Transmission Properties of Laminated Clogston Type Conductors

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The transmission properties of ideal laminated conductors of the Clogston type are discussed by introducing the concepts of equivalent inductance, capacitance and resistance values which are analogous to their corresponding counterparts in the treatment of ordinary transmission lines. From these constants the attenuation, phase constant, and speed of propagation are obtained using conventional transmission line theory, and the results compared with those for ordinary coaxial conductors.

This paper is divided into two parts. In the first part a general discussion is given of Clogston cables and a comparison made with the conventional coaxial cable. This is illustrated with a few numerical examples, based on formulas which are developed in the second part of this paper.

INTRODUCTION

The discovery that deep penetration of the current can be obtained in laminated conductors, when the speed of propagation is made constant over the entire cross-section of the cable, is described in an earlier issue of this magazine.[1] The theoretical study of the problem was based on Maxwell's field equations dealing with a stack of parallel plates of alternate conducting and insulating layers. When applied to concentric laminated tubes, this method results in a set of extremely complex equations. S. P. Morgan has given a rigorous solution for the case when the laminated layers are of infinitesimal thickness.[2]

The present paper uses a different approach which leads to simpler approximate formulas. Available theoretical results are combined with simplifying approximations and certain somewhat arbitrary assumptions

[1] Clogston, A. M., Reduction of Skin Effect Losses by the use of Laminated Conductors. Bell System Tech. J., **30**, pp. 491–529, July, 1951.

[2] Morgan, S. P., Mathematical Theory of Laminated Transmission Lines. Bell System Tech. J., Part I, **31**, pp. 883–949, Sept., 1952, and Part 2, **31**, pp. 1121–1206, Nov., 1952.

in such a way that formulas for the counterparts of the usual transmission line properties of inductance, capacitance and resistance are obtained. The approximate formulas for attenuation, phase constant and speed of propagation are then derived using conventional transmission line theory.

Some computations of attenuation are presented which illustrate the interesting transmission properties of Clogston cables under ideal conditions. Exploratory work on this type of conductor is still in the early research stage, with some very difficult problems imposed by the need for a large number of thin layers with very close tolerances.

Part I — General Discussion of Clogston Cables and a Comparison with Conventional Coaxial Cable

1. Skin Effect

An alternating current transmitted over a solid conductor has the tendency to crowd toward the surface of the wire. This phenomenon is known as the *skin effect* and the depth of penetration of the current is usually referred to as the *skin depth*. The skin depth is defined as the distance, measured from the surface toward the center of the wire, where the current density is reduced to $1/e = 0.367$. For a copper conductor it is given by:

$$\delta = \frac{2.61}{\sqrt{F_{mc}}}, \tag{1}$$

where

$$\delta = \text{Skin depth in mils.}$$

$$F_{mc} = \text{Frequency in megacycles.}$$

When the skin depth is a fraction of the wire radius, the ac resistance of the wire increases about as the square root of the frequency.

Laminated conductors disclosed by Clogston have the property that the ac resistance will remain very nearly equal to the dc resistance over a wide band of frequencies, if the conducting and insulating layers can be made thin enough and sufficiently uniform. The dc resistance of a Clogston conductor will be higher than the dc resistance of a solid conductor of the same over-all dimension by a factor $(w + t)/w$, where w and t are the thicknesses of the conducting and insulating layers respectively. As discovered by Clogston, the depth of penetration in a laminated conductor is much greater than in a conductor of solid copper if the speed of propagation is constant over the entire cross-section. A coaxial cable having an inner laminated conductor and an outer laminated sheath must obey the following relation to obtain the desired effect:

$$\mu_I \epsilon_I = \mu\epsilon\left(1 + \frac{w}{t}\right), \tag{2}$$

where:

μ_I = Permeability in space between inner and outer conductor.
μ = Permeability of laminated conductors.
ϵ_I = Dielectric constant of insulation between inner and outer conductor.
ϵ = Dielectric constant of insulating layers in the laminated conductors.
w = Thickness of copper layers.
t = Thickness of insulating layers.

In (2) the expression $\epsilon(1 + w/t)$ is of course the mean dielectric constant of the laminated conductor. Since $1/\sqrt{\mu_I \epsilon_I}$ is the speed of propagation in the main dielectric, equation (2) indicates that the speed of propagation is the same over the entire cross section of the cable. Equation (2) must be satisfied to a high degree of accuracy, otherwise deep penetration is not possible.

2. DEFINITION OF CLOGSTON CABLES

Two laminated conductors arranged as a coaxial cable are shown in Fig. 1. The inner conductor consists of a solid copper wire of diameter d_1, over which a large number of alternate layers of insulation and copper are arranged as concentric thin tubes. The over-all diameter of the inner conductor is D_1. The outer conductor of the coaxial cable consists of a laminated tube of inner diameter d_2 and outer diameter D_2. The space between D_2 and d_2 is filled with thin concentric tubes of copper and insulation of the same thicknesses as for the inner conductor. The outside of the outer conductor is covered with a solid copper sheath for protection, shielding and energizing purposes. This type of cable has been named Clogston I.

By adding more layers to the outside of the inner conductor and more layers to the inside of the outer conductor, the space between them is completely filled when $d_2 = D_1$. Such a cable is shown in Fig. 2, and has been named Clogston II.

Clogston I may be thought of as a physical variant of the conventional

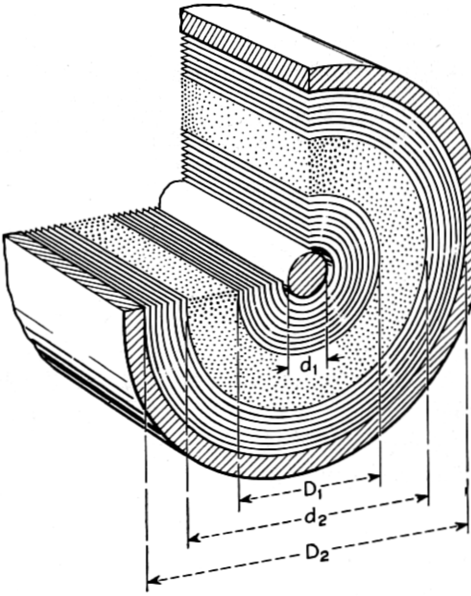

Fig. 1 — Clogston I cable.

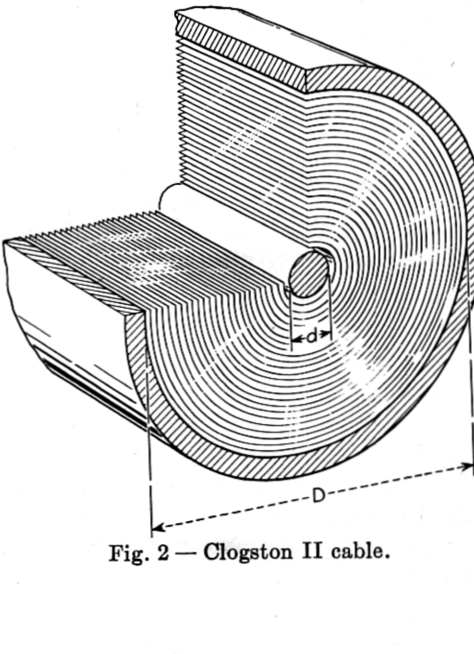

Fig. 2 — Clogston II cable.

coaxial cable, in that when one of the conductors carriers a current in one direction, the other will carry a current in the opposite direction. In Clogston II there is also a reversal of currents somewhere between the outermost layers and the innermost. It is therefore a kind of a two conductor cable, but the point of division between the conductors is determined by the electromagnetic field configuration of the situation. This point has been worked out by S. P. Morgan and will be referred to in the second part of this paper.

3. OPTIMUM PROPORTIONING

The cross-sectional aspect of Clogston II is completely characterized by that proportion of the diameter D which is occupied by the laminations. This proportion is called the *Fill Factor* and is defined by:

$$\phi_{II} = (D - d)/D \qquad \text{Clogston II.} \tag{3}$$

The fill factor is also a useful parameter for Clogston I, though it is not sufficient to determine its geometry. It is defined by:

$$\phi_{I} = (D_1 - d_1 + D_2 - d_2)/D_2 \qquad \text{Clogston I.} \tag{4}$$

The additional parameters which, with the outer diameter D_2, will completely determine the geometry, are the ratio of the over-all thickness of the inner laminated conductor to the over-all thickness of the outer conductor, and a parameter which locates the inner diameter d_1 of the inner conductor. These parameters are defined by:

$$\begin{aligned} T &= (D_1 - d_1)/(D_2 - d_2), \\ U &= d_1/D_2. \end{aligned} \tag{5}$$

In a conventional coaxial cable, shown in Fig. 3, the optimum value of attenuation[3] is obtained when $D/d = 3.59$. In Clogston cables no such optimum values exist.

S. P. Morgan, however, has shown that there are useful relative optimum relations in Clogston I, which direct the choice of T and U for the cables which are illustrated in this paper. For example, for a fill factor of one-half, there is a broad optimum of attenuation when $T = 1.96$ and $U = 0.0842$. Thus the over-all thickness of the inner conductor is about twice that of the outer conductor, and the diameter d_1 of the inner core is about one-twelfth of the outer diameter D_2. With

[3] Green, E. I., F. A. Leibe and H. E. Curtis, The Proportioning of Shielded Circuits for Minimum High-Frequency Attenuation, Bell System Tech. J., **15**, pp. 248–283, April, 1936.

these dimensions the cross sectional area of the outer conductor is nearly twice that of the inner conductor, so that the optimum arises from other causes than matching the conductivity of the two conductors.

The problem of determining the relative optimum values of Clogston I, in general, is complicated, but numerical studies indicate that for values of the fill factor other than one-half, the values for T and U are not greatly different.

Another factor in Clogston cables which can be optimized is the ratio of the layer thicknesses of the conducting and the insulating layers. Clogston[4] has shown that in the frequency range where the attenuation is substantially flat with frequency, this optimum ratio is equal to:

$$w/t = 2. \tag{6}$$

For this condition, the dc resistance of the laminated conductor is increased by $(w + t)/w$ or 3/2 over a solid conductor. The dielectric constant of the main insulation, according to (2) above must equal $\epsilon_I = 3\epsilon$, which reduces the speed of propagation by $\sqrt{3}$, assuming $\mu_I = \mu$.

At frequencies where the attenuation begins to increase, other optimum values of w/t can be obtained, and the ratio will depend upon what top frequency is considered.

In a practical case a fill factor of unity will probably not be used. A little space in the center will be made available for a solid conductor for energizing or other purposes.

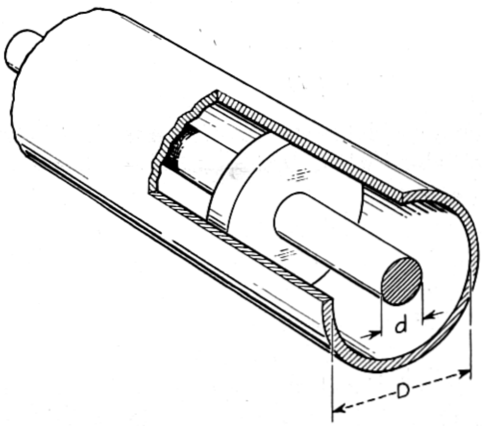

Fig. 3 — Conventional coaxial cable.

[4] Loc. cit.

4. ATTENUATION

The attenuation of a transmission circuit at high frequencies, where $\omega L \gg R$ and $\omega C \gg G$, is usually given in the following form:

$$a = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}, \tag{7}$$

where R is the total ac resistance of both conductors, L, C and G the inductance, capacitance and leakance of the circuit. It will be assumed that the insulation consists of polyethylene or some other material having a very low leakance. Thus as a first approximation the second term in (7) can be neglected.

In a conventional coaxial cable, in the frequency range considered, R will increase in proportion to the square root of frequency, so that neglecting leakance, (7) may be written as follows:

$$a = \frac{K_1}{D}\sqrt{F_{\text{mc}}}, \tag{8}$$

where K_1 is a constant depending upon the dielectric constant of the insulating material and the resistivity of the conductors. D is the inside diameter of the sheath, and F_{mc} the frequency in megacycles.

In the second part of this paper it is shown that the attenuation of Clogston I or II cables can be written in the following form:

$$a_c = \frac{K_2}{D_2} + K_3 w^2 F^2{}_{\text{mc}}, \tag{9}$$

where D = Over-all diameter of laminated cable.

w = Copper layer thickness.

K_2 and K_3 are constants, different for Clogston I and Clogston II, which depend upon the geometry of the cables, the dielectric constant of the insulating material and the resistivity of the conducting layers.

The first term in (9) gives a constant loss independent of frequency. The second term contributes little to the attenuation provided w is small enough. In fact, the attenuation will remain constant within p % of the first term provided:

$$w \leqq \frac{1}{10}\sqrt{\frac{pK_2}{K_3}\,\frac{1}{DF'_{\text{mc}}}}. \tag{10}$$

This equation (10) determines the copper layer thickness, which will result in a "flat" attenuation within p per cent up to a frequency F'_{mc}.

By neglecting the second term in (9) and comparing the result with (8) it can be seen that the attenuation of a conventional coaxial cable and a Clogston cable are equal at a frequency given by:

$$F''_{mc} = \left[\frac{K_2}{K_1 D}\right]^2 \tag{11}$$

The numerical examples given later in this paper indicate that a Clogston cable will have higher attenuation than a conventional coaxial cable at frequencies below F''_{mc}, and less attenuation at higher frequencies. At frequencies sufficiently higher than F''_{mc}, the attenuation of a Clogston cable will increase rapidly which is evident from the second term in (9). It is in the region between F''_{mc} and frequencies where the second term in (9) becomes important that Clogston cables can theoretically provide less attenuation than a conventional coaxial cable.

5. IMPEDANCE, PHASE CONSTANT AND SPEED OF PROPAGATION

The equivalent impedances of Clogston I and Clogston II cables are developed in the second part of this paper and are equal to:

$$\begin{aligned} Z_I &= \left[1 + \frac{L_{in}}{L_{ex}}\right]\sqrt{\frac{L_{ex}}{C_{ex}}} \qquad &\text{Clogston I,} \\ Z_{II} &= \frac{L_{in}}{\sqrt{L_{ex}C_{ex}}} \qquad &\text{Clogston II.} \end{aligned} \tag{12}$$

In these expressions L_{in}, L_{ex} and C_{ex} are the internal and external inductances and capacitances respectively. For conventional coaxial cables they are discussed by S. A. Schelkunoff in "Electromagnetic Waves" (Van Nostrand, 1943). For the Clogston analogy, Part II of the present paper gives the reasoning adopted in defining them. In a Clogston II cable the external inductance goes to zero and the external capacitance to infinity, but the product $L_{ex}C_{ex}$ nevertheless remains constant.

The impedance of a conventional coaxial cable, in the frequency range considered, is given by:

$$Z = \sqrt{\frac{L}{C}} \tag{13}$$

where L and C are the external inductance and capacitance of the conventional coaxial cable.

The equivalent impedance of a Clogston cable is lower than that of conventional coaxial cable of the same outer diameter. Numerical evaluations using the above formulas indicate an impedance of about 23 ohms for a half-filled Clogston I cable, and about 11½ ohms for a completely filled Clogston II cable, assuming polyethylene insulation with ϵ equal to 2.3 and copper conducting layers having a thickness twice that of the insulating layers ($w/t = 2$). These values compare with about 76 ohms for a conventional coaxial cable having air dielectric and the same outer diameter and 51 ohms for a corresponding coaxial with solid polyethylene dielectric.

The phase constants of both Clogston cables in the frequency range where the attenuation is nearly flat, are equal and independent of the geometry of the cables. They are given by:

$$\beta_{\mathrm{I}} = \beta_{\mathrm{II}} = \omega\sqrt{L_{\mathrm{ex}}C_{\mathrm{ex}}}, \tag{14}$$

assuming uniform layer thicknesses.

For a conventional coaxial cable the phase constant, neglecting leakance, is given approximately by:

$$\beta = \omega\sqrt{LC}\left[1 - \frac{1}{2}\left(\frac{R}{2\omega L}\right)^2\right]. \tag{15}$$

The computed speed of propagation, which is equal to ω/β, is about 71,000 mi/sec for Clogston cables with polyethylene insulating layers. This compares with 123,000 mi/sec for a conventional coaxial cable with polyethylene insulation and 186,000 mi/sec for a coaxial with pure air dielectric.

6. COMPARISON WITH A CONVENTIONAL COAXIAL CABLE

To illustrate the effect of the various parameters involved in the attenuation of a Clogston cable, and to compare the result with a convential coaxial cable, a few numerical examples have been evaluated.

A one-half filled Clogston I and a completely filled Clogston II have been selected arbitrarily for comparison purposes. Fig. 4 shows the attenuation characteristics of these cables for several values of copper layer thicknesses. In each case, polyethylene insulation ($\epsilon = 2.3$) is assumed, with $w/t = 2$, i.e., the insulating layers have one-half the thickness of the conducting layers. In the same figure, the attenuation characteristics of two conventional coaxial cables of the same outer diameter, one with air dielectric and one with polyethylene insulation, are shown also. The regions where Clogston cables have in theory less attenuation than conventional coaxial cables of the same outer diameter can be seen in this figure.

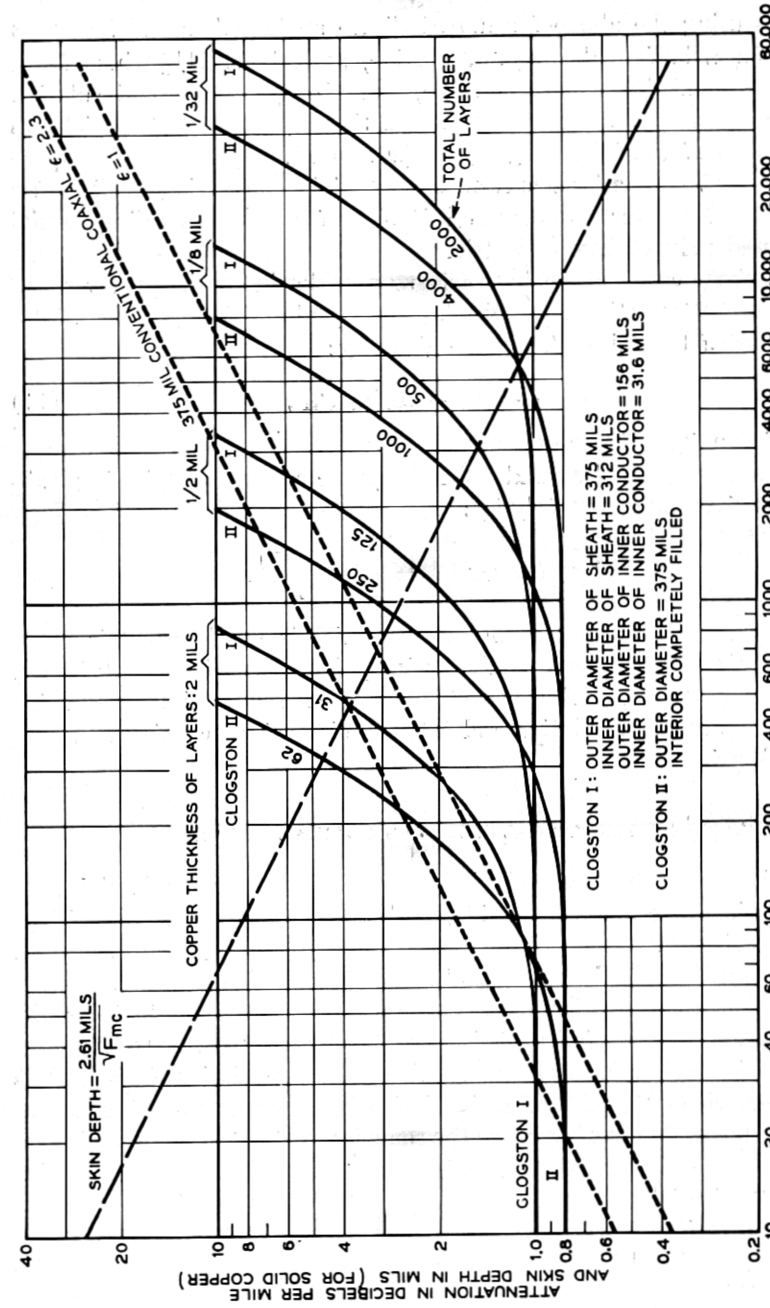

Fig. 4 — Comparison of attenuation — conventional coaxial, Clogston I and Clogston II.

The dotted curve shown on Fig. 4 gives the skin depth in solid copper. It will be noted that the copper layer thicknesses become smaller and smaller fractions of the skin depth as the frequency increases. This is also evident from (1) and (10) above, which show that the copper layer thicknesses are inversely proportional to the frequency, while the skin depth is inversely proportional to the square root of the frequency.

The effect of fill is illustrated in Fig. 5 for a 375-mil Clogston I cable, for fill factors of one-eighth, one-quarter, and one-half. It will be noted that the attenuation increases rapidly with decrease in fill, accompanied by an increase in the frequency band over which the attenuation is flat. The attenuation of a completely filled Clogston II cable is also shown for comparison.

The above estimates are based on ideal conditions; that is, it is assumed that the laminated structures are perfectly uniform. The effects of departures from ideal are not shown by the approximate methods used in this paper, but it has been shown by Morgan[5] that even small departures from ideal conditions will result in increases and irregularities in attenuation, and a decrease in the band over which the attenuation is approximately uniform.

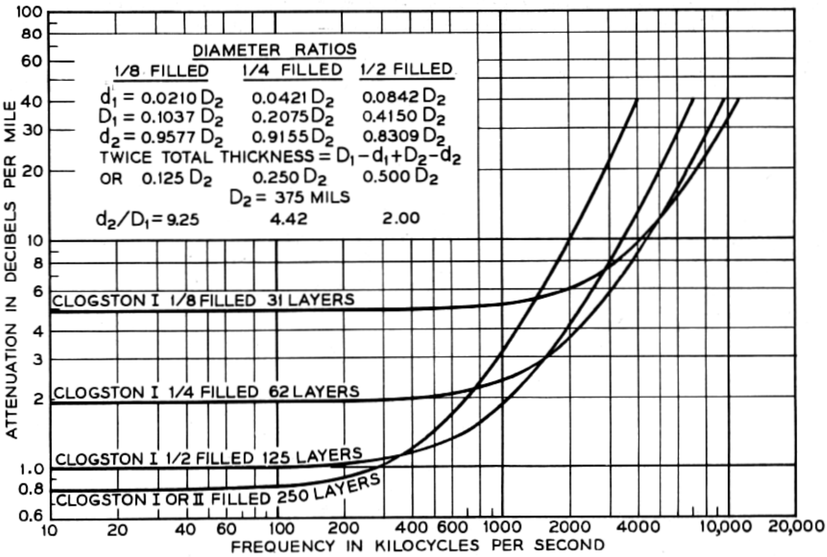

Fig. 5 — Attenuation of Clogston I.

[5] Loc. cit. Part II, pages 1161–1201.

PART II — DERIVATION AND EVALUATION OF FORMULAS

In this part of the paper, approximate formulas for the counterparts of the usual primary constants used in transmission line computations are derived for Clogston I and II type cables. From these formulas the various constants entering into the expressions for attenuation, impedance and phase as given in Part I are evaluated in terms of the dimensions of the cables and the frequency.

In deriving the formulas, certain simplifying assumptions have been made as explained in the test. The effects of these assumptions are examined by comparing the results with those obtained by more rigorous methods.

1. RESISTANCE IN GENERAL

A Clogston I cable consists of a laminated inner conductor and a laminated outer conductor as shown in Fig. 1. Each of these may be represented by the laminated conductor shown in Fig. 6. At present, we have no exact formula for the ac resistance of such conductors over a wide range of frequencies. However, S. P. Morgan has shown that for a stack of parallel plates, the resistance per unit cross-sectional area is equal to:

$$R_{ac} = \mathrm{R}_{dc} \frac{w + t}{w} \left[1 + \frac{(n^2 - 1/5)w^4}{412} F_{mc}^2 + \ldots . \right], \qquad (16)$$

where R_{dc} is the dc resistance of the stack, when completely filled with

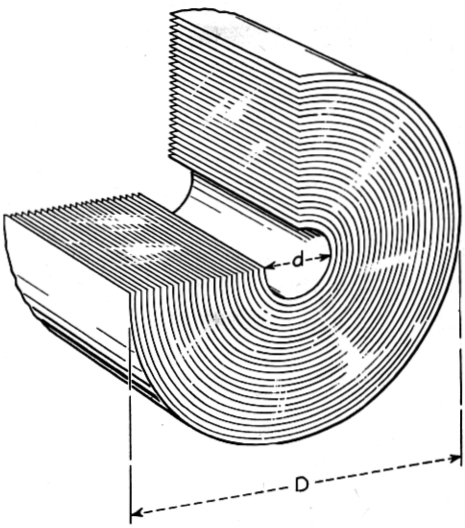

Fig. 6 — Laminated conductor.

copper. The other parameters are:

w = Thickness of copper layers in mils.
t = Thickness of insulating layers in mils.
n = Total number of layers.
F_{mc} = Frequency in megacycles.

With curvature disregarded, equation (16) also gives the ac resistance of the laminated conductor shown in Fig. 6. For only one copper layer, $n = 1$, and $t = 0$, (16) reduces to:

$$R_{ac} = R_{dc}\left[1 + \frac{w^4 F_{mc}^2}{515} + \ldots\right], \tag{17}$$

which is exactly the expression for a copper tube at frequencies where the ac resistance begins to depart from the dc resistance and $\frac{1}{2}(D - d) = w$. For a single layer, the effect of curvature is very small[6] and can be disregarded. It will be assumed that (16) also holds to a fair degree of accuracy for a laminated conductor made up of a large number of layers. Since n is large, the small fraction one-fifth can be neglected. For the optimum condition (minimum attenuation) Clogston[7] has shown that:

$$w = 2t. \tag{18}$$

The total number of layers can be obtained from the following expression:

$$n = \frac{D - d}{2(w + t)} = \frac{D - d}{3w} \text{ for } w = 2t. \tag{19}$$

With (18) and (19) substituted in (16) the ac resistance is given by:

$$R_{ac} = \frac{82080}{D^2 - d^2}\left[1 + \frac{w^2(D - d)^2 F_{mc}^2}{3710} + \ldots\right], \tag{20}$$

where the diameters and the copper layer thickness are given in mils and the frequency in megacycles. The resistivity of copper is taken to be 1.724×10^{-6} ohms/cm³.

2. CLOGSTON I CABLE

2.1 *Resistance*

The ac resistances of the inner and outer laminated conductors of Clogston I cable, shown in Fig. 1, can be obtained from (20) above by substituting D_1 and d_1 for the inner conductor and D_2 and d_2 for the

[6] Schelkunoff, S. A., The Electromagnetic Theory of Coaxial Transmission Lines and Cylindrical Shields. Bell System Tech. J., **13**, pp. 532–579, Oct., 1934. BSTJ, October, 1934, by S. A. Schelkunoff.
[7] Loc. cit.

outer conductor and adding the result. Thus:

$$R_{ac1} + R_{ac2} = \frac{82080 A_1}{D_2^2}\left[1 + \frac{B_1 w^2 D_2^2 F_{mc}^2}{3710} + \ldots\right] \text{ ohms/mi,} \tag{21}$$

where:

$$A_1 = \frac{D_2^2}{D_1^2 - d_1^2} + \frac{D_2^2}{D_2^2 - d_2^2}, \tag{22}$$

$$B_1 = \frac{2(D_1 D_2 - d_1 d_2)(D_1 - d_1)(D_2 - d_2)}{(D_1^2 - d_1^2 + D_2^2 - d_2^2)D_2^2}. \tag{23}$$

From (4) and (5) in the first part of this paper, it is possible to express A_1 and B_1 wholly in terms of ϕ_I, T and U. Thus, A_1 and B_1 are independent of D_2, and it follows that the first term in (21) is inversely proportional to D_2^2, while the second term, when multiplied out, is independent of D_2, assuming fixed values of ϕ_I, T and U.

2.2 *Impedance, Inductance and Capacitance*

In a coaxial cable the flux in the space between the two conductors gives the external inductance. The internal inductance is obtained from considering the flux within the walls of the conductors themselves but not that in the space between them. The effective inductance is then the sum of the two. Analogous considerations apply to the external and internal capacitance and the effective over-all capacitance is the value of the two acting in series.

Similarly in the frequency range where $\omega L \gg R$ and $\omega C \gg G$, but where the ac resistance is nearly equal to the dc resistance, the internal inductance and capacitance of the laminated conductors must be taken into account. Since they are in series with the external components they tend to increase the total inductance and decrease the total capacitance. The impedance of the circuit can therefore be expressed as follows:

$$Z_I = \sqrt{\frac{L}{C}} = \sqrt{\frac{L_{in} + L_{ex}}{\left[\dfrac{C_{in} C_{ex}}{C_{in} + C_{ex}}\right]}} \text{ ohms,} \tag{24}$$

$$L_{ex} = \frac{\mu_I}{2\pi} \ell n \left(\frac{d_2}{D_1}\right) \qquad \text{henries/cm,} \tag{25}$$

$$C_{ex} = \frac{2\pi\epsilon_I}{\ell n \left(\dfrac{d_2}{D_1}\right)} \qquad \text{farads/cm,} \tag{26}$$

where: $\epsilon_{\mathrm{I}} = \epsilon(1 + w/t)$ = Dielectric constant of insulation between inner and outer conductors.

At the present time no exact formulas for the internal components of either L or C are available. The internal inductance, however, must be nearly equal to that of a solid wire for the inner laminated conductor and to that of a solid sheath for the outer laminated conductor, and will be assumed so in this paper.

It should be remembered that deep penetration of the currents will be obtained when Clogston condition of constant speed of propagation over the entire cross section of the cable is satisfied. The speed of propagation over the laminated conductors is equal to $1\sqrt{L_{\mathrm{in}}C_{\mathrm{in}}}$ and in the main dielectric between the two conductors equal to $1\sqrt{L_{\mathrm{ex}}C_{\mathrm{ex}}}$. Thus to obtain the Clogston condition the following relation must hold:

$$L_{\mathrm{in}}C_{\mathrm{in}} = L_{\mathrm{ex}}C_{\mathrm{ex}} . \tag{27}$$

By solving for the unknown quantity C_{in} and substituting the result in (24), the impedance of Clogston I cable is found to be:

$$Z_{\mathrm{I}} = \left[1 + \frac{L_{\mathrm{in}}}{L_{\mathrm{ex}}}\right] \sqrt{\frac{L_{\mathrm{ex}}}{C_{\mathrm{ex}}}} \text{ ohms.} \tag{28}$$

Formulas for L_{in}, L_{ex} and C_{ex} in units convenient for numerical evaluation are given later in the section summarizing the formulas.

2.3 *Attenuation and Phase*

The attenuation of Clogston I cable neglecting leakance is obtained from the following expression:

$$\alpha_I = \frac{R_{\mathrm{ac1}} + R_{\mathrm{ac2}}}{2Z_I} . \tag{29}$$

The phase constant, at the frequencies considered, and where the ac resistance of the conductors does not depart appreciably from the dc value, is equal to:

$$\beta_{\mathrm{I}} = \omega \sqrt{LC} = \omega \sqrt{(L_{\mathrm{in}} + L_{\mathrm{ex}}) \frac{C_{\mathrm{in}}C_{\mathrm{ex}}}{C_{\mathrm{in}} + C_{\mathrm{ex}}}}, \tag{30}$$

which with (27) above reduces to:

$$\beta_{\mathrm{I}} = \omega\sqrt{L_{\mathrm{ex}}C_{\mathrm{ex}}} . \tag{31}$$

3. CLOGSTON II CABLE

3.1 *Resistance*

A Clogston II cable may be looked upon as having an "inner" and "outer" conductor which are separated by an infinitesimal amount in which R_{ac} given by (20) above is an expression for the parallel connection of R_{ac1} and R_{ac2} of a Clogston II cable having an outer diameter of D and an inner diameter of d. That is:

$$R_{ac} = \frac{R_{ac1}R_{ac2}}{R_{ac1} + R_{ac2}}. \tag{32}$$

It will now be assumed that (1) the currents flowing in one direction through the "inner" conductor and in the opposite direction in the "outer" conductor, will separate in such a way that R_{ac1} is equal to R_{ac2}, and (2) that the currents are uniformly distributed over the cross sections of the conductors. With these assumptions the respective cross sections would then be equal, and the reversal of the current would take place in a completely filled ($d = 0$) Clogston II cable at a radius equal to:

$$r = \frac{D}{2\sqrt{2}} = 0.3535D. \tag{33}$$

By substituting (32) and (33) in (20) it can be shown that:

$$R_{ac1} + R_{ac2} = \frac{328320}{D^2 - d^2}\left[1 + \frac{B_2 w^2 D^2 F_{mc}^2}{3710} + \cdots\right] \text{ohms/mi}, \tag{34}$$

where

$$B_2 = 1 + \frac{d^2}{D^2} - \left[1 + \frac{d}{D}\right]\sqrt{\frac{1}{2}\left[1 + \frac{d^2}{D^2}\right]}. \tag{35}$$

The above assumptions relating to division and distribution of the current are not exact. S. P. Morgan has shown that the current distribution is not uniform and that the reversal of the current takes place at a radius equal to 0.3138D. As shown in a later section of this paper, the error resulting from these simplifying assumptions is not large.

3.2 Impedance, Attenuation and Phase

In a Clogston II cable, the main dielectric insulation between "inner" and "outer" conductor has vanished. Thus the external inductance approaches zero and the external capacitance becomes infinitely large.

From (25) and (26) above it is evident, however, that the product of $L_{ex}C_{ex}$ remains constant, since it is independent of the diameter ratio d_2/D_1, which in a Clogston II cable approaches unity. With (27) inserted in (24) and with $L_{ex} = 0$ and $C_{ex} = \infty$, the impedance of a Clogston II cable may be written:

$$Z_{II} = \sqrt{\frac{L_{in}}{C_{in}}} = \frac{L_{in}}{\sqrt{L_{ex}C_{ex}}} \text{ ohms.} \tag{36}$$

The internal inductance of a Clogston II cable is not known, but will be taken equal to 0.1609×10^{-3} Henries/mi, which is the internal inductance of a pair of wires at low frequency. Thus:

$$Z_{II} = \frac{17.35}{\sqrt{\epsilon}} \text{ ohms,} \tag{37}$$

where ϵ is the dielectric constant of the insulating layers.

The attenuation is obtained by dividing $R_{ac1} + R_{ac2}$ from (34) by $2Z_{II}$, where leakance is disregarded.

The phase constant, in the frequency range considered, is equal to the phase constant of a Clogston I cable since $L_{ex}C_{ex}$ is a constant value.

4. *Comparison of Results with Values Obtained from Rigorous Formulas*

S. P. Morgan[8] has developed rigorous formulas for the attenuation of Clogston cables, assuming infinitesimal thickness of the layers but retaining a fixed ratio of copper to insulating layer thicknesses. A correction term gives the increase in attenuation with frequency for layers of finite thickness.

The attenuation of a one-half filled Clogston I cable computed by the approximate formulas given in the present paper was 1.1 per cent higher than the value computed using Morgan's rigorous formulas. Similar computations on a completely filled Clogston II cable gave values 8.6 per cent higher. This decrease in accuracy with increase in fill is in line with the expectation that uniform distribution of the current is more closely approximated with low percentages of fill.

5. SUMMARY OF FORMULAS

The formulas developed in the second part of this paper and those for which the derivation has been indicated are summarized below. Conducting layers of copper, and insulating layers with a dielectric constant

[8] Loc. cit.

of ϵ and a thickness half that of the conducting layers ($w/t = 2$) are assumed throughout.

5.1 *Clogston I Cable*

$$R_{\mathrm{I}} = \frac{82080A}{D_2^2}\left[1 + \frac{B_1 w^2 D_2^2 F_{mc}^2}{3710} + \cdots\right] \text{ ohms/mile,} \tag{38}$$

$$L_{in} = 0.741 \times 10^{-3} \frac{D_2^2}{D_2^2 - d_2^2}\left[\frac{D_2^2}{D_2^2 - d_2^2}\log_{10}\frac{D_2}{d_2} - 0.2172\right] \text{ henries/mile,} \tag{39}$$

$$L_{ex} = 0.741 \times 10^{-3} \log_{10}\frac{d_2}{D_1} \text{ henries/mile,} \tag{40}$$

$$C_{ex} = \frac{0.0388 \times 10^{-6}\epsilon_{\mathrm{I}}}{\log_{10}\dfrac{d_2}{D_1}} \text{ farads/mile,} \tag{41}$$

$$\epsilon_{\mathrm{I}} = \epsilon(1 + w/t), \tag{42}$$

$$Z_{\mathrm{I}} = \left[1 + \frac{L_{in}}{L_{ex}}\right]\sqrt{\frac{L_{ex}}{C_{ex}}} \text{ ohms,} \tag{43}$$

$$\beta_{\mathrm{I}} = \omega\sqrt{L_{ex}C_{ex}} \text{ radians/mile,} \tag{44}$$

$$\alpha_{\mathrm{I}} = \frac{K_2}{D_2^2} + K_3 w^2 F_{mc}^2 \text{ nepers/mile,} \tag{45}$$

where:

$$K_2 = \frac{41040}{Z_{\mathrm{I}}}\left[\frac{D_1^2}{D_1^2 - d_1^2} + \frac{D_2^2}{D_2^2 - d_2^2}\right], \tag{46}$$

$$K_3 = \frac{22.124}{Z_{\mathrm{I}}} \frac{D_1 D_2 - d_1 d_2}{(D_1 + d_1)(D_2 + d_2)}. \tag{47}$$

5.2 *Clogston II Cable*

$$R_{\mathrm{II}} = \frac{328320}{D^2 - d^2}\left[1 + \frac{B_2 w^2 D^2 F_{mc}^2}{3710} + \cdots\right] \text{ ohms/mile,} \tag{48}$$

$$Z_{\mathrm{II}} = \frac{17.35}{\sqrt{\epsilon}} \text{ ohms,} \tag{49}$$

$$\beta_{\mathrm{II}} = \omega\sqrt{L_{ex}C_{ex}} \text{ radians/mile,} \tag{50}$$

$$\alpha_{II} = \frac{K_2'}{D^2} + K_3' w^2 F_{mc}^2 \text{ nepers/mile,} \tag{51}$$

$$K_2' = \frac{164160}{Z_{II}} \frac{D^2}{D^2 - d^2}, \tag{52}$$

$$K_3' = \frac{44.248}{Z_{II}} \frac{D^2 + d^2 - (D + d)\sqrt{\frac{1}{2}(D^2 + d^2)}}{D^2 - d^2}, \tag{53}$$

$$A_1 = \frac{D_2^2}{D_1^2 - d_1^2} + \frac{D_2^2}{D_2^2 - d_2^2}, \tag{54}$$

$$B_1 = \frac{2(D_1 D_2 - d_1 d_2)(D_1 - d_1)(D_2 - d_2)}{(D_1^2 - d_1^2 + D_2^2 - d_2^2)D_2^2}, \tag{55}$$

$$B_2 = 1 + \frac{d^2}{D^2} - \left(1 + \frac{d}{D}\right)\sqrt{\frac{1}{2}\left(1 + \frac{d^2}{D^2}\right)}. \tag{56}$$

The parameters in (38) to (56) are defined as follows:

D_2 = Outside diameter of outer laminated conductor in a Clogston I cable, in mils.
d_2 = Inside diameter of outer laminated conductor in a Clogston I cable, in mils.
D_1 = Outside diameter of inner laminated conductor in a Clogston I cable, in mils.
d_1 = Inside diameter of inner laminated conductor in a Clogston I cable, in mils.
D = Outside diameter of a laminated Clogston II cable, in mils.
d = Inner diameter of a laminated Clogston II cable, in mils.
w = Thickness of copper layers in mils.
F_{mc} = Frequency in megacycles.
ϵ_I = Dielectric constant of insulation between inner and outer laminated conductors in a Clogston I cable.
ϵ = Dielectric constant of insulating layers.

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