The L3 Coaxial System

Quality Control Requirements

By H. F. DODGE, B. J. KINSBURG and M. K. KRUGER

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Economic solution of the equalization problem in the L3 system required limitation of the excursions of the transmission characteristics from the design center. To implement this, a pattern of distribution requirements for component elements of the system was worked out utilizing basic quality control techniques. An analysis of the several methods employed is presented with particular emphasis on the intent of the choices which were made and the operating characteristics of the resulting procedures. One of the novel features is the three-cell selection method which insures that the product delivered has the kind of distribution that is wanted even while the process is in trouble distribution-wise.

1.0 Reason for Requirements

1.1 equalization problems

The L3 system is a long distance carrier system designed to transmit either 1,860 telephone channels or 600 telephone channels plus one television channel. The band width is approximately 8 mc. The repeaters are spaced about 4 miles apart and in a 4,000-mile route, counting both line and office amplifiers there will be about 1,200 amplifiers in tandem.

There are two equalization problems. The first is equalization proper, i.e., delivery of a satisfactory signal to the customer from the transmission characteristic point of view. The television equalization design objective of the system is to meet a signal-to-echo ratio of 40 db. This corresponds to a uniform sinusoidal ripple of 1/10 db or a complex deviation pattern several times larger in amplitude. The telephone equalization objective is more lenient and allows deviations as large as 1 to 2.5 db, depending on length of circuit and number of links in tandem. Though care was exercised to make basic design decisions which would tend to ease the equalization problem, still the reduction of the accumu-

lated deviation arising in 1,200 amplifiers to a few tenths of a db is a formidable problem.

The second problem is concerned with the effect of equalization on the signal-to-noise performance. As deviations creep into a repeatered circuit, the transmission levels deviate more and more from the normal or design levels. This effect is lumped in one term, misalignment. The result of misalignment is degradation in signal-to-noise performance. Periodic equalization helps to limit misalignment in the succeeding repeater sections but does not eliminate the increase in noise or modulation which has occurred in the preceding repeaters. Thus, the objective is not only to equalize the over-all circuit, but also to keep the deviations all along the transmission line within the specified bounds in order not to exceed signal-to-noise margins.

1.2 NEED FOR CONTROLLING DEVIATIONS AT THEIR SOURCE

Let us examine what accounts for the magnitude of the gain-versus-frequency deviations arising in any one repeater. Let us assume that a particular element deviates +1 per cent from the design objective. Is this good or bad? The answer to this question may be had only if a deviation study of the repeater is made and its sensitivity to the deviation of the element under consideration is ascertained. There are two factors which contribute to the equalization problem: (a) the deviation sensitivity of the repeater to a given deviation of the element from its prescribed value, and (b) the actual deviation of the element itself due to all causes, including manufacture, temperature, aging, etc. To simplify our terms, factor (a) will be called the sensitivity of the element, and factor (b), the deviation of the element. The sensitivity is a function only of the circuit design and is independent of the performance of the individual element. The deviation of the element is a function only of its design and manufacture.

1.3 USE OF STATISTICAL QUALITY CONTROL METHODS TO ASSURE A CONTROLLED DISTRIBUTION

When the design of the L3 system was initiated, it was realized that the effect of the variability of the component elements could be materially reduced by the application of statistical quality control techniques to design and manufacture. Once a circuit design is available and a deviation sensitivity study is made, it is comparatively easy to formulate the desired limits on the variability of components. Actually the process of arriving at an individual tolerance objective is more complex

since there is a large area of give and take between the circuit and the component element designers.

Let us assume that the process of arriving at a satisfactory component element design has been completed and that the spread of the manufacturing limits has been set at $\pm y$ per cent. How will the gain deviations due to this element grow, as more and more repeaters, each containing one unit of this element, are placed in tandem? It is evident that the cumulative magnitude of gain deviations will depend on the distribution pattern describing the departure of individual units of this element from the prescribed value.

If all units of this element have a systematic deviation (equal in magnitude and sign) from the prescribed value, the cumulative gain deviation will be

$$n\alpha_1 \text{ db}$$

where n = number of repeaters in tandem, and

α_1 = gain deviation in one repeater due to y_1 per cent deviation of the element.

If the units of this element have a Normal distribution whose average coincides with the prescribed value, and whose extreme limits (say 3-sigma limits) are $\pm y_2$ per cent, then the averages of random groups of elements in n repeaters will be described by another Normal distribution, the corresponding limits of which will be $\pm y_2/\sqrt{n}$ per cent. Thus the over-all limits of gain deviation for n repeaters will be

$$\pm \frac{n\alpha_2}{\sqrt{n}} \text{ db} = \sqrt{n}\alpha_2 \text{ db}$$

If, however, the average of the distribution of individual units does not coincide with the prescribed value but is displaced by y_1 per cent, then the over-all gain deviation limits for n repeaters in tandem will be

$$(n\alpha_1 \pm \sqrt{n}\alpha_2) \text{ db}.$$

Thus, it is of first importance that the average of the individual units be controlled as closely as possible to the prescribed aimed-at value.

Of course, distributions other than Normal are possible. However, it is sufficient for most practical purposes to consider only the Normal distribution since a combination of a large number of distributions, which individually are not Normal, will tend to approximate a Normal distribution. This assumption is a reasonable one to make because of the large* number of different elements which are used to make up an L3

* There are over 100 component elements in the L3 amplifier of which about 12 have large element sensitivities and are therefore critical in evaluating performance. There are over 30 additional elements the sensitivities of which are also sufficiently large to require application of distribution requirements.

repeater. It should be kept in mind that this assumption would not be valid if the contribution of one element to the deviation pattern of the repeater becomes dominant, say, larger than all the other elements combined.

In the L3 system, the first stages of equalization are spaced about 25 repeaters apart. In accordance with the above considerations, a distribution of individual element variations, the average of which is controlled close to the nominal, will result in cumulative gain deviation which will be substantially smaller than if no restrictions were placed on the average. Thus, a desired objective for critical L3 component elements is to provide a stabilized production process giving a distribution of individual values (a) having an *average* that is maintained consistently close to a desired nominal, and (b) having a pattern of variation around the average that is Normal (or nearly so). If this is attained, comparatively wide limits for the individual units are acceptable. Furthermore, assembly of component elements into amplifiers can be made on a random basis, and the problem of maintaining equipment in the face of replacement of parts failing in service will be greatly simplified.

From the very nature of the over-all problem the best approach to this objective has appeared to be through the application of statistical quality control methods, both in the design of and in the production of the component elements that are important from an equalization point of view.

2.0 Basic Features of Distribution Requirements

2.1 General Plan

For each of the important component elements, then, interest centers on closely controlling the collective quality of the product, especially the average of the individual values. This can hardly be accomplished merely by specifying and securing compliance with the usual type of requirements, expressed as maximum and minimum limits for individual units. Something more is needed. Consideration must be given to ways and means of placing requirements on the distribution of individual values from the successive increments of the product turned out day after day.

Accordingly, a general plan using quality control methods has been developed, specific features of which will be discussed in this paper with particular emphasis on the intent of certain choices that were made and on the procedures selected to meet the general objective. Further development work on some of these features may of course be found warranted as experience with them is gained.

The general plan has been implemented by imposing on important component elements certain so-called "distribution requirements" which incorporate quality control procedures for assuring a high degree of statistical uniformity in the quality of product delivered for service. The aim of the distribution requirements is to place a continuing limitation on the pattern and the spread of measured values (of a final critical characteristic of the product) around their average and to impose close limits on the departures of the average from a desired nominal value. To obtain these ends, close cooperation between the element designer and the production engineer is essential. In fact, compatibility of the specification requirements and the process capability is one of the basic provisions of the general plan. This should be established, if at all possible, in the design stage.

In some cases where distribution requirements have been applied to a final characteristic of an element, the production engineer has found it advantageous to introduce quality control techniques in some of the earlier manufacturing steps, as for example, on materials, piece-parts, or process operations which are found to have a major effect on end quality.* The character of such controls can rarely be planned in advance, but must be tailor-made to fit the particular process being used. Often too, a major difficulty encountered has been not the process itself but the precision and accuracy of the measuring equipment. The resolving of such problems during the design and the early production stages has been one of the aims of the general plan.

2.2 SELECTION OF CHARACTERISTICS TO BE CONTROLLED

The general procedure calls for imposing distribution requirements on not more than one characteristic of any component element. Before assigning limits on an individual component, therefore, it is important that the characteristic selected for control be the key characteristic. This statement seems to be trite, but its importance cannot be overemphasized. Controlling all characteristics of a component is not only inherently uneconomical but may be found impossible in practice. In the case of an inductor, for example, it should be ascertained which characteristic is important to the circuit designer. It may be the value of inductance, of Q, of temperature coefficient, or of parasitic capacity. In addition, of course, requirements should be specific and apply, for instance, at a given frequency or within a definite temperature range.

* R. F. Garrett, T. L. Tuffnell and R. A. Waddell, The L3 Coaxial System — Application of Quality Control Requirements in the Manufacture of Components, see pp. 969–1006 of this issue.

Where it is desirable to exercise some control over one or more characteristics in addition to the key characteristic, a procedure is provided which requires that control charts be maintained on the additional characteristics. This procedure does not require a controlled distribution for such characteristics, but it does give a statistical record which shows the dynamic behavior of the process and indicates when remedial action is desirable.

2.3 COMPATIBILITY OF SPECIFICATION REQUIREMENTS AND MANUFACTURING PROCESSES

As mentioned above, special effort has been made in the L3 project to provide specification limits and manufacturing processes for individual component elements that are mutually compatible.

In order to determine realistic limits on the value of a particular quality characteristic, it is necessary to collect a reasonable quantity of data from the proposed process to show what it can do if brought into a state of statistical control. This is an area in which close cooperation between the element designer and the production engineer is necessary. Here it is convenient to define the "natural tolerance" of a process as the extreme range of variation to be expected among individual units of product made in relatively short periods of time, such as in single batches or production lots; mathematically it is taken to be equal to 6σ, where σ is the basic standard deviation of the process as estimated from the average spread for a *series* of samples, each selected from a different segment of production.

If it is found, for instance, that the natural tolerance of the process (6σ) is wider than the expected or desired specified limits, then a fundamental change either of the process or of the basic design of the component or both is called for, if mutual compatibility is to be attained. Of course, one way to avoid a major change would be to widen the specification limits. If the needs of the system, however, demand the closer limits, such a simple solution is not possible, and a manufacturing process or design change must be made. In many cases examined in connection with the L3 repeater, the economics of the situation — balancing the component cost against the saving in equalization gear — justified additional effort to improve designs and manufacturing processes to obtain limits for individual component elements narrower than those which initial processes appeared capable of meeting.

2.4 THE PROBLEM OF MEASUREMENT

The precision of measurement may have an important influence on the determination of the process capability. Let us assume that the

universe of true values for an element may be described by Curve A in Fig. 1, having a spread of $2r$. Let us also assume the distribution of errors of measurement representing the precision of the measuring device (assumed to be unbiased) is described by Curve B of Fig. 1, having a spread of $2s$. If, for the purposes of discussion, these distributions are Normal, the resulting apparent distribution of the individual values (the distribution of measured values) is a composite of the distribution A and B, as shown by Curve C of Fig. 1. The spread of this distribution will be $2q$, where $q = \sqrt{r^2 + s^2}$. If $s = \frac{1}{2}r$, as shown in Fig. 1, then $q = \sqrt{r^2 + 0.25r^2}$ or $q = 1.118r$. Thus, the apparent distribution has a spread which is about 12 per cent greater than the true distribution. Similar computations for other values of measuring precision in relation to the true distribution give the following:

Ratio s/r	Ratio q/r
1	1.414
0.5	1.118
0.2	1.020
0.1	1.005

Measurements normally made to determine process capabilities include the effect of random errors but not necessarily of systematic errors.

The effect of systematic error or bias of the measurements is quite different. Bias tends to cause unknown and unwanted displacement of the process average from the aimed-at value. This, in turn, can be re-

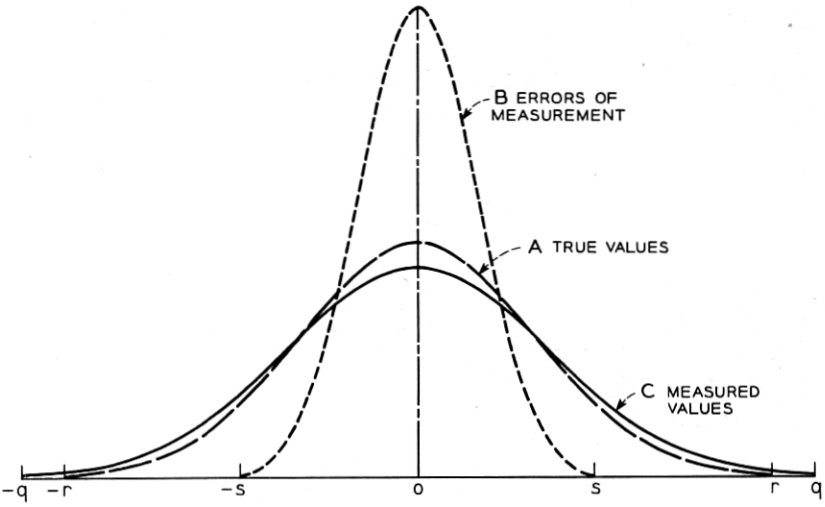

Fig. 1 — Effect of measuring errors.

sponsible for systematic deviations which are so undesirable in the L3 system. One way to combat or, rather, circumvent large bias is to rely on comparison standards in the calibration of test sets. The task of furnishing and maintaining reliable comparison standards has many pitfalls. It is an art or perhaps a science in itself and is mentioned here only because of its importance to the objective of restricting variations in the process average.

Considering the effect of both random and systematic errors of measurement, it has been found essential to place comparatively tight requirements on the accuracy of the test methods to be used. In many instances meeting these accuracy requirements was made possible by the development of new or radically improved measuring equipment for both laboratory and production purposes.

2.5 ALLOWABLE MARGIN FOR DRIFT OF PROCESS AVERAGE

Having determined σ, the basic standard deviation of an acceptable process for a quality characteristic, then the minimum spread of specified limits, to be compatible with the process, would be $\pm 3\sigma$ around the nominal (N). However, product having this σ could be expected to meet such limits practically all the time provided only that the average of the process were controlled at the nominal. Accordingly, provision was made to allow the process average itself to vary within a band of $\pm \frac{1}{3}\sigma$ around the nominal. This allowance is somewhat arbitrary and represents an estimate of relative importance to the L3 system of systematic changes

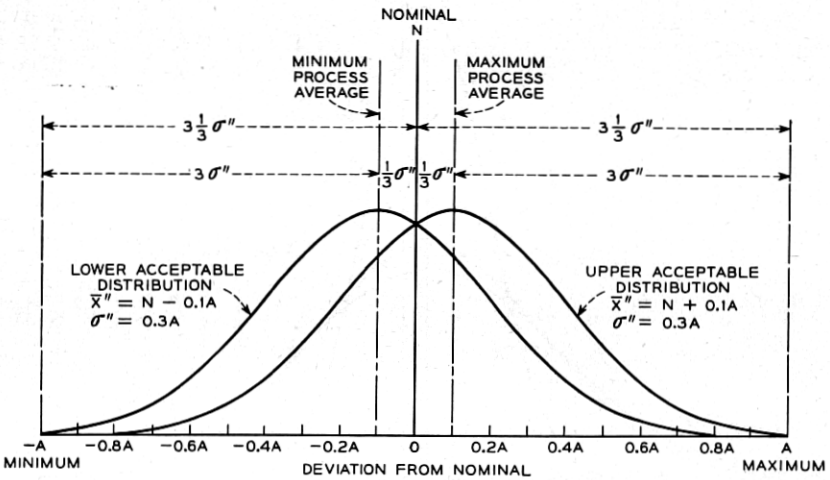

Fig. 2 — Basis of specified limits.

in the process average. Thus, two limiting acceptable distributions, Normal in shape, are defined as indicated in Fig. 2, both having a standard deviation, σ'', equal to the above-mentioned basic standard deviation of the process, one having an average, $\bar{X}''$, located $\frac{1}{3}\sigma''$ below nominal and the other having an average $\frac{1}{3}\sigma''$ above nominal. The over-all limits for individual units then become $N \pm 3\frac{1}{3}\sigma''$. If we designate the distance from the nominal to either limit by A, then the permissible variation in the process average is $\pm 0.1A$.

Our discussions here will be confined to the case of characteristics having substantially Normal distributions and having both maximum and minimum specified limits. In the over-all plan provisions have also been made for characteristics having skew distributions or having a single specified limit, maximum or minimum.

2.6 EXPRESSION OF DISTRIBUTION REQUIREMENTS

With the decision to use distribution requirements, the question arose as to how such requirements should be expressed. Would it be adequate to write:

"The distribution of the individual values of characteristic X of the product shall meet the requirements: (1) average, not outside $(N \pm a)$, and (2) standard deviation, not greater than b."? Should a clause be added saying that characteristic X "shall be statistically controlled?"

Recognizing that a requirement is not what is written but what it is interpreted to be, it appeared that such an expression or equivalent is not sufficient to explain the intent nor does it provide criteria for determining when any segment of production may be considered conforming to the intent. What is meant by "the product?" Does the limiting value on the "average" (or the "standard deviation") apply to each production segment of 10 units, each 50 units, each day's production, each six month's production? Can sampling be used? What criteria are to be used for judging whether the distribution of quality of a portion of product is satisfactory? What should be done with the product if it does not meet the criteria?

As with any requirement on the collective quality of an aggregation of like articles comprising a product, the use of distribution requirements brings in special problems on the clarification of the intent. What is really wanted is a flow of product such as would be obtained in a series of random samples of units from a process whose average is continually maintained within stated limits $(N \pm 0.1A)$ and whose standard deviation does not exceed $0.3A$.

With this in mind, the product specification prescribes for any given key characteristic a nominal value, N, maximum and minimum limits for individual units, $N \pm A$, and adds the clause "subject to the distribution requirements of (Y)." This supplementary specification (Y) gives a statistical description of the intent, together with control procedures to be followed in the inspection of the product.

3.0 Control Procedures Associated with Distribution Requirements

Distribution requirements can be given operationally definite meaning by providing procedures (of the nature of inspection procedures) that define (a) the character and quantity of evidence needed regarding the collective quality of the product as it is made day by day, (b) the criteria for judging when such product may be considered conforming to the intent of the specification, and (c) the treatment or disposition of product units when these criteria are not met.

Three such procedures have been prepared to meet the several conditions that may be encountered in the production of L3 component elements:

1. control chart method,
2. batch method, and
3. three-cell method.

The first two methods permit the use of sampling. For both of these methods the criteria have been so selected that product should be found to be conforming to the distribution requirements practically all of the time if the process is so controlled as to maintain a distribution of individual units with (a) an average within the band, $N \pm 0.1A$, and (b) a standard deviation not greater than $0.3A$. The third method requires 100 per cent inspection, and while this method may be used at any time at the option of the manufacturer, its use is mandatory whenever a failure to meet the criteria of the other methods is encountered.

To insure that the product shipped continually meets the intent of the distribution requirements, a provision is made for packaging the output in groups of 5 units. This in effect furnishes the user either with random sample groups of 5 from a process which has been shown to be in satisfactory control (control chart and batch methods), or with specially selected groups of 5, the units in each of which have been chosen to meet a particular distribution pattern (three-cell method).

The following sections give the general character of the statistical models that have been set up for the three methods.

3.1 CONTROL CHART METHOD

The control chart method is intended for application where production comprises a reasonably steady succession of individual units or small groups of units from a common source; so that units or groups, as produced, may be kept in the order of their production and control chart techniques applied to test results. Under this method control charts are maintained for averages and ranges of samples of 5.

At the outset it is necessary to demonstrate an adequate degree of control of the product in order to be considered eligible for application of this method. Once eligibility has been established, a second and somewhat more lenient set of conditions is used to judge whether this eligibility is maintained. For convenience of reference these two sets of conditions are designated (a) Criterion I, for establishing eligibility (or for reestablishing it), and (b) Criterion II, for maintaining eligibility.

The control charts used in this section are an adaptation of the well-known Shewhart control charts for sample averages, $\bar{X}$, and sample ranges, R, for "control with respect to a given standard." Two modifications of the techniques customarily used in applying the control chart for $\bar{X}$ have been introduced: (a) a central band rather than a central line has been provided, and (b) a non-parametric requirement has been imposed on seven successive sample averages to limit the excursions of the product average from the nominal value. These modifications are related to the two acceptable distributions referred to in Fig. 2 and reproduced as dotted lines in Fig. 3.

The PA limits of Fig. 3 are the desired minimum and maximum values of the process average, and are the averages of the two acceptable distributions. As indicated in Fig. 2, the PA limits for the process average give the boundaries of the band, $N \pm \frac{1}{3}\sigma''$, where σ'' is the standard deviation of the two acceptable distributions.

To determine control limits of the control charts for $\bar{X}$ and R, consideration is given to the sampling distributions of averages of samples of 5 drawn from such acceptable distributions, as shown in Fig. 3, as well as to the sampling distribution of ranges of samples of 5 from these distributions.

The A5 limits of Fig. 3 (limits to be met by the average of a sample of 5) are 3-sigma control limits for averages of 5, given by

$$N \pm \left(\tfrac{1}{3}\sigma'' + 3 \frac{\sigma''}{\sqrt{5}} \right).$$

The R5 limit (limit to be met by the range of a sample of 5) is the

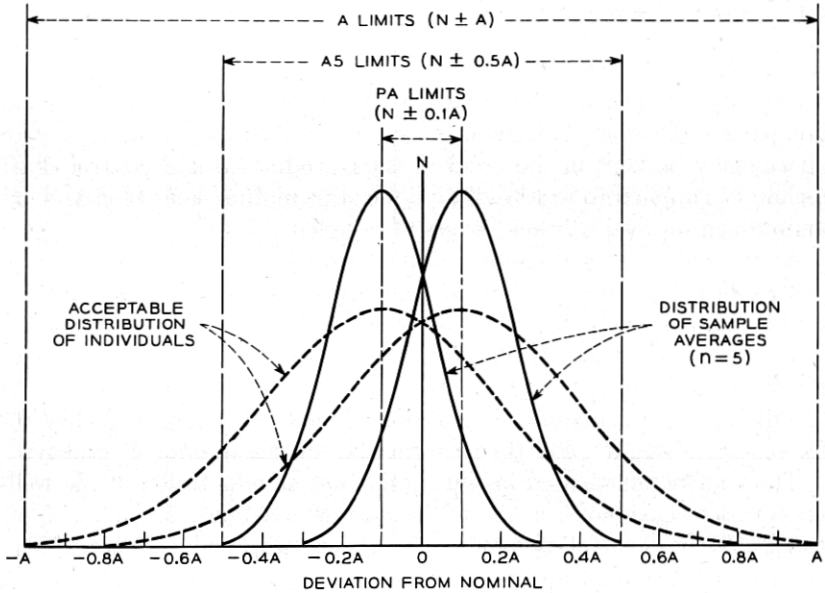

Fig. 3 — Basis of A5 limits and PA limits for sample averages.

customary upper 3-sigma control limit for ranges, $D_2\sigma''$ (where $D_2 = 4.918$ for samples of 5). Since the relation between the specified limits for individual units and σ'' is expressed by $A = \frac{1}{3}\sigma''$, the above limits are related to the specification limits as follows:

$$\text{PA limits} = N \pm 0.1A$$

$$\text{A5 limits} = N \pm 0.5A$$

$$\text{R5 limits} = 1.48A, \text{ max.}$$

At the outset and whenever eligibility to use the control chart method is lost, a 100 per cent inspection rate is required and acceptance is based on the three-cell method (discussed later). Units in groups of 5 are tested as successive samples, and subjected to the following criterion:

Criterion I — Establishing Eligibility

Eligibility for use of the control chart method is established as soon as 7 consecutive samples or groups of 5 satisfy all the following conditions:

(a) The averages all meet the A5 limits; and

(b) The ranges all meet the R5 limit; and

(c) The seven consecutive averages are not all outside the same PA limit (not all above the upper PA limit or all below the lower PA limit).

When eligibility to use the control chart method is established and so long as it is maintained, sampling inspection may be used. For this inspection, periodic samples of 5 are selected in accordance with a schedule which normally calls for measurement of about 10 per cent of the units produced. The lots represented by such samples are considered as conforming to the intent of the distribution requirements, and hence acceptable for this feature. Provisions are made for further reducing the inspection rate when consistently good control performance is evidenced. During the period of sampling, the following criterion applies to the sampling results:

Criterion II — Maintaining Eligibility

Eligibility for use of the control chart method is maintained so long as the results of the current sample satisfy all of the following conditions:

(a) The average either (1) meets the A5 limits; or (2) fails to meet the A5 limits but at the same time all of the 6 preceding consecutive averages meet the A5 limits; and

(b) The range either (1) meets the R5 limit; or (2) fails to meet the R5 limit but at the same time all of the 6 preceding consecutive ranges meet the R5 limit; and

(c) Seven consecutive averages (for the current sample and the 6 preceding samples) do not all fall outside the same PA limit.

(d) No major change is made in raw material, machine set-up or personnel, which may have a significant effect on the quality of the product.

Fig. 4 gives a control chart for averages and ranges of samples of 5 such as might be obtained under the control chart method. The first 20 points (Series A) indicate what might be expected in a series of samples if the process were controlled with its average, $\bar{X}'$, at N and its standard deviation, σ', equal to the standand value, $0.3A$. The next 20 points (Series B) indicate the expected pattern of points if the process average suddenly shifted to a level about $0.25A$ above the nominal, while σ' remained unchanged. Both sets of points represent the result of random sampling experiments simulating the conditions stated. During Series A the first seven points meet Criterion I and would have established eligibility for using the control chart method. Eligibility would have been maintained for the balance of Series A. Starting with Series

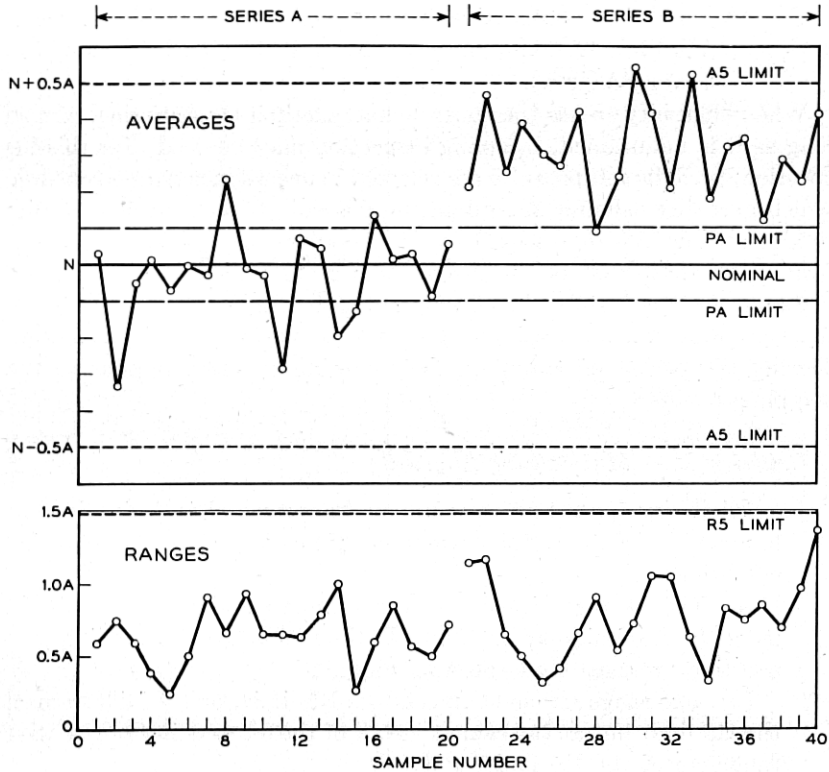

Fig. 4 — Control charts for averages and ranges, samples of 5 from a normal universe.

B, however, a failure to meet Criterion II would have been encountered on the seventh plotted point, which would have caused loss of eligibility to continue using the control chart method. This would have required a reversion to 100 per cent inspection and acceptance by the three-cell method until such time as Criterion I was again met.

Individual units in a lot accepted by the control chart method are to be selected at random from the lot in groups of 5 each, and the five units of each group are to be placed in a single package or otherwise associated so as to remain physically together until delivered to the user.

3.2 BATCH METHOD

For some component elements, units are produced intermittently in relatively large groups or batches and subjected collectively to the same manufacturing processes. Under these conditions individual units

or small groups of units do not follow one another through the process in time sequence. Here "order of production" has no meaning for individual units — the only order is the sequence of successive batches.

For this situation a batch method is provided whereby a substantial sample, normally 50 units, is selected at random from each batch. The average, $\bar{X}$, and the standard deviation, σ, of the sample are computed and the satisfactoriness of the distribution of the batch is determined by comparing (1) the sample average, and (2) the sample standard deviation with certain limits of allowable variation which have been established. As in the case of the control chart method, consideration is given to the results to be expected in samples from the two acceptable distributions already defined.

The A50 limits, the limits to be met by a sample average, are 3-sigma control limits for averages on either side of the process average band given by $N \pm (\frac{1}{3}\sigma'' + 3\sqrt{\sigma''}/\sqrt{50})$, which after substituting $\sigma'' = 0.3A$, gives

$$\text{A50 limits} = N \pm 0.23A.$$

The limit to be met by a sample standard deviation is the upper 3-sigma limit of a sampling distribution of standard deviations for samples of 50 from a universe having a standard deviation, σ''. However, in practice the standard deviation is difficult to calculate. In order to simplify calculations, an estimated standard deviation is used instead, computed as follows: Divide the 50 values into random subgroups of 5; find the range, R, of each of the subgroups; compute the average range, $\bar{R}$, and multiply by 0.43. Thus σ (estimated) $= 0.43\bar{R}$. Considering the sampling distribution of this statistic (0.43 times the average range for 10 samples of 5) as a substitute for σ, we make use of known theoretical relations between the distribution of $\bar{R}$ for samples of 5 and the standard deviation of the sampled universe. The upper 3-sigma limit of the sampling distribution of $\bar{R}$ (for 10 samples of 5) is given* by

$$2.326\sigma'' + 3\left(\frac{0.864\sigma''}{\sqrt{10}}\right)$$

which, after substituting $\sigma'' = 0.3A$, gives $0.95A$. From this the limit to be met by the sample standard deviation (estimated as $0.43\bar{R}$) is

$$\text{S50 limit} = 0.41A, \text{ max.}$$

The limits for the sample average must be met by each batch, but

A.S.T.M. Manual on Quality Control of Materials, Am. Soc. for Test. Mat., Phila., 1951; see related formula D_2, p. 114, and factors d_2 and d_3, p. 115.

provision is made for an occasional failure to meet the limit for the sample standard deviation. Thus the batch represented by a sample will be considered conforming to the intent of the specified distribution requirements, and hence acceptable for this feature if the sample meet the following criterion:

Criterion III — Batch Acceptability.

(a) The average, $\bar{\bar{X}}$, meets the A50 limits; and

(b) The standard deviation, σ, either (1) meets the S50 limit, or (2) fails to meet the S50 limit but at the same time all of the six preceding consecutive standard deviations meet the S50 limit.

If a batch fails to meet this criterion, the batch must be inspected 100 per cent and acceptance is based on the three-cell method.

Packaging is handled in the same manner as for the control chart method; individual units in a batch accepted by the batch method are to be selected at random from the batch in groups of 5 each and packaged as such for delivery to the user.

3.3 THREE-CELL METHOD

Even with the best of conditions things may go wrong from time to time due to any one of a number of causes — changes in raw materials, irregularities in manual operations, faulty performance of processing equipment, etc. As a result, samples taken under either the control chart method or the batch method will sometimes fail to meet the criteria of the methods. In such times of trouble one solution would be to stop production, find the assignable cause, rectify it, and then resume manufacture. This solution, though perhaps ideal in one sense, may not be practical for several reasons:

(a) Considerable time may elapse before the assignable cause is found and corrected;

(b) Manufacturing schedules may be disrupted; and

(c) No answer is provided to the question of what to do with the uncontrolled product already made.

What is needed, therefore, is a procedure for dealing with the finished units when the process is in trouble distribution-wise, a procedure which will permit shipment of some of the product and at the same time assure that the portions shipped will have a proper distribution.

To this end a selection procedure referred to as the three-cell method has been provided. Under this method each unit of product is measured for the characteristic in question and the conforming units are classified

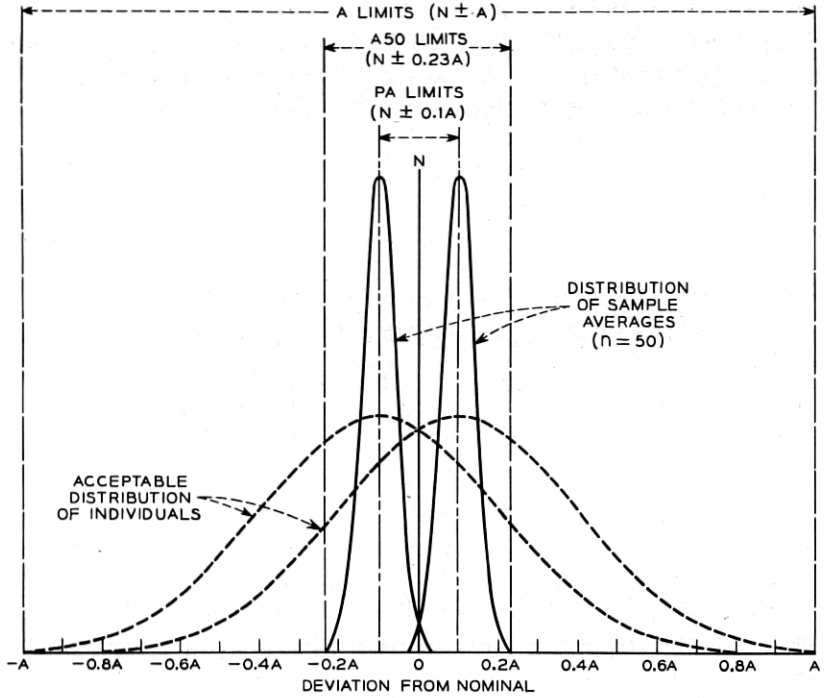

Fig. 5 — Basis of A50 limits for sample averages, batch method.

and sorted into three cells: lower, middle and upper, as shown in Fig. 6. Units are then selected from the three cells in groups of 5 and packaged, the five units in each package to be distributed among the three cells in accordance with one of the two distributions shown at the top of Fig. 6. These selected groups of five are maintained as packages of 5 in merchandise stock and in deliveries to the user.

Units in any cell which are in excess supply at any given time during production and which, therefore, cannot be packaged may be included with subsequent production provided each package of five satisfies one of the two distributions shown in Fig. 6.

For the three-cell method several matters were open to choice — the relative width of the three cells, the number of units in a package, and the required distribution of units in a package. Sampling experiments were run with groups of 4, 5, 6 and 12 and the net effects studied probability-wise for various possible kinds of quality situations that might be encountered in production.

With appropriate choices of these items it is possible to provide as-

surance that the units furnished from various segments of production will approximate the deviation pattern to be expected in random samples from an appropriately controlled process. The fundamental objective is to provide a parade of product-segments under the three-cell method that will continue to have distributions that meet the basic intent of the distribution requirements as deliveries or replacements are made. At the same time, it is desirable to make the three-cell method moderately more restrictive than the control chart method and the batch method in order to provide an incentive to attain a degree of control during production that will permit sampling. The choice of three equal cells, packages of 5, and the package distributions indicated in Fig. 6 were selected with these things in mind.

4.0 Operating Characteristics of Control Procedures

In the preceding section a description was given of the control procedures that are associated with specified distribution requirements. The most important question to be answered is: "What are the operating characteristics of these procedures?" In other words, how well will these procedures discriminate between quality that is good or bad, distribution wise? How effectively will they assure realization of the objectives for which they were set up?

For the sake of simplicity the statistical models used to evaluate the expected performance of the control methods are limited to Normal dis-

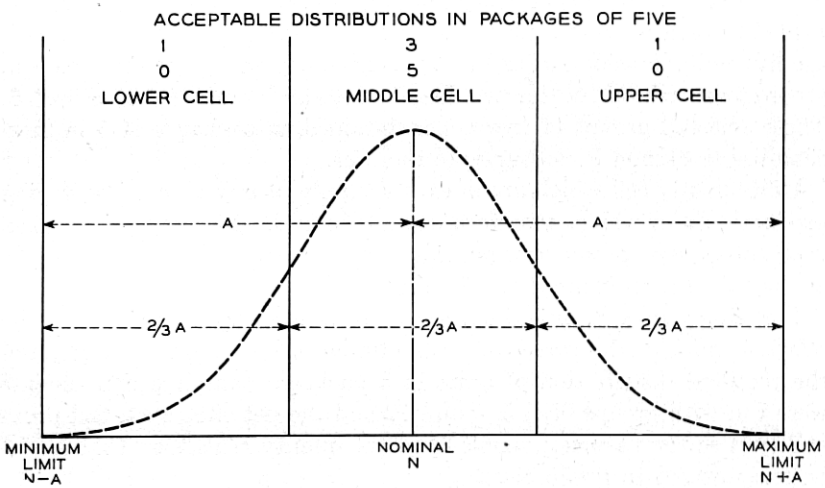

Fig. 6 — Acceptable distributions of units in packages of 5, three-cell method.

tributions.* Since a primary objective is to limit the displacement of the process average, $\bar{X}'$, from the nominal value, N, this parameter will be used as the independent variable against which the performance of the methods is computed. The curves to be shown will be referred to as "operating characteristic curves" or "OC curves" for the respective criteria of these methods, a term† that is applied generally to the related "probability of acceptance" curves for sampling inspection plans.

4.1 OPERATING CHARACTERISTICS OF THE CONTROL CHART METHOD

First, let us assume that we have a process the output of which has a Normal distribution with a standard deviation, σ', equal to $0.3A$, which is the same as the standard value, σ'', used in setting the specified limits. For this case the limits $N \pm A$ are $N \pm 3\frac{1}{3}\sigma'$. Ideally, the process average, $\bar{X}'$, should be centered at N, but by design it is agreed that a displacement of $\bar{X}'$ by an amount $0.1A$ from N should be acceptable. Conceivably the displacement could well be considerably larger than this, and the question arises as to how the two criteria of the control chart method will function for different magnitudes of displacement. With the model assumed, it is possible to work out an analytic answer to this question. For example, for any given displacement of $\bar{X}'$, from N, the resulting formula for P_{I}, the probability of meeting Criterion I, is:

$$P_{\text{I}} = \left[(1 - P_2^+ - P_2^-)^7 - (P_1^+)^7 - (P_1^-)^7\right]\left[(1 - P_4)^7\right] \qquad (1)$$

and the formula‡ for P_{II}, the probability of meeting Criterion II is

$$\begin{aligned} P_{\text{II}} = {} & \left[P_0 + P_1^+\left\{1 - (P_2^+ + P_1^+)^6\right\}\right. \\ & + P_1^-\left\{1 - (P_2^- + P_1^-)^6\right\} + P_2^+\left\{(1 - P_2^+ - P_2^-)^6 - (P_1^+)^6\right\} \\ & \left. + P_2^-\left\{(1 - P_2^+ - P_2^-)^6 - (P_1^-)^6\right\}\right]\left[1 - P_4 + P_4(1 - P_4)^6\right] \end{aligned} \qquad (2)$$

* For other distributions this limitation is unimportant for those portions of the criteria that relate to sample averages since averages of samples from a non-Normal universe may ordinarily be considered to be distributed Normally. See W. A. Shewhart, *Economic Control of Quality of Manufactured Product*, D. Van Nostrand Co., New York, 1931, pp. 180–184. But due among other things to lack of independence of $\bar{X}$ and R for skew distributions, the results given here should be considered only as reasonably close approximations for the degrees of non-Normality that may be encountered in practice.

† A term first used in the late 1930's by Capt. H. H. Zornig, of the Ballistic Research Laboratories, at Aberdeen Proving Ground.

‡ P_{II} is an unconditional probability in the sense that it does not involve the condition that previously there was a sequence of 7 samples satisfying Criterion I and no intervening sequence of 7 samples not satisfying Criterion II. P_{II} has been used as an approximation to, although possibly somewhat less than, the corresponding conditional probability. A similar consideration applies to P_{I}.

where each of the above P symbols designates the probability that a sample average (average of a random sample of 5 units) will fall in the band associated with that symbol in the following tabulation:

Symbol	Band
P_2^+	Above $+0.5A$
P_2^+	$+0.5A$ to $+0.1A$
P_0	$+0.1A$ to $-0.1A$
P_2^-	$-0.1A$ to $-0.5A$
P_2^-	Below $-0.5A$

and P_4 = probability that a sample range (range of a random sample of 5 units) will fall above its upper control limit, $1.48A$. ($P_4 = 0.0044$ for $\sigma' = 0.3A$.)

The OC curves for Criterion I and Criterion II computed from these formulas are given in Fig. 7(a), when $\sigma' = 0.3A$. These show how the control chart method serves on a probability basis as a band-pass filter for a manufacturing process, permitting the introduction of and allowing the continuance of sampling so long as the process average is maintained reasonably close to the nominal, and imposing an increasingly higher barrier to acceptance by sampling when the displacement of the process average from N is increased, thus forcing the use of the three-cell method. Examination of these curves shows that Criterion I is more stringent than Criterion II, as it should be.

Suppose for the moment that both Criterion I and Criterion II omitted condition (c) relating to seven successive sample averages. The OC curves in this case are shown in Fig. 7(b) with the designation "I, II, less c." It is seen that this requirement relating to seven successive averages is most important. Without it the criteria, particularly Criterion II, would be very ineffective in controlling excursions of the process average.

A second question of importance is: "What happens if the specified dispersion limits (A limits) are improperly set or if the process dispersion changes to produce this effect?" In other words, what are the operating characteristics of the procedure when $\sigma' \neq 0.3A$? This is shown in Figs. 7(c) and 7(d) for Criterion I and Criterion II respectively, where the values of process standard deviation (σ') are expressed as fractional values of A.

4.2 OPERATING CHARACTERISTICS OF THE BATCH METHOD

For the batch method the procedure is essentially a lot acceptance procedure and except for permitting an occasional failure of the sample

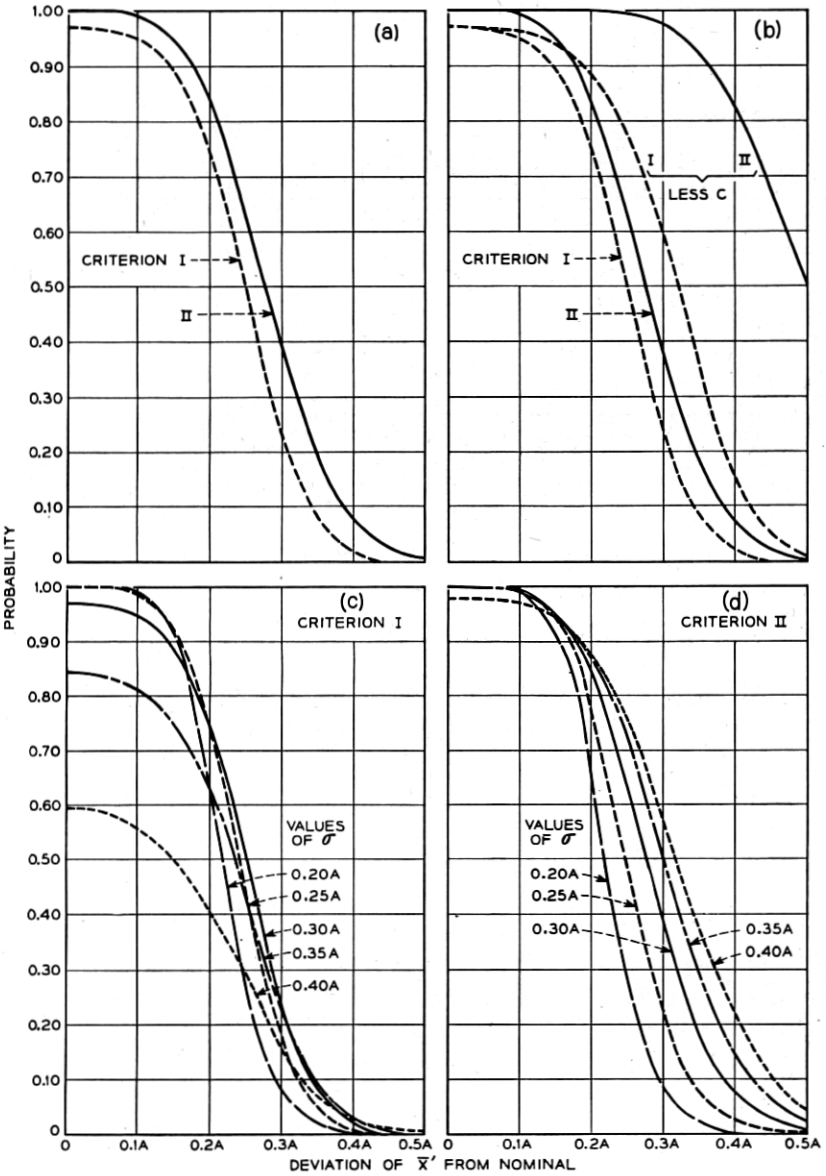

Fig. 7 — Operating characteristic curves for control chart method.

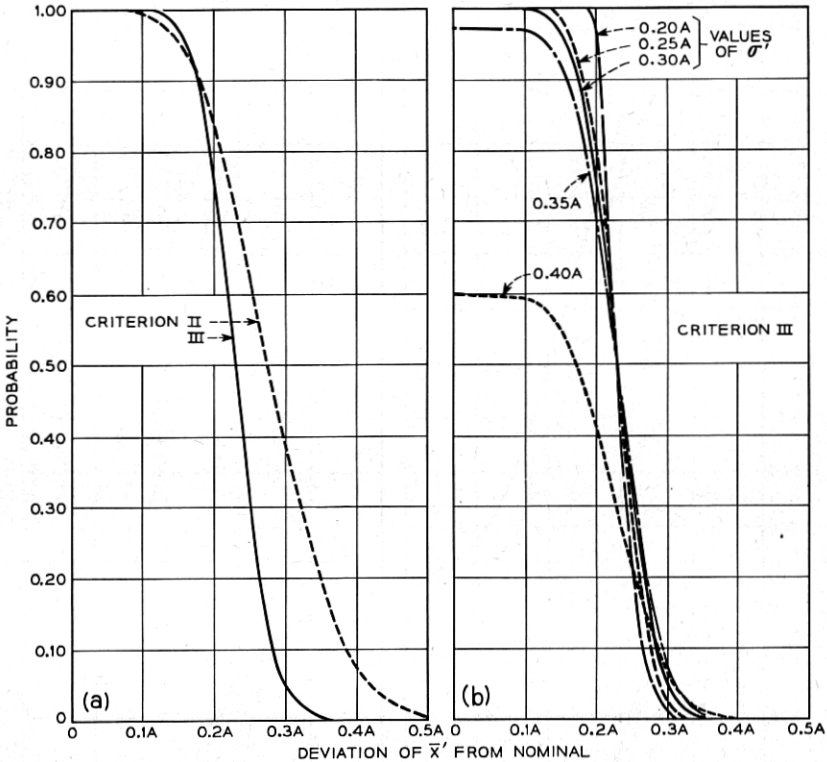

Fig. 8 — Operating characteristic curves for batch method.

standard deviation to meet its limit, each lot is judged solely on the data obtained from the sample from the lot. The OC curve for the batch method (Criterion III) is shown in Fig. 8(a) for the case where $\sigma' = 0.3A$. For comparison purposes the corresponding curve for Criterion II of the control chart method is also shown. It is noted that the batch method gives a somewhat sharper discrimination between good and bad distributions than Criterion II of the control chart method, due primarily to the use of a relatively larger sample.

Fig. 8(b) shows how the OC curve is modified for other values of process standard deviation, σ'. It is seen that the batch method is relatively less sensitive than the control chart method to changes in σ' provided the deviation from a standard value of $\sigma'' = 0.3A$ is not too great.

4.3 CHARACTERISTICS OF THE THREE-CELL METHOD

The operating characteristics of the three-cell method cannot be evaluated probability-wise in the manner given for the other two methods.

However, the manner in which the three-cell method serves as a continuous corrective influence over the distribution of delivered product can be indicated by a few diagrams, for all of which a Normal distribution with $\sigma' = 0.3A$ is assumed.

The running average of small segments of product delivered in packages of 5 is held closer to the nominal by the three-cell method than by the control chart method or the batch method, even when the process is statistically controlled at the nominal. A comparison with the control chart method is illustrated in Fig. 9. In the upper chart are shown averages of random samples of 5 units each, plotted on a control chart with A5 and PA limits. These are samples obtained experimentally from a Normal distribution whose average, $\bar{X}'$, was at the nominal for the first 20 samples (Series A). For the next 20 samples (Series B) the average was 0.15A above the nominal and for the last 20 samples (Series C) the average was 0.15A below the nominal. The same units were then classified and packaged by the three-cell method. In this experiment, as the units of each sample were classified, as many units were packaged in 1–3–1 or 0–5–0 distributions as possible. Of the first 100 units (20 groups of 5), 95 were packaged. After 200 units (40 groups of 5) were sorted into cells,

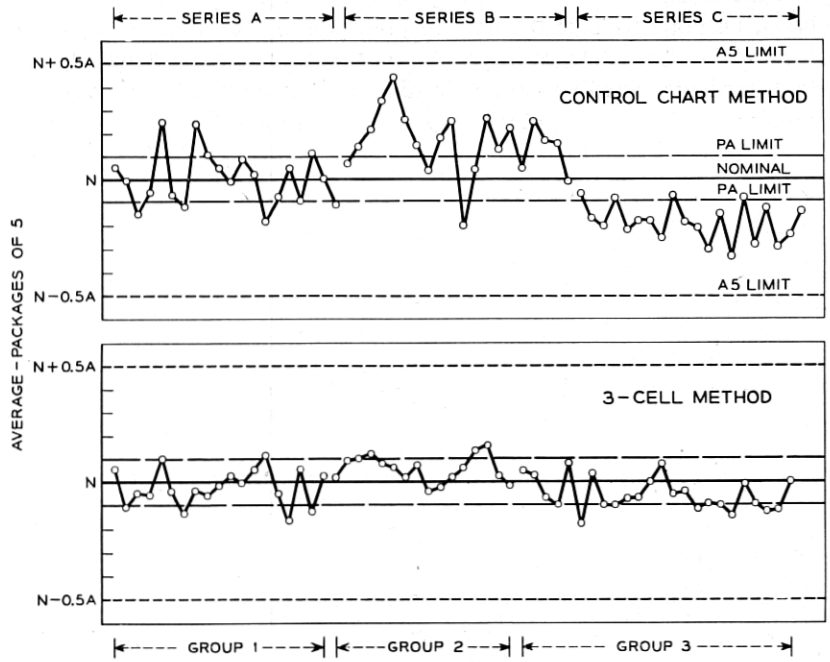

Fig. 9 — Comparison of control chart and three-cell method, averages of packages of 5.

175 were packaged. Of the 25 units not packaged, 24 were in the upper cell and 1 unit did not meet the A limits. At the end of the experiment 295 of the 300 units had been packaged. Of the 5 units not packaged, 3 remained in the upper cell, and 2 did not meet the A limits. The averages of the packages are shown in the lower chart of Fig. 9. For comparison, the PA limits are also shown on this chart. It is apparent from these two charts that the three-cell method yields packages whose averages are held closer to the nominal than are averages for packages from the control chart method.

The corrective effect of the three-cell method is further illustrated by Fig. 10, which shows the average of product packaged by the three-cell method as a function of the process average. This curve is for "long term" conditions, that is, it represents the expectancy for any given level of process average. This corrective effect is purchased at the expense of not packaging a portion of the product while the process average is not at the nominal value. However, as already noted, the unpackaged portion may be packaged with subsequent product if the process average subsequently deviates from the nominal in the opposite direction.

The percentage that can be packaged is also shown in Fig. 10 as a function of the deviation of the process average from the nominal. It should be noted that this curve also represents the expectancy for any given level of process average. Of course, continued production at a fixed level other than nominal would result in a steadily growing accumulation of unpackaged units, a situation that would call for corrective

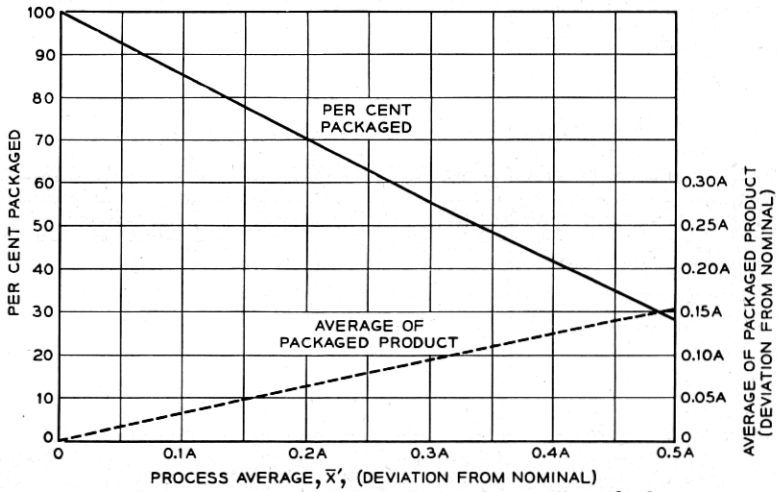

Fig. 10 — Expectancy curves for three-cell method.

action on the process. Close to 100 per cent packaging can be assured by introducing a negative bias in the process to compensate for the effect of a prior positive bias, and vice versa.

5.0 Conclusions

The L3 system's need for holding transmission performance close to the design center, both within short segments and over the full span of the transcontinental line, has called for a high degree of statistical uniformity of critical characteristics of component elements. The statistical quality control methods are imposed from the point of view of the user in the interests of the over-all economy of system design. The control procedures are designed to provide at all times a parade of suitably distributed batches of production units, and at the same time to furnish incentives for controlling manufacturing processes at the design center.

Any enterprise of this kind, involves the closest of interplay and adjustment between design and production interests. Many cases of incompatibility of design desires and production capabilities had to be cleared in the early stages of the work. Intensive process quality control work and the development of a number of ingenious processing techniques on the part of the Western Electric Company have contributed greatly to what has been achieved. Experience will undoubtedly indicate the need for some refinements or adjustments in the plan.

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