwound. If we subtract that value from the initial twist, the time-angle curve is very similar to that for spring steel and indicates that no additional relaxation occurs in the nickel silver terminal.

The conclusion from these experiments is that stress relaxation for the value of strain used in the wrapped solderless connections follows a similar activation energy pattern to that followed for smaller strains except that instead of a single process with a single activation energy we are dealing with many separate processes having an activation energy range from 12.5 kilocalories to 40 kilocalories. For each stage of the process a different activation energy is effective. For example, for a ratio of relaxed stress to initial stress of 0.9 the curves of Fig. 21 indicate an activation energy of about 12.5 kilocalories. With this value of activation energy the time required to relax this amount of stress at room temperature of 25°C (77°F) is 2.98×10^5 seconds or 0.0095 years as shown by Fig. 22. For a ratio of relaxed to initial stress of 0.8, the activation energy is 15.3 kilocalories and the time at room temperature is 0.126 years. Similar values can be calculated for the other relaxation ratios and the complete relaxation ratio and time curves are shown by Fig. 22 for temperatures of 77°F and 135°F. To reach a value of 0.5 of the initial hoop stress requires 2500 years at 77°F and about forty years at 135°F. The circles show the measured values at 77°F carried out in a tempera-

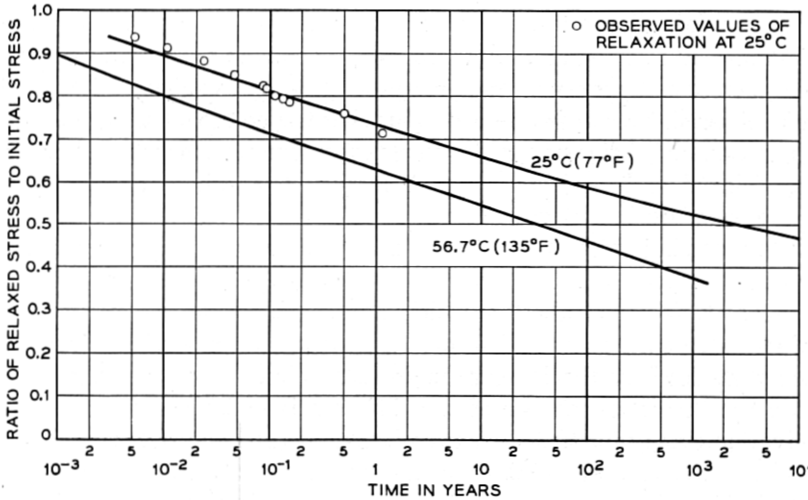

Fig. 22 — Aging at room temperature and at 135°F plotted as ratio of angle to initial angle as a function of time.

ture controlled air conditioned room and these agree well with the calculated values. 135°F is in general the maximum temperature that wrapped solderless connections will be subjected to. For cases of very high temperature it is planned to use copper covered soft iron wires for the wrapping wires since, as shown by Equation (17), iron has a much higher activation energy than copper and can be expected to maintain its hoop stress for forty years even under an ambient temperature of 200°C.

The question arises as to whether the hoop stress of a small part of the initial value, that has been shown to continue for a long time by the data of Fig. 21, is sufficient to maintain a good contact. This question may be important if aluminum is to be used as a wrapping wire since with an activation energy of only 34.5 kilocalories, 0.5 of the hoop stress will be maintained for forty years only for temperatures lower than 100°F. The problem then is whether corrosion can occur between the wrapping wire and the inside terminal when the relaxable component of hoop stress has been relaxed. A very sensitive test for this question is obtained by winding aluminum wire on an aluminum terminal since if any break occurs in the contact between the wire and the terminal, oxidation of the aluminum surface takes place very rapidly and should affect the resistance of the solderless connection. Accordingly, a number of aluminum-aluminum solderless connections were wound up and their resistances were measured. They were then put in an oven and heated to a temperature of 200°C for twelve hours, which was sufficient to relax all the stress that can be relaxed. On remeasuring the resistance, it was found that there was no change within the experimental error of 1 per cent, which corresponds to a resistance of 3×10^{-5} ohms. A similar result is found by studying the corrosion of surfaces of copper and nickel silver in solderless wrapped connections when they are fully relaxed and subjected to a corrosive atmosphere as discussed in the paper by Van Horn.

The stripping force of the aluminum-aluminum connection subjected to a temperature of 200°C for 12 hours has actually been found to increase by a factor of about 2 which suggests that the two parts have diffused into each other and formed a permanent connection. This has been confirmed by the increase in force required to unwrap the wire. Since the activation energy of self diffusion is about the same as the activation energy for stress relaxation, then as the hoop stress is relaxed at high temperatures, solid state diffusion takes place and a diffusion joint is formed in aluminum. The same process, both for stress relaxation and diffusion, should take place at a much lower rate at lower temperatures and as the hoop stress is relaxed, a diffusion connection between the two parts is formed so that no decrease in the conductivity of the connection occurs and an actual increase in the strength of the connection results. The same process should result between any two materials provided the energy of self diffusion from one into the other is less or equal to the activation energy of stress relaxation for the weakest component.

It is planned to tin plate both terminals and wires for all of the wrapped solderless connections used in the telephone system. In order to find how the two processes of stress relaxation and self diffusion, which progress as a function of time and temperature, affect the two fundamental properties electrical conductivity and mechanical strength of the connections, a large number of connections were wound up and subjected to temperatures of 200°C for times corresponding to 0.9, 0.8 etc., of the initial stress as determined from Fig. 21. The electrical resistances of the connections were determined before and after the treatment and the stripping force was also measured. Within the experimental error the resistances of the connections remained the same while the average stripping force for twenty connections for each point are shown by Fig. 23. No significant change in the stripping force occurred out to values of stress relaxation less than 0.2 times the initial hoop stress. These experiments show that as the hoop stress is relaxed by time and temperature, self diffusion occurs between the two parts of the connection in such a way that the mechanical strength and conductivity are maintained unchanged with time.

In summary it has been shown that

1. The connections are sufficiently intimate to permit solid state diffusion but the strains are not high enough to cause cold welding of the connection.

2. The hoop stress relaxes as a function of time and temperature according to well known activation energy equations with an activation energy for copper wires on nickel silver connections varying from 12.5 to

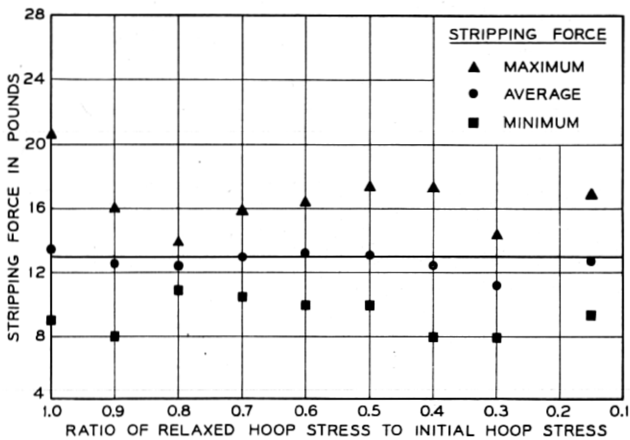

Fig. 23 — Stripping force plotted as a function of stress relaxation.

40 kilocalories. It requires about 2500 years to relax half the hoop stress at 77°F (25°C) and 40 years at 135°F.

3. The twin processes of stress relaxation and self diffusion occur in such a way as to maintain or increase the strength of the connection and to leave the resistivity of the connection unchanged with time.

4. The conclusion from all of these tests is that the life span of the solderless wrapped connection appears to be ample for meeting any of the likely requirements. This conclusion is reinforced by life tests of a large number of wrapped solderless connections in service that have been carried out over two years without a failure and by the long and satisfactory tests of the screw connection whose success also depends on the stress relaxation, diffusion processes. Such connections have been shown to be satisfactory over periods of time in excess of twenty years.

Appendix

METHOD FOR ANALYZING STRESSES AND STRAINS FROM PHOTOELASTIC PICTURES

The methods for analyzing photoelastic pictures are given in detail in a number of books and other publications[7] and only a brief summary of the method used here will be given.

When polarized light is sent normal to a plane of photoelastic material that is strained in the plane, the light is broken up into an ordinary and an extraordinary ray that travel with different velocities. It is shown[7] that the birefringence, which is defined as the difference between the two indices of refraction μ_1 and μ_2 (i.e. the index of refraction is the ratio between the velocity of light for one of the rays in a vacuum to the velocity in the medium) is given by

$$B = \mu_1 - \mu_2 = C'\sqrt{(T_1 - T_2)^2 + 4T_6^2}\,, \tag{19}$$

where C′ is a constant called the relative stress optical constant, T_1 is the tensional stress along the X axis, T_2 the tensional stress along the Y axis and T_6 the shearing stress in the XY plane. If we change the direction that we call the X axis until we reach the direction of maximum stress, the relations between this stress and the tensional stress at right angles to it are given by the equations

$$\begin{aligned} T_1' &= T_1 \cos^2\theta + 2 \sin\theta \cos\theta T_6 + T_2 \sin^2\theta, \\ T_2' &= T_1 \sin^2\theta - 2 \sin\theta \cos\theta T_6 + T_2 \cos^2\theta, \end{aligned} \tag{20}$$

where

θ is the angle between the X axis and the axis of maximum tension.

If θ is chosen so that T_1' is a maximum, we find

$$\tan 2\theta = \frac{2T_6}{T_1 - T_2} \tag{21}$$

and

$$\begin{aligned} T_1' &= \frac{T_1 + T_2}{2} + \tfrac{1}{2}\sqrt{(T_1 - T_2)^2 + 4T_6^2} \\ T_2' &= \frac{T_1 + T_2}{2} - \tfrac{1}{2}\sqrt{(T_1 - T_2)^2 + 4T_6^2} \end{aligned} \tag{22}$$

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[7] Coker and Filon, Photoelasticity, Cambridge University Press, 1931. M. Hetenyi, Handbook of Experimental Stress Analysis, John Wiley and Sons, Chap. 17, 1950. R. D. Mindlin, J. Applied Physics, April 1939, pp. 222–241 and May 1939, pp. 273–294. W. P. Mason, Electrooptic and Photoelectric Effects in Crystals, Bell System Tech. J., **29,** pp. 161–188, April, 1950.

and hence

$$T_1' - T_2' = \sqrt{(T_1 - T_2)^2 + 4T_6^2}. \tag{23}$$

Hence, the birefringence is directly proportional to the difference between the principal stresses.

The retardation R is the difference between the path length for the fast and slow rays when they are transmitted through the thickness of the specimen. If h is the thickness of the specimen

$$h = v_2 t, \qquad h - R = v_1 t \tag{24}$$

and hence

$$\frac{R}{h} = \frac{v_2 - v_1}{v_2} \qquad \text{and} \qquad \frac{c}{v_1}\frac{R}{h} = \left(\frac{v_2 - v_1}{v_2}\right)\frac{c}{v_1} = \left(\frac{c}{v_1} - \frac{c}{v_2}\right) = B. \tag{25}$$

Hence, the retardation is given by

$$\frac{R}{h} = \frac{C'c}{v_1}(T_2' - T_1') = C(T_2' - T_1'). \tag{26}$$

If λ is the wavelength of light used for the measurement,

$$\frac{N\lambda}{h} = \frac{R}{h} = C(T_2' - T_1'), \tag{27}$$

or the number of fringes is related to the difference of the principal stress by

$$N = \frac{Ch}{\lambda}(T_2' - T_1'). \tag{28}$$

For photoelastic bakelite for the green mercury line 5461 Å, the fringe constant λ/C is 88 pounds per square inch/inch/fringe. Hence the difference between the principal stresses is given by

$$(T_2' - T_1') = \frac{N}{h}(88) \text{ pounds/sq in.} \tag{29}$$

The isochromatic lines such as shown on Fig. 3 are taken with quarter-wave plates in addition to the crossed polaroids. These lines give directly the multiple number of times the stress is greater than the value $(88/h)$ pounds per square inch. In order to know the exact multiple, one has to know the starting point or the points of zero stress difference, called isotropic points. These are determined by using white light and locating the gray fringes. The zero order fringe is gray because for this case, all the wavelengths of light are delayed the same amount and hence no color effects appear. A first order fringe will have red (absence of violet) nearest the zero order fringe and violet (absence of red) furthest from the zero order fringe. High order fringes will not appear at all in white light since they are the resultant of a number of colors which add up to white light.

Having located the zero order fringe, a simple count will give the number of times the factor (88/h) has to be multiplied by to obtain the number of pounds per square inch. This stress will, however, only be a stress difference and in order to resolve this into stresses along X and Y axes and a shearing stress in the XY plane, other information is necessary.

One part of the information is obtained when the isoclinic directions are obtained. These directions are the directions of the principal stress axes and these are obtained by taking out the quarter wave plates and rotating the axes of the polaroids (keeping them crossed) until the polarization axes coincide with the principal stress axes at any point. When this occurs, the picture will be black because if no model were there, the polarized light passed by the polarizer would be cancelled by the analyzer. If the principal axis of the stress ellipsoid coincides with the direction of polarized light from the polarizer, only one ray will be generated whose plane of polarization coincides with that of the polarized light and hence this will be cancelled in the analyzer. The isoclinic lines show up much better if a white light source is used. Hence, the isoclinic lines locate the direction of the principal axes of the stress ellipsoid. From equations (20) and (23) we have

$$T_6 = \left(\frac{T_1' - T_2'}{2}\right) \sin 2\theta, \qquad T_1 - T_2 = (T_1' - T_2') \cos 2\theta. \quad (30)$$

Hence, if we know θ (the direction of the principal stress axes with respect to the axes for which the stresses are to be analyzed) the shearing stress T_6 and the difference between T_1 and T_2 can be obtained from equation (30).

The other necessary relation can be obtained from the equilibrium stress relations that have to be satisfied by any stationary body, namely

$$\frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} = 0; \qquad \frac{\partial T_2}{\partial y} + \frac{\partial T_6}{\partial x} = 0. \quad (31)$$

Integrating these equations

$$T_1 = T_{1_0} - \int \frac{\partial T_6}{\partial y}\, dx; \qquad T_2 = T_{2_0} - \int \frac{\partial T_6}{\partial x}\, dy. \quad (32)$$

For example, the isoclinic lines for the square plate of Fig. 3 are shown by Fig. 24. All the isoclinic lines converge on the positions of the "eyes" which is a characteristic to be expected of an isotropic point $(T_1' - T_2') = 0$. For the line through the center of the "eyes", which we designate as the X direction, the isoclinic line lies along the X axis and hence there is no shearing stress along this axis and $T_1 - T_2 = T_1' - T_2'$. At either edge the stress in the X direction is zero and hence at this point the stress T_2 (which is a compression) is obtained by counting the number of lines from the isotropic point to the edge (in this case 20) so that the stress at the edge is

$$T_2 = \frac{20 \times 88}{0.95} = 1850 \text{ pounds/square inch} \qquad (33)$$

To determine the shearing stresses at any point, one needs to fit the isoclinic lines over the isochromatic lines. For example, for any point along the isoclinic line marked 15, the direction of the principal axis is

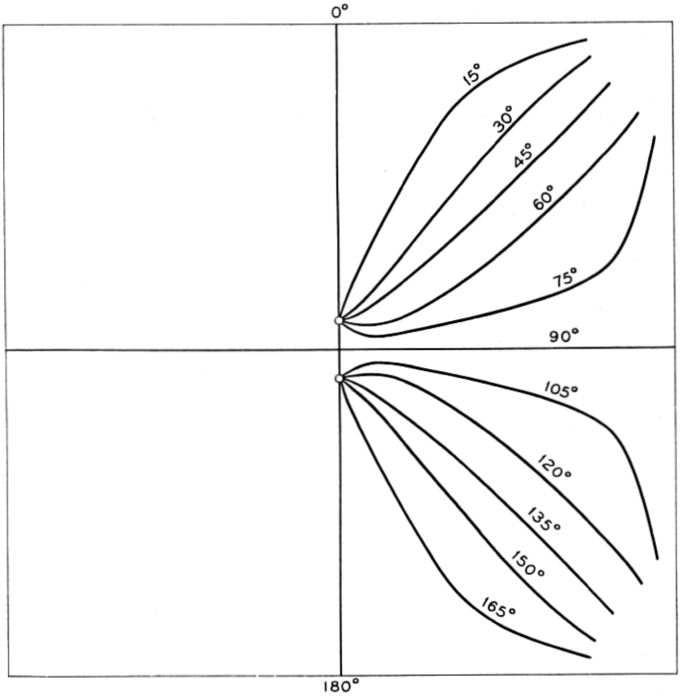

Fig. 24 — Isoclinic lines for a square terminal whose isochromatic lines are shown by Fig. 3.

15° from the X axis, since it was obtained by setting the analyzer and polarizer at 15° from the direction X. Counting the number of isochromatic lines from the "eye", the principal stress difference $T_1' - T_2'$ is known and using equation (30), T_6 is at once calculated. To obtain $\partial T_6/\partial y$, it is sufficient to divide T_6 by the distance y that the point is above the X axis. Proceeding this way, values of $\partial T_6/\partial y$ are plotted as a function of X. Then from the first of equations (32) with $T_{1_0} = 0$ at the edge, one can obtain T_1 by integrating $\partial T_6/\partial y \, dx$ over the X axis. The result is the curve marked T_1 of Fig. 5. Having T_1, T_2 can be obtained from the isochromatic lines which determine $T_2 - T_1$ and this is plotted on the upper part of Fig. 5.