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Design Theory of Junction Transistors

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The small signal ac transmission characteristics of junction transistors are derived from physical structure and bias conditions. Effects of minority carrier flow and of depletion layer capacitances are analyzed for a one dimensional model. The ohmic spreading resistance of the base region of a three dimensional model is then approximated. Short circuit admittances representing minority carrier flow, depletion layer capacitances, and ohmic base resistance elements are then combined into an equivalent circuit. Theoretical calculations are compared to observations for two typical designs.

1.0 INTRODUCTION

1.1 General

Junction transistors have been in commercial production for nearly a year. A detailed understanding of their behavior is necessary both for the increasingly exacting requirements of modern circuit engineering and for the wise design of improved types. Design theory, by relating function to structure, can serve both these needs.

The principal object of this paper is to develop in logical fashion a design theory for junction transistors. The product of the development is an equivalent circuit, founded on device physics, which predicts the circuit characteristics of junction devices in a simple and intelligible fashion. Although attention is concentrated on small signal transmission performance, some large signal aspects are also examined.

1.2 Method and Assumptions

The usefulness of the junction transistor derives primarily from the flow of holes or electrons across two closely-spaced *p-n* junctions, one of which is biased in the forward or conducting direction while the other is biased in the reverse or non-conducting direction. Development of design theory begins quite properly with analysis of this mechanism, which is considered, for simplicity, as a problem in the flow of holes and electrons in one dimension, at right angles to the *p-n* junctions. In the analysis, it is assumed that these carriers are controlled largely by the voltages applied to the junctions and that they move principally by diffusion. The dependence of the diffusion currents on the junction voltages is reduced to a set of two terminal-pair short-circuit admittances, which form the initial and most important segment of the equivalent circuit model for the junction transistor.

Practical transistors have not only the very useful transisting mechanism mentioned above, but also passive capacitances across the charge depletion layers which separate the p and n regions at each junction. These capacitances limit the useful frequency range of transistors and must be considered in any practical theory. In the synthesis of the equivalent circuit, these capacitances are placed in parallel with the short-circuit input and output admittances which represent the flow of diffusing holes and electrons.

A further limitation on performance is imposed by the ohmic or body spreading resistance of the base region. The base current of the transistor, in flowing from the region between the emitter and collector to the base contact, develops a base contact to emitter voltage which seriously limits the frequency response. Calculation of these effects requires the assumption of flow paths for the base current. The circuit elements representing base spreading resistance effects appear in series in the base leg of the equivalent circuit.

1.3 Existing Design Theory

W. Shockley's classic paper* announcing the junction transistor also initiated the design theory. Diffusion effects for dc and low frequencies were analyzed, and formulae for depletion layer capacitances were developed. The mechanism of the frequency cutoff of the current transmission (alpha) was reported in a subsequent article†, and the effects of

* W. Shockley, The Theory of *p-n* Junctions in Semiconductors and *p-n* Junction Transistors, B.S.T.J., **28,** p. 435.

† W. Shockley, M. Sparks, and G. K. Teal, The *p-n* Junction Transistors, Phys. Rev., **83,** p. 151, July, 1951.

ohmic resistance of the base region were discussed briefly. Still more recently, the dependence of base thickness on collector voltage was used to explain output and feedback effects*. The present paper is both a consolidation and an extension of the earlier works and borrows freely from them. The diffusion current analysis of Appendix A is patterned after Shockley's.

1.4 Scope

The design theory developed here is not complete, even for small signal ac transmission. In particular, effects of large carrier emission densities are not considered, nor are the effects of non-parallel junction arrangements. Despite these omissions, it is hoped that the theory developed will be both useful and instructive to those engineers charged with transistor device and transistor circuit design.

2.0 METHODS AND ASSUMPTIONS

2.1 General

In developing the design theory, it is convenient to break the transistor down into several internal electronic functions and to consider their dependence on structure and materials individually. These functions are then fitted together and used to predict the terminal electrical characteristics. With this approach, it seems proper to describe separately the methods and assumptions used in analyzing each of the functions.

2.2 List of Symbols

The symbols listed here are used in the body of the paper. A separate list for Appendix A appears at the end of that section.

Emitter and collector currents are assumed to flow inward at the corresponding terminals, in accord with the convention usually used for transistors.

a = gradient of $(N_d\text{-}N_a)$, usually given in atoms/cm^4.

a_{ce} = short-circuit forward current transfer constant for theoretical one-dimensional transistor.

C_c = collector to base capacitance with emitter open-circuit ac.

C_{se}, C_{sc} = hole storage or diffusion capacitances at emitter and collector. These capacitances are directly related to the current trans-

* J. M. Early, Effect of Space-Charge Layer Widening in Junction Transistors, I.R.E., Proc., **40**, pp. 1401–1406, Nov., 1952.

mission cutoff frequency and may be used as an alternative characterization of that quantity.

C_{Te}, C_{Tc} = theoretical depletion layer capacitances of emitter and collector.

D, D_p, D_n = diffusion constants for minority carriers, usually given in cm^2/sec.

$f_\alpha = D/\pi w_0^2$ = current transmission or alpha cutoff frequency.

g_{ee}, g_{ce}, g_{ec}, g_{cc} = low-frequency conductance components of y's given below.

h's = set of two terminal-pair parameters, defined by Guillemin, Communication Networks, **2,** p. 137, John Wiley and Sons.

h_{11} = short circuit input impedance.

h_{21} = short circuit forward current transfer ratio.

h_{12} = open circuit feedback voltage ratio.

h_{22} = open circuit output admittance.

I_b = average or dc base current.

I_{pe}, I_{ne}, I_{pc}, I_{nc} = hole and electron components of average or dc emitter and collector currents.

I_{peo} = emitter reverse current when collector is also reverse biased.

J_e = emitter current density in amperes/cm^2.

k = Boltzmann's constant.

kT/q = average thermal energy per carrier, approximately 0.026 electron-volts at 25°C.

L, L_p, L_n = diffusion length or average distance a minority carrier will diffuse before recombining; average distance diffused in one lifetime (τ).

N_A, N_D = concentration of acceptor and donor atoms in semi-conductor, usually in atoms/cm^3.

n = concentration of electrons/cm^3.

n_i = electron concentration which would exist in the semi-conductor at thermal equilibrium if donor and acceptor concentrations were zero.

n_p = thermal equilibrium concentration of electrons in p-region.

p = hole concentration/cm^3.

p_i = hole concentration which would exist in the semi-conductor at thermal equilibrium if donor and acceptor concentrations were zero.

p_n = thermal equilibrium concentration of holes in n-region.

q = electronic charge, 1.6×10^{-19} coulombs.

q/kT = see kT/q.

r_b', r_{b1}', r_{b2}' = ohmic spreading resistances of base region, specifically, the effective base to emitter feedback resistances for diffusion currents and for collector capacitance currents.

r_1, r_2, r_3 = geometrical radii in transistor of Fig. 2(b).

T = temperature in °K.

V_c = average or dc collector to base voltage.

V_c' = electrostatic potential across collector depletion region.

V_s = electrostatic potential across emitter depletion layer at therma equilibrium (no biases applied).

v_c = small signal ac collector to base voltage.

w, w_0 = base region thickness.

w_1, w_2, w_3 = base region thicknesses in transistor of Fig. 2(b).

x_m = thickness of collector depletion region.

y_{ce}, y_{ce}, y_{ec}, y_{cc} = theoretical short circuit input, forward transfer, feedback, and output admittances for one-dimensional transistor.

α, α_0 = short-circuit emitter to collector current transfer ratio and its low-frequency value.

α^*, α_0^* = collector junction current multiplication ratio and its low-frequency value.

β, β_0 = current transport ratio across base region and its low-frequency value.

γ, γ_0 = current emission ratio at emitter and its low-frequency value.

ϵ_0 = dielectric constant of vacuum, 8.854×10^{-14} farad/cm.

k = relative dielectric constant, ϵ/ϵ_0.

ρ, ρ_b = resistivity, base region resistivity.

σ_{nc}, σ_{pc} = conductivities produced by electrons and holes in collector region.

τ, τ_n, τ_p = lifetimes of minority carriers.

μ_{bc} = constant of feedback generator used to characterize modulation of dc base spreading resistance.

$\omega = 2\pi f$ = angular frequency in radians.

$\omega_\alpha = 2\pi f_\alpha = 2D/w_0^2$ = alpha or current transmission cutoff frequency in radians.

2.3 Minority Carrier Admittances

An admittance representation of minority carrier diffusion is a way of writing the dependence of the diffusion currents on the junction potentials. To obtain this dependence analytically, the minority carrier densities on both sides of each of the two depletion layers (emitter and collector) are assumed to be exponential functions of the junction voltages. This exponential dependence is a result of the normal thermal distribution of hole and electron energies. The carrier diffusion currents are computed directly from the gradients of the minority carrier densities at the depletion layer surfaces. Since the gradients of the carrier densities are affected by many conditions besides the junction voltages, additional assumptions are necessary. Their nature and pertinence may be seen from consideration of the normal operation of a junction transistor.

The three principal regions of a junction transistor, the emitter, the base or control, and the collector, are indicated in Fig. 1. These regions are separated by transition regions in which the conductivity type changes either gradually or abruptly from p-type to n-type. Roughly coincident with these transition regions are the emitter and collector depletion layers across which the emitter and collector voltages appear

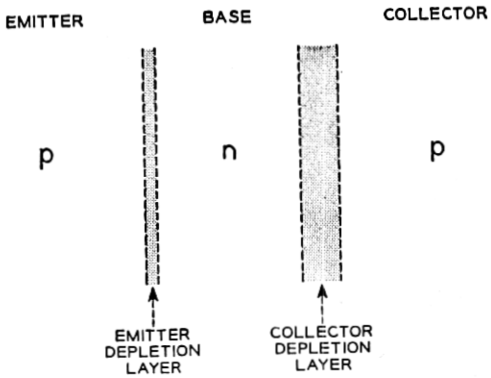

Fig. 1 — p-n-p transistor.

when the unit is biased. In normal operation for the *p-n-p* transistor shown, the emitter is biased positive with respect to the base so that a current of holes is injected into the base from the emitter. The collector is biased negative with respect to the base so that the holes diffusing across the base from the emitter are collected whenever they reach the edge of the collector depletion layer.

In the analysis each of the three major regions is assumed to have a uniform resistivity, ρ; a diffusion constant for minority carriers, D, which is a measure of the speed with which injected carriers will diffuse; and a lifetime for minority carriers, τ. This lifetime is the average time which a minority carrier remains free before recombining with a majority carrier. The minority carrier density in each region is assumed to have a thermal equilibrium value in the absence of applied potentials. The density is increased or decreased exponentially from this value by the applied potentials. The base layer is assumed to have a thickness, w, which is dependent on the collector voltage V_c. An increase of collector voltage increases the collector depletion region thickness, x_m, thus decreasing the base thickness. The rate at which base thickness changes with collector voltage is determined by the nature of the transition from base to collector. For gradual transitions, the rate of the transition is important, while for abrupt or step transitions the rate of change of base thickness with collector voltage is determined by the base region and collector region resistivities.

Determination of the diffusion currents from minority carrier density gradients requires determination of minority carrier densities everywhere in the three principal regions of Fig. 1. These are obtained by solving a continuity equation for carrier flow in each region, subject to the applied junction potentials and other assumptions described above. It must be pointed out that, in normal operation, there may be a significant flow of electrons to the emitter and from the collector in the *p-n-p* transistor of Fig. 1. Small signal ac diffusion currents are determined by assuming small signal variations of the junction voltages and discarding all but first-order ac terms from the diffusion currents.

Results of the analysis are given in Section 3.0, and the analysis appears in Appendix A.

2.4 Depletion Layer Capacitances

In a *p-n* junction with no bias potential applied, there is a tendency for holes to diffuse into the *n*-region and for electrons to diffuse into the *p*-region. This creates a slight unbalance of charge in the two regions and the resulting electrostatic potential keeps each type of carrier in its own region. The potential appears across a thin layer separating the two regions. In this depletion layer, the hole density is lower than in the p-region and the electron density is lower than in the n-region, and there is a net charge density. Acceptor and donor atoms are not neutralized by mobile charge as they are in the p- and n-regions, but instead serve to terminate the field of the electrostatic potential. Application of external potential across the junction changes the electrostatic potential, and by exposing more or fewer fixed (donor and acceptor in equal number) charges widens or narrows the depletion layer.

The passive capacitance of this region is simply that of a parallel plate condenser having a plate spacing equal to the layer thickness. Calculation of this capacitance is explained in Section 3.0, following the discussion of the minority carrier diffusion admittances.

*2.5 Base Spreading Resistance**

Implicit in the one-dimensional analyses described above is the assumption that the base region is everywhere at the same potential. Actually, since the emitter and collector currents are not equal, current must flow through the base region parallel to the junctions. Because the base region has finite, rather than zero resistivity, this current produces transverse voltage drop in the base region.

It is assumed that the most important effect of these voltage drops is the feedback produced between the base contact and the emitter junction. In consequence, each of the ohmic base resistances studied is defined as the quotient of an average voltage between base contact and emitter junction divided by the current producing it. The need for defining more than one feedback base spreading resistance results from the fact that the base current has two principal ac components. One of these is the difference between the emitter and collector minority carrier diffusion currents. The other is the collector depletion layer capacitance current. The feedback effects of these two currents on the emitter junction are the same only when the flow paths of the two currents through the base region are the same. Consequently, the representation of base resistance effects is somewhat more complicated in transistors where the flow paths differ than in those where they are identical or nearly so.

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* The majority carrier resistance of the base region for base current flow parallel to the junctions. The word "spreading" was suggested by the base contact geometry of Fig. 2(a) and readily distinguishes this resistance from the "base resistance" of the familiar tee network, which was long believed to be identical with it. It is not. See Sections 5.2 and 5.3.

Fig. 2(a) shows a structure for which the flow paths to the base contact are substantially the same for all components of the base current. Both the collector capacitance current and the diffusion loss base current enter the base region substantially uniformly over the entire area and follow the same path to the base contact. In Fig. 2(b) these two currents have quite different flowpaths and the associated feedback resistances are likewise very different. The general method of calculation is, however, the same in both cases.

Another important effect is associated with modulation of the dc voltage drop in the base region. The base current ordinarily has a dc as well as an ac component, and a dc voltage drop occurs between the base contact and the emitter junction. Since the base region thickness changes when collector voltage changes, the dc resistance of the base region is modulated by the collector voltage, producing a modulation of the dc voltage between base contact and emitter.* This effect is most easily represented by an ac voltage generator in series with the base

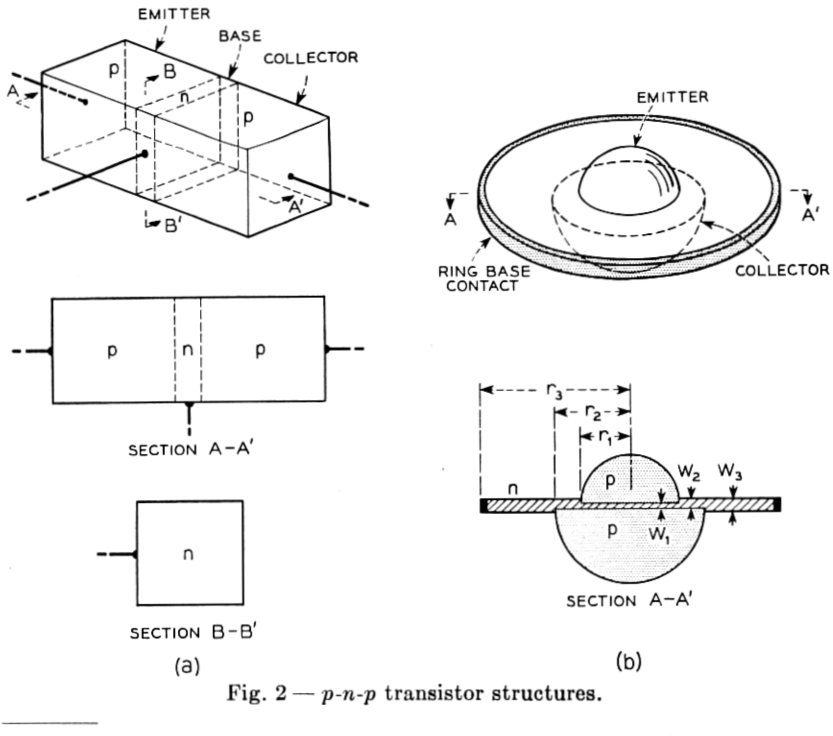

Fig. 2 — *p-n-p* transistor structures.

* This effect was first pointed out by J. N. Shive.

contact. The voltage is computed as the product of the dc base current and the modulation of the dc base resistance. This modulation can be calculated from the base region resistivity and the dependence of base thickness on collector potential.

2.5 Summary of Methods

In developing the design theory, simple physical assumptions are made concerning the behavior of the charge carriers in the semiconductor. The transistor is studied as a one-dimensional problem and the per unit area electrical characteristics of the one dimensional structure are computed. The effects of current flow within the base region parallel to the junctions are then calculated for a three-dimensional model. Finally, the equivalent circuit representations of these electronic functions are combined in structural fashion to give the terminal electrical characteristics of the junction transistor triode.

It should be noted that the base region thickness between emitter and collector is assumed uniform, and that design theory has not been extended here to cover the case of non-uniform thickness. Likewise, edge effects at the emitter and surface effects in general are neglected. These omissions were made for mathematical simplicity and are necessary omissions in a one-dimensional analysis. The place of surface leakage among the electronic functions is discussed at the end of Section 4.0. Analysis of the effects of sharp discontinuities in base layer thickness requires new solutions to the continuity equation but gradual changes in thickness can be accounted for by averaging over the active area of the transistor the short circuit admittances which are the subject of the next section.

3.0 ONE-DIMENSIONAL TRANSISTOR

3.1 General

This section deals with the small signal transmission electronics of the structure of Fig. 1. It is assumed that the emitter is biased to provide a flow of carriers into the base and that the collector is reverse biased sufficiently so that no majority carriers can diffuse out of the collector region into the base region (a reverse voltage of 0.5 volts is more than enough to prevent this). The four admittances associated with minority carrier flow and the two depletion layer capacitances are indicated in Fig. 3. In each case, the design expressions are given first in their most exact form and are progressively simplified. For convenience in discussion and comparison, current densities per unit area rather than

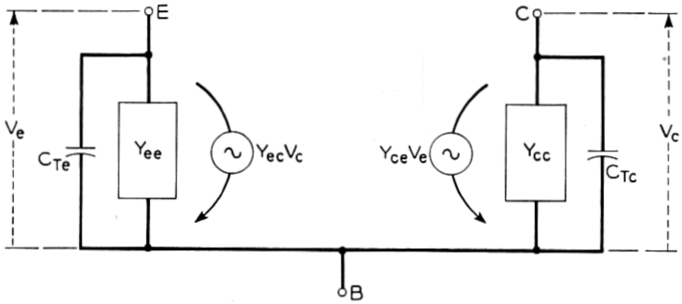

Fig. 3 — Theoretical equivalent circuit for "one-dimensional" transistor.

currents are used and the admittances and barrier capacitances are written on a per square centimeter basis. It will be noted that transverse voltage drops resulting from base spreading resistance are ignored in developing the expressions of this section.

A number of physical mechanisms are involved in the admittances for minority carrier flow. The forward current across the emitter junction rises as an exponential of the emitter to base region voltage. This is the result of the thermal energy distribution of minority carriers and is common to all thermionic emission. A natural effect of this exponential dependence is that a given change in the voltage results in a fixed per cent change in the current, thus producing an ac admittance which is proportional to the average or dc emitter current. For several reasons not all of the current which flows through the emitter junction is collected. First, some of the emitter current consists of electrons diffusing into the emitter body and of displacement current through the emitter depletion layer capacitance. These effects are expressed in the emission factor or γ, which is the ratio of the hole current injected into the base region to the total emitter current. Further, some of the injected holes recombine in the base layer. Those which are collected suffer a transit delay which results in a phase difference between emitter and collector currents. These two effects are summed up in the forward current transport factor or β, which is the ratio of the minority carrier current reaching the collector to that injected by the emitter. Finally, the current of holes entering the collector from the base gives rise to a much smaller flow of electrons from the collector to the base, thus producing a collector multiplication factor or α^*, which is the ratio of total carrier current crossing the collector junction to the hole current entering it from the base.

Although the most important minority carrier flow originates at the emitter, this flow is altered by changes in collector reverse voltage. As the collector reverse potential is increased, the base region becomes narrower because of widening of the collector depletion layer. This reduction in base thickness permits more emitter current to flow for a fixed emitter to base voltage, so that both emitter and collector currents are increased, thus producing output and feedback admittances. However, since large changes in collector potential are required to produce small changes in base thickness, relatively small changes in the junction currents are produced and the admittances are far smaller than those associated with change of emitter potential.

3.2 Diffusion Current Admittances

The short circuit two terminal-pair admittances associated with the diffusion of minority carriers in the structure of Fig. 1 are

$$y_{ee} = \frac{q}{kT}\left[(I_{pe} - I_{peo})\frac{(1 + i\omega\tau_p)^{1/2}\tanh w_0/(D_p\tau_p)^{1/2}}{\tanh[(1 + i\omega\tau_p)^{1/2}w_0/(D_p\tau_p)^{1/2}]} + I_{ne}(1 + i\omega\tau_{ne})^{1/2}\right]$$

$$y_{ce} = -\frac{q}{kT}\left[(I_{pe} - I_{peo})\left[\frac{(1 + i\omega\tau_p)^{1/2}\tanh w_0/(D_p\tau_p)^{1/2}}{\sinh[(1 + i\omega\tau_p)^{1/2}w_0/(D_p\tau_p)^{1/2}]}\right]\left[1 + \frac{\sigma_{nc}}{\sigma_{pc}}\right]\right.$$

$$y_{ec} = -\frac{\partial w}{\partial V_c}\frac{(1 + i\omega\tau_p)^{1/2}}{(D_p\tau_p)^{1/2}\sinh[(1 + i\omega\tau_p)^{1/2}w_0/(D_p\tau_p)^{1/2}]}I_{pc}$$

$$y_{cc} = -\frac{\partial w}{\partial V_c}\frac{(1 + i\omega\tau_p)^{1/2}}{(D_p\tau_p)^{1/2}\tanh[(1 + i\omega\tau_p)^{1/2}w_0/(D_p\tau_p)^{1/2}]}I_{pc}\left[1 + \frac{\sigma_{nc}}{\sigma_{pc}}\right]$$

in which I_{pe}, I_{ne}, and I_{pc} are average hole and electron currents at the emitter and collector junctions and I_{pe0} is the emitter reverse current measured with both junctions reverse biased. w_0 is the (time) average base region thickness and V_c is the average collector voltage. The depletion layer capacitances which shunt the input and output diffusion admittances, y_{ee} and y_{cc}, are discussed in Section 3.3.

Some general features of the admittances may be noted at once. The expressions are similar to those for a section of lossy transmission line, but differ significantly in the ordering of the magnitudes of the terms. Each of the admittances is proportional to a dc current, and in fact, since I_{pc} is ordinarily ninety per cent or more of I_{pe}, approximately the same dc current. This effect, which results from the exponential dependence of emitter current on emitter potential, can be related to the transmission line analogy by the argument that the increase in minority carrier density in the base region which accompanies increase of dc current lowers the characteristic impedance of the transmission line.

Finally, the last two terms, the feedback and output admittances, are smaller than the first two by approximately

$$\frac{kT}{qw_0} \frac{\partial w}{\partial V_c}$$

which is usually 10^{-3} or less, but the four form a rather symmetrical set.

The symmetry of the terms can be seen by removing the dissymmetries. The term I_{ne} $(1 + i\omega\tau_{ne})$ in y_{ee} results from diffusion of electrons into the emitter from the base and is the emission loss term which makes γ less than unity. The factor $(1 + \sigma_{nc}/\sigma_{pc})$ which appears in y_{ce} and y_{cc} is the collector multiplication factor α^* which results from the flow of electrons out of the collector body. If γ and α^* are assumed to be unity, the admittances assume a more symmetrical form. In that case $y_{ce}/y_{ee} = y_{ec}/y_{cc} = -\beta$, the base transport factor, and the forward and reverse current transmission ratios are identical. This can be seen more clearly if the hyperbolic expressions are replaced by the first two terms of their polynomial expansions, yielding

$$y_{ee} = g_{ee} \frac{(1 + i\omega/\omega_\alpha)}{(1 + i\omega/3\omega_\alpha)}$$

$$y_{ce} = \frac{-\beta_0 g_{ee}}{1 + i\omega/3\omega_\alpha}$$

$$y_{ec} = \frac{-\beta_0 g_{cc}}{1 + i\omega/3\omega_\alpha}$$

$$y_{cc} = g_{cc} \frac{(1 + i\omega/\omega_\alpha)}{1 + i\omega/3\omega_\alpha}$$

where

$$g_{ee} = qI_e/kT, \qquad g_{cc} = -\frac{1}{w}\frac{\partial w}{\partial V_c} I_{pc}\left(1 + \frac{1}{2}\frac{w_0^2}{D_p\tau_p}\right),$$

$$\omega_\alpha = \frac{2D_p}{w_0^2} \qquad \text{and} \qquad \beta_0 = 1 - \frac{1}{2}\frac{w_0^2}{D_p\tau_p}$$

is the low frequency value of the base transport factor. A lumped parameter equivalent circuit for these simplified admittances is shown in detail in Fig. 4(a).

It is apparent here that a single parameter, $\omega_\alpha = 2D_p/w_0^2$, specifies the frequency variation of all four admittances. This frequency dependence, which has been commonly measured as the alpha cutoff frequency

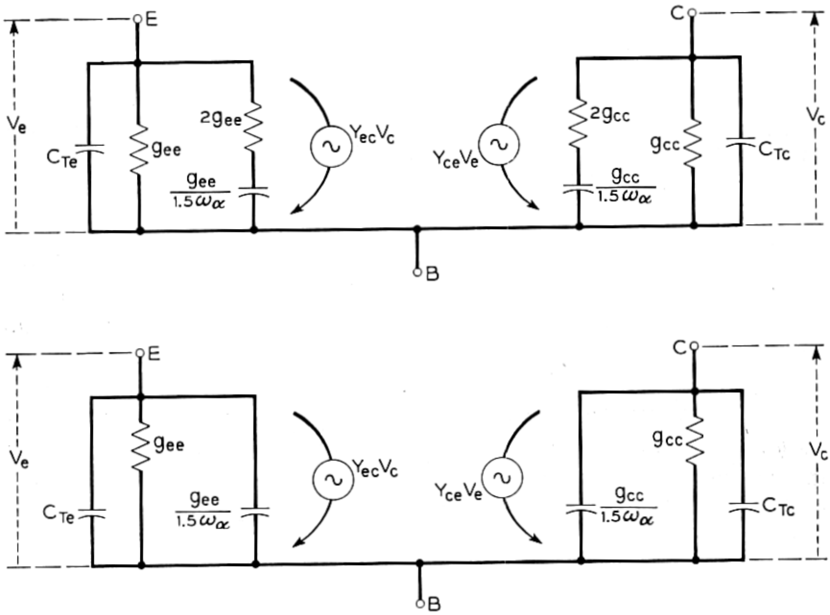

Fig. 4 — Simplified equivalent circuits for "one-dimensional" transistor.

or three db point† of the current transmission amplitude, appears in the admittances in the form of storage capacitance or as transfer delay. These result from minority carrier storage in the base region and are of the same nature as hole storage capacitances in semi-conductor diodes.

If desired, further simplication can be obtained by eliminating the effective power loss in the storage capacitances as shown in Fig. 4(b). In this arrangement, as in the more rigorous forms preceding it, the forward transfer admittance y_{ce} is a little smaller than the input admittance y_{ee} and the feedback admittance y_{ec} is not quite as large as the output admittance y_{cc}. This makes the circuit delta $(y_{ee}y_{cc} - y_{ce}y_{ec})$ greater than zero so that the structure is inherently stable when γ and α^* are unity.

3.3 Depletion Layer Capacitances

Even in the absence of applied potentials, there exists at useful *p-n* junctions a depletion layer in which the density of mobile carriers is low,

† R. L. Pritchard has pointed out that this three db point is 22 per cent larger than ω_a as defined here. See R. L. Pritchard, Frequency Variations of Current-Amplification Factor for Junction Transistors, Proc. I.R.E., **40,** p. 1476, Nov., 1952.

and across which there is a small electrostatic potential. The potential difference across the region exists primarily because of the difference in energy levels between the conduction band in which the mobile electrons exist and the valence bond band in which the mobile holes move. The potential difference depends on the densities of holes and electrons in the p and n regions, but it cannot exceed the energy gap or difference in band levels, so long as the junction is at equilibrium.

If a reverse voltage is applied to a junction, the applied voltage appears principally across the depletion region, where it strengthens the electric field by widening the barrier region so as to bring more fixed donor and acceptor charges into the field. It is obvious that the depletion region has a capacitance, since an electric field exists across it. This capacitance decreases with increase of reverse voltage, since the capacitance is charged not by bringing mobile carriers to fixed electrodes but rather by widening the region to include new fixed charges from the semiconductor regions on each side.

Both emitter and collector capacitances may be calculated easily for the principal cases of published engineering interest, the graded transition and the abrupt or step transition. For graded transitions, the depletion layer is in a region of linearly changing fixed charge density (zero at the center of the layer). The step junction has the barrier layer almost entirely in either a p or an n region of uniform fixed charged density since the fixed charged density in the other region is usually so large that the field effectively terminates at its surface.

The general expression for barrier capacitance is

$$C = \frac{\kappa\epsilon_0}{x_m}$$

where x_m is barrier thickness and the other symbols have conventional meanings. In the case of graded junctions, this becomes

$$C = \frac{\kappa\epsilon_0}{2}\left(\frac{2qa}{3\kappa\epsilon_0 V_c'}\right)^{1/3}$$

where a is the rate of change of fixed charge density in charges per cm^3 per cm. For step junctions, the relation is

$$C = \kappa\epsilon_0\left(\frac{q(N_d - N_a)}{2\kappa\epsilon_0 V_c'}\right)^{1/2}$$

where (N_d-N_a) is the fixed charge density in the low charge density region, ordinarily the base region in transistors. The potential V_c is the electrostatic potential across the depletion layer and under bias conditions is the sum of the equilibrium barrier potential and whatever part of an applied external potential is not lost in IR drops in the p and n regions. For reverse bias conditions, the sum of the equilibrium potential and the entire applied potential is almost always a satisfactory approximation, but for equilibrium and forward bias conditions, the barrier potential may be computed more easily from the minority carrier densities immediately adjacent to the depletion layer.

The equilibrium barrier potential may be calculated by

$$V_s \simeq \frac{kT}{q} \ln \frac{n_i}{n_p} \cdot \frac{p_i}{p_n}$$

in which n_n, p_p and n_i, p_i are, respectively, equilibrium and intrinsic carrier densities in the two regions. The total depletion region potential for a forward biased emitter junction may be calculated by the same expression but the hole density in the base region side of the emitter depletion layer is not the equilibrium value p_n but rather

$$p = \frac{w_0 J_e}{q D_p}$$

which is simply a modified form of the equation for diffusion current. Because of the large difference in the voltages across the barriers, emitter depletion region capacitance per unit area is ordinarily 2–20 times larger than the collector depletion region capacitance per unit area. It should not be assumed that emitter depletion region capacitance is the more important. It contributes but a small fraction of the emitter admittance, while the collector depletion layer capacitance contributes greatly to collector admittance.

3.4 Summary

A one-dimensional study of the small signal transmission properties of the junction transistor shows two terminal-pair short circuit admittances which are closely proportional to the dc bias currents. The input and output admittances of this set are shunted by depletion layer capacitances which are essentially passive circuit elements. A major feature of the diffusion current admittances is the presence of a single frequency determining factor, the alpha cutoff frequency. Comparison of the diffusion or storage capacitances with the depletion region capacitances shows that the collector depletion region capacitance is usually much larger than its storage capacitance, while at the emitter the reverse is true. This results from the fact that the output and feedback minority carrier admittances are many orders of magnitude smaller than the input and forward transfer terms. The output and feedback admittances decrease with increase of collector reverse voltage in much the same way that collector capacitance decreases, while the input and forward transfer admittances are but little affected.

This simple picture of the junction transistor is incomplete in that it ignores the important effects of majority carrier resistance in the base region. These effects are discussed in the next section.

4.0 EFFECTS OF BASE REGION RESISTANCE

4.1 General

The resistance of the base region to the flow of currents parallel to the emitter and collector junctions is important primarily because of voltage developed between the base contact and the emitter junction. Necessarily associated with this is power dissipation which is usually, however, of less importance than the feedback effect of the voltage.

The primary factors in determination of base region feedback voltages are the materials and geometry of the base region and the flow paths for the transverse currents moving through the base region to the base contact. Reduction of the base region resistance effects to equivalent circuit elements permits them to be incorporated in the equivalent circuit obtained in the previous section. This circuit is then an essentially complete model for the electronic mechanisms or functions in junction transistor triodes.

4.2 Base Region Currents

Two of the three principal components of the base current have nearly identical origin and flow paths, while the third component differs in origin and may differ greatly in flow paths to the base contact. The dc component of the base current arises principally from recombination of injected holes in the portion of the base between emitter and collector,* as does also the ac component associated with the diffusion admittances. The ac component of base current required to charge the collector barrier capacitance, however, is introduced into the base uniformly over the surface of the collector. The ac component required to charge the emitter capacitance is both small and similar in flow paths to the first components discussed.

* In some transistors, much of the recombination occurs on the exposed surface of the base region. This surface recombination may be replaced by a reduction of volume lifetime for a one-dimensional analysis. This is not exact, but is a fair approximation.

For the structure of Fig. 2(a), all current components are introduced into the base layer relatively uniformly over its area. The dc component and the $(1 - \alpha)$ or diffusion component originate within the layer volume, while the collector capacitance current is introduced from the collector side of the layer, but this difference is of little consequence. The transverse current density is largest at the base contact and diminishes monotonically to zero at the layer edge opposite the contact. The feedback voltage to the emitter junction consequently rises most rapidly close to the contact and becomes nearly constant as the opposite side of the base region is reached.

For the structure of Fig. 2(b), significant differences in flow paths are apparent. The transverse current densities are zero at the center of the unit and rise out to the edge of the emitter. Here the dc and $(1 - \alpha)$ components of current density begin to decrease because of the increasing cross-section of the base region, while the collector capacitance components continues to increase because of the additional contribution from the larger collector area. All components decrease in density beyond the collector circumference. It is apparent that two feedback resistances are required to describe the separate electronic functions in this case.

4.3 *Effective Feedback Resistances*

The resistance effects of importance are not the series resistances to the flow of the various current components through the base region to the base contact, but rather the feedback resistances defined by the quotient of the average base contact to emitter junction voltage by the current component producing the voltage. The calculations necessary to determine such resistances are detailed in Appendix II.

For the structure of Fig. 2(a), a single equivalent resistance given by

$$r_b' = f\left(\frac{\rho_b}{w}\right) \simeq \frac{\rho_b}{w}$$

is sufficient. Both $(1 - \alpha)$ and C_c current components pass through this resistance en route to the base terminal. In the structure of Fig. 2(b), the $(1 - \alpha)$ or diffusion current component has an effective feedback resistance of†

$$r_{b1}' \simeq \rho_b \left[\frac{1}{8\pi w_1} + \frac{1}{2\pi w_2} \ell n(r_2/r_1) + \frac{1}{2\pi w_3} \ell n(r_3/r_2)\right]$$

while the collector capacitance component of base current has associated

† This value of r_{b1}' is an upper bound, since the assumptions are pessimistic.

with it a resistance

$$r_{b_2}' = \rho_b \left(\frac{1}{2\pi w_3} \ln(r_3/r_2) + (r_1/r_2)^2 \left[\frac{1}{2\pi w_2} \ln \frac{r_2}{r_1} + \frac{1}{8\pi} w_1 \right] + \left[1 - \left(\frac{r_1}{r_2} \right)^2 \right] \frac{1}{4\pi w_2} \right)$$

For the proportions shown, the ratio of r_{b_1}' to r_{b_2}' is approximately 1.5 to one. Reasonable variations in the proportions can make this ratio as small as unity or as large as five.

A common feature in these resistances is the presence of the base region thickness w in the denominator. Since low values of base spreading resistance are desirable, this requirement opposes directly the primary requirement for a high alpha cutoff frequency — a thin base layer.

4.4 *Base Resistance Modulation Feedback*

As was mentioned previously, the dc voltage drop between the base contact and the emitter junction is modulated by the widening and narrowing of the collector depletion layer which alternately increases and decreases the dc base resistance. The resulting ac voltage may be represented by a voltage generator in series with the base resistance, as

$$\mu_{bc} v_c = I_b \frac{\partial r_b'}{\partial w} \frac{\partial w}{\partial V_c} v_c$$

where I_b is the dc base current, V_c is the dc collector voltage, v_c is the ac collector voltage, and $\mu_{bc} v_c$ is the feedback voltage.

For the structure of Fig. 2(a), the expression given earlier for r_b' may be differentiated to obtain the $\partial r_b'/\partial w$. The calculation of $\partial w/\partial V_c$ has been indicated in Section 3.0. The resulting expression for μ_{bc} is approximately

$$\mu_{bc} \simeq I_b \frac{r_b'}{w} \frac{\partial w}{\partial V_c}$$

Determination of this feedback effect for the structure of Fig. 2(b) is more difficult. Only those portions of the base region adjacent to the collector barrier are affected by the barrier widening, so that only the first two terms in the expression for r_b', given above enter into the calculation. The value of μ_{bc} is

$$\mu_{bc} \simeq I_b \rho_b \left(\frac{1}{8\pi w_1^2} + \frac{1}{2\pi w_2^2} \ln \frac{r_2}{r_1} \right) \frac{\partial w}{\partial V_c}$$

This feedback term cannot be reduced to an expression containing r_b', in the same simple way possible for the structure of Fig. 2(a), since the base resistance between the collector circumference and the base contact is not changed by collector depletion region thickness changes.

The details of the base contact may change the magnitude of this feedback greatly. In particular, poor placement of a bonded base contact may sometimes result in very large values of μ_{bc}. A point of general interest is that the sign of the feedback is determined directly by the direction of base current flow, while the phase is otherwise exactly that of the collector voltage.

4.5 Equivalent Circuit with Base Resistance Effects

The base resistance effects discussed above can be combined with the admittance circuits obtained in the one-dimensional study of Section 3.0 to give equivalent circuits for the three-dimensional structures of Fig. 2. The resulting representations, shown in Fig. 5, reflect the geometry of these structures since the base resistance elements are shown in a branch through which the base current must flow to reach the base terminal. All of the effects can be seen to give feedback, which is to say, coupling from the output circuit of the collector to the input loop of the emitter. This leads to complication of the electrical characteristics of the transistor, whose characteristics would otherwise be given by the electronic functions developed in the previous section.

Fig. 5(a) is an equivalent circuit which represents the small signal transmission properties of the structure of Fig. 2(a). It should be noted that a single base resistance is needed, through which all of the base current passes. The effects of surface leakage are indicated by the admittance Y_l from collector terminal to base resistance. The indicated uncertainty of placement of this leakage effect with respect to r_b' reflects the fact that the feedback resulting from the leakage depends on the position of the leakage with respect to the base contact.

The structure of Fig. 2(b) is represented by the circuit of Fig. 5(b). The separate base resistances r_{b_1}' and r_{b_2}' mirror the physical fact that the collector region is, on the average, closer to the base contact than is the emitter region. Leakage effects may be added to this circuit in the same manner as before.

Although the circuits of Fig. 5 represent primarily small signal ac transmission functions in the transistor, very similar representations may be employed for large signal functions. The principal changes occur in the diffusion admittances, which should be replaced by the diffusion currents written as functions of the barrier voltages, and in the capaci-

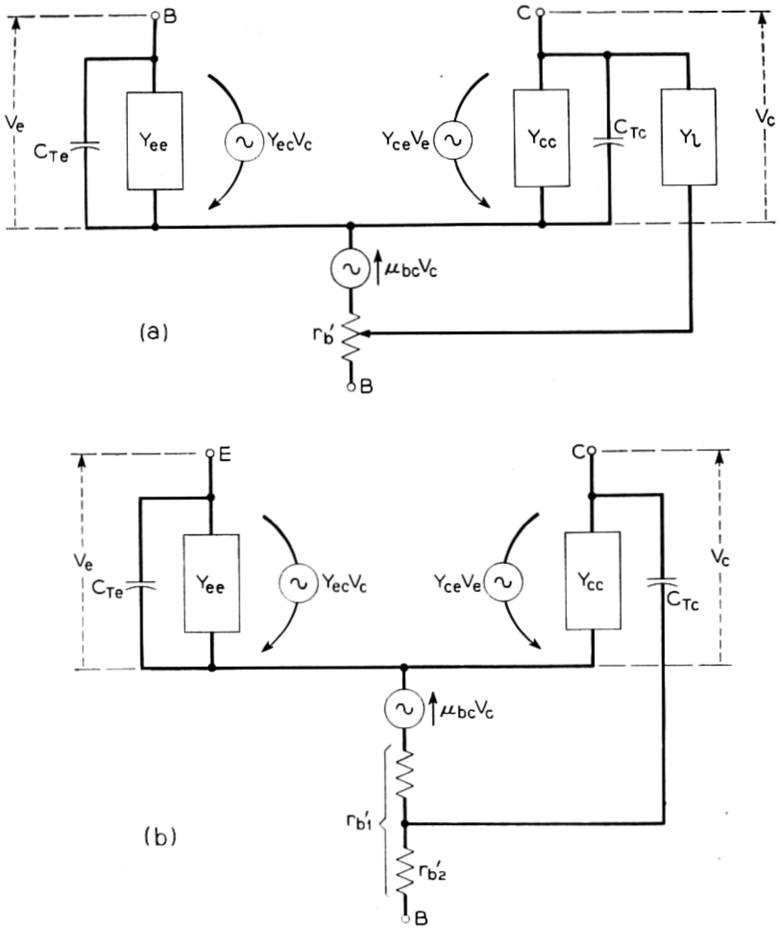

Fig. 5 — Equivalent circuits for junction transistors.

tances, whose variation with barrier potential must now be considered explicitly.

5.0 DESIGN CALCULATIONS

5.1 General

The understanding of junction transistors is now sufficiently advanced that this type of device is designable — the engineer may work from requirements to choice of structure and material. The limitations on designability are now primarily lack of adequate control over manufacturing variables together with the inherent characteristics of the semi-conductor materials available to the designer. The designability of the major small signal transmission parameters of junction devices is illustrated below by a comparison of calculated and measured characteristics of both grown and fused junction devices.

The characteristics calculated are the theoretical short circuit conductances and current transmission ratios (alphas), the current transmission three db cutoff frequencies, the collector capacitances, the base region spreading resistances, and the low-frequency values of the "h" parameters. The latter are computed because of their ease of measurement and interpretation and are compared with measured values.

5.2 n-p-n Drawn Junction Transistor

The properties assigned to this unit are believed to be reasonable, but a close check on many of them is very difficult. Base region thickness was chosen as a reasonable value for current practice. The physical structure is that of Fig. 2(a), with the bar cross-section a square 20 mils on a side. It should be noted that the assumptions made are consistent; e.g., the grading of the collector junction is consistent with the collector and base region resistivities.

The physical parameters of interest are:

V_c Collector voltage, 4.5 volts
I_e Emitter current, -1 ma
w Base thickness (not including depletion layers), 3.6×10^{-3} cm or 1.41 mil
ρ_b Base resistivity, 1 ohm-cm
τ_b Electron lifetime in base, 10 μ sec
ρ_e Emitter resistivity, 0.01 ohm-cm
τ_e Hole lifetime in emitter, 0.45 μ sec
ρ_c Collector resistivity, 1.7 ohm-cm
τ_c Hole lifetime in collector, 30 μ sec
a Concentration gradient in junctions, 3×10^{18} atoms/cm^4
A Junction areas, 0.0025 cm^2

The parameters of this unit are then

$$g_{ee} = \frac{q}{kT} I_e = \frac{10^{-3}}{0.026} = 0.039 \text{ mhos}$$

$$\gamma_0 \simeq \frac{2}{1 + \frac{\rho_e}{\rho_b} w/\sqrt{D_p\tau_e}} = \frac{1}{1 + 0.008} = 0.992$$

$$\beta_0 \simeq \left[1 - \frac{1}{2}\frac{w^2}{D_n\tau_b}\right] = 1 - 0.0072 \simeq 0.993$$

$$\text{at } 25°\text{C } \alpha_0^* \simeq \left[1 + \frac{1}{2}\frac{\sigma_{pc}}{\sigma_{nc}}\right] = 1 + 0.0003 \simeq 1.0003$$

$$70°\text{C } \alpha_0^* \simeq 1.03$$

$$25°\text{C } a_{ce} = \gamma_0\beta_0\alpha_0^* = 0.992 \times 0.993 \times 1.0003 = 0.985$$

$$70°\text{C } a_{ce} = 1.015$$

$$\text{Now } \frac{\partial w}{\partial V_c} = \frac{x_m}{6V_c} = \frac{6 \times 10^{-4}}{6 \times 6} = 1.6 \times 10^{-5} \text{ cm/volt}$$

$$g_{cc} = -\frac{1}{w}\frac{\partial w}{V_c\partial} I_c \cosh(w/L_n)$$

$$\simeq \frac{kT}{qw}\frac{\partial w}{\partial V_c} g_{ee} = 0.026 \times 1.6 \times 10^{-5} \times 0.039$$

$$= 4.5\ \mu\text{mhos}$$

The parameters which determine frequency response are

$$f_\propto = \frac{\omega_\propto}{2\pi} = \frac{D_n}{\pi w^2} = \frac{90}{\pi(3.6 \times 10^{-3})^2} = 2.45 \text{ mcps}$$

$$C_T \simeq 2400\ \mu\mu fd/\text{cm}^2 \text{ for collector}$$

$$C_T \simeq 6500\ \mu\mu fd/\text{cm}^2 \text{ for emitter}$$

so that

$$C_{T_c} = 6\ \mu\mu fd$$

$$C_{T_e} = 16\ \mu\mu fd$$

The electron storage capacitances of the emitter and collector diffusion admittances of this *n-p-n* transistor are:

$$C_{se} = \frac{g_{ee}}{1.5\omega_\alpha} = \frac{0.039}{1.5 \times 2.45 \times 2\pi \times 10^6} = 1690\ \mu\mu fd$$

$$C_{sc} = \frac{g_{cc}}{1.5\omega_\alpha} = 0.2\ \mu\mu fd$$

It is obvious that diffusion effects are most important at the emitter, while depletion layer capacitance is important at the collector.

The ohmic base spreading resistance of this unit is given (probably within a factor of two) by

$$r_b' \simeq \frac{\rho b}{w} = \frac{1.0}{3.6 \times 10^{-3}} = 278 \text{ ohms}$$

The low-frequencies values of the hybrid ("h") parameters may be calculated from the conductances, etc.. given above, but more direct methods are available and are used. The expressions required for both types of computation are given, however.

$$h_{21} \simeq -a_{ce} \simeq -0.985$$

$$h_{12} \simeq (g_{cc}/g_{ee}) + h_{22}r_b' + \mu_{bc}$$

$$\simeq \frac{kT}{qw}\frac{\partial w}{\partial V_c} \simeq 1.15 \times 10^{-4}$$

$$h_{22} \simeq \frac{g_{cc}g_{ee} - g_{ec}g_{ce}}{g_{ee}} = g_{cc}[1 - a_{ec}a_{ce}]$$

$$g_c' = h_{22} = \frac{I_e}{w}[2(1 - \beta_0) + (1 - \gamma_0)]\frac{\partial w}{\partial V_c} = 0.11\ \mu mho$$

$$h_{11} \simeq \frac{1}{g_{ee}} + (1 - a_{ce})r_b'$$

$$\simeq 26 + (0.015)278 \simeq 30.5 \text{ ohms}$$

These parameter values are in the range commonly encountered in drawn crystal units of good quality. It is obvious that this unit may be unstable at high temperatures, since the current transmission factor (alpha) is greater than unity at 70°C. The rise in alpha is the result of the very large increase in the hole density in the collector which accompanies the temperature rise.

In terms of the conventional tee network of r_e, r_b, r_c, and α, two points are interesting. First, the collector resistance $r_c \simeq 1/h_{22} \simeq 9$ megohms may seem very high. Actually, the lower values so commonly encountered are primarily the result of high leakage conductance, rather than a large electronic conductance. Second, the tee network base resistance $r_b \simeq h_{12}/h_{22} \simeq 1000$ ohms is nearly four times the high frequency feedback base resistance of 278 ohms.

5.3 p-n-p Fused Junction Transistor

The physical structure assumed for this unit is that of Fig. 2(b). The ring base contact is of particular importance. The material and structure are chosen to facilitate comparison with a specific development model for which a large amount of data is available. The emitter diameter is 15 mils and the collector diameter 30 mils. The base contact ring diameter is 40 mils.

The physical parameters of interest are:

V_c	Collector voltage, -4.5 volts
I_e	Emitter current, 1 ma
w_3	Wafer thickness, 3.5 mils
w_2	Collector to surface thickness, 2.1 mils
w_1	Collector to emitter thickness (not including depletion layers), 3.6×10^{-3} cm or 1.41 mil
ρ_b	Base resistivity, 1.5 ohm cm
τ_b	Hole lifetime in base region, 20 μ sec
ρ_e, ρ_c	Emitter and Collector resistivities unknown but very low, probably 0.001 ohm cm
τ_e, τ_c	Electron lifetime in emitter and collector unknown but very low, probably 0.1 μ sec
A_e	Emitter area, 0.00114 cm^2
A_c	Collector area, 0.00456 cm^2

The transmission parameters are then

$$g_{ee} = 0.039 \; mho$$

$$\gamma_0 \text{ probably } \geq 0.995$$

$$\beta_0 = 0.993$$

$$\alpha_0^* \simeq 1.0000$$

$$a_{ce} \geq 0.988$$

$$\frac{\partial w}{\partial V_c} = \frac{x_m}{2V_c} = \frac{2.6 \times 10^{-4}}{2 \times 4.5} = 2.89 \times 10^{-5} \text{ cm/volt}$$

$$g_{cc} \simeq 8.15 \; \mu mhos$$

The parameters which determine frequency response are:

$$f_\alpha = \frac{\omega_\alpha}{2\pi} = \frac{D_p}{\pi w_1^2} = \frac{44}{\pi(3.6 \times 10^{-3})^2} = 1.08 \; mcps$$

$$C_T = 5{,}000 \; \mu\mu fd/cm^2 \text{ for collector}$$

$$C_T \simeq 20{,}000 \; \mu\mu fd/cm^2 \text{ for emitter}$$

so that

$$C_{Tc} = 22.8 \; \mu\mu fd$$

$$C_{Te} = 22.8 \; \mu\mu fd$$

The effective hole storage capacitances of the emitter and collector are:

$$C_{se} = 3840\ \mu\mu fd$$

$$C_{sc} = 0.8\ \mu\mu fd$$

By the equations given in Section 3.0

$$r_b{'}_1 = 55 \text{ ohms}$$

$$r_b{'}_2 = 35 \text{ ohms}$$

The low frequency values of the "h" parameters are

$$h_{21} \simeq -0.988$$

$$h_{12} \simeq 2.09 \times 10^{-4}$$

$$h_{22} \simeq 0.17\ \mu mho$$

$$h_{11} \simeq 26 + (0.012)55 \simeq 26.7$$

These theoretical values are compared with observed values in Table I. The major discrepancy in h_{21} is charged to surface recombination of injected holes, which was ignored in the calculation. It should be noted that h_{22} becomes 0.45×10^{-6} *mho* if the calculated value is corrected by the ratio 0.032/.012, which is the ratio of the measured and calculated $(1 - \alpha)$'s or $(1 + h_{21})$'s. The difference between computed and measured h_{11} is the sum of a number of effects. First, the actual $(1 - \alpha)$ is greater than the computed value. Next, the junction temperature was probably greater than the assumed 25°C. Finally, the carrier injection level is high enough to modify the emitter diode properties in this direction.

The difference between calculated and observed current transmission cutoff is greater than appears from the data, since the theoretical three db response frequency is about 22 per cent higher than the "alpha

TABLE I

	Calculated	Measured
h_{21}	$-$ 0.988	$-$0.968
h_{22}	0.17×10^{-6} mho	0.48×10^{-6} mho
h_{12}	2.09×10^{-4}	1.85×10^{-4}
h_{11}	26.7 ohm	33 ohm
C_c	22.8 $\mu\mu fd$	24.7 $\mu\mu fd$
f_α	1.08 mcps	0.95 mcps
$r_b{'}_1$	55 ohm	55 ohm
$r_b{'}_2$	35 ohm	63 ohm

cutoff" frequency calculated here. The difference is believed to be the result of the fact that holes emitted around the emitter periphery have much longer transit paths than do those emitted into the region directly between the electrodes and consequently reduce significantly the current cutoff frequency.

The serious discrepancy in $r_b{}'_2$ is probably the result of the $r_b{}'_1$ calculation being very pessimistic because of neglect of peripheral emission effects and of w_2 and w_3 being somewhat smaller than the assumed values. Again it can be seen that the equivalent tee base resistance $r_b = h_{12}/h_{22}$ is 375 ohms, nearly seven times the high frequency resistance of 55 ohms.

5.4 Qualitative Comparison

As might be expected, the qualitative agreement between theory and observation is better than the quantitative. For example, C_c , h_{22} , and h_{12} vary approximately as $(V_c')^{-1/2}$ in fused junction units. Alpha cutoff frequency increases as collector reverse bias is increased—a natural result of the narrowing of the base region. The qualitative discrepancies that are found are usually associated with large experimental deviations from the assumptions of the analysis.

5.5 Review of Design Calculations

Numerical analysis of both drawn junction and fused junction transistors has shown rather good agreement of theory and experiment. It is necessary, however, to modify some of the results empirically because of lack of full understanding of some effects, such as leakage and surface recombination.

Qualitative agreement of measurements and theory is, of course, much better than the quantitative correlation. For example, the dependence of all of the parameters on emitter current and collector voltage is almost exactly that expected from theory. Some of the dependence on emitter currents involves high carrier injection level theory which has been omitted from this study.

6.0 SUMMARY

6.1 Transmission Theory

Design theory of the small signal transmission parameters of junction transistors is relatively complete.

A one-dimensional analysis of minority carrier diffusion currents in terms of short circuit admittances has been combined with a similar analysis of depletion layer capacitances and an approximate three-dimensional analysis of ohmic base region spreading resistance. The resulting equivalent circuit has characteristics in good agreement with experimental observations. In particular, collector capacitance, ohmic base region spreading resistance and the current transmission three db cutoff frequency may be computed with fair accuracy. The low frequency values of the common base hybrid parameters may also be calculated, but neglect of surface recombination and surface leakage results in serious errors in the short circuit current transmission factor h_{21} and the open circuit collector conductance h_{22}. The deviations of these two parameters from calculated values are, however, both reasonable and mutually consistent. The ohmic base layer spreading resistance, *which is the only base resistance of importance at high frequencies*, is very often much smaller than the low frequency base resistance appearing in an equivalent tee network.

Qualitative agreement of theory and measurement is excellent. The variation of all parameters with emitter current and collector voltage is within a few per cent that predicted from theory.

6.2 State of the Art

Since this paper was deliberately limited in scope, it is pertinent both to review its objectives and to point out significant omissions. The principal objective sought was presentation of small signal transmission design theory. No attempt was made to give a simple explanation of the junction transistor, relating both its large signal and its transmission characteristics to simple physical assumptions. While the design theory presented consolidates in one place some already published information, much remains to be done in assembling and integrating such knowledge from its present widely scattered locations.

In addition there exists a more detailed understanding of junction transistor characteristics than can be found in the literature. For example, units are found occasionally with negative h_{12}. This is a result of an easily modulated high resistance between base region and base contact (high μ_{bc}). Publication of such information can reduce by a few db the amount of head-scratching done by production engineers.

Other phenomena for which explanations have been developed are surface recombination and high carrier injection level effects. Despite this, much work remains to be done.

ACKNOWLEDGEMENTS

The general point of view taken here has been much influenced by discussions with J. A. Morton. Comments and criticisms by R. M. Ryder have been particularly helpful in the preparation of this material.

Appendix A

1.0 GENERAL

This study is an extension of Shockley's analysis of the junction transistor to include high-frequency effects and the voltage dependence of base-layer thickness. Shockley's paper* and the later paper by Shockley, Sparks, and Teal† contain the following of interest here:

(a) analysis of the dc steady state of a junction transistor*;
(b) analysis of the low-frequency small-signal parameters r_e, α, C_c*†; and
(c) analysis of frequency dependence of the transport factor β†.

In addition to repeating the above, this study gives these new results:

(a) analysis of steady-state small-signal ac operation [dc biases present]; and
(b) the small-signal ac short-circuit admittances y_{ee}, y_{ce}, y_{ec}, and y_{cc}.

1.1 ASSUMPTIONS

Semiconductor.

The *p-n-p* type structure assumed is shown in Fig. 6. The emitter, base, and collector regions may each be characterized by a resistivity and a minority carrier diffusion length. The emitter, collector, and base contacts have no effect on the currents which flow at the junctions. Injected carriers pass through the base layer by diffusion and through

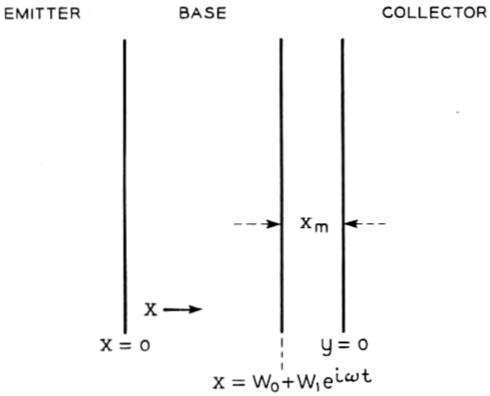

Fig. 6 — *p-n-p* junction transistor.

* W. Shockley, The Theory of *p-n* Junctions in Semiconductors and *p-n* Junction Transistors, B.S.T.J., **28**, p. 435.

† W. Shockley, M. Sparks, and G. K. Teal, The *p-n* Junction Transistors, Phys. Rev., **83**, p. 151, July, 1951.

the barrier layers* by drift. Base-layer thickness w depends on collector potential V_c through variation of the collector barrier thickness x_m. Variations of emitter barrier thickness are unimportant. The junctions are parallel.

Currents and Potentials

The currents and potentials studied are those at the collector and emitter barriers. Unless otherwise specified, "potential" implies difference in majority carrier Fermi levels, i.e., externally applied potential, rather than difference in electrostatic potential. The collector reverse potential is assumed to be many multiples of kT/q(e.g. $>$ 0.5 volts), so that the classical p-n diode reverse conductance may be neglected. At 0.5 volt, this is of the order of 10^{-11} mhos. It decreases in magnitude one decade per sixty millivolts of bias potential.

Base Resistance

Majority carrier resistance in the base layer is not considered here.

Other Assumptions

Surface effects are excluded from consideration. In addition, several mathematical approximations of little physical consequence appear in the text as needed.

1.2 METHOD

The procedure employed is substantially that used by Shockley with some additions. First, minority carrier concentrations on both sides of each barrier are related to the barrier potentials.

The dc minority carrier distribution in each of the three transistor regions is then computed from these boundary conditions with the aid of the continuity equation.

Next, small-signal ac perturbations of the barrier potentials and of the minority carrier densities at the barriers are used in the same way to find ac distributions of the minority carriers. The effects of voltage dependence of base-layer thickness are found by means of a small-signal ac perturbation of the position of the collector side of the base layer. The resulting ac distribution of minority carriers is computed as before.

Finally, dc and ac currents at emitter and collector barriers are com-

* I.e., charge depletion layers.

puted from the gradients of the minority carrier densities at the barriers.

In the ac analysis, the forward rotating time function $e^{i\omega t}$ is used. In general, the first-order solution for small signals is obtained by assuming that the disturbance associated with $e^{i\omega t}$ is small, by neglecting its powers and harmonics, and by using first-order expansions, e.g.,

$$e^{\frac{w_0}{L} + \frac{w_1}{L} e^{i\omega t}} \approx \left(1 + \frac{w_1}{L} e^{i\omega t}\right) e^{\frac{w_0}{L}}$$

The ac magnitudes such as w_1, p_{e1}, n_{c1}, represent complex phasors of the form $ae^{j\phi}$. A list of symbols is given in Section 1.5.

1.3 ANALYSIS

In the base layer at $x = 0$,

$$p = p_{e0} + p_{e1}e^{i\omega t} = p_n e^{qV_e/kT} \tag{1a}$$

at $x = w_0$,

$$p = p_{c0} + p_{c1}e^{i\omega t} = p_n e^{qV_c/kT} \tag{1b}$$

in which

$$V_e = V_{e0} + V_{e1}e^{i\omega t}$$

$$V_c = V_{c0} + V_{c1}e^{i\omega t}$$

and $V_{e1} \ll V_{e0}$ and $V_{c1} \ll V_{c0}$ so that

$$p_{e0} \approx p_n e^{qV_{e0}/kT} p_{e1} \approx p_{e0} \frac{q}{kT} V_{e1}$$

$$p_{co} \approx p_n e^{qV_{c0}/L} p_{c1} \approx p_{co} \frac{q}{kT} V_{c1}$$

P_{e0}, P_{c_0}, V_{e_0}, and V_{c0} are average values while p_{e1}, p_{c1}, V_{e1}, and V_{c1} are ac phasors.

The continuity equation for holes for one-dimensional flow is:

$$D_p \frac{\partial^2 p}{\partial x^2} - \left(\frac{p_n - p}{\tau}\right) = \frac{\partial p}{\partial t} \tag{2}$$

in which τ is hole lifetime. A solution to equation (8–18) is:

$$p = p_n + Ae^{x/L} + Be^{-x/L} + Ce^{sx/L+i\omega t} + De^{-sx/L+i\omega t} \tag{3}$$

where $L = \sqrt{D\tau}$ and $s = (1 + i\omega\tau)^{1/2}$.

Application of the boundary values of equation (1a) and (1b) to equation (3) gives hole density in the base layer:

$$
\begin{aligned}
p_0(t,x) = p_n &+ \left[\frac{(p_{c0} - p_n) - (p_{e0} - p_n)e^{-w_0/L}}{2 \sinh (w_0/L)}\right]e^{x/L} \\
&- \left[\frac{p_{c0} - p_n) - (p_{e0} - p_n)e^{w_0/L}}{2 \sinh (w_0/L)}\right]e^{-x/L} \\
&+ \left[\frac{p_{c1} - p_{e1}e^{-sw_0/L}}{2 \sinh (sw_0/L)}\right] e^{sx/L+i\omega t} \\
&- \left[\frac{p_{c1} - p_{e1}e^{sw_0/L}}{2 \sinh (sw_0/L)}\right] e^{-sx/L+i\omega t}
\end{aligned}
\tag{4}
$$

Since up to this point w has been assumed constant $[w = w_0]$, equation (4) does not include effects of voltage dependence of base layer thickness. To introduce these, a new set of boundary conditions is used:
at $x = 0$,

$$p = p_{e0} + p_{e1}e^{i\omega t} \tag{5a}$$

and at $x = w_0 + w_1 e^{i\omega t}$,

$$p = p_{c0} + p_{c1}e^{i\omega t} \tag{5b}$$

in which $w_1 \leqq w_0$ and is a phasor.

$$w_1 = \frac{\partial w}{\partial V_c} V_{c1}$$

It can be seen that conditions are as before except that the collector side of the base layer swings about position w_o at angular frequency ω.

A solution of equation (31) with conditions (5a) and (5b) is given by

$$p_1(t,x) = p_0(t,x) + \tilde{p}(t,x) \tag{6}$$

in which $p_0(t,x)$ is given by equation (4) and $\tilde{p}(t,x)$ is the perturbation associated with $w_1e^{i\omega t}$.

If equations (5a) and (5b) are rewritten in terms of $\tilde{p}(t,x)$, they become, using first order expansions:

$$\tilde{p}(t,o) = 0 \tag{7a}$$

$$
\begin{aligned}
\tilde{p}(t,w_0 + w_1e^{i\omega t}) = [(p_{e0} - p_n) \operatorname{csch} (w_0/L) \\
- (p_{c0} - p_n) \coth (w_0/L)] \frac{w_1}{L} e^{i\omega t}
\end{aligned}
\tag{7b}
$$

Since $\tilde{p}(t,x)$ is an ac solution of the continuity equation (2), it has the form:

$$\tilde{p}(t,x) = Ee^{sx/L+i\omega t} + Fe^{-sx/L+i\omega t} \tag{8}$$

Use of equations (7a) and (7b) leads to:

$$p(t,x) = \frac{\sinh(sx/L)}{\sinh(sw_o/L)} [(p_{e0} - p_n)\,\mathrm{csch}\,(w_0/L) - (p_{c0} - p_n)\coth(w_0/L)] \frac{w_1}{L} e^{i\omega t} \tag{9}$$

The complete solution for hole density in the base layer is:

$$\begin{aligned} p_1(t,x) = p_n &+ \left[\frac{(p_{c0} - p_n) - (p_{e0} - p_n)e^{-w_0/L}}{2\sinh(w_0/L)}\right] e^{x/L} \\ &- \left[\frac{(p_{c0} - p_n) - (p_{e0} - p_n)e^{w_0/L}}{2\sinh(w_0/L)}\right] e^{-x/L} \\ &+ \left[\frac{p_{cl} - p_{e1}e^{-sw_0/L}}{2\sinh(sw_0/L)}\right] e^{sx/L + i\omega t} \\ &- \left[\frac{p_{cl} - p_{e1}e^{sw_0/L}}{2\sinh(sw_0/L)}\right] e^{-sx/L + i\omega t} \\ &+ \frac{w_1}{L} e^{i\omega t} \frac{\sinh(sx/L)}{\sinh(sw_0/L)} [(p_{e0} - p_n)\,\mathrm{csch}\,(w_0/L) - (p_{c0} - p_n)\coth(w_0/L)] \end{aligned} \tag{10}$$

The hole-current density in the base layer is found from equation (10) by the use of the equation for diffusion current

$$I_p = -qD_p \frac{dp}{dx} \tag{11}$$

which yields

$$\begin{aligned} I_p = -q\frac{Dp}{L}\Bigg((p_{c0} - p_n)&\frac{\cosh(x/L)}{\sinh(w_0/L)} - (p_{e0} - p_n) \cdot \frac{\cosh[(x - w_0)/L]}{\sinh(w_0/L)} + sp_{c1}\frac{\cosh(sx/L)}{\sinh(sw_0/L)} e^{i\omega t} \\ &- sp_{e1}\frac{\cosh[(sx - sw_0)/L]}{\sinh(sw_0/L)} e^{i\omega t} + s\frac{w_1}{L} e^{i\omega t}\frac{\cosh(sx/L)}{\sinh(sw_0/L)} \\ &[(p_{e0} - p_n)\,\mathrm{csch}\,(w_0/L) - (p_{c0} - p_n)\coth(w_0/L)] \end{aligned}$$

Hole-current densities at emitter and collector may be found by substitution of $x = 0$ and $x = w_0$ respectively. The first two terms in equation (12) give dc current components* which may be attributed to

* With some labor, these terms may be converted to Shockley's equation (5.6).

collector and emitter potentials in that order, while the last three are ac terms resulting from collector potential, emitter potential, and base layer thickness variation respectively.

The hole-current densities may be related to the ac potentials by use by use of the approximations given at the beginning of paragraph 1.3.

$$p_{e1} \approx p_{e0} \frac{q}{kT} V_{e1} \qquad\qquad p_{c1} \approx p_{c0} \frac{q}{kT} V_{c1}$$

and the relation

$$w_1 = \frac{\partial w}{\partial V_c} V_{c1}$$

However, the average collector potential V_{c0} is negative and many times kT/q so that $p_{c0} \simeq 0$ and $pc_1 = 0$. The ac emitter hole-current density is therefore

$$\begin{aligned} I_{pe1}e^{i\omega t} = \frac{qDp}{L} \Bigg(& V_{e1} \frac{q}{kT} sp_{e0} \coth (sw_0/L) \\ & - \frac{V_{c1}}{L} \frac{\partial w}{\partial V_c} s \operatorname{csch} (sw_0/L)[(p_{e1} - p_n) \operatorname{csch} (w_0/L) \\ & - (q_{c0} - p_n) \coth (w_0/L)]\Bigg) e^{i\omega t} \end{aligned} \tag{13a}$$

If $p_{e0} > p_n$, equations (13a) and (13b) becomes very closely

$$I_{pe1} = \left[I_{pe0} \frac{q}{kT} \frac{s \tanh (w_0/L)}{\tanh (sw_0/L)} V_{e1} + I_{pc0} \frac{\partial w}{\partial V_c} \cdot \frac{sV_{c1}}{L \sinh (sw_0/L)} \right] \tag{13b}$$

In equation (13b), I_{pe0} and I_{pc0} are average emitter and collector hole-current densities. The entire coefficients of V_{e1} and V_{c1} in equation (13b) are the input and the feedback short-circuit admittances associated with hole flow in the transistor.

Similarly, forward transfer and output short-circuit admittances associated with hole flow may be found from equation (12) by stubstitution of $x = w_0$ and use of the approximation $p_1 \simeq p_1 - p_n$, i.e., $p_1 \gg p_n$. In calculating collector current, the sign of equation (12) must be reversed, since equation (12) gives current flow in the x-direction while collector current is assumed to flow in the negative x-direction. The admittances are given in the summary at the end.

Next, there are admittances associated with electron flow in the *p-n-p* transistor. Flow of electrons from base to emitter gives rise to an input admittance term, while electrons flowing from collector to base give rise to output and forward transfer admittance terms. An outline of the derivation of these terms follows. The terms are given in the summary.

For electrons in the emitter:

at $x = 0$,

$$n_e = n_{e0} + n_{e1}e^{i\omega t} = n_p e^{qV_e/kt}$$

at $x = -\infty$,

$$n_e = n_p$$

in which, again, $V_e = V_{e0} + V_{e1}e^{i\omega t}$ and first-order expansions are used. $x = -\infty$, $n_e = n_p$, is chosen as a boundary condition in order to eliminate effects of the emitter contact. Solution of the continuity equation results in:

$$n_{0e}(t,x) = n_p + (n_{0e} - n_p)\, e^{x/L} + n_{01}e^{-sx/L+i\omega t} \tag{14}$$

in which the τ is $s = (1 + i\omega\tau)^{1/2}$ now implies electron lifetime in the emitter body. Electron diffusion current at the emitter is computed by means of the equation corresponding to equation (11), giving

$$I_{ne} = \frac{qD_n}{L_n}\, ((n_{e0} - n_p) + n_{e1}e^{i\omega t} \tag{15}$$

The ac admittance associated with the last term appears in the summary, Section 1.4.

The ac electron current from the collector is not a diffusion current, but rather a drift current resulting from the hole current flowing in the collector body. Since the ac electron current is directly proportional to the ac hole current in the collector, the result is an effective multiplication of the output and forward transfer admittances associated with hole current in the collector body.*

For electrons in the collector the boundary conditions are:

at $y = 0$,

$$n = n_p e^{qV_c/kT} = 0 \tag{16a}$$

at $y = \infty$,

$$n = n_p \tag{16b}$$

The condition $n = 0$ at the edge of the barrier region results from V_c

* Space-charge layer widening effects are neglected since they are usually very small and are difficult to analyze.

being negative and many times kT/q. The distribution of electrons in the collector is governed by conditions (16a) and (16b) and the modified continuity equation* developed by W. van Roosbroeck.

$$\frac{\partial n}{\partial t} = -\frac{n - n_p}{\tau} + \frac{\mu_n E_a}{M_0}\frac{\partial n}{\partial y} + D_0 \frac{\partial^2 n}{\partial y^2} \tag{17}$$

in which E_a is the electric field which would be associated with the total current if $n \equiv n_p$. $M_0 = (p_p + bn_0)/(p_p + n_p)$ and $D_0 = (p_p + n_p)/(p_p/D_n + n_p/D_p)$.

If n is assumed to be given by $n(t,x) = n_0(y) + n_1(y)e^{i\omega t}$ and E is assumed to be

$$E_a(t,x) = E_0 + E_1 e^{i\omega t} = \rho_c I_{c0} + \rho_{pc} I_{pc1} e^{i\omega t}$$

in which I_{c0} is average collector current and I_{pc1} is the ac hole current of the collector, equation (17) may be reduced to two equations

$$0 = -\left(\frac{n_0 - n_p}{D_0 \tau}\right) + \frac{\mu_n E_0}{D_0 M_0}\frac{\partial n_0}{\partial y} + \frac{\partial^2 n_0}{\partial y^2} \tag{18a}$$

$$-\frac{\mu_n E_1}{M_0 D_0}\frac{\partial n_0}{\partial y} = -\frac{1 + i\omega\tau}{D_0 \tau} n_1 + \frac{\mu_n E_0}{D_0 M_0}\frac{\partial n_1}{\partial y} + \frac{\partial^2 n_1}{\partial y^2} \tag{18b}$$

The solution of equation (18a) under condtiions (16a) and (16b) is

$$n_0 = n_p(1 - e^{m_2 y}) \tag{19}$$

where

$$m_2 = -\frac{\mu_n E_0}{2D_0 M_0} - \sqrt{\left(\frac{\mu_n E_0}{2D_0 M_0}\right)^2 + \frac{1}{D_0 \tau}}$$

Inspection of equation (19) shows that

$$\frac{\partial n_0}{\partial y} = -m_2 n_p e^{m_2 y} \tag{20}$$

It is then apparent that n_1 must be of the form

$$n_1 = n_p e^{m_2 y} f(y) \tag{21}$$

Substitution of equation (2) and equation (21) into (18b) leads to

$$\frac{\partial^2 f}{\partial y^2} + \left(m_2 + \frac{\mu_n E_0}{M_0 D_0}\right)\frac{\partial f}{\partial y} - \frac{i\omega}{D_0} f = \frac{n_p m_2 \mu_n e_2}{M_0 D_0} \tag{22}$$

A general solution of equation (22) is

$$f(y) = \frac{n_p m_2 \mu_n E_1}{-i\omega M_0} + Be^{r_1 y} + Ce^{r_2 y} \tag{23}$$

* W. van Roosbroeck; private communication.

where

$$r_1 \,.\, r_2 = -\frac{1}{2}\left(m_2 + \frac{\mu_n E_0}{D_0 M_0}\right) \pm \sqrt{\left[\frac{1}{2}\left(m_2 + \frac{\mu_n E_0}{D_0 M_0}\right)\right]^2 + \frac{i\omega}{D_0}}$$

It may be shown that the boundary conditions on $f(y)$ are:

$$\text{at } y = 0, \quad f = 0; \qquad \text{at } y = \infty, \quad f = 0.$$

Application of these values to equation (23) results in

$$f(y) = \frac{n_p m_2 \mu_n E_1}{i\omega M_0} [1 - e^{r_2 y}] \tag{24}$$

in which r_2 is obtained using the negative square root.

The electron density in the collector is now given by

$$n = n_p(1 - e^{m_2 y}) + \frac{n_p m_2 \mu_n E_1}{i\omega M_0} [1 - e^{r_2 y}] e^{m_2 y} e^{i\omega t} \tag{25}$$

The electron current is given by

$$I_n = n_p q D_n \left[-m_2 e^{m_2 y} + e^{i\omega t} \frac{\mu_n E_1 m_2}{i\omega M_0} (m_2 - m_2 e^{r_2 y} - r_2 e^{r_2 y}) e^{m_2 y} \right]. \tag{26}$$

At the collector junction, the ac component reduces to

$$I_{nc1} = \frac{n_p q D_n \mu_n m_2 r_2 E_1}{i\omega M_0} \tag{27}$$

Now, since $E_1 = \rho_{pc} I_{pc1}$, the collector multiplication factor $(I_{pc1} + I_{nc1})/I_{pc1}$

is

$$\alpha^* = 1 + \frac{n_p q D_n \mu_n m_2 r_2}{i\omega M_0} \rho_{pc} \tag{28a}$$

$$\alpha^* = 1 + \frac{\sigma_{nc}}{\sigma_{pc}} \frac{D_n m_2 r_2}{i\omega M_0} \tag{28b}$$

The effect of collector multiplication as given in equations (28a) and (28b) is included in the general admittance expressions given later.

Finally, no mention has been made of the admittances associated with barrier capacitances. Since the currents which charge these are majority carrier currents, there are no input-output interactions except those associated with majority carrier resistance of the base layer [an effect not analyzed in this study]. These capacitances add directly to input and output admittances. Shockley* gives methods for calculating these capacitances.

* Loc. cit., vol. 28, page 435.

1.4 SUMMARY

In paragraph 1.3, derivations were given or outlined for each of the four small-signal short-circuit admittances associated with the hole flow, electron flow, and barrier capacitances in junction transistors. The terms appear in that order in the expressions which follow.

$$y_{ee} = \left[\frac{qp_nD_ps_p}{L_p} \coth (s_pw_0/L_p) + qn_pD_ns_{ne}/L_{ne}\right] \frac{q}{kT} e^{qV_{e0}/kT} + i\omega C_{Te} \tag{29}$$

$$y_{ce} = -\left[q\,\frac{p_nD_ps_p}{L_p} \operatorname{csch} (s_pw_0/L_0)\right] \frac{q}{kT} e^{qV_{e0}/kT} \left(1 + \frac{\sigma_{nc}}{\sigma_{pc}}\frac{D_nm_2r_2}{i\omega M_0}\right) \tag{30}$$

$$y_{ec} = -\frac{\partial w}{\partial V_c}\frac{s_p}{L_p \sinh (s_pw_0/L_p)}\frac{qp_nD_p}{L_p}[(e^{qV_{e0}/kT} - 1)\cdot \operatorname{csch} (w_0/L_p) + \coth (w_0/L_p)] \tag{31}$$

$$y_{cc} = \left(\frac{\partial w}{\partial V_c}\frac{s_p}{L_p \tanh (s_pw_0/L_p)}\frac{qp_nD_p}{L_p}[(e^{qV_{e0}/kT} - 1)\cdot \operatorname{csch} (w_0/L_p) + \coth (w_0/L_p)] + i\omega C_{Te}\right)\left(1 + \frac{\sigma_{nc}D_nm_2r_2}{\sigma_{pc}i\omega M_0}\right) \tag{32}$$

in which all symbols are defined in Section 1.5. It should be noted that collector multiplication operates on the current to the barrier capacitance since the latter current is a hole current in the collector body.

The term $\partial w/\partial V_c$ is the same for both *p-n-p* and *n-p-n* structures. It is: for step junctions

$$\frac{\partial w}{\partial V_c} = -\frac{x_m}{2V_c} \tag{33}$$

and for graded junctions

$$\frac{\partial w}{\partial V_c} = -\frac{x_m}{6V_c} \tag{34}$$

V_c in equations (33) and (34) means dc electrostatic potential difference across the collector barrier.

Equations (29) through (32) may be manipulated into many forms. One of these sets which may be employed as a starting point for the approximate forms given in the body of this chapter is:

$$y_{ee} = \frac{q}{kT}\left[I_{pe0}\frac{s_p \tanh (w_0/L_p)}{\tanh (s_pw_0/L_p} + I_{ne}s_{se}\right] + i\omega C_{Te} \tag{35}$$

$$y_{ce} = -\frac{q}{kT} I_{pe0} \frac{s_p \tanh (w_0/L_p)}{\sinh (s_p w_0/L_p)} \left[1 + \frac{\sigma_{nc}}{\sigma_{pc}}\right]^* \tag{36}$$

$$x_{cc} = \frac{\partial w}{\partial V_c} \frac{s_p}{L_p \sinh (s_p w_0/L_p)} I_{pc0} \tag{37}$$

$$y_{cc} = \left[\frac{-\partial w}{\partial V_c} \frac{s_p}{L_p \tanh (s_p w_0/L_p)} I_{pc0} + i\omega C_{Te}\right] \left[1 + \frac{\sigma_{nc}}{\sigma_{pc}}\right]^* \tag{38}$$

The change in signs which occurs in going from equations (31) and (32) to equations (37) and (38) takes place because the current replaced by I_{pc} had the opposite assigned positive sense.

1.5 SYMBOLS USED IN THE APPENDIX

C_c, C_{Tc} = collector barrier capacitance.
C_{Te} = emitter barrier capacitance.
D_n, D_p = diffusion constants for electrons and holes
$D_0 = (p_p + n_p)/(p_p/D_n + n_p/D_p)$
$E_a = i\rho$ = electric field associated with current at thermal equilibrium carrier densities.
E_1, E_0 = ac and dc components of E_a
I_{ne}, I_{ne0}, I_{ne1}, I_{nc}, I_{nc0}, I_{nc1} = total, average, and ac emitter and collector electron currents
I_{pe}, I_{pe0}, I_{pe1}, I_{pc}, I_{pc0}, I_{pc1} = total, average, and ac emitter and collector hole currents
kT/q = thermal energy of carriers = 0.026 electron-volt
L_p, L_n = diffusion lengths for holes and electrons
$m_2 = -\frac{\mu_n E_0}{2D_0 M_0} - \sqrt{\left(\frac{\mu_n E_0}{2D_0 M_0}\right)^2 + \frac{1}{D_0 \tau}}$ decay constant for average electron density in the collector
$M_0 = (p_p - bn_p)/(p_p = n_p)$
n_p, n_n = thermal equilibrium electron densities in p and n regions
n_{e0}, n_{e1} = dc and ac components of electron density at emitter junction
p_n, p_p = thermal equilibrium hole densities in n and p regions
p_{e0}, p_{e1}, p_{c0}, p_{c1} = dc and ac components of hole density at emitter and collector junctions
q = electronic charge, 1.6×10^{-19} coulombs
r_e = ac emitter resistance

$$r_1, r_2 = -\frac{1}{2}\left(m_2 + \frac{\mu_n E_0}{D_0 M_0}\right) \pm \sqrt{\left[\frac{1}{2}\left(m_2 + \frac{\mu_n E_0}{D_0 M_0}\right)\right]^2 + \frac{i\omega}{D_0}}$$

* The factor $a^* = (1 + \sigma_c/\sigma_{pc})$ is current dependent. At small average collector currents, it is $(1 + \sigma_{nc}/2\sigma_{pc})$ and rises to $(1 + \sigma_{nc}/\sigma_{pc})$ at high current densities.

$s = (1 + i\omega\tau)$
T = temperature in degrees Kelvin (absolute temperature)
V_e, V_{e0}, V_{e1}, V_c, V_{c0}, V_{c1} = total, average, and ac emitter and collector potentials.
V_c = average collector potential
w, w_0, w_1 = total, average, and ac variation of base-layer thickness
x = distance from emitter barrier
x_m = thickness of collector barrier layer
y = distance from collector side of collector barrier layer
y_{ee}, y_{ce}, y_{ec}, y_{cc} = short-circuit ac admittances
α = current amplication factor
β = base-layer transport factor
ρ_c = resistivity of collector region
ρ_{pc} = resistivity for holes in collector region
σ_{pc}, σ_{nc} = conductivities for holes and electrons in collector region
τ = minority carrier lifetime
μ_n = electron mobility
μ_p = hole mobility
ω = angular frequency in radians

Appendix B

BASE LAYER SPREADING RESISTANCE

Significance

The bulk resistance of the base layer or base layer spreading resistance (r_b') is important because base current passing through r_b' produces a base contact to emitter junction negative feedback voltage $i_b r_b'$. Reduction of r_b' is an important objective in improvement of junction units.

Types

Since feedback voltage to the emitter junction is produced by two separately measurable base current components having very different flow paths, two separately measurable base layer spreading resistances ($r_b'{}_1$ and $r_b'{}_2$) may be defined.

The current $(1\text{-}\alpha)i_e$ originates in the base layer between the emitter and collector junctions and flows radially through the base layer to the base contact producing a feedback voltage at the emitter; $r_b'{}_1$ is defined as the ratio of this feedback voltage to the current.

The collector capacitance current $j\omega C_c v_c$ enters the base layer uniformly over the entire area of the collector junction resulting in a feedback voltage at the emitter; $r_b{'}_2$ is defined as the ratio of this feedback voltage to this current.

Calculations

The resistances $r_b{'}_1$ and $r_b{'}_2$ for the transistor of Fig. 2(*b*) may be computed with the help of three formulas which give the feedback voltages for the three geometrical problems involved in this transistor. Each expression gives the voltage V developed at electrode C by a current I entering through electrode A and leaving through electrode B. The formulas are in terms of sheet resistance ρ_b/w (resistance per square in ohms) and the radii involved.

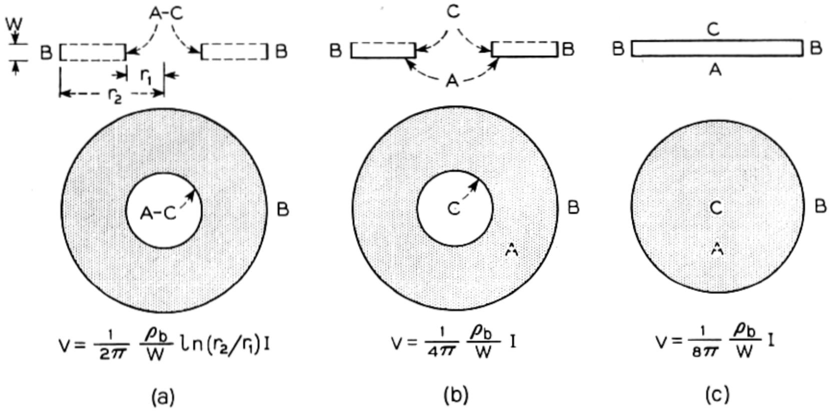

Fig. 7 — Feedback voltage for three geometries.

The simplest situation and its formula are shown in Fig. 7(a). Electrodes A and C are the same and expression gives the resistance of an annular ring to radial current flow.

Fig. 7(b) also shows an annular ring, but the current I is introduced uniformly over one of the flat surfaces, while the voltage V is measured from outside edge to inside edge.

Fig. 7(c) shows current introduced uniformly over the surface of a disc, while voltage V is the average voltage developed along the surface of the disc. It should be clear that electrodes A and C do not conduct parallel to the disc surface (i.e., they are not equipotentials).

Example

The $r_b{'}s$ for the transistor of Fig. 2(b) will be calculated.

A. $r_b{'}_1$ — The current $I = (1\text{-}\alpha)i_e$ is assumed to originate uniformly in the region between emitter and collector in Fig. 2(b). It flows radially outward to the base contact.

$$V = I\left[\frac{1}{8\pi}(\rho_b/w_1) + \frac{1}{2\pi}(\rho_b/w_2)\ln(r_2/r_1) + \frac{1}{2\pi}(\rho_b/w_3)\ln(r_3/r_2)\right]$$

$$r'_{b1} = V/I = \rho_b\left[\frac{1}{8\pi w_1} + \frac{1}{2\pi w_2}\ln\frac{r_2}{r_1} + \ln\left(\frac{r_3}{r_2}\right)\right]$$

B. $r_b{}'_2$ — Inspection of Fig. 7 shows that the collector capacitance current originating opposite the emitter junction (I_1) has the same flow path as the $(1\text{-}\alpha)i_e$ current. The remainder of the collector current which enters the base from the collector outside the emitter (I_2) likewise flows radially out to the base contact. The total feedback voltage developed between emitter junction and base contact is

$$V = I_2 r'_{b1} + I_2\left[\frac{1}{4\pi}(\rho_b/w_2) + \frac{1}{2\pi}(\rho_b/w_3)\ln(r_3 r_2)\right]$$

$$r'_{b2} = \frac{V}{I_1 + I_2} = \rho\left\{\frac{1}{2\pi w_3}\ln\left\{\frac{r_3}{r_2}\right\} + \left\{\frac{r_1}{r_2}\right\}^2\left[\frac{1}{2\pi w_2}\ln\left\{\frac{r_2}{r_1}\right\} + \frac{1}{8\pi w_1}\right] + \left[1 - \left\{\frac{r_1}{r_2}\right\}^2\right]\frac{1}{4\pi w_2}\right\}$$

C. Numerical Results

For $\rho_b = 1.5$ ohm-cm
$w_1 = 0.5$ mil
$w_2 = 1.0$ mil
$w_3 = 2.0$ mil

$r_2/r_1 = 2$
$r_3/r_2 = 2$

$r_b{}'_1 = 147$ ohms
$r_b{}'_2 = 96$ ohms