Theory of Magnetic Effects on the Noise in a Germanium Filament

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A magnetic field will influence the current noise in a germanium filament. This fact bears out the hypothesis that at least part of the noise arises from minority carriers emitted in random bursts and recombining at the surfaces. A quantitative theory of this effect is given.

INTRODUCTION

In a series of fundamental experiments, H. C. Montgomery[1] has established that minority carriers play an important part in the current-noise associated with semiconductors. He found that on the one hand, the noise voltage is usually proportional to the biasing current, suggesting fluctuations in the conductivity, and hence the carrier concentration. On the other hand the spectrum of the noise suggested a rather coarse-grained time variation, not likely to be caused by fluctuations in the normal carrier density. One might conclude, therefore, that the noise is caused by a distribution of sources emitting or absorbing minority carriers in random bursts. Such carriers would be subject to the same laws of motion and of recombination as intentionally injected carriers. Montgomery was, in fact, able to verify that the noise along a filament showed marked correlation over a distance roughly equal to that through which minority carriers could drift in the biasing field before recombination.

W. Shockley has pointed out another corollary of this theory: A magnetic field transverse to the filament should have a pronounced effect on the noise. This conclusion, too, Montgomery was able to verify experimentally.[1] His results are in good qualitative agreement with theory. Complete quantitative agreement was perhaps not to be expected, since technical difficulties prevented attainment of the idealized conditions assumed by the theory. This paper gives an account of that theory. On it are based the computed curves in Montgomery's paper showing the change in noise power with magnetic field.

To see how such a change comes about, we imagine the magnetic field applied normal to one pair of the long faces of a rectangular filament. This field, and the longitudinal drift current used to measure the noise, yield a sidewise thrust on the carriers, directed at right angles to the other pair of long sides. As a result the density distribution over the cross section is distorted, the minority carriers tend to accumulate near one of those sides, while the neighborhood of the opposite side is depleted. But for the usual conditions the recombination of carriers occurs mainly near the surface, and is proportional to their density there. Hence the magnetic field will change their lifetime.[2, 3] Clearly the amount of noise is dependent on the length of time carriers are able to contribute to the change in conductivity, that is, dependent on their lifetime. Therefore, the magnetic field should change the noise power. In simple extreme cases one can even make a semiquantitative argument for the maximum variation to be expected on the basis of such considerations.[1]

FORMULATION OF THE PROBLEM

In order to make an exact calculation, we require a few preliminaries: The conductivity g is supposed to undergo a small time-dependent fluctuation $\Delta g(t)$ about its mean value.

The fluctuation arises from certain sources each of which, for macroscopic purposes, may be considered to emit a noise-current $J(t)$ of minority carriers. Thus in a small time-interval dt' near t' the excess charge injected is $J(t')\,dt'$. This charge decays by recombination. Let $r(t - t')$ denote the fraction of carriers left over at time $t(>t')$. Then at time t there remains a charge $r(t - t')\,J(t')\,dt'$ of the original injection. Now provided the excess density is small compared with the mean density, $\Delta g(t)$ is proportional to the excess charge at time t, due to all the previous emissions added together. Therefore

$$\Delta g(t) \propto \int_{-\infty}^{t} r(t - t')J(t')\,dt'. \qquad (1)$$

In practice we do not literally plot $\Delta g(t)$ as a function of t, but rather its frequency component $\Delta g(f)$ in a narrow range df of frequencies near f. In other words, we single out for observation the contribution to Δg from that part $J(f)$ of the injected current $J(t')$ which varies as $e^{-2\pi i f t}$ Suppose now that $1/f$ is large compared with the time over which $r(t)$

is appreciably different from zero (that is, let $1/f$ be much greater than the lifetime). Then, in the integral (i), $r(t - t')$ will have gone from unity to zero long before $J(f)e^{-2\pi ift}$ has changed appreciably from its value at $t' = t$. Therefore, for purposes of observation at frequencies much smaller than the reciprocal lifetime, we can rewrite (1) as

$$\begin{aligned}\Delta g(t) &\propto J(t) \int_{-\infty}^{t} r(t - t')\, dt' \\ &= J(t) \int_{0}^{\infty} r(t)\, dt.\end{aligned} \tag{2)*}$$

The integral in (2) can be interpreted as the average lifetime of carriers. For, by definition, the rate of recombination at time t is $-\, dr(t)/dt$, so that $-\,(dr/dt)dt$ is the number of carriers recombining between time t, $t + dt$. Hence the average lifetime is

$$\begin{aligned}\tau = -\int_{0}^{\infty} t\, \frac{dr(t)}{dt}\, dt &= -[tr(t)]_{0}^{\infty} + \int_{0}^{\infty} r(t)\, dt \\ &= \int_{0}^{\infty} r(t)\, dt\end{aligned}$$

since $tr(t) \to 0$ as $t \to \infty$.

If $1/f$ is not large compared with τ one cannot simplify the integral (1) in this way. One then has to consider separately each frequency component $\Delta g(f)e^{-2\pi ift}$ due to the current $J(f)e^{-2\pi ift}$. Then

$$\begin{aligned}\Delta g(f)e^{-2\pi ift} &= J(f) \int_{-\infty}^{t} r(t - t')e^{-2\pi ift'}\, dt' \\ &= J(f)e^{-2\pi ift} \int_{0}^{\infty} r(t')e^{2\pi ift'}\, dt'\end{aligned}$$

or

$$\Delta g(f) = J(f)\tau(f)$$

where

$$\tau(f) = \int_{0}^{\infty} r(t')e^{2\pi ift'}\, dt'.$$

The calculation of $\tau(f)$ is more complicated than that of $\tau = \tau(0)$, and

* From here on the equality sign will replace the proportionality sign. The resulting change of units is of no consequence in the final results which are only concerned with ratios of conductivity modulations.

at the present time the experimental situation does not call for refinements of this kind. Therefore we shall restrict ourselves to the calculation of τ.

To evaluate τ it is not necessary to consider a time-dependent case at all. In our experiment, it is the mean square conductivity fluctuation which is actually observed. Hence from (2)

$$<\Delta g^2> = <J^2(t)> \tau^2.$$

If the emission processes are stationary in time, $<J^2(t)>$ is time independent:

$$<\Delta g^2> = <J_0^2> \tau^2.$$

Now τ can be written as

$$\int_{-\infty}^{t} r(t - t')\, dt',$$

which is simply the total concentration at the present time t due to a constant injection from time $-\infty$ to the present.

Therefore the problem is reduced to finding the total carrier concentration in the filament due to a distribution of sources of constant strength $\sqrt{<J_0^2>}$.

Let $w(x, y, z; x_1, y_1, z_1)$ denote the carrier concentration at x, y, z due to a steady unit source at x_1, y_1, z_1. Then the total carrier concentration is

$$\tau(x_1, y_1, z_1) = \int w(x, y, z; x_1, y_1, z_1)\, dx\, dy\, dz.$$

The reason for the dependence on x_1, y_1, z_1, is that the recombination process takes place largely on the surface. Therefore a source near the surface will yield a smaller concentration than one well inside the filament. (Volume recombination will be neglected throughout this paper.)

The mean square conductivity modulation due to many statistically independent sources at x_r, y_r, z_r $(r = 1, 2 \cdots)$ is then

$$<(\Delta g)^2> = \Sigma \tau^2(x_r, y_r, z_r) <J^2(x_r, y_r, z_r)>.$$

The behavior of w is governed by the diffusion equation, subject to the boundary conditions expressing the recombination process, and subject to a suitable singularity at x_r, y_r, z_r, expressing the injection of a unit current. But in two and three dimensions the solution is not available in closed form, or at any rate not in terms of the elementary transcendental functions. The infinite series for the solution is not easy to handle computationally. It is therefore desirable to simplify the experimental conditions to a point where the problem becomes almost one-dimensional. A solution for w can then be found in closed form.

Consider a very long uniform rectangular filament with one pair of sides very much wider than the other pair. Suppose that the y and z directions are respectively parallel to the wide sides and to the length of the filament (Fig. 1).

Consider sources located anywhere on a plane $x = \xi$, which is parallel to the wide sides of the filament. If the recombination properties of the filament are uniform in the $y - z$ directions, the lifetime due to a unit source anywhere in that plane is independent of the location of the source on that plane, and depends only on ξ. Hence the conductivity modulation due to sources of strength

$$J(\xi, y_r, z_r) \qquad (r = 1, 2 \cdots)$$

in that plane is simply

$$\tau(\xi) \sum_r J(\xi, y_r, z_r)$$

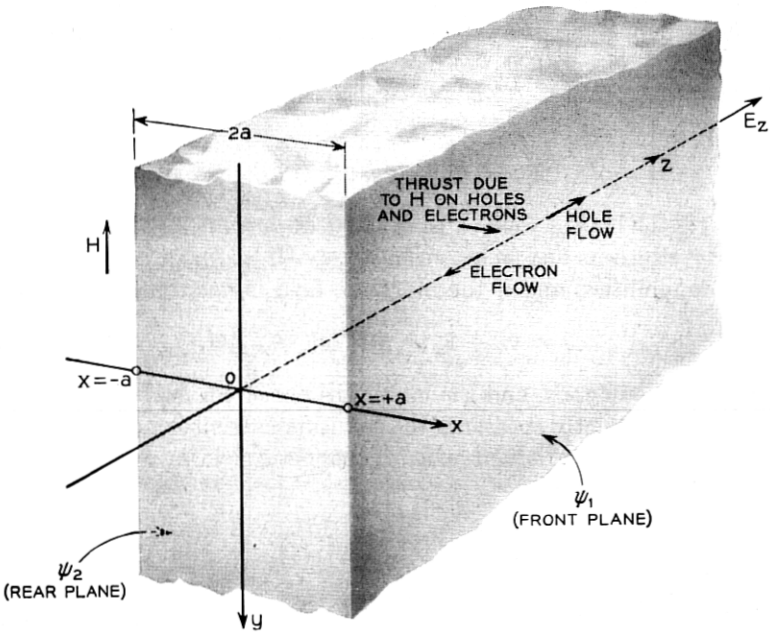

Fig. 1 — Geometry of the filament, and disposition of the fields.

and is the same as that due to an infinite source of strength

$$\sum_r J(\xi, y_r, z_r)$$

uniformly distributed over the plane x-ξ.

But the density w due to such a source will be a function of x and ξ only, and will be the solution of the one-dimensional diffusion equation. Hence for a geometry approaching that of figure 1 sufficiently closely, the problem is one-dimensional.

The Evaluation of τ

We now have to write down the one-dimensional diffusion equation in the presence of a magnetic field along 0_y, which combines with the drift velocity of the carriers so as to force them towards one of the surfaces $x = \pm a$. If F_x is the effective field arising in this manner, and D is the diffusion coefficient, the equation is

$$D \frac{d^2 w}{dx^2} - \mu F_x \frac{dw}{dx} = 0, \tag{3}$$

which expresses the fact that the diffusion current $-Dq\, dw/dx$ plus the drift current $q\mu F_x w$ must be constant since the carrier density cannot build up indefinitely. μ is the mobility of the minority carriers: μ_n for electrons, μ_p for holes. As is shown elsewhere[2] the effective field F_x is given by

$$F_x = (\theta_n + \theta_p)\, E_z = \theta E_z ,$$

where E_z is the biasing field causing the drift current, θ_n, θ_p are the Hall angles for electrons and holes, respectively. If μ_n, μ_p are the electronic and hole-mobilities, and if the magnetic field is not too large,

$$\theta = \theta_n + \theta_p = 10^{-8}(\mu_n + \mu_p)H,$$

where H is in oersteds, and the mobilities are in cm^2/volt-second, and θ is in radians. (Strictly speaking, the diffusion current is not in the direction of the density gradient when a magnetic field is present.[4] As the result mixed derivatives

$$\frac{\partial^2 w}{\partial x\, \partial y}$$

occur in the diffusion equation. But in the reduction to one dimension these terms integrate out. All that remains is a small correction to D, negligible for ordinary values of H.) It is convenient to specify a dimensionless parameter in the same notation as H. C. Montgomery.

$$\Phi = \frac{2a\mu F_x}{D} = \frac{2a\theta E_z}{D/\mu}$$

By the Einstein Relation $D/\mu = kT/q$ this may be written

$$= \frac{2a\theta E_z}{kT/q}$$

where q is the absolute value of the electronic charge. Φ is the ratio of the voltage corresponding to the transverse field to the thermal voltage kT/q. In terms of Φ, equation (3) can be rewritten

$$\frac{d^2w}{dx^2} - \frac{\Phi}{2a}\frac{dw}{dx} = 0 \tag{4}$$

The integral of this equation has the form

$$w = Ae^{\Phi(x/2a)} + B \tag{5}$$

where A and B are two constants. Because of the existence of a singularity at $x = x_0$, say, the constants A, B take on different values for $x < x_0$ and $x > x_0$. To see what these values are, we first write the solution (5) in the form

$$w_1 = A_1e^{\Phi(x-x_0)/2a} + B_1 \qquad x > x_0,$$

$$w_2 = A_2e^{\Phi(x-x_0)/2a} + B_2 \qquad x < x_0.$$

At $x = x_0$ the density w must be continuous. Hence

$$A_1 + B_1 = A_2 + B_2 \tag{6}$$

Further, the discontinuity at x_0 must be such that the difference of the currents on the two sides is just unity, the strength of the injected current. Now the total current is

$$-D\left(\frac{dw}{dx} - \frac{\Phi}{2a}w\right)$$

(i.e., the diffusion current plus the drift current), and w is continuous at x_0. Hence the difference between the current on the two sides of x_0 is

$$\begin{aligned} 1 &= -D \lim_{h\to 0}\left(\left(\frac{dw_1}{dx}\right)_{x_0+h} - \left(\frac{dw_2}{dx}\right)_{x_0-h}\right) \\ &= \frac{\Phi}{2a}D(A_2 - A_1). \end{aligned} \tag{7}$$

So far we have two relations (6) and (7) between the four constants A_1, A_2, B_1, B_2. To determine these constants, we need two more equations. These are supplied by the boundary conditions. We assume that the recombination rate is proportional to the density at the boundaries ($x = \pm a$). The recombination then has the formal appearance of a current through the surface, the factor of proportionality s playing the part of a "recombination velocity."[2, 3] That current through the surface must equal the current arriving at the surface from the interior of the filament, under steady-state conditions. The boundary conditions are thus:

$$-D\left(\frac{dw_1}{dx} - \frac{\Phi}{2a} w_1\right)_{x=+a} = s_1 w(+a),$$

$$-D\left(\frac{dw_2}{dx} - \frac{\Phi}{2a} w_2\right)_{x=-a} = -s_2 w(-a),$$

where s_1, s_2 are the surface recombination velocities of the faces $x = +a$, $-a$ respectively. The minus sign on the right-hand side of the second of these equations is due to the fact that the recombination current at $x = -a$ is along the $-x$ direction. Defining the recombination parameters

$$\psi_{1,\,2} = \frac{s_{1,\,2}\,a}{D}$$

and substituting the solutions w_1, w_2 in the last two equations we obtain

$$B_1 \frac{\Phi}{2} = \psi_1(A_1 e^{\Phi(1-x_0/a)/2} + B_1)$$
$$B_2 \frac{\Phi}{2} = \psi_2(A_2 e^{-\Phi(1+x_0/a)/2} + B_2) \tag{8}$$

Equations (6), (7) and (8) suffice to determine A_1, A_2, B_1, B_2. We easily find that

$$A_1 = -e^{-\Phi(1-x_0/a)/2}\,\frac{2a}{\Phi D}\,\frac{\alpha_1(1 - \alpha_2 e^{\Phi(1+x_0/a)/2})}{\Delta}$$

$$A_2 = +e^{+\Phi(1+x_0/a)/2}\,\frac{2a}{\Phi D}\,\frac{\alpha_2(1 + \alpha_1 e^{-\Phi(1-x_0/a)/2})}{\Delta}$$

$$B_1 = -\frac{2a}{\Phi D}\,\frac{(1 - \alpha_2 e^{\Phi(1+x_0/a)/2})}{\Delta} \tag{9}$$

$$B_2 = -\frac{2a}{\Phi D}\,\frac{(1 + \alpha_1 e^{-\Phi(1-x_0/a)/2})}{\Delta}$$

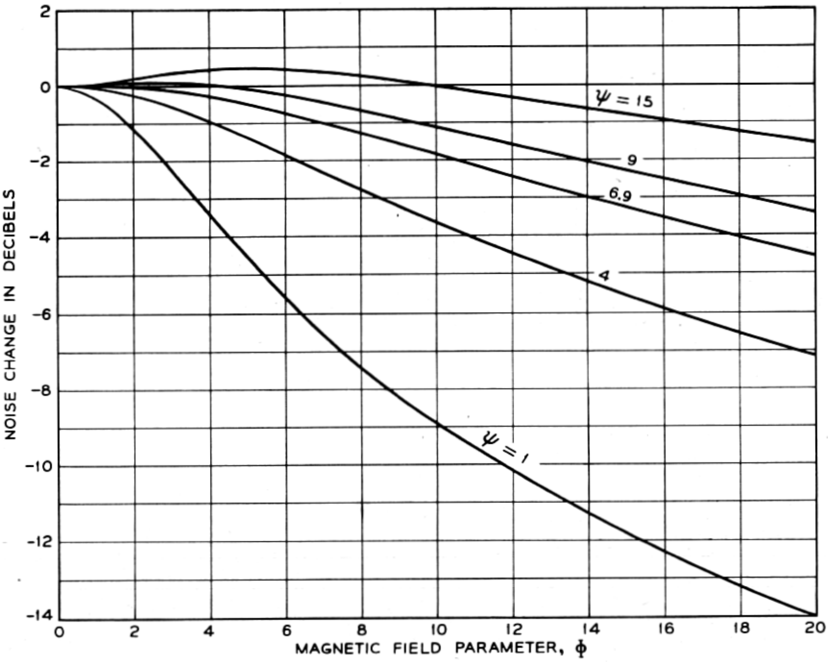

Fig. 2 — Noise change for a filament with equal surfaces as a function of Φ for various ψ. Surface generation.

where

$$\alpha_1 = \left(\frac{\Phi}{2} - \psi_1\right)/\psi_1 \; ; \qquad \alpha_2 = \left(\frac{\Phi}{2} + \psi_2\right)/\psi_2 \; ;$$

$$\Delta = \alpha_1 e^{-\Phi(1-x_0/a)/2} + \alpha_2 e^{\Phi(1+x_0/a)/2}.$$

Now we are in a position to calculate $\tau_H(x_0)$, the life-time in a magnetic field H (contained in Φ), due to an emission at x_0:

$$\tau_H(x_0) = \int_{-a}^{x_0} w_2(x, x_0)\, dx + \int_{x_0}^{+a} w_1(x,x_0)\, dx$$

$$= (B_1 + B_2)a + (B_2 - B_1)x_0 + \frac{2a}{\Phi}(A_2 - A_1) \qquad 10$$

$$\frac{2a}{\Phi}\left[A_1 e^{\Phi(1-x_0/a)/2} - A_2 e^{-\Phi(1+x_0/a)/2}\right].$$

Special Cases

With the help of (9) and (10) we can now examine special cases.

1. Surface Emission

When only the surface $x = +a$ is emitting, we set $x_0 = +a$ in (10) and get

$$\tau_H(+a) = 2B_2 a + \frac{2a}{\Phi} A_2(1 - e^{-\Phi})$$

$$= \frac{4a^2}{\Phi D}(1 + \alpha_1) \frac{\left[1 - \alpha_2 \frac{e^{\Phi}}{\Phi}(1 - e^{-\Phi})\right]}{\alpha_1 + \alpha_2 e^{\Phi}}$$

$$= -\frac{2a^2}{D\psi_1} \frac{\left(1 - \alpha_2 \frac{(e^{\Phi} - 1)}{\Phi}\right)}{\alpha_1 + \alpha_2 e^{\Phi}}.$$

Similarly, when only $-a$ is emitting, we get

$$\tau_H(-a) = -\frac{2a^2}{D\psi_2} \frac{\alpha_1 \frac{e^{-\Phi} - 1}{\Phi} - 1}{\alpha_1 e^{-\Phi} + \alpha_2}.$$

But in an actual experiment, both faces will be emitting, with mean square strengths $\langle J_1^2 \rangle$, $\langle J_2^2 \rangle$ say. The quantity that is then measured

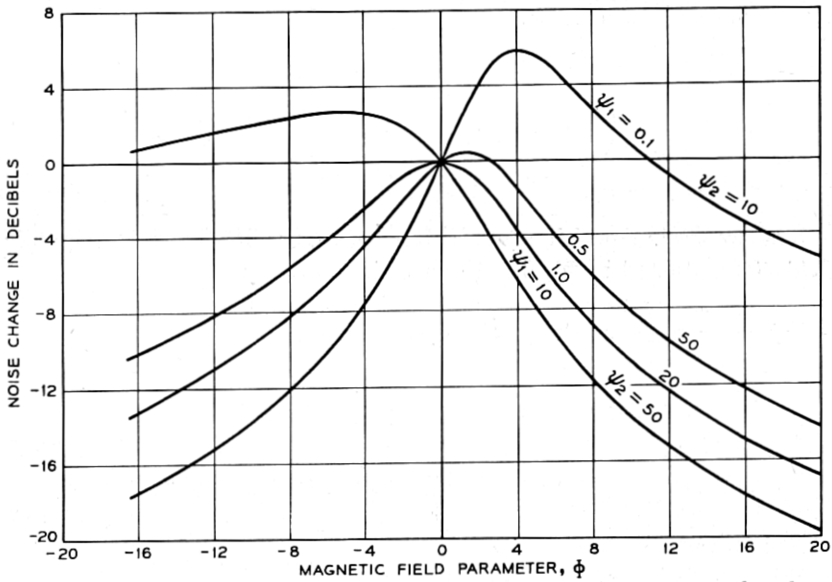

Fig. 3 — Contribution to the noise change from a unit source at the plane $x = +a$.

is the ratio

$$N_H = \frac{\Delta g_H^2}{\Delta g_{H=0}^2} = \frac{\langle J_1^2\rangle \tau_H^2(+a) + \langle J_2^2\rangle \tau_H^2(-a)}{\langle J_1^2\rangle \tau_{H=0}^2(+a) + \langle J_2^2\rangle \tau_{H=0}^2(-a)}. \tag{11}$$

Therefore we also need the lifetimes at zero H (that is, zero Φ). But at $\Phi = 0$, the τ's are indeterminate, and we therefore have to take limits. Expansion in powers of Φ shows that

$$\underset{\Phi\to 0}{lt}\ \tau_H(+a) = \frac{a^2}{D\psi_1} \frac{\left(1 + \frac{1}{\psi_2}\right)}{1 + \frac{1}{2}\left(\frac{1}{\psi_1} + \frac{1}{\psi_2}\right)},$$

$$\underset{\Phi\to 0}{lt}\ \tau_H(-a) = \frac{a^2}{D\psi_2} \frac{\left(1 + \frac{1}{\psi_1}\right)}{1 + \frac{1}{2}\left(\frac{1}{\psi_1} + \frac{1}{\psi_2}\right)}.$$

Thus we finally get

$$N_H = \frac{\Delta g_H^2}{\Delta g_0^2} = 4\left(1 + \frac{1}{2}\left(\frac{1}{\psi_1} + \frac{1}{\psi_2}\right)\right)^2 \frac{\frac{\langle J_1^2\rangle}{\psi_1^2}\left[\frac{1 - \alpha_2\left(\frac{e^{\Phi} - 1}{\Phi}\right)}{\alpha_1 + \alpha_2 e^{\Phi}}\right]^2 + \frac{\langle J_2^2\rangle}{\psi_1^2}\left[\frac{\alpha_1\left(\frac{1 - e^{-\Phi}}{\Phi}\right) + 1}{\alpha_1 e^{-\Phi} + \alpha_2}\right]^2}{\frac{\langle J_1^2\rangle}{\psi_1^2}\left(1 + \frac{1}{\psi_2}\right)^2 + \frac{\langle J_2^2\rangle}{\psi_2^2}\left(1 + \frac{1}{\psi_1}\right)^2} \tag{12}$$

There remains one small difficulty in the way of comparing experiment with theory: We do not know $\langle J_1^2\rangle$, $\langle J_2^2\rangle$. As suggested by H. C. Montgomery, we are able to overcome this difficulty as follows: We first draw a number of curves of

$$\frac{\tau_H^2(+a)}{\tau_0^2(+a)}, \qquad \frac{\tau_H^2(-a)}{\tau_0^2(-a)} \text{ versus } \Phi,$$

for various sets of parameters $(\psi_1, \psi_2.)$ (See figures 3, 4). Then we contrive to match a linear superposition

$$c_1 \frac{\tau_H^2(+a)}{\tau_0^2(+a)} + c_2 \frac{\tau_H^2(-a)}{\tau_0^2(-a)}$$

where $c_1 + c_2 = 1$ to the experimental curve. This will be possible only for one particular set of values (ψ_1, ψ_2). From c_1 and $c_2 = 1 - c_1$ and

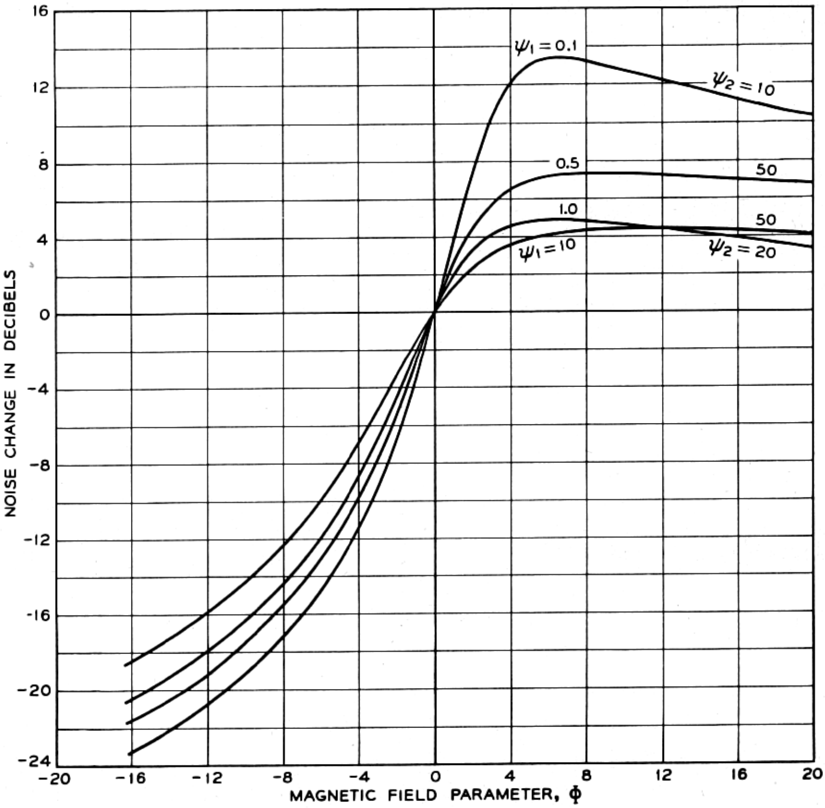

Fig. 4 — Contribution to the noise change from a unit source at $x = -a$.

from (ψ_1, ψ_2) we can determine the ratio of $\langle J_1^2 \rangle \langle J_2^2 \rangle$, for, having matched the experimental curve, we know that

$$c_1 \frac{\tau_H^2(+a)}{\tau_0^2(+a)} + (1 - c_1) \frac{\tau_H^2(-a)}{\tau_0^2(-a)} = \frac{\langle J_1^2 \rangle \tau_H^2(+a) + \langle J_2^2 \rangle \tau_H^2(-a)}{\langle J_1^2 \rangle \tau_0^2(+a) + \langle J_2^2 \rangle \tau_0^2(-a)},$$

which is satisfied for all Φ if

$$\frac{c_1}{\tau_0^2(+a)} = \frac{\langle J_1^2 \rangle}{\langle J_1^2 \rangle \tau_0^2(+a) + \langle J_2^2 \rangle \tau_0^2(-a)},$$

$$\frac{(1 - c_1)}{\tau_0^2(-a)} = \frac{\langle J_2^2 \rangle}{\langle J_1^2 \rangle \tau_0^2(+a) + \langle J_2^2 \rangle \tau_0^2(-a)}.$$

These equations are self-consistent and give

$$\frac{c_1}{1 - c_1} = \frac{\tau_0^2(+a)\langle J_1^2 \rangle}{\tau_0^2(-a)\langle J_2^2 \rangle} = \frac{\langle J_1^2 \rangle}{\langle J_2^2 \rangle} \frac{\left(1 + \frac{1}{\psi_2}\right)^2}{\left(1 + \frac{1}{\psi_1}\right)^2} \frac{\psi_2^2}{\psi_1^2}.$$

Thus the match of theory to experiment actually provides information about the relative emissivities of the surfaces.

When the surfaces are equal, $\psi_1 = \psi_2 = \psi$; $\langle J_1^2 \rangle = \langle J_2^2 \rangle$ and the result 12 simplifies to

$$N_H = \frac{\left(1 + \frac{1}{\psi}\right)^2 + \left(\frac{2}{\Phi} - \coth \frac{\Phi}{2}\right)^2}{\left(1 + \frac{\Phi}{2\psi} \coth \frac{\Phi}{2}\right)^2}. \tag{13}$$

$10 \log N_H$ for this case is plotted in figure 2.

It is interesting to see how N_H for equal surfaces varies for small values of Φ. $2/\Phi - \coth \Phi/2$ is regular at $\Phi = 0$ and varies as $-\Phi/6$ there. Hence the initial variation is as

$$N_{H\text{small}} = 1 + \frac{1}{4\left(1 + \frac{1}{\psi}\right)^2} \left(\frac{\Phi^2}{9} - \frac{2}{3} \frac{\Phi^2}{\psi} \left(1 + \frac{1}{\psi}\right)\right)$$

$$= 1 + \frac{\Phi^2}{12\left(1 + \frac{1}{\psi}\right)^2} \left(\frac{1}{3} - \frac{2}{\psi} \left(1 + \frac{1}{\psi}\right)\right).$$

Hence the curve rises or falls initially according as

$$\frac{1}{3} \gtrless 2 \frac{1}{\psi} \left(1 + \frac{1}{\psi}\right).$$

The noise therefore increases initially if

$$\sqrt{\frac{5}{12}} - \frac{1}{2} > \frac{1}{\psi} \qquad \text{or} \qquad \psi > 6.9 \text{ approximately}$$

and falls initially if

$$\psi < 6.9.$$

2. Volume Generation.

At first sight it may seem that if volume generation is considered, so should volume recombination (detailed balancing). This is not necessarily so, since we are not dealing with an equilibrium situation here.

Therefore the possibility of volume generation and surface recombination cannot be discarded.

This case is somewhat more difficult. Assuming all sources to be uncorrelated and uniformly distributed throughout the interval $(-a, +a)$, we have to square expression (10) and integrate it from $x_0 = -a$ to $x_0 = +a$ in order to find $\langle \Delta g_H^2 \rangle$. (We suppose that all the sources are of equal strength $\langle J^2 \rangle$). Substituting the values of the A and B from 9 and 10, we get, after some obvious cancellations

$$\tau_H(x_0) = \frac{2a}{\Phi D}\,[x_0 + Se^{-\Phi(x_0/2a)} + T]$$

where

$$S = 2a\,\frac{\left(1 + \frac{1}{2}\left(\frac{1}{\psi_1} + \frac{1}{\psi_2}\right)\right)}{\alpha_1 e^{-\Phi/2} + \alpha_2 e^{\Phi/2}},$$

$$T = \frac{2a}{\Phi}\left[\frac{\Phi}{2}\,\frac{\alpha_1 e^{-\Phi/2} - \alpha_2 e^{+\Phi/2}}{\alpha_1 e^{-\Phi/2} + \alpha_2 e^{+\Phi/2}} - 2\,\frac{\alpha_1\alpha_2 \sinh \frac{\Phi}{2}}{\alpha_1 e^{-\Phi/2} + \alpha_2 e^{\Phi/2}} - 1\right].$$

Hence

$$\langle \Delta g_H^2 \rangle = \langle J \rangle \int_{-a}^{+a} \tau_H^2(x_0)\,dx_0$$

$$= \frac{4a^2}{\Phi^2 D^2}\left[\frac{2}{3}a^3 + 2T^2 a + \frac{8TSa}{\Phi}\sinh\frac{\Phi}{2} + \frac{2aS^2}{\Phi}\sinh\Phi + \frac{8Sa^2}{\Phi}\left(\frac{\sinh\frac{\Phi}{2}}{\frac{\Phi}{2}} - \cosh\frac{\Phi}{2}\right)\right]. \tag{14}$$

Before proceeding with this general case, we first consider the limiting case $\psi_1 = \psi_2 = \infty$, when $\alpha_1 = -1$, $\alpha_2 = +1$. Then

$$T = -a\coth\frac{\Phi}{2} \qquad S = \frac{a}{\sinh\frac{\Phi}{2}}$$

and

$$\int_{-a}^{+a} \tau_H^2(x_0) = \frac{8a^5}{\Phi^2 D^2}\left[\frac{1}{3} + \coth^2\frac{\Phi}{2} - 6\,\frac{\coth\frac{\Phi}{2}}{\Phi} + \frac{8}{\Phi^2}\right].$$

To find $N_H = \langle \Delta g_H^2 \rangle / \langle \Delta g_{H=0}^2 \rangle$ we need the limit of $\int_{-a}^{+a} \tau_H^2$ as $\Phi \to 0$. After some tedious algebra, we find this limit to be

$$\tau_{H=0}^2 = \frac{4a^5}{15D^2}$$

so that

$$(N_H)_{\psi_1,\psi_2=\infty} = \frac{\int \tau_H^2}{\int \tau_0^2} = \frac{30}{\Phi^2}\left[\frac{1}{3} + \coth^2\frac{\Phi}{2} - \frac{6\coth\frac{\Phi}{2}}{\Phi} + \frac{8}{\Phi^2}\right]. \quad (15)$$

In the general case we can again take the limit of (14) as $\Phi \to 0$ in order to determine N_H, but this would be too tedious. Instead, we solve the diffusion equation directly when $\Phi = 0$. The equation is then simply

$$\frac{d^2w}{dx^2} = 0$$

and the solution subject to the correct boundary conditions and allowing for a steady unit injection at x_0 is

$$w_1 = A_1(x - x_0) + B \qquad x > x_0$$

$$w_2 = A_2(x - x_0) + B \qquad x < x_0 .$$

where

$$A_1 = -\frac{1}{D}\psi_1\left[1 + \psi_2\left(1 + \frac{x_0}{a}\right)\right] \Big/ (\psi_1 + \psi_2 + 2\psi_1\psi_2),$$

$$A_2 = \frac{1}{D}\psi_2\left[1 + \psi_1\left(1 - \frac{x_0}{a}\right)\right] \Big/ (\psi_1 + \psi_2 + 2\psi_1\psi_2),$$

$$B = \frac{a}{D}\left[1 + \psi_1\left(1 - \frac{x_0}{a}\right)\right]\left[1 + \psi_2\left(1 + \frac{x_0}{a}\right)\right] \Big/ (\psi_1 + \psi_2 + 2\psi_1\psi_2).$$

We now have

$$\tau_{H=0}(x_0) = \int_{-a}^{+a} w\,dx = \int_{-a}^{x_0} w_2\,dx + \int_{x_0}^{a} w_1\,dx$$

$$= -\frac{1}{2}\left[2ax_0(A_1 + A_2) + \frac{a^2 + x_0^2}{D}\right] + 2aB.$$

From this we can compute

$$\int_{-a}^{+a} \tau_{H=0}^2(x_0)\,dx_0 ,$$

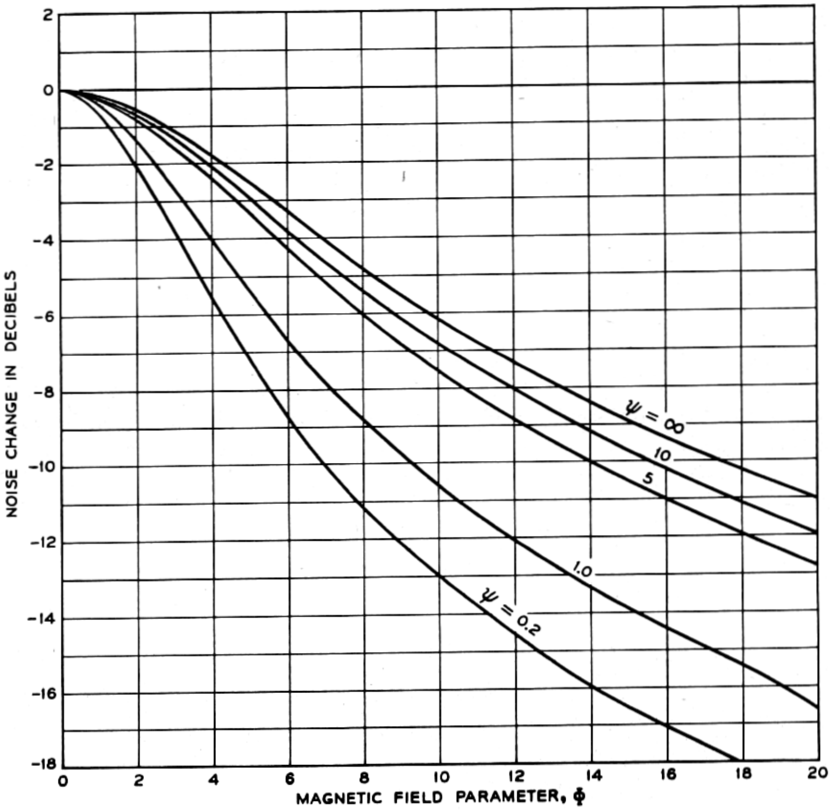

Fig. 5 — Noise change in a filament with uniform volume generation and equal surface recombination rates.

but the result is still rather complicated unless $\psi_1 = \psi_2 = \psi$. If we therefore restrict ourselves to that case we find

$$A_1 + A_2 = -\frac{\psi}{D(1+\psi)} \cdot \frac{x_0}{a},$$

$$B = \frac{1}{2D}\left[\frac{1+\psi}{\psi} - \frac{\psi}{1+\psi}\frac{x_0^2}{a^2}\right]$$

and

$$D\tau_{H=0}(x_0) = a^2\left(\frac{1}{2} + \frac{1}{\psi}\right) - \frac{x_0^2}{2}$$

and

$$\int_{-a}^{+a} \tau_{H=0}^2(x_0)\, dx_0 = \frac{2a^5}{D^2}\left(\frac{1}{45} + \left(\frac{1}{\psi} + \frac{1}{3}\right)^2\right).$$

(Note that as $\psi \to \infty$, this quantity tends to $4a^5/15D^2$, as before in our limiting process.)

When $\psi_1 = \psi_2 = \psi$, S and T also simplify somewhat, and the result is

$$N_H = \frac{\langle \Delta g_H^2 \rangle}{\langle \Delta g_{H=0}^2 \rangle}$$

$$= 2\,\frac{\frac{2}{3} + 2T^2 + \frac{8S}{\Phi}\left(\frac{2}{\Phi} - T\right)\sinh\frac{\Phi}{2} - \frac{8S}{\Phi}\cosh\frac{\Phi}{2} + \frac{2S^2}{\Phi}\sinh\Phi}{\Phi^2\left[\left(\frac{1}{\psi} + \frac{1}{3}\right)^2 + \frac{1}{45}\right]}$$

where now

$$S = \frac{1 + \frac{1}{\psi}}{\left(\frac{\Phi}{2\psi}\cosh\frac{\Phi}{2} + \sinh\frac{\Phi}{2}\right)},$$

$$T = \left(1 + \frac{1}{\psi}\right)\frac{\frac{\Phi}{2\psi} + \coth\frac{\Phi}{2}}{1 + \frac{\Phi}{2\psi}\coth\frac{\Phi}{2}}.$$

An alternative form is

$$N_H = \frac{4}{\frac{1}{45} + \left(\frac{1}{3} + \frac{1}{\psi}\right)^2}\left[\frac{1}{3\Phi^2} + \frac{4}{\Phi^3\Delta}\left(1 + \frac{1}{\psi}\right)\left(\frac{2}{\Phi} - \coth\frac{\Phi}{2}\right) + \frac{\left(1 + \frac{1}{\psi}\right)^2}{\Phi^2\Delta^2}\left(\left(\frac{\Phi}{2\psi} + \coth\frac{\Phi}{2}\right)^2 + \frac{2}{\psi} - \frac{2\coth\frac{\Phi}{2}}{\Phi}\right)\right] \tag{16}$$

where

$$\Delta = 1 + \frac{\Phi}{2\psi}\coth\frac{\Phi}{2},$$

a result which correctly tends to (15) as $\psi \to \infty$. 10 log N_H for various ψ is shown in Fig. 5.

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