Transistor Oscillator for Use in Multifrequency Pulsing Current Supply

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This paper covers the design and performance of an oscillator using a transistor, in the multifrequency pulsing of digital information over telephone transmission media. The frequencies used are in the range from 700 to 1700 cycles per second.

INTRODUCTION

A large proportion of the telephone calls made require connections to be set up in more than one central office. Where common control systems are used, this requires that information needed by the second central office be transmitted to it from the first central office which in turn has received its information from the calling subscriber. The "language" used in some cases for transmitting the information is in the form of short pulses of alternating current. Each pulse consists of a combination of two of six available frequencies. Twelve combinations of the six frequencies make up the total "vocabulary". Ten are required for digits and two for special signals sent at the beginning and end of pulsing. This is known as multifrequency, MF, pulsing. The device which controls the pulses is called an MF sender. The device which receives the MF pulses and translates the information received for use by other equipment is called an MF receiver.

An operator may also use multifrequency pulsing when transmitting information by means of a key set to a distant office. Control of the frequencies used is obtained by contacts on the key depressed. Twelve keys are used.

The previous source of ac was a circuit capable of supplying both operator positions and senders. This equipment is however quite expensive. If only a few senders in an office require the MF current the cost per

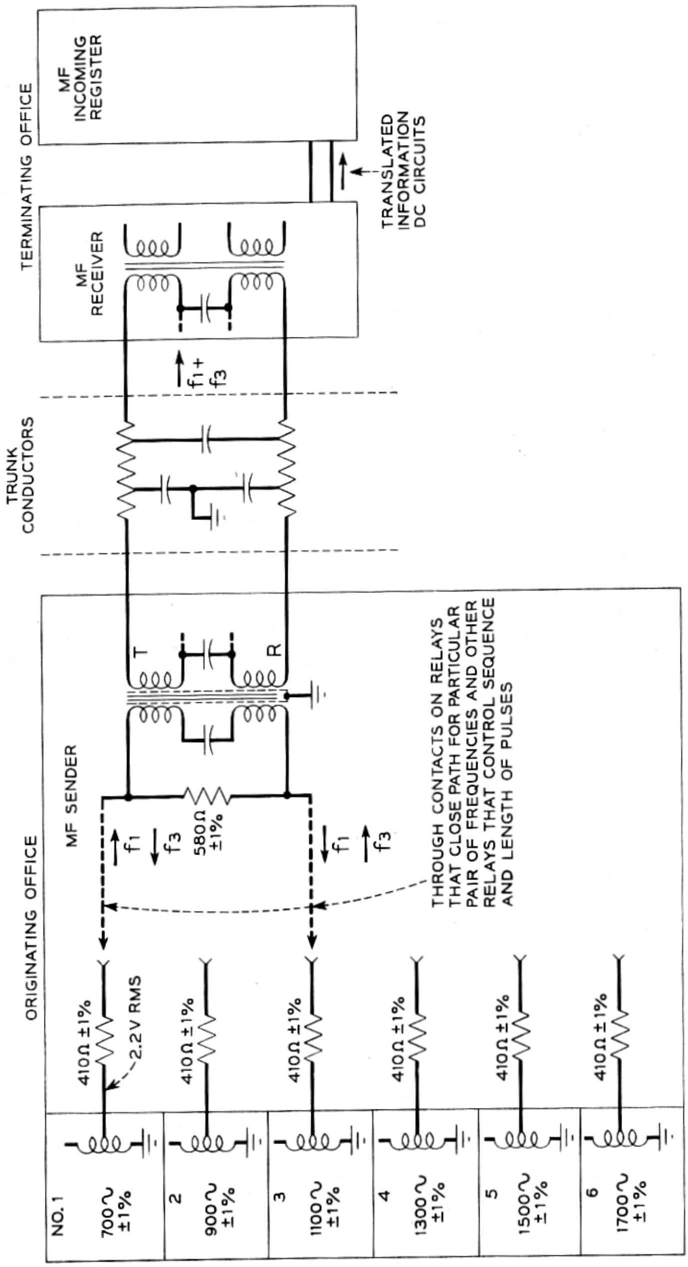

Fig. 1 — Simplified diagram of MF pulsing circuits.

sender becomes very large. It was for this reason that transistor oscillators were developed for use as individual current supplies.

An individual sender will handle a large number of calls during a day but the aggregate time during which the multifrequency current is required is normally less than two hours. The ability of transistors to operate the instant that power is applied is therefore a distinct advantage in saving power. The low voltage that is required for their operation and the long life expectancy are further advantages.

The method used is illustrated in Fig. 1. To transmit a pulse the output windings of two oscillators are connected in series through two 410-ohm resistors and the parallel combination of a 580-ohm resistor and windings on a transformer. The other windings on the transformer are connected to the trunk conductors. At the distant office the trunk is terminated by a similar transformer. The output of this transformer is connected to an amplifier, filter circuits and detectors in an MF receiver. Relays associated with the detectors convert the ac signal into a dc signal for the register circuit.

The 580-ohm resistor across the transformer winding serves as a termination for the trunk between pulses. This resistor in combination with the two 410-ohm resistors in the oscillator circuits serve as the trunk termination during pulsing.

REQUIREMENTS FOR OSCILLATOR

This oscillator was designed to be used in place of an existing piece of equipment. It therefore must have the properties of the existing equipment on which the design of equipments that are associated with it have been predicated.

The requirements are then:

1. Operate at 700, 900, 1100, 1300, 1500 or 1700 cycles per second ± 1 per cent.
2. Furnish ac signals at a level of -3 dbm ± 1 db at the input to the trunk.
3. The level difference at all frequencies must be no greater than 1 db.
4. Attain normal output level between the time the sender is seized and the time the first pulse is sent. This time is approximately one half second.
5. Meet all requirements with dc input voltage limits of 45–50 volts.
6. Meet all requirements with ambient temperatures of $+40°$F to $+135°$F.
7. Have level of second harmonic a minimum of 35 db below the fundamental.

8. Provide an output impedance sufficiently low that the differences in impedance of the associated oscillator will not appreciably affect either output level or frequency.

9. Meet frequency requirements when working into trunks that have impedances with phase angles of $+15°$ to $-45°$.

10. Be unaffected by transient disturbances on trunk conductors coupled to the oscillator. These disturbances are caused by lightinng or troubles on closely associated power lines.

DISCUSSION

There are many ways of making a transistor oscillator. One of the simplest and most stable of these uses an LC circuit for controlling the frequency. Additional windings inductively coupled to the tuned circuit inductance provides the necessary feedback to the transistor and the connection to the load. This is the type of circuit adopted for this application. A schematic of the circuit is shown in Fig. 2. The transistor controls power supplied to the oscillatory circuit during only one half the operating cycle. The amplitude of the ac voltage produced across winding 1–2 is almost equal to the dc voltage applied to the collector.

The transistor acts as an amplifier or it may be considered as a negative resistance. Control of the transistor is obtained by current flowing in the emitter that is in phase with the collector voltage. To build up oscillations, the power put into the circuit must be greater than that expended. Stability is obtained when the power put in equals the power expended.

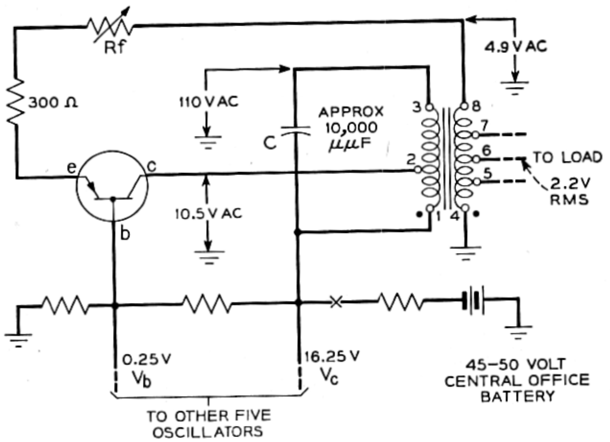

Fig. 2 — Oscillator circuit for MF signaling showing approximate values for voltages.

In the following sections the functions of a resonant circuit are first discussed. Then, the characteristics of a point contact transistor that make it suitable for use in an oscillator are considered. This is followed by a discussion of the behavior of the two when combined to form an oscillator with consideration given to the effect various factors have on performance.

RESONANT CIRCUIT

A resonant circuit has several properties of interest when one considers it as part of an oscillator. These are:

1. The input impedance of a parallel circuit consisting of an inductance, L, and a capacitance, C, has the property of a resistance at approximately frequency,[1]

$$f_0 = \omega_0/2\pi.$$

That is, input impedance,

$$Z = RQ^2\left(1 - j\frac{1}{Q}\right),$$

where R = effective resistance of the inductor and

$$Q = \frac{\omega_0 L}{R}$$

when Q is high, $Z \doteq RQ^2 = R_0$ the equivalent resistance at resonance.

2. The decrement of a parallel resonant circuit when disconnected from sources of power or loads is:[2]

$$\begin{bmatrix}\text{numerical decrement in cur-}\\ \text{rent or voltage per cycle}\end{bmatrix} = 1 - e^{-\frac{R}{2Lf_0}}. \tag{1}$$

When the effective resistance R becomes zero the decrement likewise becomes zero.

In the oscillatory circuit considered the decrement is made zero by the use of a negative resistance equal to the effective positive resistance of the turned circuit. Additional loads on the turned circuit are accommodated in the same way.

If the negative resistance in the simple circuit considered becomes greater than the equivalent resistance of the tuned circuit and its loads, power will be supplied by the tuned circuit at a rate that will maintain equality between the power put into the circuit and the power taken out. As a corollary to this, if the negative resistance is less than the positive resistance, the voltage will increase across the circuit. For the oscillator this would be:

$$\begin{bmatrix}\text{numerical increment in cur-}\\ \text{rent or voltage per cycle}\end{bmatrix} = e^{\pi/Q_1} - 1. \qquad (2)$$

The subscript on Q is used to denote the condition where R is negative. The equation is obtained from the following relationships.

$$\begin{aligned} Q &= \frac{\omega_0 L}{R} = \frac{L}{R/\omega_0} = \frac{1/2 I_m^2 L}{1/2 I_m^2 R/\omega_0} \\ &= \frac{\text{Energy stored in resonant circuit}}{\text{Rate of change of energy stored}}. \end{aligned} \qquad (3)$$

Since $R = \dfrac{\omega_0 L}{Q}$, a negative R, $(-R) = -\dfrac{\omega_0 L}{Q_1}$,

$$e^{-\frac{R}{2Lf_0}} \text{ (from equation 1)} = e^{\frac{-(-R)}{2Lf_0}} = e^{\frac{-(-2\pi f_0 L)}{(2Lf_0 Q_1)}} = e^{\frac{\pi}{Q_1}}. \qquad (4)$$

I_m = peak value of current flowing in resonant circuit.

The relationship of energy stored to rate of change of energy stored is a convenient relationship for computing the value of Q_1 in this particular case.

The ability of a transistor to satisfy the requirements of the resonant circuit for sustaining oscillation as well as building oscillation will be discussed later.

Energy which is large in comparison to the power taken by the load is stored in the tuned circuit in order to minimize:

(a) The effect on oscillator frequency of small changes in load current phase angle with trunks of different types, connected.

(b) The effect on oscillator frequency of different values of inductance in the output windings of the associated oscillator circuit.

(c) The effect of surges on the transistor that are transmitted over the connected trunk to the oscillator.

(d) The harmonics produced by the method of supplying power to the tuned circuit. This includes the use of an incorrect feedback adjustment.

The Q for the transformers varied from 60 to 85. The stored energy was then 60 to 85 times the energy dissipated per radian at the oscillaing frequency for the particular transformer. By adjustment of the ratio of the turns between windings 1–2 and 1–3 a dissipation in the tuned circuits of from 6 to 10 milliwatts was obtained.

In the six-frequency supply three different coil designs are used. One design is used for the two lower frequencies, a second for the two intermediate frequencies and a third for the last two frequencies. Data on these coils are given in Appendix I.

The frequency of oscillation for each coil is determined by the capacitance used with it and is given approximately by the formula:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}.$$

Sufficient inductance is used in these coils so that mica condensers can be economically used with them. Mica condensers are used because of their low temperature coefficient.

Leakage reactance and capacitance between windings make the circuit resonant at more than one frequency. To minimize the current fed back to the emitter under such parasitic conditions, the 4–8 winding was placed next to the core with terminal 8 next to terminal 1. This placed an ac ground next to the end of the winding connected to the emitter. As a further precaution against parasitic oscillations the resistance between emitter and transformer is kept high.

TRANSISTOR OPERATION

It is customary to consider the transistor as an amplifier working into a load represented by the tuned circuit. However since the current in the collector and that in the emitter are intimately related during that part of the cycle when power gain is obtained, the collector circuit can be considered equally well as a negative resistance of a value established by the feedback used and the emitter circuit simply as a load. At best either method is only an approximation due to the nonlinearity of the transistor characteristics for large signals. Representation of the transistor as a resistance puts the requirements in terms of values readily obtained from the static characteristics of a transistor. This form of treatment is therefore used.

The regions in which positive or negative resistance is obtained is illustrated in Fig. 3. The characteristics shown are those for an ideal transistor. That is, the ratio of an incremental change in collector current to an incremental change in emitter current, α, with a constant collector voltage is a uniform value in region 2. Also, that in regions 1 and 3 the slope of the lines in each region is constant. That this is not very different from that obtained from some transistors can be seen by comparison with the actual characteristics shown in Fig. 4.

The division line between regions 1 and 2 represents the magnitude of the dc voltage applied between collector and base, V_c-V_b. On the left hand side of Fig. 3 the phase relationship and magnitude of ac voltage applied to the collector and the emitter current are shown. The condition which limit the ac voltage on the collector to this value will be discussed later.

In region 1 the net voltage on the emitter to base circuit is negative and it has no effect on the current which flows in the collector. The current which flows in the collector will therefore depend purely on the reverse resistance of the collector acting as a diode and the voltage applied between collector and base. This is a positive resistance but it is not always linear. Since the only voltage active in this region is that obtained from the tank circuit, this resistance acts as a load on the tank circuit.

In region 2 the voltage applied to the emitter circuit is positive and the current that flows as a result of this voltage exercises control over the collector current. The external resistance that is used in the emitter circuit is sufficiently high so that the small changes that occur in the emitter input resistance as the emitter current is varied are insignificant. The current that flows in the emitter circuit can therefore be considered vary linearly with the ac voltage. Since for each incremental change in collector voltage a proportional change in emitter current will be obtained, a plot of this relationship for a given ratio of emitter to collector ac voltage will result in a straight line. The slope will be determined by the amount of feedback (emitter voltage). Since a decrease in the absolute voltage on the collector results in an increase in collector current, the line has a negative slope. It therefore represents a negative resistance.

In region 3 the emitter current exercises very little control over the collector current.

A negative resistance represented by $-R2$ is therefore obtained from 0 to π of the ac wave. If a second transistor were added of identical charac-

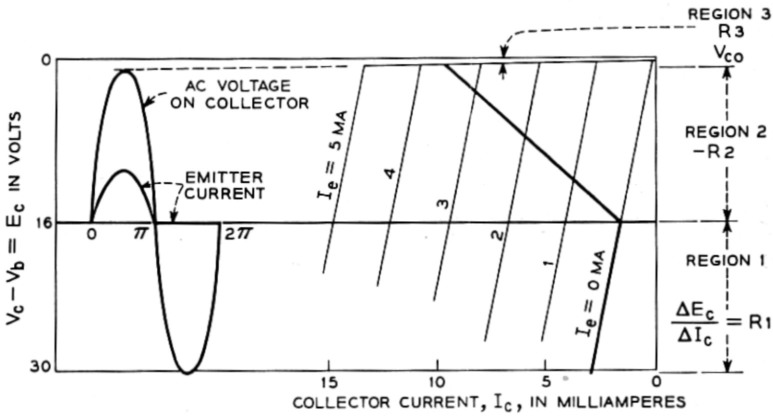

Fig. 3. — Idealized transistor characteristics with operating regions for oscillator indicated.

teristics so connected (push-pull circuit) that a negative resistance would be obtained from π to 2π of the ac wave, the effect of a negative resistance of constant value would be obtained. In order to reduce the transistors to the terms of a two-pole device having a negative resistance the positive resistances represented by the emitter circuits and regions 1 of the transistors would also have to be included.

It is more convenient however to combine all elements that produce a loss when using the static characteristics of a transistor to determine if the transistor will satisfy the circuit requirements for oscillation. This method is therefore used when considering the circuit operation.

There is one important difference between the characteristics of a transistor and the ideal characteristics shown. That is, for very small values of emitter current bias and a constant E_c, the ratio $\Delta Ic/\Delta Ie$, or, α, drops rapidly from three or more to approximately one as the emitter current approaches zero. To eliminate this change in the dynamic negative resistance for very low voltage changes on the collector, a small dc current is supplied in the emitter circuit. The need for this will be discussed later.

CIRCUIT OPERATION

The negative resistance of the transistor is effectively connected in parallel with the resistances representing the various elements that make up the total loss by transformer action. The requirements for oscillation are therefore met when $-R2$ for a complete cycle is less than the positive resistance representing all losses. If the length of time that $-$R2 is effective is only one half cycle the value of $-R2$ must of course be cut in half.* Since a single transistor could meet the requirement for $-R2$ only one was used in the working circuit as is shown in Fig. 2.

The transistor adopted for this use was the 1729 type, now in production, which has been given the RTMA designation 2N25. The 1729 type was used because its characteristics are least affected by changes in temperature, and in addition the allowable power dissipation was approximately twice that of other comparable transistors.

The various factors that when combined make up the load and their normal variation are given in Table I. All losses are in terms of power into the 1–2 winding.

The corresponding value of load resistance is,

$$R_{TL} = \frac{(V_c - V_b - V_{co})^2 1000}{2 \times 2 \times W_t}.$$

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* See Appendix II.

TABLE I

Source of Loss	Power in Milliwatts		
	Min.	Avg.	Max.
Sustaining stored energy in resonant circuit	6.0	8.0	10.0
Output load	4.5	4.5	4.5
Region 1 of collector operation	2.8	7.0	14.0
Loss in emitter	4.0	8.5	13.4
Margin for stability of adjustment	12.0	12.0	12.0
Total, W_t	29.3	40.0	53.9

This would be 1920-ohms, 1400-ohms and 1045-ohms respectively. For purpose of illustration the average value is plotted on the characteristics of an average transistor in Fig. 4 along with $-R2$ for the transistor. It is evident from this that $-R2$ is lower in value hence the circuit will oscillate and build up to the required voltage. Minor corrections in the emitter current would normally be required in order to meet test requirements. The potentiometer that is shown in Fig. 2 provides the means for adjustment.

The oscillogram shown in Fig. 5 illustrates the condition described above. The characteristics of the transistor used are shown in Fig. 4. The oscillogram is a multiple exposure from which $R1$, $-R2$ and $R3$ (see Fig. 3) for several values of feedback may be obtained. The normal condition of adjustment is illustrated in Fig. 6 with normal load. Four times normal load is a test condition.

When the extra load that is applied during test is removed the output voltage should go up since power input exceeds the power expended.

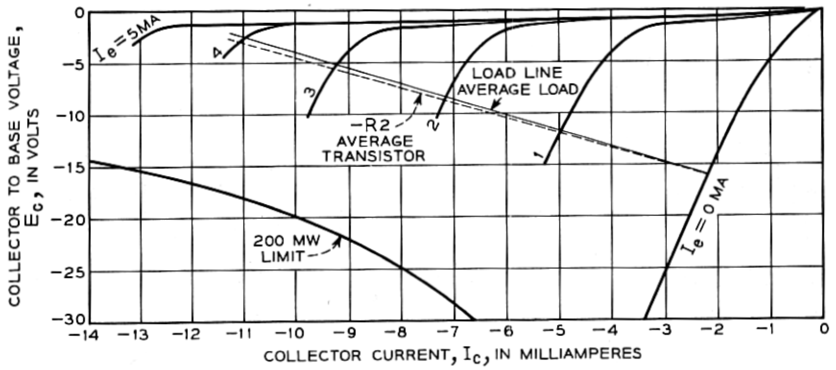
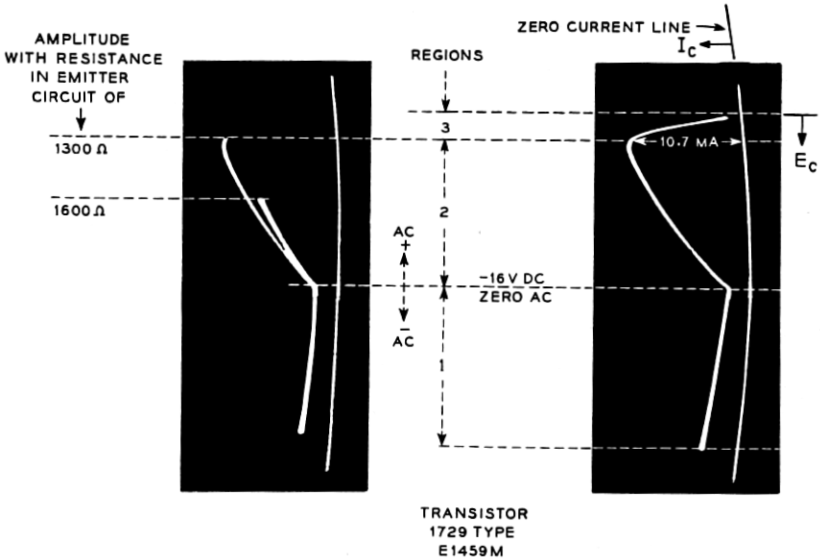


Fig. 4 — Characteristics of representative 1729 type transistor with negative resistance values plotted for the average condition given in table. A load line is also shown for the average condition. In plotting $-R2$ a line is drawn from $I_e = 0$, $E_c = 16$ to point corresponding to maximum I_e. Maximum $I_e \doteq W_e \times \dfrac{1.414 \times 2}{E_{erms}}$ at $E_c = 1.5$ volts. W_e is loss in emitter circuit given in table .



Figs. 5 and 6 — Oscillograms showing the relationship of collector current to collector voltage during a complete cycle for several operating conditions. Fig. 5 (left) is a multiple exposure made to illustrate the ability to oscillate at a lower output level with decreased feedback. This is due to the much higher alpha obtained with low emitter currents. Fig. 6 is for normal operating conditions.

This increase in voltage is however very small since the losses will increase as the square of the voltage and the rate at which energy is supplied to the circuit will decrease. The decrease in rate is due to the change from a negative resistance to a positive resistance in region 3. This causes the average negative resistance for the complete cycle to have a higher value.

Feedback which is far greater than is required can in some cases cause the peak value of the ac voltage to exceed the dc voltage. Power is drawn from the energy stored in the tank circuit when this occurs. This effectively limits any further increase in output voltage.

A change will also be introduced in the emitter circuit due to operation in region 3. That is, the voltage feedback introduced in the emitter circuit by collector current flowing in the common base resistance is reversed in phase in region 3.[3] This is due to a reduction in collector current when the voltage applied to the emitter circuit is still rising. This feedback is sufficient in many cases to cancel the increase in emitter voltage. The emitter current in such a transistor will therefore remain nearly constant in this region. In region 2 however the feedback is in such a direction as to aid the flow of emitter current. The result is that the voltage drop across the emitter resistance is approximately canceled by the voltage across the base resistance. Due to this relationship the transistor may appear to have zero resistance in the emitter to base circuit or a small positive or negative resistance.

It is evident from the foregoing that the ability of a transistor to fulfill the requirements for oscillation is therefore dependent upon both the average α along the load line and, $R1$, resistance in region 1. The relationship of these two factors is shown in Fig. 7. Dots on this chart represent the 1729 type transistors tested that met the requirements for the 2N25 transistor. The ambient temperature was +135°F. It is inadvisable to use transistors having a resistance of much less than 4000-ohms in region 1 since both the average dissipation rate and the peak dissipation rate would exceed allowable limits for continuous operation. This will tend to cause the transistor characteristics to change at a more rapid rate.

The effect of different values of feedback on the output of the oscillator is shown in Fig. 8. Fig. 8 also shows the variation in output obtained with several different values of load resistance. This was done to illustrate the use of increased load in determining the proper point for adjusting the feedback resistance. The proper adjustment is the minimum feedback with which the output changes only approximately 1 db in going from normal load to four times normal load. This degree of margin

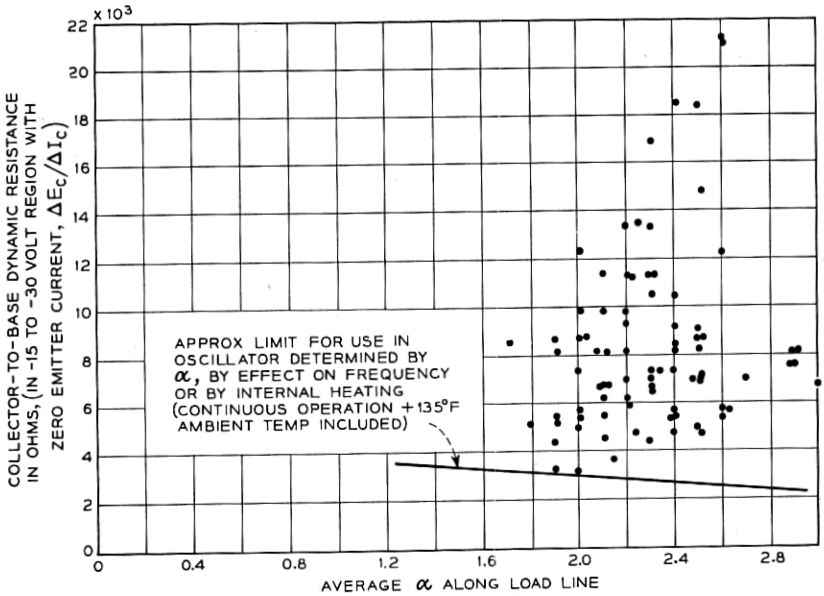


Fig. 7 — Plot of α versus collector to base resistance for representative group of transistors meeting 2N25 transistor requirements.

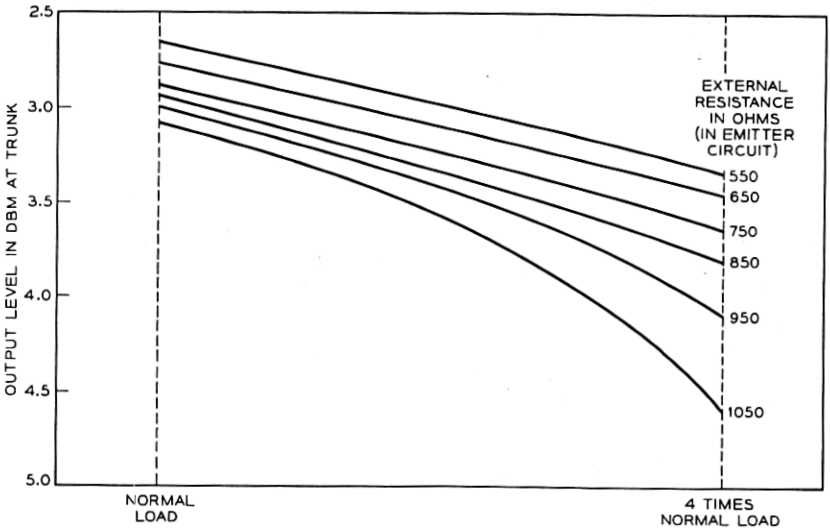


Fig. 8 — Effect of load and value of feedback resistance on the level of the output.

permits some deterioration in transistor characteristics before the change in output is sufficient to require readjustments.

Variation in the absolute level of output due to variations in V_{co} (see Fig. 3) between transistors and to differences in coils is taken care of by the use of taps on the output windings.

Harmonics of the fundamental frequency are created by the nonlinearity of the transistor characteristics. These harmonics are accentuated by excessive feedback. The level of the harmonics for a representative transistor are shown in Fig. 9.

The effect of variations in feedback on the frequency is shown in Fig. 10. The shift is thought to be due to several factors all small. One is the lack of perfect coupling between the transformer windings. Another is

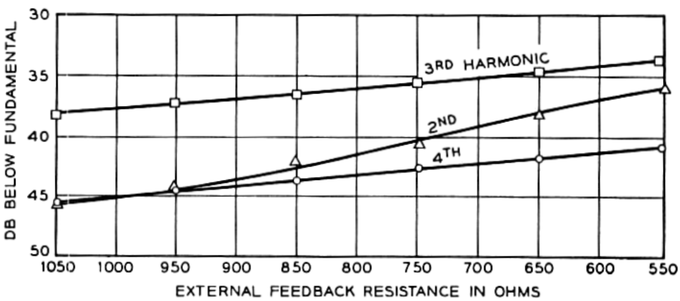


Fig. 9 — Effect of feedback on harmonics with normal load.

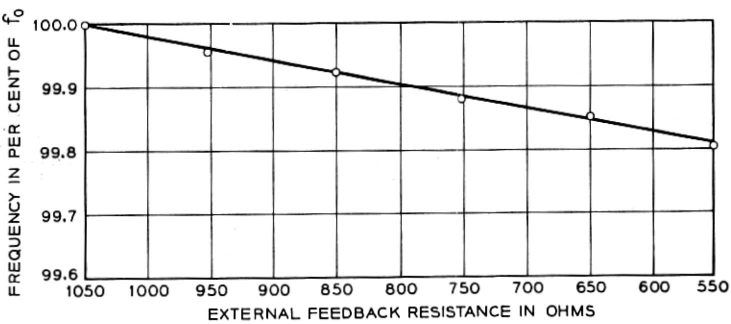


Fig. 10 — Effect of feedback on frequency.

the leakage reactance of the input winding. As the time rate of change in current is increased by increased feedback these factors become increasingly greater although never very large. Fewer turns are required on the coils used for operation at the higher frequencies hence these effects are reduced.

The output level of the oscillator will vary almost directly with the variations in the dc voltage since the amplitude of ac voltage across collector to base is almost equal to the dc voltage applied. Hence, a variation of approximately 0.9 db will be obtained in the output when the central office battery is reduced from 50 volts to 45 volts due to power failure conditions.

The over-all output variation from all causes is shown in Fig. 11. This is based on data obtained using the transistors having the distruibtion in characteristics shown in Fig. 7.

Decreases in the value of R1 with temperature is normally compensated by a corresponding increase in α. However small positive or negative voltage changes that alter the level of output do occur in the cut-off voltage. This is minimized by keeping the dc voltage as high as permissible and still meet the 200 milliwatt dissipation limit for the 2N25 transistor.

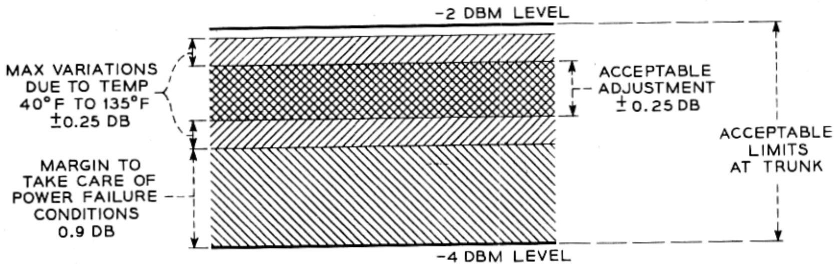
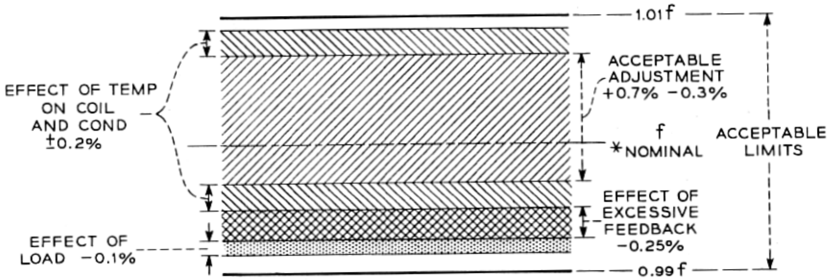


Fig. 11 — Output level with effect of various factors that may alter the level indicated.

The effect of the various factors mentioned before on the frequency of operation are shown in Fig. 12. Since several of the factors causing a shift in frequency were in the negative direction only, the adjustment limits were set correspondingly higher. The over-all frequency variations could be reduced by reducing adjustment tolerances.

The starting condition is important in this type of circuit since energy must be introduced into the oscillatory circuit before the dynamic characteristics of the circuit become effective. This means that the build up time is dependent upon the amount of energy introduced into the system at the start. In this application energy is introduced by the current which flows when the dc voltage is applied to the collector circuit. The value of this current is largely dependent upon the collector to base dc resistance



*(700, 900, 1100, 1300, 1500 OR 1700~)

Fig. 12 — Frequency of output with effect of various factors that may alter the frequency indicated.

which is in turn affected by the ambient temperature. This resistance is between approximately 4,000 and 20,000 ohms.

The oscillograms shown in Fig. 13 illustrate this effect. Both are for the same circuit operating under normal conditions of adjustment. Q_1 for this condition is approximately 18. Oscillogram (a) is for the application of voltage in the normal fashion to the voltage divider circuit. The closure occurs at the point oscillation starts. Oscillogram (b) shows the build up obtained when no impulse is applied to start oscillation except for minor irregularities in the dc voltage applied. Starting is prevented in this case by a short circuit on 4–5 widing that was removed approximately 2 ms after the start of the trace. The amplitude of oscillation is so low however for the first few cycles that the start is difficult to distinguish. The build up time for (a) is approximately 27 milliseconds and for (b) it is approximately 37 milliseconds. The exponential build up of amplitude is modified greatly by the rapid change in the transistor's α as the voltage approaches the cutoff region.

It should be noted also that oscillation would not have started under

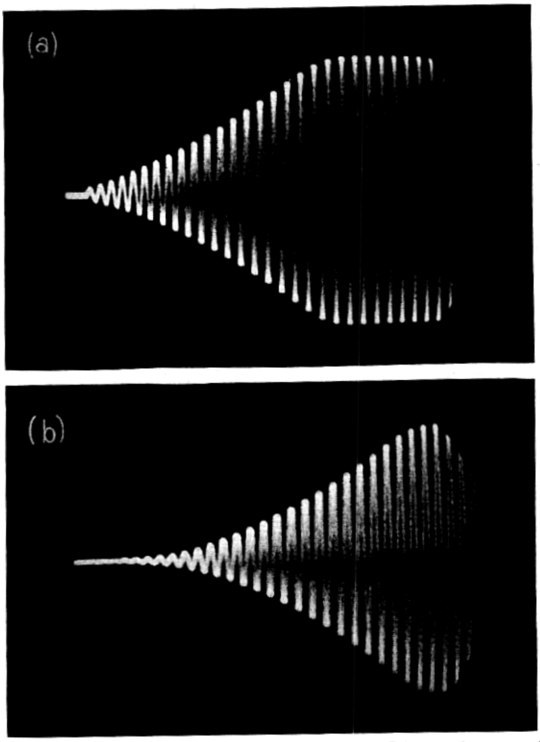


Fig. 13 — Build-up of oscillation in resonant circuit. Normal operating conditions.

the condition for oscillogram (b) with some transistors if the small emitter bias current had not been provided.

TRANSIENT EFFECTS

The trunk conductors are balanced with respect to ground. Voltages set up in these conductors due to electrostatic or magnetic coupling to the source of the interference will cause longitudinal currents to flow. An electrostatic shield in the transformer, shown in Fig. 1, effectively prevents such longitudinal currents from reaching the oscillator circuit.

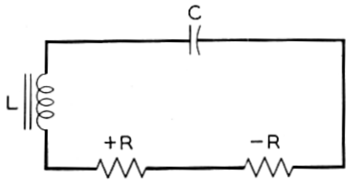


Fig. 14 — Equivalent resonant circuit.

If however the voltage becomes sufficient to breakdown one of the protector blocks that are connected between each trunk conductor and ground, a voltage comparable to the breakdown potential of the protector blocks (400 to 600 volts) would then be impressed across the output windings of the transformer. The usual cause of this condition is lightning. However, neither artificially simulated lightning nor transients of longer duration were capable of raising the voltage on the tank circuit to the point where a transistor was damaged. This is due both to

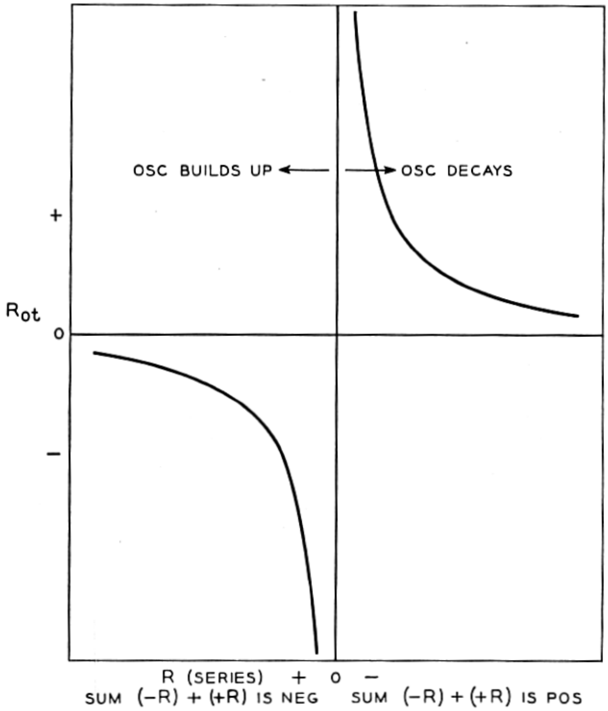


Fig. 15 — Relationship of shunt to series resistance.

the high energy level of the tank circuit and the isolation furnished between repeating coil and oscillator by the series resistors.

SUMMARY:

The transistor oscillator adequately fulfills the requirements for a source of current for multifrequency pulsing over telephone transmission circuits. Adjustments are provided so that the requirements for frequency stability, harmonic level and output level can be met with transistors having a wide range of characteristics. Sufficient margin is provided by design and by initial adjustment so that an appreciable change in transistor characteristics can be tolerated before readjustment is required.

ACKNOWLEDGMENT

The transformer used in the oscillator was designed by H. E. Vaiden and A. M. King. The transistor used was developed by R. J. Kircher and N. J. Herbert. D. J. Houck assisted in testing the circuit.

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Appendix I

Transformers moly-permalloy dust core 1.57 O.D.

700- and 900-cycle operation

Winding (1–2) = 500 turns   Q at 700-cycles, 60
Winding (4–5) = 107 turns   Q at 900-cycles, 68
Winding (4–6) = 115 turns
Winding (4–7) = 123 turns
Winding (4–8) = 220 turns
Winding (1–3) = approximately 5560 (adjusted to meet inductance requirements of 5.2H ± 1 per cent)

1100- and 1300-cycle operation

Winding (1–2) = 372 turns   Q at 1100-cycles, 73
Winding (4–5) = 80 turns   Q at 1300-cycles, 75
Winding (4–6) = 86 turns
Winding (4–7) = 92 turns
Winding (4–8) = 164 turns
Winding (1–3) = approximately 4080 (adjusted to meet inductance requirements of 2.8H ± 1 per cent)

1500- and 1700-cycle operation

Winding (1–2) = 305 turns   Q at 1500-cycles, 82
Winding (4–5) = 66 turns   Q at 1700-cycles, 85
Winding (4–6) = 71 turns
Winding (4–7) = 75 turns
Winding (4–8) = 135 turns
Winding (1–3) = approximately 3360 (adjusted to meet inductance requirements of 1.9H ± 1 per cent)

Appendix II

The equivalent circuit for the oscillator is shown in Fig. 14. For stable operation from equation (1), since $f_0 = 1/t$,

$$1 - e^{-\frac{(+R)t}{2L}} = e^{-\frac{(-R)t}{2L}} - 1.$$

The negative resistance, $(-R)$, must therefore be equal to the positive resistance $(+R)$. If however $(-R)$ is active for only half the time, $(-R)$ must be equal in magnitude to $2(+R)$ in order to satisfy the requirements for equality. This assumes that boundary effects are negligible. This assumption was borne out by experiment.

The equivalent resistance of the resonant circuit is

$$R_0 \doteq \left(\frac{\omega_0 L}{R}\right)^2 R \doteq K \frac{1}{R},$$

where

$$k = (\omega_0 L)^2.$$

A plot of this relationship when positive and negative resistances are combined, is shown in Fig. 15.

In the actual circuit the negative resistance is connected across the 1–2 winding. The 1–3 winding is in the resonant circuit. The equivalent resistance ($R_{0(1\text{-}2)}$ of the resonant circuit across winding 1–2, is determined as follows:

$$R_{0(1\text{-}2)} = \frac{(\text{turns, winding 1–2})^2}{(\text{turns, winding 1–3})^2} \times R_0 .$$