DC Field Distribution in a "Swept Intrinsic" Semiconductor Configuration

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This paper contains an analysis of the dc field intensity distribution in an idealized one-dimensional n-*intrinsic*-p *semi-conductor configuration biased in reverse. It gives some quantitative insight into the progressive penetration of the field into the intrinsic region as the magnitude of the bias voltage is increased.*

INTRODUCTION

Possible applications have been suggested for semi-conductor configurations involving intrinsic regions adjacent simultaneously to n- and p-type extrinsic regions. The basic idea behind some of these proposals is that a suitably large reverse bias voltage (n-regions positive with respect to p- regions) will set up a substantial electric field in the interior of the intrinsic region. This field would sweep most of the mobile carriers out of the intrinsic material, producing a region of material ("swept intrinsic") supporting a large field and having a high resistivity.

This paper contains a dc analysis of an idealized one-dimensional swept intrinsic structure with abrupt transitions from strongly n-type to highly intrinsic to strongly p-type material. It gives some quantitative insight into the penetration of the electric field into the intrinsic region as the bias voltage is progressively increased.

FORMULATION OF PROBLEM

A one-dimensional structure will be considered having the distribution of excess of donor conceptration over acceptor concentration (N_d-N_a) shown in Fig. 1. It will be supposed that N/n_i and P/n_i are $\gg$ 1 and that a reverse bias voltage (Fig. 2) is applied between the bodies of the n- and p-type regions. (n_i denotes the thermal equilibrium con-

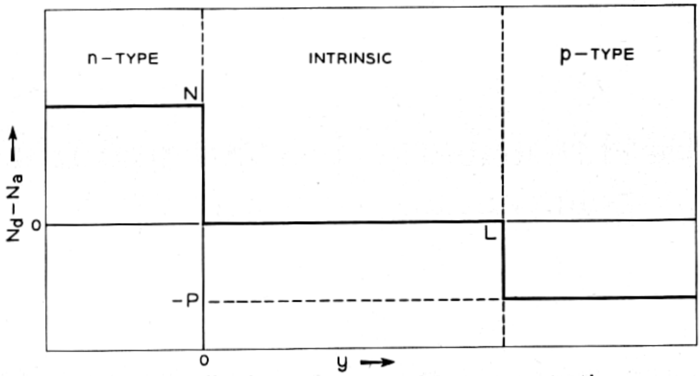

Fig. 1 — Assumed distribution of excess of donor concentration over acceptor concentration.

centration of mobile electrons — or of holes — in intrinsic material at the given temperature.)

The set of equations which (together with boundary conditions to be specified later) determines the electric field in the intrinsic region $0 < y < L$ is:

$$\frac{dE}{dy} = \frac{q}{\kappa}(p - n), \tag{1}$$

$$\frac{dp}{dy} = \frac{q}{kT} pE - \frac{i_p}{\mu_p kT}, \tag{2}$$

$$\frac{dn}{dy} = -\frac{q}{kT} nE + \frac{i_n}{\mu_n kT}, \tag{3}$$

$$\frac{d\psi}{dy} = -E, \tag{4}$$

$$\frac{di_p}{dy} = q(g - r), \tag{5}$$

$$\frac{di_n}{dy} = -q(g - r). \tag{6}$$

where E: electric field intensity, volts/m.

q : electronic charge magnitude, coulombs.

κ : absolute dielectric constant, farads/m.

p : hole concentration, m^{-3}.

n : electron concentration, m^{-3}.

k : Boltzmann's constant, joules/°K.

T : temperature, °K.
ψ : electric potential, volts.
i_p : hole current density, amps/m^2.
i_n : electron current density, amps/m^2.
μ_p: hole mobility constant, m^2/volt-sec.
μ_n: electron mobility constant, m^2/volt-sec.
g : rate of generation of hole-electron pairs, m^{-3} sec^{-1}.
r : rate of recombination of hole-electron pairs, m^{-3} sec^{-1}.

An order-of-magnitude comparison of the terms in (2) or (3) reveals that the currents probably have little influence on the field distribution. For example for

$$i_n \sim 5 \text{ amps/m}^2,$$

$$\mu_n kT = 1.44 \times 10^{-21} \text{ amp-m}^2,$$

$$N = 2 \times 10^{22} \text{ m}^{-3}, \text{ and } L = 10^{-3}\text{m}.$$

$$\int_0^L \frac{i_n}{\mu_n kT}\, dy \sim 3 \cdot 10^{21} L \sim 3 \cdot 10^{18} \text{ m}^{-3}$$

while

$$\left| \int_0^L \frac{dn}{dy}\, dy \right| = n(0) - n(L) \sim N = 2 \cdot 10^{22} \text{ m}^{-3}.$$

On this basis, the current terms in (2) and (3) can be omitted without serious error. No use then has to be made of (5) and (6), so the governing equations for the intrinsic region become:

$$\frac{dE}{dy} = \frac{q}{\kappa}(p - n), \tag{1'}$$

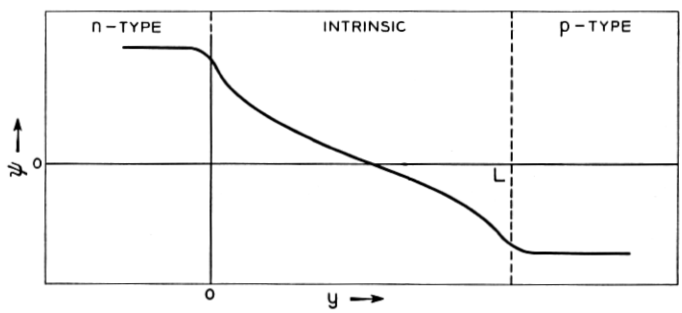

Fig. 2 — Qualitative picture of potential distribution in reverse-biased *n-intrinsic-p* structure.

$$\frac{dp}{dy} = \frac{q}{kT}\, pE\,, \tag{2'}$$

$$\frac{dn}{dy} = -\frac{q}{kT}\, nE, \tag{3'}$$

$$\frac{d\psi}{dy} = -E. \tag{4'}$$

Equations (2′)–(4′) will also be used within the n-type extrinsic region ($y < 0$) and the p-type extrinsic region ($y > L$). However, for the n-type region (1′) is replaced by

$$\frac{dE}{dy} = \frac{q}{\kappa}\,(N + p - n), \tag{1'a}$$

where N denotes the excess concentration of ionized donor over acceptor centers for $y < 0$. Similarly, for the p-type region (1′) is replaced by

$$\frac{dE}{dy} = \frac{q}{\kappa}\,(-P + p - n), \tag{1'b}$$

where P denotes the excess concentration of ionized acceptor over donor centers for $y > L$.

In order to solve the equation set (1′)–(4′) for the intrinsic region $0 < y < L$, it will be necessary partially to solve the sets (1′a) (2′)–(4′) and (1′b) (2′)–(4′) governing the two extrinsic regions because only in this way can a sufficient number of appropriate boundary conditions be imposed.

Deep inside the extrinsic regions the electric field intensity will be negligible and the mobile carrier concentrations will have their equilibrium values. This leads to the conditions

$$E = 0, \qquad np = n_i^2, \qquad n - p = N \text{ at } y = -\infty \tag{7}$$

and

$$E = 0, \qquad np = n_i^2, \qquad p - n = P \text{ at } y = +\infty. \tag{8}$$

It will further be supposed that there is no infinite charge concentration at the extrinsic-intrinsic interfaces so that the electric field intensity is continuous at the interfaces. The concentration of the (local) majority carrier will also be assumed continuous at an interface. In short,

$$E \text{ and } n \text{ are continuous at } y = 0 \tag{9}$$

and

$$E \text{ and } p \text{ are continous at } y = L. \tag{10}$$

Finally, we choose the reference level for the electric potential in the intrinsic region so that

$$\psi = 0 \text{ for } n = p \tag{11}$$

and regard the potential at the interface $y = 0$ as a prescribed parameter,

$$\psi = \frac{kT}{q} \cdot U \text{ at } y = 0. \tag{12}$$

The two conditions (11) and (12) apply directly to the solutions for the intrinsic region. The conditions (7)–(10) indirectly imply the two additional restraints necessary to determine a unique solution of (1′)–(4′) in $0 < y < L$.

NORMALIZED VARIABLES AND EQUATIONS

It is convenient to introduce dimensionless normalized variables before proceeding further with the mathematical analysis. As reference voltage it is natural to adopt the *Boltzmann voltage*

$$\psi_B \equiv \frac{kT}{q}, \tag{13}$$

the voltage equivalent of the mean kinetic energy of an electron at temperature T. (At room temperature the Boltzmann voltage is about 1/40 of a volt.) As reference quantity for carrier concentrations we choose the geometric mean of the majority carrier excess concentrations for the two extrinsic regions. i.e.,

$$\text{reference concentration} = (\text{NP})^{1/2}. \tag{14}$$

The reference voltage and carrier concentration having been so chosen, it is natural to select as reference length the *mean Debye length*

$$\mathfrak{L} \equiv \left[\frac{kT/q}{2\frac{q}{\kappa}\,(NP)^{1/2}}\right]^{1/2}. \tag{15}$$

This mean Debye length is related by

$$\mathfrak{L} = (\mathfrak{L}_n\,\mathfrak{L}_p)^{1/2} \tag{16}$$

to the n-region and p-region Debye lengths defined respectively by

$$\mathfrak{L}_n = \left[\frac{kT/q}{2\frac{q}{\kappa}N}\right]^{1/2} \tag{17}$$

and

$$\mathfrak{L}_p = \left[\frac{kT/q}{2\frac{q}{\kappa}P}\right]^{1/2}. \tag{18}$$

We now use the reference quantities defined in (13)–(15) to introduce the *normalized distance*

$$\hat{y} \equiv \frac{y}{\mathfrak{L}}, \tag{19}$$

the *normalized thickness* of the intrinsic layer

$$\hat{L} \equiv \frac{L}{\mathfrak{L}}, \tag{20}$$

the *normalized concentrations* of positive and negative mobile carriers

$$\hat{p} \equiv \frac{p}{(NP)^{1/2}} \quad \text{and} \quad \hat{n} \equiv \frac{n}{(NP)^{1/2}}, \tag{21, 22}$$

the *normalized potential*

$$\hat{\psi} \equiv \frac{\psi}{\psi_B}, \tag{23}$$

and the *normalized electric field intensity*

$$\hat{E} \equiv \frac{E}{\psi_B/\mathfrak{L}}. \tag{24}$$

In terms of these normalized variables, the governing equations [(1′)–(4′)] for the intrinsic region become:

$$\frac{d\hat{E}}{d\hat{y}} = \frac{1}{2}(\hat{p} - \hat{n}), \tag{25}$$

$$\frac{d\hat{p}}{d\hat{y}} = \hat{p}\hat{E} \tag{26}$$

$$\frac{d\hat{n}}{d\hat{y}} = -\hat{n}\hat{E}, \tag{27}$$

$$\frac{d\hat{\psi}}{d\hat{y}} = -\hat{E}. \tag{28}$$

For the n-type region, (25) should be replaced by

$$\frac{d\hat{E}}{d\hat{y}} = \tfrac{1}{2}(\Lambda + \hat{p} - \hat{n}) \tag{25a}$$

where

$$\Lambda \equiv \left(\frac{N}{P}\right)^{1/2}. \tag{29}$$

For the p-type region, (25) should be replaced by

$$\frac{d\hat{E}}{d\hat{y}} = \tfrac{1}{2}(-\Lambda^{-1} + \hat{p} - \hat{n}). \tag{25b}$$

FORMAL SOLUTION OF EQUATIONS FOR INTRINSIC REGION

Division of (25)–(27) by (28) yields, after evident rearrangements of factors,

$$\frac{d\hat{E}^2}{d\hat{\psi}} = \hat{n} - \hat{p}, \tag{30}$$

$$\frac{d \ln \hat{p}}{d\hat{\psi}} = -1, \tag{31}$$

$$\frac{d \ln \hat{n}}{d\hat{\psi}} = 1. \tag{32}$$

From (31) and (32) follow

$$\hat{p} = Ae^{-\hat{\psi}} \tag{33}$$

and

$$\hat{n} = A_1 e^{\hat{\psi}}, \tag{34}$$

where A and A_1 are constants of integration. The condition (11) that $\psi = 0$ for $p = n$ implies that

$$A_1 = A.$$

Substitution of (33) and (34) into (30) yields

$$\begin{aligned} \frac{d\hat{E}^2}{d\hat{\psi}} &= A(e^{\hat{\psi}} - e^{-\hat{\psi}}) \\ &= 2A \sinh \hat{\psi}. \end{aligned} \tag{35}$$

Integration of (35) now leads to

$$\hat{E} = [2A\ (\cosh \hat{\psi} + B)]^{1/2} \tag{36}$$

where B is another integration constant. Substitution of (36) into (28) in the form

$$\frac{d\hat{\psi}}{\hat{E}} = -d\hat{y}$$

yields after another integration

$$\int_0^{\hat{\psi}} \frac{ds}{(\cosh s + B)^{1/2}} = (2A)^{1/2}(C - \hat{y}), \tag{37}$$

where C is the fourth integration constant.

In order to express in terms of tabulated functions the relationship between $\hat{\psi}$ and $\hat{y}$ defined by (37) we shall consider two cases: $-1 < B \leq 1$ and $B \geq 1$. (It is not necessary to consider $B < -1$ because A is essentially positive [see (33)] so that $B < -1$ would imply an imaginary field strength [see (36)] at the plane in the intrinsic region where $\hat{\psi} = 0$.)

The changes of variable of integration

$$s = 2 \sinh^{-1} \cot \lambda \text{ for } -1 < B \leq 1,$$

$$s = 2 \sinh^{-1} \tan \theta \text{ for } B \geq 1$$

permit the carrying out of the integration indicated on the left side of (37). This gives

$$2^{1/2}\left(K\left[\left(\frac{1-B}{2}\right)^{1/2}\right] - F\left[\left(\frac{1-B}{2}\right)^{1/2}, \sin^{-1} \operatorname{sech} \frac{\hat{\psi}}{2}\right]\right) = (2A)^{1/2}(C - \hat{y}) \tag{38a}$$

or

$$\hat{\psi} = \ln \frac{1 + cn\left(\left(\frac{1-B}{2}\right)^{1/2}, K\left[\left(\frac{1-B}{2}\right)^{1/2}\right] - A^{1/2}(C - \hat{y})\right)}{1 - cn\left(\left(\frac{1-B}{2}\right)^{1/2}, K\left[\left(\frac{1-B}{2}\right)^{1/2}\right] - A^{1/2}(C - \hat{y})\right)} \tag{38b}$$

for $-1 < B \leq 1$

and

$$\frac{2}{(B+1)^{1/2}} F\left[\left(\frac{B-1}{B+1}\right)^{1/2}, \sin^{-1} \tanh \frac{\hat{\psi}}{2}\right] = (2A)^{1/2}(C - \hat{y}) \tag{39a}$$

or

$$\hat{\psi} = \ln \frac{1 + sn\left(\left(\frac{B-1}{B+1}\right)^{1/2}, \left(\frac{B+1}{2}\right)^{1/2} A^{1/2}(C - \hat{y})\right)}{1 - sn\left(\left(\frac{B-1}{B+1}\right)^{1/2}, \left(\frac{B+1}{2}\right)^{1/2} A^{1/2}(C - \hat{y})\right)} \tag{39b}$$

for $B \geq 1$.

Note:

$$F[k, \phi] \equiv \int_0^{\phi} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

is the Elliptic Integral of First Kind, usually tabulated for

$$0 \leq \phi \leq \frac{\pi}{2}, \qquad 0 \leq k \leq 1.$$

$$K[k] \equiv F\left[k, \frac{\pi}{2}\right]$$

is the Complete Elliptic Integral of First Kind.

$$sn[k, F] \equiv \sin \phi$$

and

$$cn[k, F] \equiv \cos \phi$$

are Jacobian Elliptic Functions associated with $F[k, \phi]$.

When A, B, and C are prescribed, (38b) or (39b) gives the normalized potential $\hat{\psi}$ as a function of the normalized distance $\hat{y}$. The normalized hole concentration $\hat{p}$, electron concentration $\hat{n}$, and field intensity $\hat{E}$ are then given as functions of $\hat{y}$ by (33), (34), and (36).

In a following section it will be shown that, subject to the reasonable assumptions

$$\Lambda e^{-U/2}, \Lambda^{-1} e^{-U/2} \ll 1, \tag{40a, b}$$

$$\hat{L} \gg \Lambda^{-1/2}, \Lambda^{1/2}, \tag{40c, d}$$

$$U < \frac{1}{20} \Lambda^{1/2} L, \frac{1}{20} \Lambda^{-1/2} \hat{L}, \tag{40e, f}$$

A, C, and B are given in terms of the normalized potential at $y = 0$, the normalized thickness of the intrinsic region, and the parameter $\Lambda \equiv (N/P)^{1/2}$ by

$$A \approx \Lambda e^{-U-1}, \tag{41}$$

$$C \approx \tfrac{1}{2}\hat{L} + e^{1/2}(\Lambda^{1/2} - \Lambda^{-1/2}), \tag{42}$$

and

$$\Phi(B) \approx \frac{1}{2}\left(\frac{\Lambda}{e}\right)^{1/2} \hat{L}e^{-U/2} \tag{43}$$

where

$$\Phi(B) \equiv \begin{cases} K\left[\left(\dfrac{1-B}{2}\right)^{1/2}\right] & \text{for } -1 < B < 1 \\ \dfrac{\pi}{2} & \text{for } B = 1 \\ \left(\dfrac{2}{B+1}\right)^{1/2} K\left[\left(\dfrac{B-1}{B+1}\right)^{1/2}\right] & \text{for } B > 1. \end{cases}$$

Thus $\hat{\psi}$ $\hat{p}$, $\hat{n}$, and $\hat{E}$ can be computed as functions of $\hat{\psi}$ for any choice of U, $\hat{L}$, and Λ consistent with (40).

DESCRIPTION OF SAMPLE FIELD DISTRIBUTION COMPUTATIONS

The results of the foregoing analysis were used to compute $\hat{\psi}(\hat{y})$ and $\hat{E}(\hat{y})$ for $\Lambda = 1$ (symmetric case, $N = P$) and the combinations of U and $\hat{L}$ indicated by the following table:

		$\hat{L}$	
U	4,000	40,000	400,000
7	X	X	X
20	X	X	X
200	X	X	X
2,000		X	X
20,000			X

The results of these computations are presented in Figs. 3–5 (pages 675 to 677) as plots of $\hat{\psi}/U$ versus $\hat{y}/\hat{L}$, and in Figs. 6–8 (pages 678 to 680) as plots of $\hat{E}/(2U/\hat{L})$ versus $\hat{y}/\hat{L}$. ($2U/\hat{L}$ is the average reduced field strength for the intrinsic region.) Selected curves from the above figures are replotted on non-logarithmic axes in Figs. 9 and 10 (pages 681 and 682). The curves need only be plotted for $0 \leq \hat{y}/\hat{L} \leq \frac{1}{2}$ because, by symmetry for $\Lambda = 1$, $\hat{E}(\hat{L} - \hat{y}) = \hat{E}(\hat{y})$ and $\hat{\psi}(\hat{L} - \hat{y}) = -\hat{\psi}(\hat{y})$. The ordinates and abscissas of Figures 3–10 are given in terms of the original unnormalized variables by

$$\frac{\hat{\psi}}{U} = \frac{\psi}{\psi(0)}, \qquad \frac{\hat{y}}{\hat{L}} = \frac{y}{L} \qquad \text{and} \qquad \frac{\hat{E}}{2U/\hat{L}} = \frac{E}{2\psi(0)/L}.$$

($2\psi(0)/L$ is the average field intensity for the intrinsic region.)

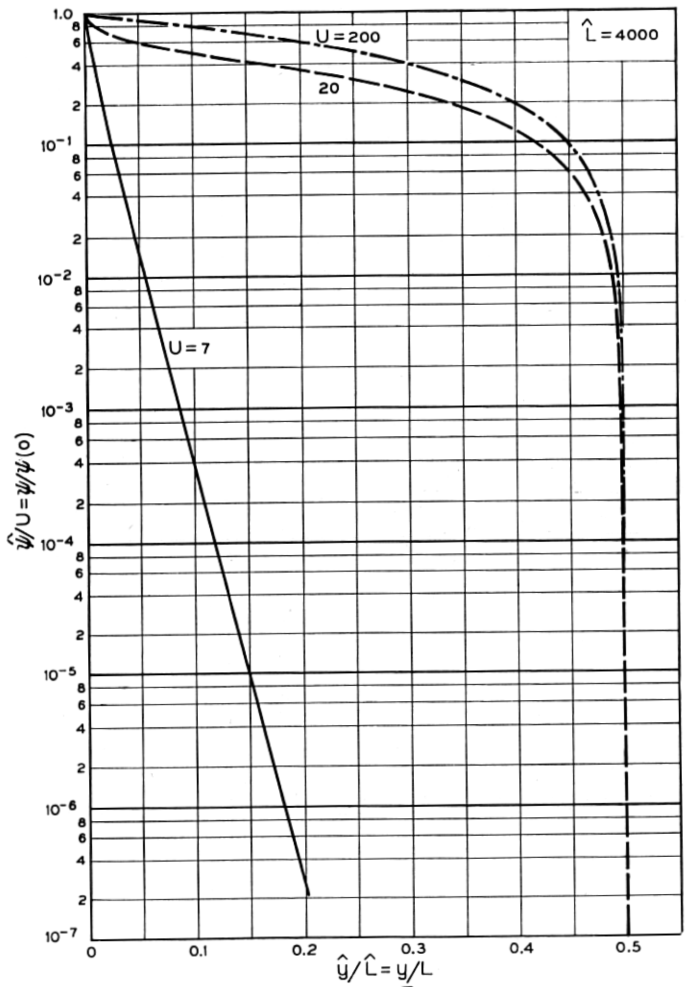

Fig. 3 — (Potential at point in intrinsic layer/Potential of *n-intrinsic* junction) versus (Distance from *n-intrinsic* junction/Intrinsic layer thickness) for $\hat{L}$ = (Intrinsic layer thickness/Mean Debye length) = 4,000 and several values of U = (Potential of *n-intrinsic* junction/Boltzmann voltage).

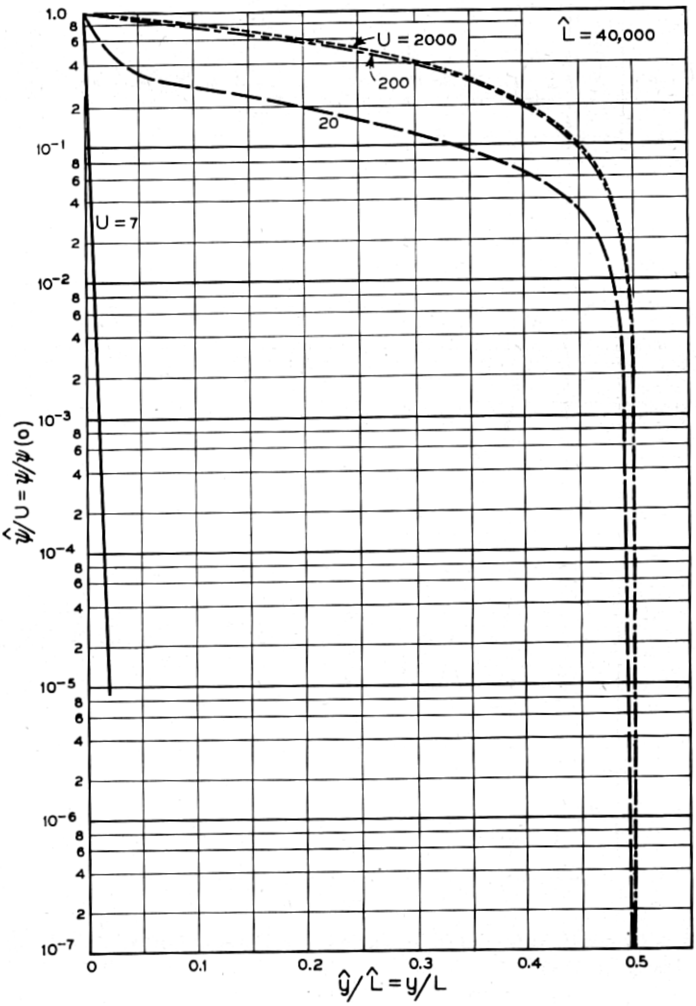

Fig. 4 — Same as Fig. 3 except for $\hat{L}$ = 40,000.

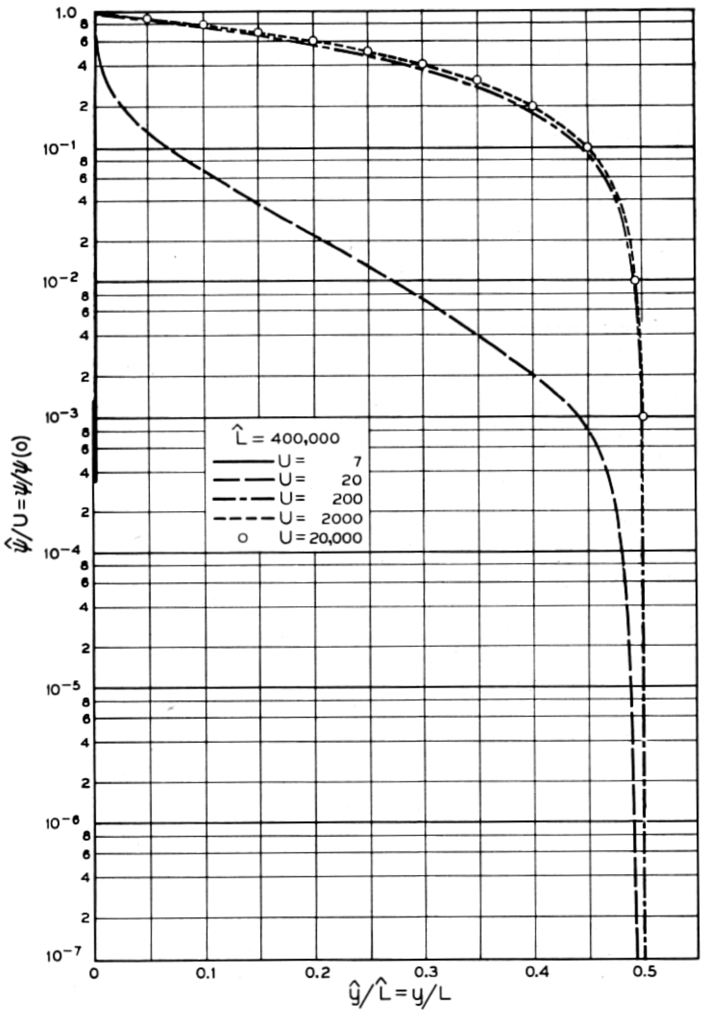

Fig. 5 — Same as Fig. 3 except for $\hat{L}$ = 400,000.

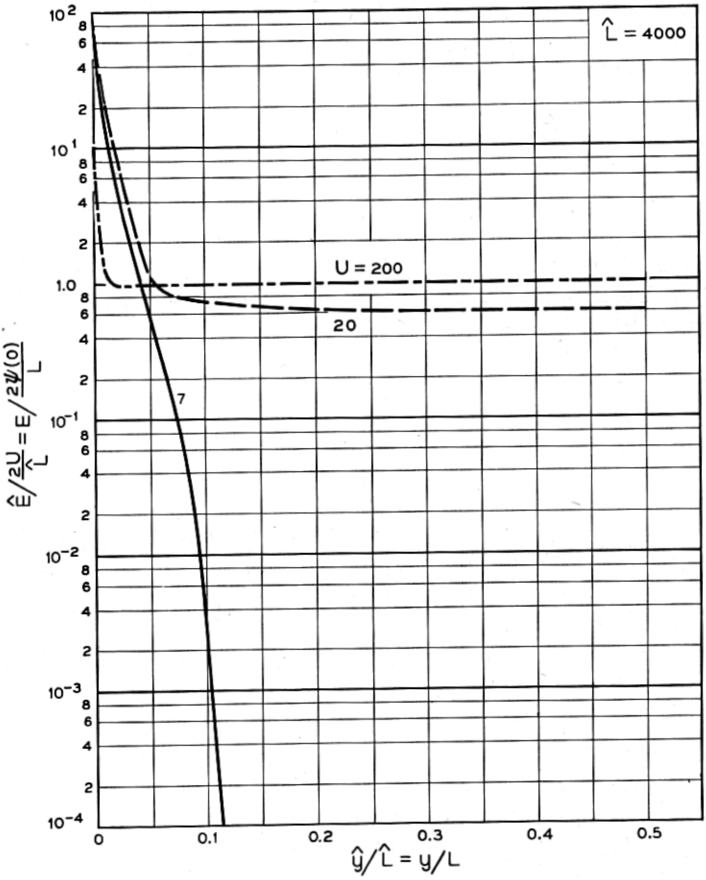

Fig. 6 — (Field intensity at point in intrinsic layer/Average field intensity over intrinsic layer) versus (Distance from *n-intrinsic* junction/Intrinsic layer thickness) for $\hat{L}$ = (Intrinsic layer thickness/Mean Debye length) = 4,000 and several values of U = (Potential of *n-intrinsic* junction/Boltzmann voltage).

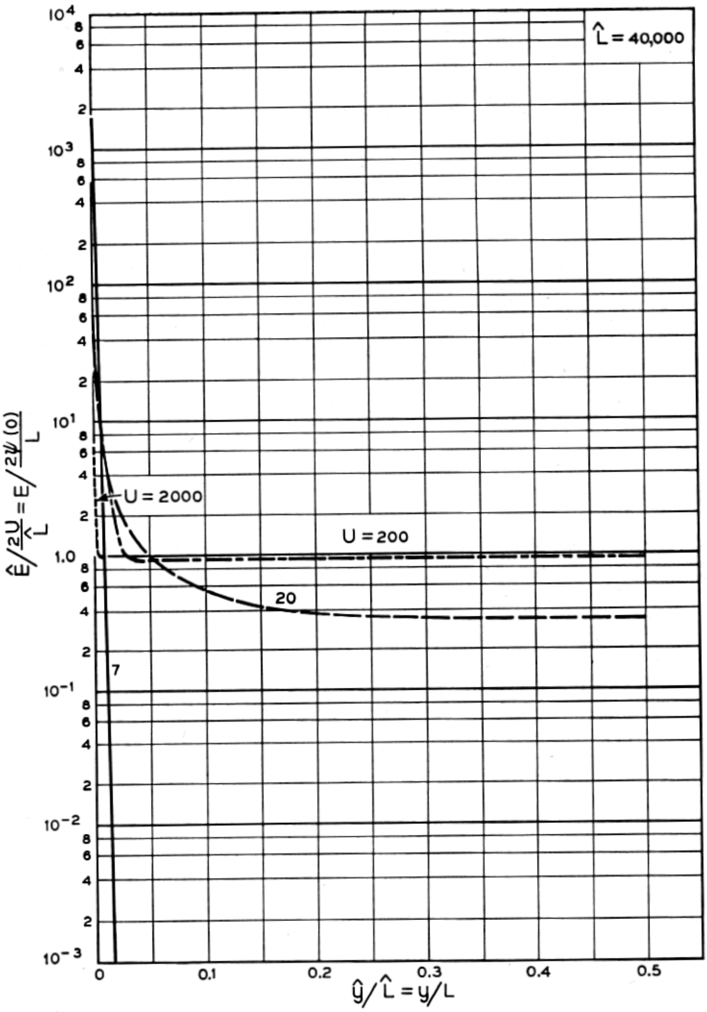

Fig. 7 — Same as Fig. 6 except for $\hat{L}$ = 40,000.

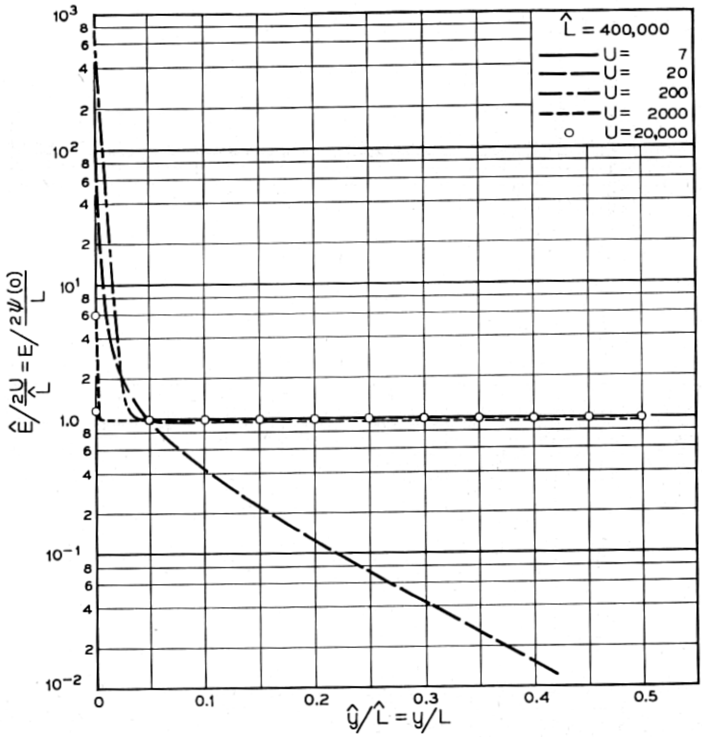

Fig. 8 — Same as Fig. 6 except for $\hat{L}$ = 400,000.

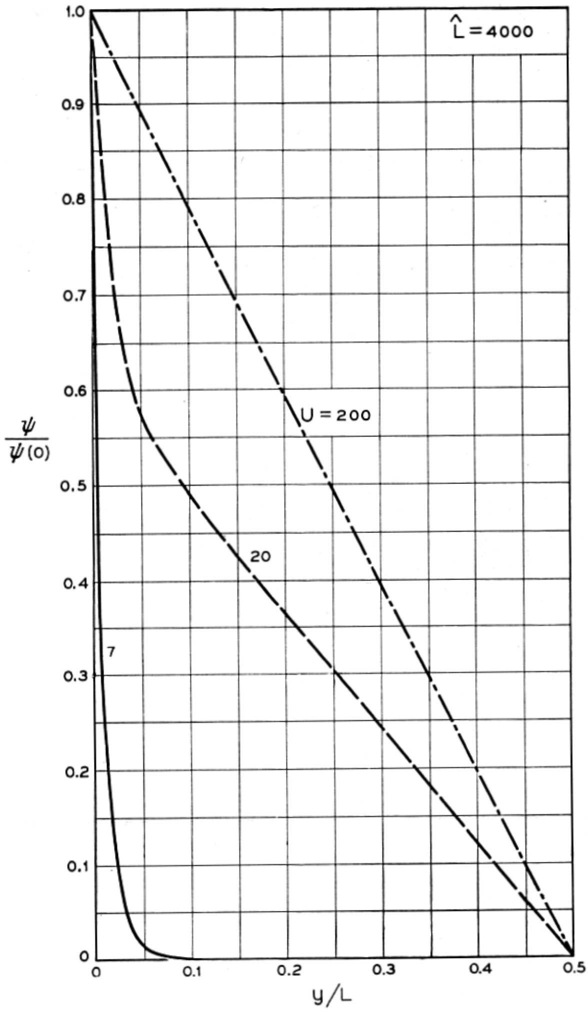

Fig. 9 — Same as Fig. 3 except with non-logarithmic potential scale.

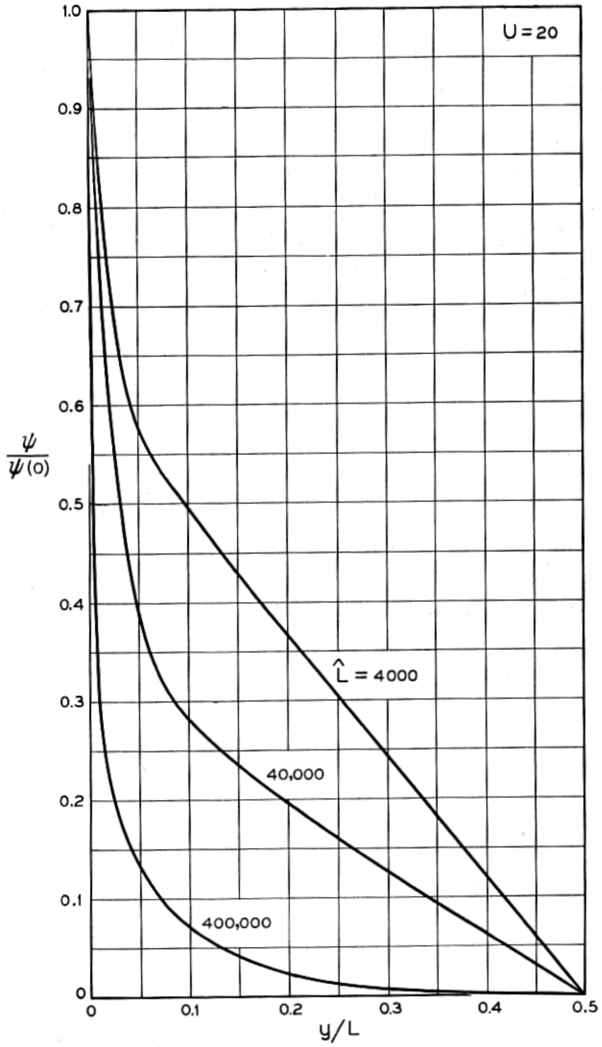

Fig. 10 — Similar to Fig. 9 except for U = (Potential of *n-intrinsic* junction/Boltzmann voltage) = 20 and several values of $\hat{L}$ = (Intrinsic layer thickness/Mean Debye length).

Because of the extreme values of the parameters encountered in these computations, it was necessary to make extensive use of the following expansions to supplement available tables of elliptic functions:

for $k^2 \ll 1$

$$K[k] = \frac{\pi}{2}\left[1 + 2\left(\frac{k^2}{8}\right) + 9\left(\frac{k^2}{8}\right)^2 + \cdots\right],$$

$$sn[k, v] = \sin v - \frac{k^2}{4}\cos v(v - \sin v \cos v) + \cdots$$

$$cn[k, v] = \cos v + \frac{k^2}{4}\sin v(v - \sin v \cos v) + \cdots$$

for $k' \equiv (1 - k^2)^{1/2} \ll 1$

$$K[k] = \ell n\frac{4}{k'} + \left(\ell n\frac{4}{k'} - 1\right)\frac{k'^2}{4} + \cdots$$

$$sn[k, v] = \tanh v + \frac{k'^2}{4}\operatorname{sech}^2 v(\sinh v \cosh v - v) + \cdots$$

$$cn[k, v] = \operatorname{sech} v - \frac{k'^2}{4}\tanh v \operatorname{sech} v(\sinh v \cosh v - v) + \cdots$$

Also useful were:

for $\phi \ll 1$

$$\ell n\frac{1 + \phi}{1 - \phi} = 2\phi\left(1 + \frac{1}{3}\phi^2 + \cdots\right)$$

for $\phi \gg 1$

$$\ell n\frac{1 + \phi}{1 - \phi} = \ell n\frac{2}{1 - \phi} - \frac{1 - \phi}{2} - \frac{1}{2}\left(\frac{1 - \phi}{2}\right)^2 - \cdots$$

In the determination of B from (43) the problem arises of solving the transcendental equation

$$k'K[(1 - k'^2)] = a \ll 1$$

where a is a given positive quantity. This equation must be solved by iteration or plotting and a reasonably good estimate of the root saves a great deal of labor. Making use of the approximation valid for $k' \ll 1$

$$K[(1 - k'^2)^{1/2}] \approx \ell n 4/k'$$

leads to

$$k' \ln 4/k' \approx a.$$

Now set $k' \equiv 4e^{-r}$ to obtain

$$4re^{-r} \approx a$$

or

$$f(r) \equiv r - \ln r - \ln 4/a \approx 0.$$

A rough approximation to r is obtained by neglecting $\ln r$ in comparison to r, it is $r \approx \ln 4/a$. This can be used to compute a much better approximation from Newton's formula:

$$r - r_0 \approx -\frac{f(r_0)}{f'(r_0)}.$$

The resulting useful formula is

$$r \approx \ln 4/a \left[1 + \frac{\ln \ln 4/a}{\ln 4/a - 1}\right].$$

DISCUSSION OF FIELD DISTRIBUTION CURVES

For the symmetric case ($N = P$) for which numerical computations were made, the electric field is a minimum in the middle of the intrinsic region, and rises to symmetrical maxima at the extrinsic-intrinsic interfaces. For fixed $\hat{L}$ and relatively small U the field is very small except quite near the interfaces. That is, practically all the potential drop takes place in thin "space charge layers" near the intrinsic boundaries at $\hat{y} = 0$ and $\hat{y} = \hat{L}$. As U is increased (for given $\hat{L}$) the region of appreciable field strength increases in width and the minimum field at $y = L/2$ increases. As U is made very large the minimum field strength approaches the average field over the intrinsic region. In this limit the potential distribution approaches linearity across the intrinsic region.

For $N \neq P$, the qualitative behavior of the field is the same except that the minimum field ($\hat{\psi} = 0$) moves away from $\hat{y} = \hat{L}/2$ and the maxima at $\hat{y} = 0$ and $\hat{y} = \hat{L}$ are no longer equal. This minimum field (see (38b), (39b)) occurs at

$$y = C \approx \tfrac{1}{2} \hat{L} + e^{1/2} (\Lambda^{1/2} - \Lambda^{-1/2})$$

Unless the asymmetry is very pronounced, this will not differ appreciably from $\hat{L}/2$ because of (40 c, d). It will be shown in a following section that the normalized voltage at the p-intrinsic interface is given by

$$V \equiv -\hat{\psi}(\hat{L}) \approx U - 2 \ln \Lambda. \tag{44}$$

The average normalized field intensity over the intrinsic region is then given by

$$\hat{E}_{av} \equiv \frac{U + V}{\hat{L}} \approx \frac{2U}{\hat{L}}\left(1 - \frac{\ln \Lambda}{U}\right). \tag{45}$$

For $\Lambda = 1$, of course

$$\hat{E}_{av} = \frac{2U}{\hat{L}}.$$

Unless the degree of asymmetry is great,

$$\frac{\ell n \Lambda}{U}$$

will be small compared to unity and $\hat{E}_{av}$ will still be almost

$$\frac{2U}{\hat{L}}.$$

In order to present a simple quantitiative picture of the penetration of the field into the interior of the intrinsic region it is convenient to introduce the *field penetration parameter* η defined by

$$\eta = \frac{\text{minimum field intensity}}{\text{average field intensity}}.$$

For fixed intrinsic region thickness and small applied voltages, $\eta \ll 1$, indicating localization of the regions of high field intensity. For large applied voltage, $\eta \to 1$, implying substantial uniformity of field throughout the intrinsic region. Subject to the assumptions (40) relations will now be derived among the applied voltage, the intrinsic layer thickness, the asymmetry parameter Λ, and the field penetration parameter η.

The minimum field intensity occurs where $\hat{\psi} = 0$. Therefore, from (36) and (45)

$$\eta \approx \frac{[2A(B + 1)]^{1/2}}{2\hat{L}^{-1}(U - \ell n \Lambda)}. \tag{46}$$

Eliminating A from (46) by the use of (41) and $\hat{L}$ by the use of (43) leads to

$$\eta \approx \frac{2^{1/2}(B + 1)^{1/2}\Phi(B)}{U - \ell n \Lambda}$$

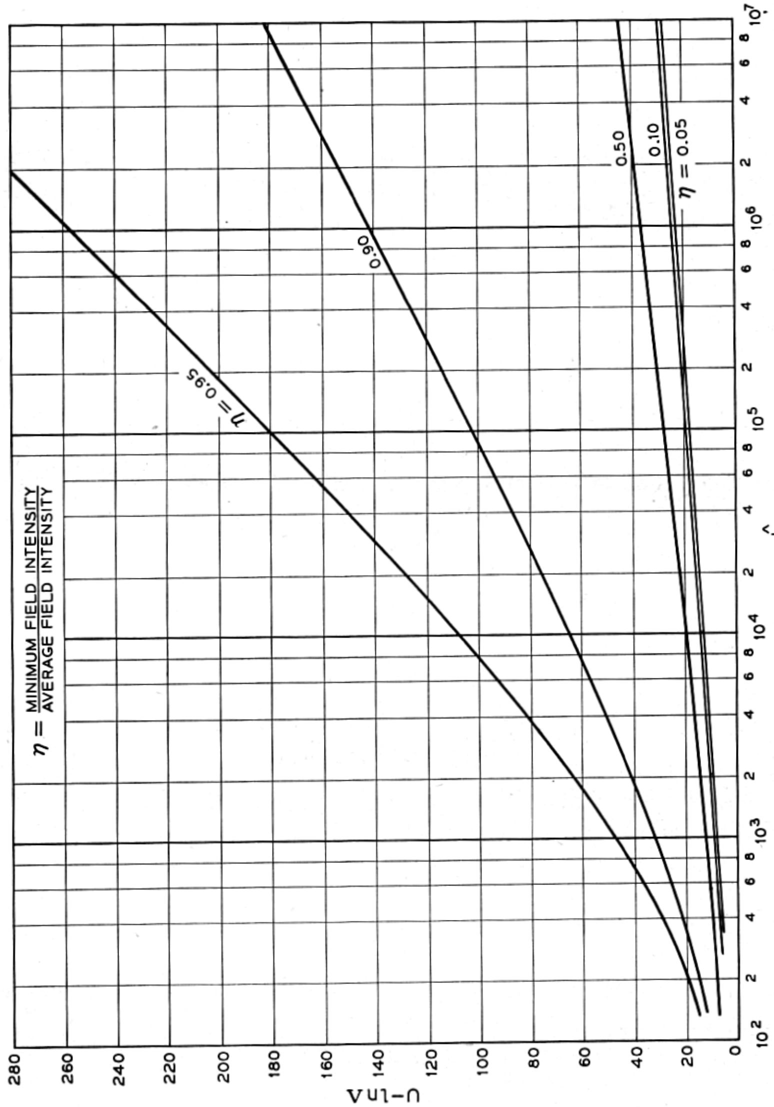

Fig. 11 — Half of voltage drop across intrinsic layer versus normalized intrinsic layer thickness for several values of the field penetration parameter η.

or

$$U - \ell n\,\Lambda \approx \frac{2}{\eta}\left(\frac{B+1}{2}\right)^{1/2}\Phi(B). \tag{47}$$

Then substitution of (47) into (43) yields

$$\hat{L} \approx 2e^{1/2}\Phi(B)\exp\left[\frac{1}{\eta}\left(\frac{B+1}{2}\right)^{1/2}\Phi(B)\right]. \tag{48}$$

For fixed η, (47) and (48) are parametric equations of a function $U_\Lambda(\hat{L})$. These equations were used to compute the curves of Fig. 11 in which $U - \ell n\Lambda$ is plotted against $\hat{L}$ for $\eta = 0.05, 0.1, 0.5, 0.9$ and 0.95. This figure gives a quantitative picture of the dependence of the field penetration parameter η on impressed voltage and intrinsic region thickness. [The ordinate $U - \ell n\Lambda$ is ½ the total voltage drop across the intrinsic layer in (kT/q) units.].

The foregoing analysis clarifies the progressive elimination of the low field region near the center of the intrinsic material as the applied voltage is increased for fixed L. Now the high field regions near $y = 0$ and $y = L$ will be described. Making use of (41), together with

$$\cosh U \approx \tfrac{1}{2}eU$$

and

$$2AB \ll \Lambda, \Lambda^{-1}$$

implied by (40), in (36) leads to

$$\hat{E}(0) \approx e^{-1/2}\Lambda^{1/2} \tag{49}$$

and to

$$\frac{d\hat{E}}{d\hat{y}}(0) \approx -\tfrac{1}{2}\,e^{-1}\Lambda. \tag{50}$$

Hence a length characterizing the "space charge layer thickness" at $y = 0$ is

$$\frac{\hat{E}(0)}{-\dfrac{d\hat{E}}{d\hat{y}}(0)} \approx 2e^{1/2}\Lambda^{-1/2}. \tag{51}$$

Similarly, for the p-intrinsic interface at $\hat{y} = \hat{L}$, we have

$$E(\hat{L}) \approx e^{-1/2}\Lambda^{-1/2}, \tag{52}$$

$$\frac{d\hat{E}}{d\hat{y}}(L) \approx -\tfrac{1}{2}e^{-1}\Lambda^{-1}, \tag{53}$$

and

$$\frac{\hat{E}(\hat{L})}{-\frac{d\hat{E}}{d\hat{y}}(\hat{L})} \approx 2e^{1/2}\Lambda^{1/2}. \tag{54}$$

If it is noted that the n-region and p-region Debye lengths are related to the mean Debye length $\mathfrak{L}$ and the asymmetry parameter Λ by

$$\mathfrak{L}_n = \Lambda^{-1/2}\mathfrak{L}$$

and

$$\mathfrak{L}_p = \Lambda^{1/2}\mathfrak{L}$$

it is seen that (49)–(54) can be written:

$$E(0) \approx e^{-1/2}\left[\frac{kT}{q} \text{ per } \mathfrak{L}_n\right], \tag{49'}$$

$$E(L) \approx e^{-1/2}\left[\frac{kT}{q} \text{ per } \mathfrak{L}_p\right], \tag{52'}$$

$$\frac{dE}{dy}(0) \approx -\tfrac{1}{2}e^{-1}\left[\frac{kT}{q} \text{ per } \mathfrak{L}_n \text{ per } \mathfrak{L}_n\right], \tag{50'}$$

$$\frac{dE}{dy}(L) \approx -\tfrac{1}{2}e^{-1}\left[\frac{kT}{q} \text{ per } \mathfrak{L}_p \text{ per } \mathfrak{L}_p\right], \tag{53'}$$

$$\frac{E(0)}{-\frac{dE}{dy}(0)} \approx 2e^{1/2} \qquad [\text{times } \mathfrak{L}_n], \tag{51'}$$

$$\frac{E(L)}{-\frac{dE}{dy}(L)} \approx 2e^{1/2} \qquad [\text{times } \mathfrak{L}_p]. \tag{54'}$$

Equations (49′)–(54′) show that the maximum field intensity and the "space charge layer thickness" at either extrinsic junction is dependent only on the Debye length of the adjacent extrinsic material.

Similarly, the concentrations of the majority carrier at either interface is found to be substantially independent of U and L, and determined by the neighboring extrinsic material:

$$\frac{n(0)}{N} \approx e^{-1}, \tag{55}$$

$$\frac{p(L)}{P} \approx e^{-1}. \tag{56}$$

The minority carrier concentration at the interfaces depends too, on the neighboring extrinsic material, but is also U dependent:

$$\frac{p(0)}{N} \approx e^{-1}e^{-2U}, \tag{57}$$

$$\frac{n(L)}{P} \approx e^{-1}e^{-2U}. \tag{58}$$

At the point of minimum field intensity, $\hat{\psi} = 0$ and

$$\frac{n}{N} = \frac{p}{N} \approx e^{-1}e^{-U}. \tag{59}$$

The extremely low carrier concentrations given by (57)–(59) are not really meaningful, of course, because the analysis has neglected the carrier concentrations due to thermally generated hole-electron pairs and to saturation currents injected through the biased junctions. While these latter concentrations are negligible compared to (55) and (56), they are undoubtedly large compared to (57) and (58). Therefore, although they can be neglected in determining the electric field intensity distribution, they are of principal importance in determining the small residue carrier concentrations in the "swept" region. A computation of these concentrations can be made by regarding the fields determined in the present analysis as impressed and studying the resulting motion of the generated and injected carriers.

APPENDIX I

EVALUATION OF INTEGRATION CONSTANTS A, B, AND C.

In this section the conditions described in (7)–(12) will be used to evaluate the integration constants A, B, and C in terms of the prescribed parameters L, U, and Λ.

First a partial integration of the differential equations for the extrinsic regions will be performed to obtain from (7)–(10) relations between $\hat{E}$ and $\hat{n}$ at $\hat{y} = 0$ and $\hat{E}$ and $\hat{p}$ at $\hat{y} = \hat{L}$. Division of (25a) by (27) gives (for the n-region $\hat{y} < 0$)

$$\frac{d\hat{E}^2}{d\hat{n}} = 1 - \frac{\hat{p}}{\hat{n}} - \frac{\Lambda}{\hat{n}}. \tag{60}$$

Addition of $\hat{n}$ times (26) to $\hat{p}$ times (27) yields

$$\frac{d}{d\hat{y}}(\hat{p}\hat{n}) = 0,$$

whence

$$\hat{p} = \frac{\nu}{\hat{n}}, \tag{61}$$

where ν is a constant. Condition (7_2) then implies

$$\nu = \frac{n_i^2}{NP}. \tag{62}$$

Substituting (61) into (60) and integrating we obtain

$$\hat{E}^2 = \hat{n} - \Lambda \ln \hat{n} + \frac{\nu}{\hat{n}} + D,$$

where D is a constant of integration. Now by (7_3), *for* $\hat{y} = -\infty$ $E = 0$ and

$$\hat{n} = \Lambda + \hat{p} = \Lambda + \frac{\nu}{\hat{n}}$$

$$= \Lambda\left(1 + \frac{\nu}{\Lambda\hat{n}}\right).$$

Or, since

$$\frac{\nu}{\Lambda\hat{n}} \equiv \frac{n_i^2}{nN} \approx \left(\frac{n_i}{N}\right)^2 \ll 1,$$

$$\hat{n} \approx \Lambda \text{ for } \hat{E} = 0.$$

Therefore

$$D \approx -\Lambda + \Lambda \ln \Lambda - \frac{\nu}{\Lambda},$$

giving

$$\hat{E}^2 \approx (\hat{n} - \Lambda)\left(1 - \frac{\nu}{\Lambda\hat{n}}\right) + \Lambda \ln \frac{\Lambda}{\hat{n}},$$

or, for $(1 - \hat{n}/\Lambda)$ positive but not $\ll 1$,

$$\hat{E}^2 \approx \hat{n} + \Lambda \ln \frac{\Lambda}{e\hat{n}} \qquad [\text{for } \hat{y} = 0]. \tag{63}$$

A directly parallel computation for the p-region ($\hat{y} > \hat{L}$) leads to

$$\hat{E}^2 \approx \hat{p} + \Lambda^{-1} \ln \frac{\Lambda^{-1}}{e\hat{p}} \qquad [\text{for } \hat{y} = \hat{L}]. \tag{64}$$

Upon substituting (34) and (36) into (63) and setting $\hat{\psi} = U$ (12), we obtain

$$Ae^{-U} + 2AB = \Lambda \ln \frac{\Lambda}{Ae^{U+1}}, \tag{65}$$

or, because both terms on the left are negligible under the assumptions (40)*,

$$A \approx \Lambda e^{-U-1}. \tag{66}$$

Similarly, substitution of (35) and (36) into (63) and setting of $\hat{\psi} = -V$ yields

$$Ae^{-V} + 2AB = \Lambda^{-1} \ln \frac{\Lambda^{-1}}{Ae^{V+1}}, \tag{67}$$

or, by virtue of (40),

$$A \approx \Lambda^{-1} e^{-V-1}. \tag{68}$$

Combining (66) and (68) we obtain

$$V \approx U - 2 \ln \Lambda. \tag{69}$$

This formula gives the normalized potential magnitude at the p-intrinsic interface in terms of that (U) at the n-intrinsic interface and the asymmetry parameter Λ.

Now the condition $\hat{\psi} = U$ for $\hat{y} = 0$ requires (from 38a and 39a) that

$$K\left[\left(\frac{1-B}{2}\right)^{1/2}\right] - F\left[\left(\frac{1-B}{2}\right)^{1/2} \sin^{-1}_{(1)} \operatorname{sech} \frac{U}{2}\right] = A^{1/2}C \tag{70a}$$

for $-1 < B \leq 1$,

or

$$\left(\frac{2}{B+1}\right)^{1/2} F\left[\left(\frac{B-1}{B+1}\right)^{1/2}, \quad \sin^{-1}_{(1)} \tanh \frac{U}{2}\right] = A^{1/2}C \tag{70b}$$

for $B \geq 1$.

In addition, the condition $\hat{\psi} = -V$ for $\hat{y} = \hat{L}$ requires

$$K\left[\left(\frac{1-B}{2}\right)^{1/2}\right] - F\left[\left(\frac{1-B}{2}\right)^{1/2}, \quad \sin^{-1}_{(2)} \operatorname{sech} \frac{V}{2}\right] = A^{1/2}(C - \hat{L}) \tag{71a}$$

for $-1 < B \leq 1$,

* It will be shown later that (40) implies $AB \ll 1$.

or

$$\left(\frac{2}{B+1}\right)^{1/2} F\left[\left(\frac{B-1}{B+1}\right)^{1/2}, \quad \sin_{(4)}^{-1} \tanh\left(\frac{-V}{2}\right)\right] = A^{1/2}(C - \hat{L}) \tag{71b}$$

for B ≥ 1.

Fortunately, because of the assumptions (40a, b) the formidable relations (70a–71b) can be simplified to

$$\Phi(B) \approx A^{1/2}C + 2e^{-U/2} \tag{72}$$

and

$$\Phi(B) \approx A^{1/2}(\hat{L} - C) + 2e^{-V/2}, \tag{73}$$

where

$$\Phi(B) \equiv \begin{cases} K\left[\left(\frac{1-B}{2}\right)^{1/2}\right] & \text{for } -1 < B < 1 \\ \frac{\pi}{2} & \text{for } B = 1 \\ \left(\frac{2}{B+1}\right)^{1/2} K\left[\left(\frac{B-1}{B+1}\right)^{1/2}\right] & \text{for } B > 1. \end{cases}$$

Subtracting (73) from (72) we obtain

$$O = 2A^{1/2}C - A^{1/2}\hat{L} + 2e^{-U/2} - 2e^{-V/2},$$

whence, substituting (66) and (69),

$$C \approx \frac{1}{2}\hat{L} + e^{1/2}(\Lambda^{1/2} - \Lambda^{-1/2}). \tag{74}$$

Finally, substitution of (66) and (74) into (72) gives

$$\Phi(B) \approx [\tfrac{1}{2}\Lambda^{1/2}\hat{L} + e^{1/2}(\Lambda + 1)]e^{-U/2-1/2},$$

or, by virtue of (40c, d),

$$\Phi(B) \approx \frac{1}{2}\Lambda^{1/2}\hat{L}e^{-U/2-1/2}. \tag{75}$$

Equations (66), (69), (74), and (75) are the desired expressions for determining A, B, C, and V when values are assigned to Λ, $\hat{L}$, and U (subject to (40)). (If necessary, some or all of the restrictions (40) could be eliminated, but the transcendental equations to be solved for A, B, C, and V would become quite formidable.)

It should be noted that (75) permits an easy determination whether the formulae for $B < 1$ or those for $B > 1$ should be used in any particular case. Since $\Phi(1) = \pi/2$,

$$B \lessgtr 1 \text{ for } \Lambda^{1/2}\hat{L}e^{-U/2-1/2} \gtrless \pi. \tag{76}$$

Appendix II

CONDITIONS FOR $2AB \ll \Lambda, \Lambda^{-1}$

It has been stated without proof in the foregoing analysis that the conditions (40) imply $2AB \ll \Lambda, \Lambda^{-1}$ (and hence also $AB \ll 1$). This must now be demonstrated.

For B *not* $\gg 1$, (40a–40d) are sufficient, for (41) shows that $A \ll 1$. However, for $B \gg 1$, the product AB is not necessarily small because of $A \ll 1$ and additional limitations are required. To establish suitable additional conditions we shall consider combinations of U, $\hat{L}$, and Λ for which AB *is* very small and estimate the conditions under which this smallness begins to weaken.

By eliminating U between (41) and (43) we can write

$$\Phi(B) \approx \tfrac{1}{2}\hat{L}A^{1/2},$$

or

$$B^{1/2}\Phi(B) \approx \tfrac{1}{2}\hat{L}(AB)^{1/2}. \tag{77}$$

Now for $k \approx 1$,

$$K(k) \approx \ln \frac{4}{\sqrt{1-k^2}}.$$

Therefore, for $B \gg 1$

$$\Phi(B) \equiv \left(\frac{2}{B+1}\right)^{1/2} K\left[\left(\frac{B-1}{B+1}\right)^{1/2}\right] \approx \frac{\ln B}{(2B)^{1/2}}. \tag{78}$$

Substitution of (78) into (77) now yields

$$\ln B \approx 2^{-1/2}\hat{L}(AB)^{1/2},$$

or

$$B \approx \exp\left[2^{-1/2}\hat{L}(AB)^{1/2}\right]. \tag{79}$$

From (78) and (79)

$$\Phi(B) \approx \tfrac{1}{2}\hat{L}(AB)^{1/2} \exp\,[-2^{-3/2}\hat{L}(AB)]. \tag{80}$$

Now using the expression just obtained for $\Phi(B)$ in terms of (AB), we can eliminate $\Phi(B)$ from (43) and solve for U as a function of (AB). After some manipulations we obtain

$$\frac{2U}{\hat{L}} \approx (2AB)^{1/2} - \frac{2(1 - \ln 2)}{\hat{L}} - \frac{2}{\hat{L}} \ln\left(\frac{2AB}{\Lambda}\right). \tag{81}$$

For $2AB\Lambda$ or $2AB\Lambda^{-1}$ of the order of 10^{-2}, (40c, d) insure that the right side of (81) will not change in order of magnitude if the two terms multiplied by $\hat{L}^{-1}$ are dropped. Therefore $2AB\Lambda$ and $2AB\Lambda^{-1}$ will be of order 10^{-2} or less if

$$\frac{2U}{\hat{L}} < \frac{1}{10}\,\Lambda^{1/2},\ \frac{1}{10}\,\Lambda^{-1/2}. \tag{82}$$

Thus for a given intrinsic layer thickness there is a ceiling on the impressed voltages for which AB is negligibly small.