Estimation and Control of the Operate Time of Relays

Part I — Theory

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The dynamic equations applying to the operation and release of electromagnets are derived in a form in which the armature position and the flux linkages of the coil are the variables to be determined as functions of time. The effective magnetomotive force and pull are taken as given by the static magnetization relations. This formulation is valid if the dynamic field pattern is substantially that of the static field, and it is shown that this condition is satisfied if the effective conductance of the eddy current paths is small compared with that of the coil or other linking circuits. This is usually the case in operation, but is not the case in normal release.

Approximate solutions are obtained for both fast and slow operation, giving the time as related to the mechanical work done, the mass and travel of the moving parts, and the steady state power input to the coil. Expressions are derived for optimum coil design and pole face area, and design requirements for fast operation are discussed.

An approximate expression is derived for the release waiting time, the initial period of field decay prior to armature motion. The effect on this decay of contact protection in the coil circuit is discussed. The slow release case is treated in another article in this issue. An analysis is given of the armature motion in release, and the results of an analog computer solution to this problem reported. These show the effect of design parameters, particularly the armature mass, on the velocity attained in release motion, and hence on the amplitude of armature rebound.

INTRODUCTION

The operate and release times of most telephone relays lie in the range from 1 to 100 millisecs. (0.001 to 0.100 secs.). To use these relays in telephone switching, involving complex patterns of sequential switch and relay operation, it is necessary: (1) to be able to estimate the operate and release times of any relay for which this information is needed in circuit studies, and (2) to develop relays capable of meeting specific timing requirements, both fast and slow.

The essential basis for such design and estimation is a dynamic theory of the operation of electromagnets, of sufficient accuracy for engineering use. The theory presented here is applicable to electromagnets in general, although the applications discussed are those relating to relays. The theory presented is approximate, partly because of the difficulty of providing a more exact treatment, and partly because simplicity and generality are more important for engineering purposes than accuracy in estimation, which is in any case subject to correction by measured results. The theoretical relations are used alone only in preliminary estimation and in the initial stages of development, as discussed in Part I of this article. In advanced development and in the modification and application of existing structures, as discussed in Part II, the theory is used as a guide in the correlation and extrapolation of observed performance.

The dynamics of electromagnets involve the concurrent and interrelated phenomena of field development and armature motion, and are accordingly governed by the two differential force equations respectively applying, each containing a coupling term expressing their reaction on each other. These basic equations are formally identical with those of electromechanical transducers, such as loud-speakers, but the treatment has little else in common. The operation of an electromagnet is the transient change from one state of equilibrium to another, as distinguished from the sustained low amplitude oscillations of the transducer case.

The coupling terms represent the effect of field energy changes on the coil voltage and on the force causing armature motion respectively. In the steady state, the field energy is a function of magnetomotive force and of armature position alone. In a transient state, eddy currents in the magnetic members affect both field energy and the pattern of the field. In developing an approximate theory, it is assumed that these effects are confined to the total effective mmf, and that the pattern of the field is the same as that of the static field: i.e., that the field energy associated with any portion of the structure, such as an air gap, is fixed by the flux linkages of the coil and by the armature position. The limitations on the analysis imposed by this simplifying assumption are discussed at the end of the next section.

1 The Dynamic Equations

The notation of this article conforms to the list given on page 257.

THE ELECTRICAL EQUATION

The voltage equation for a coil of N turns linking a field of strength φ may be written in the form:

$$(i - I)R + N\frac{d\varphi}{dt} = 0, \tag{1}$$

where i is the instantaneous current, R is the circuit resistance, and IR is the constant voltage applied. Writing $\mathfrak{F}_i$ for the instantaneous mmf $4\pi\ Ni$, and $\mathfrak{F}_{Si}$ for the steady state mmf $4\pi\ NI$, the equation becomes:

$$\mathfrak{F}_i - \mathfrak{F}_{Si} + 4\pi G_i\frac{d\varphi}{dt} = 0,$$

where $G_i = N^2/R$, the coil constant, or equivalent single turn conductance. If the field φ is linked by a number of circuits, a similar voltage equation applies to each such circuit. By addition of these expressions, there is obtained:

$$\mathfrak{F} - \mathfrak{F}_S + 4\pi G\frac{d\varphi}{dt} = 0,$$

where $\mathfrak{F} = \sum\mathfrak{F}_i$, the total effective mmf, $\mathfrak{F}_S = \sum\mathfrak{F}_{Si}$, the total applied mmf, and $G = \sum G_i$, the total equivalent single turn conductance. If the dynamic field has the same pattern as the static field, the instantaneous mmf $\mathfrak{F}$ must equal $\mathcal{R}\varphi$, where $\mathcal{R}$ is the reluctance $\mathfrak{F}/\varphi$ observed in static magnetization measurements. The preceding equation may therefore be written in the form:

$$\mathcal{R}\varphi - \mathfrak{F}_S + 4\pi G\frac{d\varphi}{dt} = 0. \tag{2}$$

This relation controls the time rate of development of the flux φ. As the coil is the only circuit linking the field that has an externally applied voltage, $\mathfrak{F}_S$ is simply the steady state applied mmf $4\pi\ NI$. The conductance G includes not only the coil conductance G_C, or N^2/R, but terms for all conductive paths linking the field, including the eddy current paths. The effective conductance of the eddy current paths is denoted G_E. When a short circuited winding or sleeve is used, as in slow release relays, its conductance is denoted G_S. Thus in most cases of interest $G = G_C + G_E$; when a sleeve is used, $G = G_C + G_E + G_S$.

The static magnetization relations between $\mathfrak{F}$ and φ vary with the position of the armature, which is defined by its displacement x from the operated position. Hence the reluctance $\mathcal{R}$ is, in general, a function of both x and φ. Thus equation (2) is a differential equation in which φ, x, and $d\varphi/dt$ are the variables. With the armature at rest, x is constant, and (2) may be solved to give the initial field development in operate, or decay in release. When the armature is moving, the variation of φ and x is governed jointly by (2) and the mechanical force equation.

THE MECHANICAL EQUATION

The forces acting to accelerate the armature and associated moving parts, of effective mass m, are the magnetic pull F and the forces exerted by the contact and other springs. The total spring force may be written as dV/dx, where V is the potential energy stored in the springs, expressed as a function of x. The force equation may be written in the form:

$$F + m\frac{d^2x}{dt^2} + \frac{dV}{dx} = 0. \tag{3}$$

As x measures the gap opening, the velocity dx/dt is positive as the gap opens, and negative as it closes. The pull is directed toward the closed position and therefore tends to algebraically decrease the velocity. The spring force tends to open the gap and to algebraically increase the velocity, as V decreases with x, and dV/dx is negative.

On the assumption that the dynamic and static field patterns are the same, the pull F can be evaluated from the static magnetization relations. F is a function of φ and x, the variables of the flux development equation (2). Thus the dynamic performance is governed jointly by (2) and (3), to which the concurrent variation in φ and x with time must conform. To complete the formulation there are required explicit expressions for $\mathcal{R}$ and F in terms of φ and x.

RELUCTANCE AND PULL

As shown in a companion article,[1] the magnetization relations for most electromagnets conform to the simple magnetic circuit of Fig. 1. Through the region of linear magnetization, in which the flux density is below that producing incipient saturation, the reluctances $\mathcal{R}_0$ and $\mathcal{R}_L$ and the area A are constants. If the magnetization relations conform to this schematic, the total reluctance $\mathcal{R}$ is given by:

$$\mathcal{R} = \frac{\mathcal{R}_L\left(\mathcal{R}_0 + \dfrac{x}{A}\right)}{\mathcal{R}_L + \mathcal{R}_0 + \dfrac{x}{A}}. \tag{4}$$

The constants $\mathcal{R}_0$, $\mathcal{R}_L$, and A of Fig. 1 and equation (4) are called the equivalent values of the closed gap reluctance, the leakage reluctance, and the pole face area respectively. Procedures for estimating these quantities from the magnet's dimensions and material constants are described in the article cited above,[1] while methods for their experimental evaluation are described in another article in this issue.[2] When equation (4) applies, the pull F is given by:

$$F = \frac{(\mathcal{R}_L \varphi)^2}{8\pi A \left(\mathcal{R}_L + \mathcal{R}_0 + \frac{x}{A}\right)^2}. \tag{5}$$

In normal operation, the flux level lies in the region of linear magnetization, and equations (4) and (5) apply through the greater part of the period of flux development determining the operate time. In what fol-

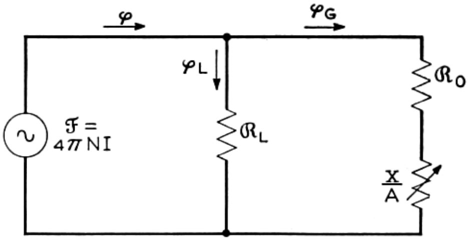

Fig. 1 — Equivalent magnetic circuit of an electromagnet.

lows, some simplification of notation is obtained by writing x_0 for $A\mathcal{R}_0$, and C_L for $(\mathcal{R}_0 + \mathcal{R}_L)/\mathcal{R}_0$, so that these equations become:

$$\mathcal{R} = \mathcal{R}_0 \frac{(C_L - 1)(x_0 + x)}{C_L x_0 + x}, \tag{4A}$$

$$F = \left(\frac{(C_L - 1)x_0}{C_L x_0 + x}\right)^2 \frac{\varphi^2}{8\pi A}. \tag{5A}$$

Thus the dynamic relations are given by (2) and (3), in which $\mathcal{R}$ and F are given by (4) and (5) respectively. The magnetic circuit constants can be evaluated by the methods cited above, and the other constant terms are known or given quantities, with the exception of the eddy current conductance G_E. Evaluation of this term requires determination of the conditions under which such a constant term can adequately represent the effects of eddy currents.

EDDY CURRENT CONDUCTANCE

Fig. 2 shows a simple electromagnet with a cylindrical core of length ℓ and diameter D. If $\mathcal{R}$ is the reluctance, the magnetomotive force $\mathcal{F}_C$ of the winding current results in a flux $\varphi_C = \mathfrak{F}_C/\mathfrak{R}$, uniformly distributed across the core cross section. The leakage paths cause some longitudinal variation in φ, which can be neglected for the present purpose. If the flux is varying, eddy currents flow in circular paths with a current density j, which varies with the radius r. These give an increment to the total mmf varying from zero at the surface to a maximum at the center, producing a corresponding variation in the density of the total flux. If $\mathfrak{F}_C$ is large compared with the maximum magnetomotive force produced by the eddy currents, however, the density is nearly constant, and its rate of change is nearly constant throughout the core cross section. To the extent that this condition is satisfied, the effect of the eddy currents can be determined as follows.

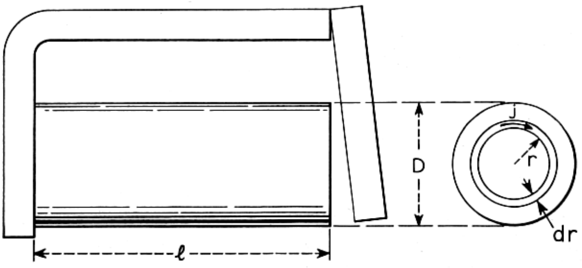

Fig. 2 — Eddy current paths in the core of an electromagnet.

The current in a cylindrical shell of radius r and differential thickness dr is $j\ell\, dr$. This links a part of the field proportional to the area enclosed, or $4r^2\varphi/D^2$, which varies, by hypothesis, at the same proportional rate as the total field. The resistance of the shell is $2\pi r\rho/(\ell\, dr.)$, where ρ is the resistivity of the material. The voltage equation for the shell circuit is therefore:

$$j\ell\, dr\, \frac{2\pi r\rho}{\ell\, dr} + \frac{4r^2}{D^2}\frac{d\varphi}{dt} = 0,$$

and hence:

$$j = -\frac{2r}{\rho\pi D^2}\frac{d\varphi}{dt}.$$

The magnetomotive force of the shell is its current multiplied by 4π. This produces a flux increment $d\varphi$ in the area enclosed inversely proportional to the reluctance of the tubes of induction within this area. This reluctance may be taken as inversely proportional to the area, as would be the case in a closed magnetic circuit of uniform cross section. Then the flux increment produced by each shell is given by:

$$d\varphi = 4\pi j \ell \, dr \, \frac{4r^2}{D^2 \mathcal{R}} .$$

On substituting the preceding expression for the current density j, and evaluating the integral $\int_0^{D/2} d\varphi$, the total flux φ_E produced by the eddy currents is found to be:

$$\varphi_E = -\frac{\ell}{2\rho\mathcal{R}} \frac{d\varphi}{dt} . \tag{6}$$

This is identical in form with the expression for one of the linking circuits assumed in deriving equation (2), with $\mathcal{R}\varphi_E$ equal to $\mathfrak{F}_E$, and $4\pi G_E$ given by $\ell/(2\rho)$. Thus to the extent that the assumption of a uniform flux distribution is valid, G_E is given by:

$$G_E = \frac{\ell}{8\pi\rho} . \tag{7}$$

This expression applies to a round core. A parallel approximation may be obtained for a rectangular rod by considering it as made up of rectangular shells of differential thickness, having their sides in the same ratio as the rod. By treating the perimeter of each shell as corresponding to the circumference of the circular shell in the cylinder case, with the area enclosed corresponding to that enclosed by the circular shell, the following expression is obtained for the conductance of a rectangular rod whose sides are in the ratio k:

$$G_E = \frac{\ell}{8\pi\rho} \frac{\sqrt{\pi k}}{1 + k} . \tag{8}$$

This shows that the effect of using a rectangular section is equivalent to increasing the resistivity of the material. A similar effect is obtained by subdividing the core section, as in laminated construction. If the core, for example, were made of two similar round rods, equation (6) would apply to each, and the effective mmf acting on both would be the value of $\mathcal{R}\varphi_E$ given by this expression, with φ in $d\varphi/dt$ equal to half the total flux, so that the resulting expression for G_E would be half that given by (6). This argument can be generalized to show that, aside from the effect of changes in section shape, the effect of subdivision is to reduce G_E in proportion to the number of subdivisions.

The approximate validity of these expressions for G_E rests on the assumption that the eddy current magnetomotive force is minor compared with that of the coil or other external circuit. From the derivation of (2), the component mmf's are proportional to the corresponding terms in G, so the preceding expressions for G_E are valid only if G_E is small compared with $G_C(N^2/R)$. In all but exceptional cases, this condition is satisfied in operation, and in release with a short circuited winding or a slow release sleeve. In these cases, the effect of eddy currents is adequately represented by a constant G_E term in (2). The exact values of G_E applying are in only approximate agreement with those given by (7) and (8), as the derivation of these expressions ignores the variation in flux density along the length of the core, and the eddy current effects in the armature and return path.

EDDY CURRENTS IN RELEASE

In normal release, the winding circuit is open, and the only effective magnetomotive force is that of the eddy currents. Evidently, the field in the outer layers must collapse almost instantly, while that in the center of the core is sustained by eddy currents in paths whose mean conductance, per line linked, is higher than that applying to a uniform field.

The relations applying to a closed path of uniform section can be formulated in differential form, and the solution for a cylindrical section has been given by Wwedensky.[3] For decay from an initially uniform field, the expression for the flux as a function of time is a series of exponential terms with progressively smaller time constants. The first term represents the most persistent part of the flux, and represents a field varying from zero at the surface to a maximum at the center, comprising 69 per cent of the initial field. Its time constant corresponds to an effective value of G_E 35 per cent larger than that given by (7). Somewhat similar relations must apply to an electromagnet, causing a time variation in the pattern of the field, not only radially, as in a closed uniform path, but in the longitudinal variation and in the division of the field between the leakage and armature paths.

An experimental study of flux development and decay in relays has been reported by M. A. Logan.[4] His results agree with this discussion in showing G_E in (2) to be effectively a constant, provided G_E/G_C is less than 0.2, as in the operation of most relays. An empirical expression is given for an effective value of $G_C + G_E$ which provides a correction factor applicable for small values of G_E/G_C. The results for normal release show the field decay to have the general character of Wwedensky's solution, and an empirical expression is given which agrees with observed results. This is primarily of interest in connection with the voltage induced in the coil and imposed on the contact opening the coil circuit. Because of the changing field pattern, the gap field which determines the pull has a different, though similar, decay rate.

These considerations, as supported by Wwedensky's relations and Logan's results, indicate that the rate of pull decay in normal release is faster, and only roughly of the same order of magnitude, as that estimated on the assumption that (2) applies, with values of G_E similar to those applying in operation.

LIMITATIONS OF THE ANALYSIS

The validity of equations (2) through (5) rests on the assumption that the pattern of the dynamic field is essentially that of the static field to which the magnetization relations apply. The above discussion of the eddy currents indicates that this assumption is valid when G_E is a minor term in G. This condition is approximately satisfied in normal operation. In open circuit release, however, the condition is not satisfied, and equation (2) is only a crude first approximation to the controlling relation.

The use of (4) as an expression for the reluctance $\mathcal{R}$ rests on the further assumption that the magnetization is linear, a condition only satisfied in the low density region. The initial and controlling stages of operation are usually complete before the field passes out of the low density region, and (4) is therefore applicable to the operate case. A different expression for $\mathcal{R}$ is required in the release case, as discussed in Section 7.

2 Character of the Operate Solution

GRAPHICAL REPRESENTATION

Some understanding of the relations applying to operation may be obtained from their graphical representation in Figs. 3 and 4. In these two figures the path followed by the variables in dyamic operation is indicated by the dotted lines, with the dots spaced to indicate equal time intervals between them. Fig. 3 shows the φ versus $\mathfrak{F}$ relation, referred to the steady state magnetization curves for various values of x, with $x = 0$ corresponding to the operated position, and $x = x_1$ to the initial unoperated position. Fig. 4 shows the dynamic F versus x relation, together with the load curve (bounding the cross hatched area V) and the steady state pull curve for the applied mmf $\mathfrak{F}_S$.

The flux and pull increase together with the armature at rest at x_1 until the pull equals the back tension at the point 1. In the earlier motion, 1–2, the velocity is small, and the reluctance (from (4)) changes slowly with x so the motion has little effect on the rate of flux develop-

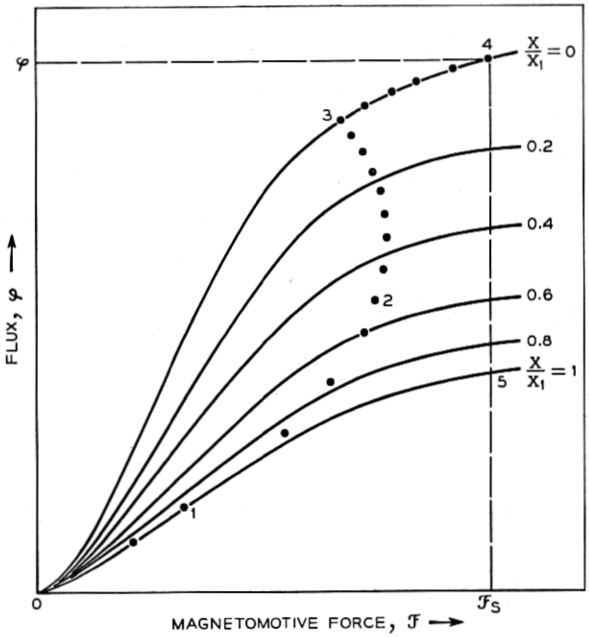

Fig. 3 — Field energy relations in the operation of an electromagnet.

ment. In the later motion, 2–3, the reluctance changes more rapidly with x, and the velocity is high, increasing $d\varphi/dt$ so as to result in a temporary decrease in $\mathfrak{F}$. Operation is complete at 3, with φ and F still below their steady state values at 4, which they then approach exponentially with $x = 0$.

The mechanical work done in operation is represented in Fig. 4 by the area under the dynamic pull curve (dotted line), and in Fig. 3 by the area bounded by 0–1–2–3–0. This, of course, is less than the work that would be done if $\mathfrak{F}$ were equal to its steady state value $\mathfrak{F}_S$ throughout the motion, represented by the area under the $\mathfrak{F}_S$ curve in Fig. 4, and by the loop 0–3–4–5–0 in Fig. 3. In Fig. 4 it can be seen that the work done exceeds the static load V by an amount represented by the shaded area T, corresponding to the kinetic energy of the armature and the parts that move with it. At the end of the stroke this energy is partly dissipated in impact, and partly transferred to vibratory motion of the relay and its parts.

The initial flux development, 0–1 is governed wholly by equation (2). This same relation dominates in controlling the performance in the early travel 1–2, in which the velocity is small, and the reluctance changes slowly. In the later travel, where the velocity is high and the reluctance changes rapidly, equation (3) is controlling, with F corresponding to a more nearly constant value of φ.

METHODS OF SOLUTION

Formally, the expression for φ given by combining (3) and (5) may be substituted in (2), eliminating φ and giving a third order non-linear differential equation in x and t. No analytical solution to this equation is known, even for the idealized case of a linear load curve. Specific solutions may be obtained by means of computing devices, but there are too many parameters involved to prepare a useful library of specific solutions that would serve as a general treatment.

Approximate analytical solutions can be obtained by a step by step process, starting from the initial stage of flux development with the armature at rest. These solutions are relatively simple in form and application, and provide an analytical statement of the relations between performance and design parameters, from which conclusions can be drawn as to design features and operating conditions favoring a desired

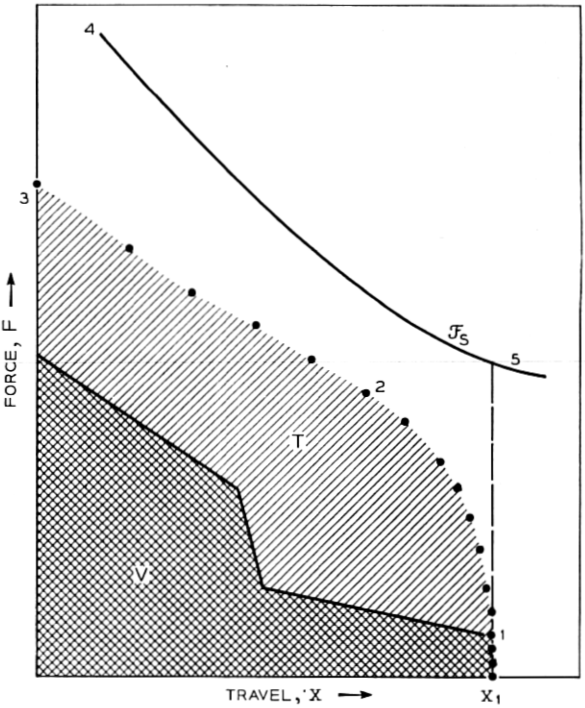

Fig. 4 — Load and pull relations in the operation of an electromagnet.

type of performance. For development purposes, an approximate solution indicating such optimum conditions is preferred to a rigorous solution in a form too complex for such use.

Three such approximate solutions for the operate case are described below. The first of these, the single stage approximation, is an application of the solution to the flux rise equation (2), neglecting the armature motion. It is applicable in cases of slow operation controlled by the rate at which the flux development provides sufficient pull to operate the load. The other two solutions are more generally applicable, particularly for relatively fast operation. One of these, the two-stage approximation, is the simpler in form, while the other, the three stage approximation, is the more accurate.

3 Single Stage Approximation

The initial stage of flux development with the armature at rest ends when the pull equals the back tension. During this stage the flux development is governed wholly by (2), with $x = x_1$, the initial gap, and $\mathcal{R} = \mathcal{R}_1$, as given by (4) for $x = x_1$. Writing $\varphi_1 \mathcal{R}_1$ for $\mathfrak{F}_S$, equation (2) becomes:

$$t = \frac{4\pi G}{\mathcal{R}_1} \int_0^{\varphi} \frac{d\varphi}{\varphi_1 - \varphi},$$

which on integration gives:

$$t = \frac{4\pi G}{\mathcal{R}_1} \ln \frac{1}{1 - v}, \tag{9}$$

where $v = \varphi/\varphi_1$, the ratio of the flux attained at time t to the steady state flux φ_1 or $\mathfrak{F}_S/\mathcal{R}_1$. The mmf ratio $\mathfrak{F}/\mathfrak{F}_S$ is also equal to v.

Equation (9) applies rigorously only while the armature is at rest. It is, however, also a close approximation for the initial motion, in which the velocity is small and x differs little from x_1. If the rate of flux development is slow ($G/\mathcal{R}_1$ large) the armature moves slowly with the pull only slightly in excess of the load curve. In this case the inertia term $m \cdot d^2x/dt^2$ in (3) is minor, and the pull F nearly equals the static load dV/dx. Thus the operate time is approximately equal to the time required to develop a pull which exceeds the static load at all points in the travel. This pull is attained at the just operate current or ampere turn value, corresponding to the minimum mmf for static operation, $\mathfrak{F}_0$. Assuming that the armature moves as the flux and pull develop, the operate time is that required for $\mathfrak{F}$ to equal $\mathfrak{F}_0$, and is given by equation

(9) for $v = \mathfrak{F}_0/\mathfrak{F}_S$. While this is a crude approximation, it provides satisfactory estimates of the time for slow operation.

EDDY CURRENT CONDUCTANCE DETERMINATION

Equation (9) is used in one experimental method for the evaluation of the eddy current conductance G_E. In this, measurements are made of the time at which motion starts for various values of the coil constant G_C. The latter may be varied by adding series resistance, and the applied voltage adjusted to maintain the current and hence $\mathfrak{F}_S$ constant in successive measurements. The time t for motion to start may be determined from shadowgraph measurements. For a constant back tension, the pull and hence the value of v or $\mathfrak{F}/\mathfrak{F}_S$ when motion starts is constant in these measurements. From (9), t is then directly proportional to $G_C + G_E$, so that a plot of t vs. G_C should be linear, with a negative intercept on the G_C axis numerically equal to G_E. Experimentally, an approximately linear relation is obtained in this way, provided the values of G_C covered are in excess of $5G_E$, in agreement with the discussion of Section 1. Values of G_E thus determined are consistent with those obtained from other measurements, and in approximate agreement with estimates obtained from (7) or (8).

COIL CONSTANT

In equation (9) G_E and $\mathfrak{R}_1$ are determined by the magnetic design. The latter, together with the load, determines $\mathfrak{F}_0$, the just operate mmf, or $4\pi(\mathrm{NI})_0$. As v equals $\mathfrak{F}_0/\mathfrak{F}_S$, or $(NI)_0/(NI)$, the only quantities not fixed by the design and the load requirement are G_C and the steady state ampere turn value NI. As the square of this latter quantity is equal to $G_C \cdot I^2R$, equation (9) may be written in the form:

$$t_0 = \frac{4\pi}{\mathfrak{R}_1}\left(G_E + \frac{(NI)_0^2}{v^2I^2R}\right)\ln\frac{1}{1-v}. \tag{10}$$

The steady state power I^2R is either determined by circuit requirements, or chosen with reference to economy of power consumption. With I^2R fixed, the only independent quantity in (10) is v, which is determined by the choice of the coil constant G_C. Neglecting G_E, the time is proportional to

$$\frac{1}{v^2}\ln\frac{1}{1-v},$$

which is shown plotted against v in Fig. 5. This figure includes a curve giving corresponding values of ln $1/(1 - v)$, which may be used in specific cases to determine the correction corresponding to the term in G_E.

In most cases of slow operation, the operate time is of little importance, and G_C is chosen to make NI greater than $(NI)_0$ in all cases: i.e., after allowing for possible variations in $(NI)_0$ resulting from variations in the load and in the magnetic characteristics. These variations result in a variation in v, and the corresponding variation in time is shown by the curve of Fig. 5. If, however, it is desired to minimize the operate time for a given power input, G_C should be chosen to give the value of v (0.715) corresponding to the minimum of the curve. As this minimum is broad, variations in v, corresponding to those in $(NI)_0$, produce little change in tne operate time.

4 Two Stage Approximation

Unless the rate of flux development is very slow, as assumed in the single stage approximation, the pull attained in the early travel is in excess of the load, and the kinetic energy T is a considerable part of the total work output $V + T$. As a first approximation to this case, the operate time can be computed as though operation occurred in two successive stages: (1) a stage of flux development with the armature at rest in the unoperated position ($x = x_1$), and (2) a stage of motion, in which

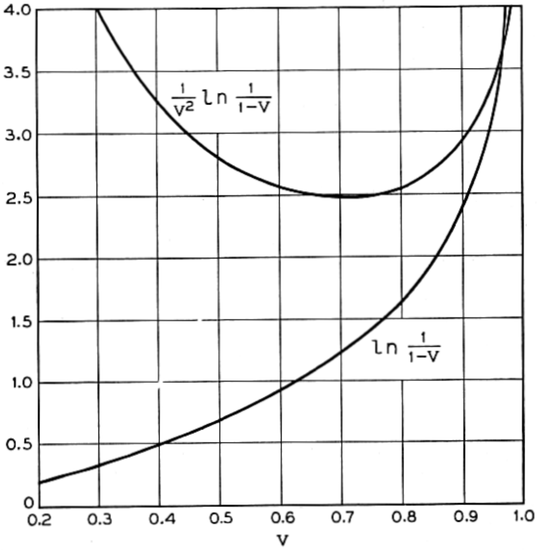

Fig. 5 — Relations for estimating time of flux development.

the flux remains constant at the value attained at the end of the first stage. This approximation necessarily over-estimates the operate time, as it takes the two stages as occurring in sequence, whereas they actually overlap.

Let t_1 be the time of the first stage, and t_2 the time of the second stage, where t_2 is the time for the motion from x_1 to some other armature position x_2, measuring the completion of operation. The first stage ends when the flux has attained some value $v\varphi_1$, where $\varphi_1 = \mathfrak{F}_S/\mathfrak{R}_1$, the steady state flux for the open gap, and v remains to be determined. Then t_1 is given by (9) for this value of v.

By hypothesis, the flux remains constant at $v\varphi_1$ throughout the second stage, and the corresponding pull is then given by (5A) for $\varphi = v\varphi_1$. The work $V + T$ is the integral of $F \cdot dx$ from x_1 to x_2, given by the integral of (5A). Hence:

$$V + T = \frac{(C_L - 1)v^2\varphi_1^2}{8\pi A} \frac{x_0^2(x_1 - x_2)}{(C_Lx_0 + x_1)(C_Lx_0 + x_2)}. \tag{11}$$

In this equation x_0/A may be replaced by the expression for $\mathfrak{R}_0$ given by (4A) for $x = x_1$, and $G_C \cdot I^2R$ substituted for $(NI)^2$ in the resulting equation, which then becomes:

$$v^2 = \frac{\mathfrak{R}_1 C_W}{2\pi G_C I^2 R}(V + T), \tag{12}$$

where C_W is given by:

$$C_W = \frac{(x_0 + x_1)(C_Lx_0 + x_2)}{(C_L - 1)x_0(x_1 - x_2)}. \tag{13}$$

The time t_2 of the second stage depends upon the difference between the pull and load curves. As a representative condition, this may be taken as approximately constant. For such uniform acceleration of the effective mass m through the distance $x_1 - x_2$, the kinetic energy T equals $2m(x_1 - x_2)^2/t_2^2$, giving an expression for t_2 in terms of T. As the time t_1 of the first stage is given by (9), the total operate time t_0, or $t_1 + t_2$ is given by:

$$t_0 = \frac{4\pi(G_E + G_C)}{\mathfrak{R}_1} \ln \frac{1}{1 - v} + \left(\frac{2m(x_1 - x_2)^2}{T}\right)^{\frac{1}{2}}. \tag{14}$$

In this approximate expression for the operate time, v and T are as yet undetermined quantities, related by equation (12), in which G_C is the value of N^2/R for the coil used. The two stage operation assumed in deriving this expression corresponds to the existence of a restraint which holds the armature at rest until it attains the flux level $v\varphi_1$. The approximation will most nearly approach the true relation if the effect of this restraint is minimized, and v taken at the value which minimizes t_0. This minimum value can be determined by trial, using the curves of Fig. 5 to compute t_0 for several values of v, and taking the minimum from the resulting plot of t_0 versus v.

In development studies, the operate time of interest is usually not that for an assigned winding, but that for the winding giving minimum time for a given power input. Substituting in (14) the expression for G_C given by (12), there is obtained the equation:

$$t_0 = \left(t_E + \frac{2C_W}{v^2}\,\frac{V+T}{I^2R}\right)\ln\frac{1}{1-v} + \left(\frac{2m(x_1-x_2)^2}{T}\right)^{\frac{1}{2}},$$

where t_E is written for $4\pi G_E/\mathcal{R}_1$, the eddy current time constant. As G_C remains to be determined, this expression contains two independent variables, v and T, which are to be so chosen as to make t_0 a minimum. Equating the partial derivative with respect to T to zero, there is obtained the following expression for the value of T for which the time is a minimum:

$$T = 2m(x_1-x_2)^2\left(\frac{v^2I^2R}{4C_W\ln\dfrac{1}{1-v}}\right)^2.$$

On substituting this expression for T, the preceding equation becomes:

$$t_0 = \left(t_E + \frac{2C_W}{v^2}\,\frac{V}{I^2R}\right)\ln\frac{1}{1-v} + \left(\frac{27C_W}{v^2}\ln\frac{1}{1-v}\,\frac{m(x_1-x_2)^2}{I^2R}\right)^{\frac{1}{3}}. \quad (15)$$

If the value of v which minimizes t_0 is determined, the resulting value of t_0 is the minimum operate time attainable by optimum coil design. In particular, if t_E is negligible, this minimum corresponds, from Fig. 5, to $v = 0.715$, for which

$$\frac{1}{v^2}\ln\frac{1}{1-v} = 2.5.$$

The corresponding value of G_C, the coil constant value making t_0 a minimum, is given by (12). In this use of equation (12), v and T are taken at the optimum values obtained as indicated above.

OPTIMUM COIL CONSTANT

As in the case of the single stage approximation, the optimum coil constant corresponds to a value for v of 0.715 when t_E is negligible. From the curves of Fig. 5, it can be seen that if t_E is not negligible, the optimum value of v is less than 0.715, and will be smaller the larger the ratio of the first term in (15), that in t_E, to the other two terms. From (12), a reduction in v corresponds to an increase in NI, and hence in the coil constant G_C (I^2R being given). Thus the optimum coil constant for fast operation is larger for electromagnets with long cores of low resistivity material, for which G_E and t_E are large, than for those with short cores of high resistivity material, for which these quantities are small.

The physical significance of the optimum coil constant can be deduced from the relations shown graphically in Fig. 4. For a given value of the power input I^2R, an increase in G_C increases both the steady state mmf $\mathfrak{F}_S$ and the time constant of flux development $4\pi G_C/\mathcal{R}_1$, increasing the upper limit to the attainable pull while reducing the rate at which this limit is approached. If the coil constant were so small that the area under the dynamic pull curve equalled the static work load V, the operate time would be infinite. On the other hand, an infinitely large coil constant would correspond to infinite inductance and an infinite operate time. Between these extremes lies the optimum value of the coil constant, giving minimum operate time. This is a broad optimum, near which a change in $\mathfrak{F}_S$ is compensated by a corresponding change in the rate of flux development, so that the realized dynamic pull curve is not affected by a small change in G_C.

In addition to the operate time, the value of G_C affects the final velocity of the armature, and thus its kinetic energy in impact with the core at the end of the stroke. This energy is dissipated in the relay and spring vibration associated with contact chatter. For values of G_C above the optimum, the higher inductance reduces the pull in the early travel, while the higher value of $\mathfrak{F}_S$ increases it in the later travel. The net effect is to increase both the operate time and the final velocity. Values of G_C above the optimum are therefore disadvantageous not only in slower operation, but in increased impact energy tending to cause contact chatter.

FACTORS CONTROLLING SPEED

Aside from the term in t_E, equation (15) shows that the minimum operate time, corresponding to the optimum value of v, is determined by the static load V, the inertia load measured by $m(x_1 - x_2)^2$, the steady state power I^2R, and the constant C_W. The latter is given by (13), and is the only quantity in (15), aside from t_E, which depends upon the magnetic design. In the range of values applying in practice, C_W is determined primarily by the leakage factor C_L, measuring the ratio of leakage to useful flux. In most practical cases, x_2 is small (zero for complete operation), C_L equals or exceeds 4 ($\mathcal{R}_L$ equals or exceeds 3 $\mathcal{R}_0$), and x_1/x_0 lies in the range from 1 to 3. For $4 < C_L < 10$ and $1 < x_1/x_0 < 3$, $1.5 < C_W < 2.6$. In practice therefore C_W has a value close to 2.0 for most electromagnets.

The term in V, the second term in (15), varies inversely as the power I^2R, while the third, or inertia term, varies as the cube root of the power. Hence the second term tends to dominate at low values of I^2R, and the third term tends to dominate at high values. For a low power input therefore the operate time is controlled by the load V, and varies inversely as the power input, while for a high power input, the operate tine is controlled by the inertia term, $m(x_1 - x_2)^2$, and varies inversely as the cube root of the power. In the load controlled case, the time varies directly as the load; in the mass controlled case it varies as the cube root of the mass and as the two-thirds power of the travel $x_1 - x_2$. The eddy current term is very nearly a constant increment to the other two terms, and is therefore relatively more important, the faster the operation.

PRELIMINARY TIMING ESTIMATES

Within the limits of accuracy to which this two stage approximation applies, the effect of the magnet design upon the operate time appears only in t_E and C_W. If the former is neglected, the optimum value of v is 0.715, and

$$\frac{1}{v^2} \ln \frac{1}{1 - v} = 2.5 .$$

Taking C_W as having the representative value of 2.0, (15) reduces to:

$$t_0 = \frac{10V}{I^2R} + \left(\frac{135m(x_1 - x_2)^2}{I^2R}\right)^{\frac{1}{3}}. \tag{16}$$

This simple approximation provides rough estimates of the operate times attainable with any electromagnet for a given load and power input. As an illustration, consider a typical relay spring load involving a travel of 40 mil-in (0.1 cm) with a level of spring force of 100 gm (10^5 dynes), so that $V = 10^4$ ergs. Let the effective mass m of the moving parts be 10 gm. For a steady state power input of 0.5 watts (5×10^6 ergs sec.), the two terms of (16) have values of 0.020 sec and 0.014 sec, respectively, so that t_0 equals 0.034 sec. For an input of 5 watts, the two terms become 0.002 sec and 0.007 sec, respectively, and t_0 equals 0.009 sec. The neglected t_E term of (15) might amount to an increment of 0.005 sec for a solid core of magnetic iron, but would be proportionally smaller for higher resistivity material. In either case, this increment would be of little consequence in the low power case, but would materially affect the operate time in the high power case.

5 Three Stage Approximation

The following analysis provides greater accuracy in estimating operate time than the two stage approximation, together with a more accurate representation of the relations between performance and the controlling variables. It differs from the two stage approximation in treating the initial motion as a separate stage of operation. The three stages of operation thus become: (1) Increase of the flux to the value at which the pull equals the back tension, with the armature at rest, (2) flux development and concurrent armature acceleration, assuming equations (2) and (3) to apply, with the approximation that the variations of $\mathcal{R}$ and $\mathfrak{F}$ with x are ignored, and (3) the later motion, which is treated in the same manner as the second stage of the two stage approximation.

Formally, the second stage should be restricted to a very small part of the total armature motion, in order to minimize the change in $\mathcal{R}$ which is ignored. Relatively minor error, however, is introduced in ignoring the variation in $\mathcal{R}$ through as much as half the total travel. The spring force is taken as constant through the second stage. In the relay case, this is approximately true for the initial travel prior to the actuation of the contacts, which results in an abrupt increase in the spring load. Thus the second stage can be taken to extend to the travel at which contact actuation first occurs. If all contacts are actuated near the same point in the travel, the end of the second stage coincides with complete operation, and the third stage need not be considered. If the contacts are spread out, the third stage coincides with the stagger time, or time between operation of the first and last contacts. The operate time therefore consists either entirely or principally of the time for the first two stages.

As before let:

x_1 be the initial (open) gap, for which $\mathcal{R} = \mathcal{R}_1$,

F_1 be the back tension (constant load in the first two stages),

t_C be the initial coil time constant, $\dfrac{4\pi}{\mathcal{R}_1} \cdot \dfrac{N^2}{\mathrm{R}}$,

t_E be the initial eddy current time constant, $\dfrac{4\pi}{\mathcal{R}_1} G_E$,

$\mathfrak{F}_S$ be the steady state mmf, $4\pi NI$, and φ_1 the corresponding flux for $x = x_1$, or $\mathfrak{F}_S/\mathcal{R}_1$.

FIRST STAGE

The pull for $x = x_1$ is given by (5A), which may be written:

$$F = \frac{(v\varphi_1)^2}{8\pi A_1}, \tag{17}$$

where A_1 is given by:

$$A_1 = \left(\frac{C_L x_0 + x_1}{x_0(C_L - 1)}\right)^2 A. \tag{18}$$

In the first stage the flux increases to $v_1\varphi_1$, where v_1 is given by (17) for $F = F_1$. Then the time t_1 of the first stage is given by (9) for $v = v_1$, or by:

$$t_1 = (t_C + t_E) \ln \frac{1}{1 - v_1}. \tag{19}$$

The solution for the first stage therefore determines v_1 and t_1.

SECOND STAGE

It is convenient to write the equations for the second stage in terms of the ratio z defined by:

$$z = \frac{t}{t_C + t_E},$$

where t is now measured from the start of the second stage. Neglecting the change in reluctance with armature motion, the ratio φ/φ_1 or v is given by (9), except that for $v = v_1$ at $t = 0$ the relation must be written:

$$v = 1 - (1 - v_1)e^{-z}. \tag{20}$$

As $\varphi_1 = \mathfrak{F}_S/\mathcal{R}_1$, or $4\pi NI/\mathcal{R}_1$, and $t_C = \frac{4\pi}{\mathcal{R}_1} \cdot \frac{N^2}{R}$,

$$\varphi_1^2 = \frac{4\pi}{\mathcal{R}_1} I^2R \cdot t_C.$$

Taking the pull in the second stage as given by the expression applying for $x = x_1$, and substituting the preceding expressions for v and φ_1, there is obtained:

$$F = \frac{I^2Rt_C}{2\mathcal{R}_1 A_1} (1 - (1 - v_1)e^{-z})^2. \tag{21}$$

For the second stage dV/dx is constant at $-F_1$ throughout, and (3) reduces to $m\, d^2x/dt^2 = F - F_1$, or:

$$\frac{d^2x}{dt^2} = \frac{I^2Rt_C}{2m\mathfrak{R}_1A_1}((1 - (1 - v_1)e^{-z})^2 - v_1^2).$$

Substituting $(t_C + t_E)\, z$ for t, this equation may be integrated with respect to t to obtain the following expressions for the velocity $\dot{x}$ and the travel x in the second stage:

$$\dot{x} = \frac{I^2Rt_C(t_C + t_E)}{2m\mathfrak{R}_1A_1} f_1(v_1, z), \tag{22}$$

$$x_1 - x = \frac{I^2Rt_C(t_C + t_E)^2}{2m\mathfrak{R}_1A_1} f_2(v_1, z). \tag{23}$$

The functions $f_1(v_1, z)$ and $f_2(v_1, z)$ are the integrals with respect to z of the bracketed term in the preceding equation. These functions can be evaluated from the curves of Fig. 6, which shows them plotted against z for several values of v_1.

For a specific case, the values of v_1 and t_1 will have been determined from the relations for the first stage. Let x_2 be the travel at the end of the second stage. Then the value of $f_2(v_1, z)$ for $x = x_2$ is given by (23), and z_2, the corresponding value of z, can be read from the curve of Fig. 6 for the value of v_1 applying. Then the time for the second stage, t_2, is given by $z_2(t_C + t_E)$. For $z = z_2$, (20) gives v_2, the value of v at the end of the second stage. This in turn gives the corresponding value of the flux $v_2\varphi_1$, and, from (21), the pull F at the end of this stage. The velocity at the end of this stage is given by (22), with $f_1(v_1, z)$ read from the curves of Fig. 6 for $z = z_2$.

THIRD STAGE

The total operate time is $t_1 + t_2 + t_3$, where t_3 is the time for the third stage. In the relay case, this is the stagger time between the operation of the first and last contacts. A first, and frequently adequate approximation to t_3, is given by assuming the velocity in the third stage to be constant at the value attained at the end of the second stage. For a more exact determination, particularly in determining the final velocity for complete operation, it may be assumed that the flux in the third stage is constant at the value $v_2\varphi_1$ attained at the end of the second stage. The mechanical output $V + T$, in the third stage is then given by equation (11), with v taken as v_2, and x_1 and x_2 replaced, respectively, by x_2 and x_3, the latter denoting the travel at the end of the third stage (zero for complete operation). Knowing the spring load V between x_2 and x_3, t_3 and the final velocity can be computed from the increase in kinetic energy T on the assumption of uniform acceleration.

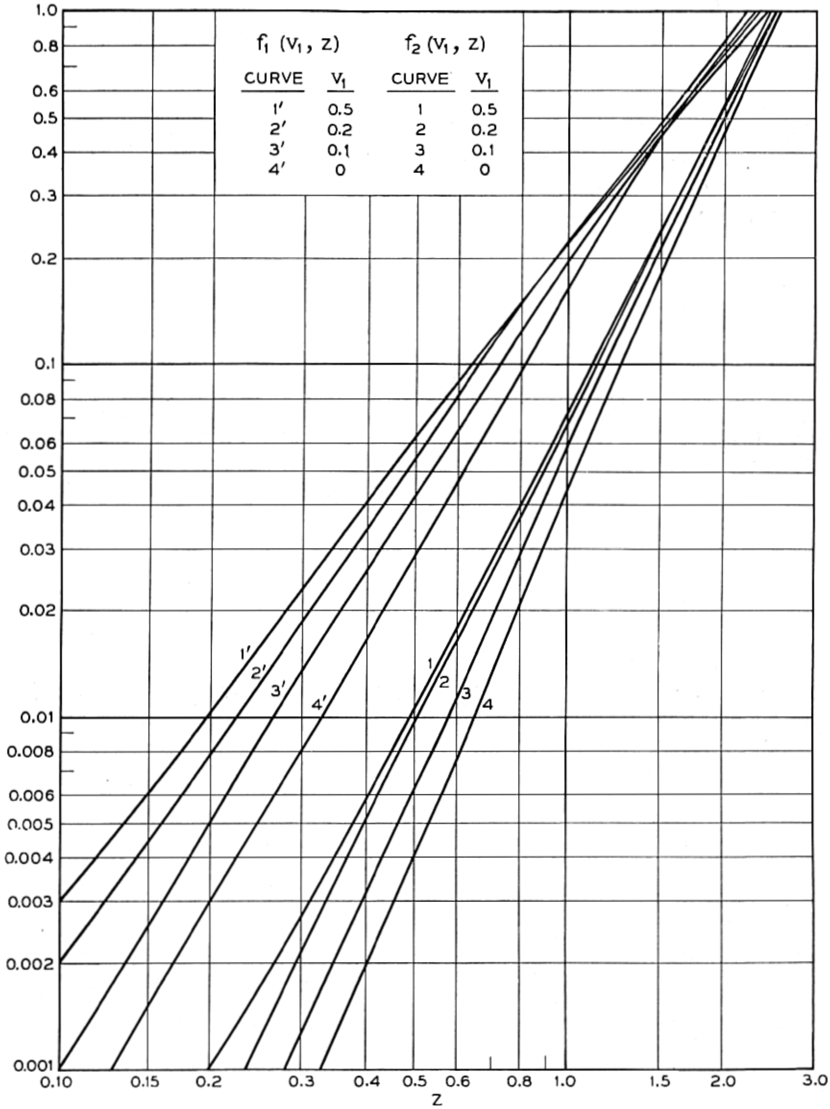

Fig. 6 — Relations for initial motion in operation.

FACTORS AFFECTING OPERATE PERFORMANCE

The dominant term in the operate time is the time t_2 for the second stage, which also determines the velocity and pull level in the third stage, and hence the final velocity. As $t_2 = z_2(t_C + t_E)$, equation (23) gives the following expression for t_2:

$$t_2^3 = \frac{2mA_1\mathcal{R}_1(x_1 - x_2)}{I^2R} \frac{t_C + t_E}{t_C} \frac{z_2^3}{f_2(v_1, z_2)}. \tag{24}$$

As can be seen in Fig. 6, $f_2(v_1, z)$ is, to a rough first approximation, proportional to z^3. To the extent that this approximation holds, t_2 is proportional to the cube root of $mA_1\mathcal{R}_1(x_1 - x_2)/(I^2R)$, as for the last term in (15) and (16), except that $(x_1 - x_2)^2$ is replaced by $A_1\mathcal{R}_1(x_1 - x_2)$. From (4A) and (18), $A_1\mathcal{R}_1$ is given by:

$$A_1\mathcal{R}_1 = \frac{(x_0 + x_1)(C_Lx_0 + x_1)}{(C - 1)x_0}.$$

Here $x_0 = A\mathcal{R}_0$, and t_2 is a minimum for that pole face area A for which $A_1\mathcal{R}_1$ is a minimum. This condition, as determined by equating the derivative of the preceding expression with respect to x_0 to zero, is given by:

$$x_1^2 = C_Lx_0^2,$$

or $x_1/A = \sqrt{C_L} \cdot \mathcal{R}_0$, where A is the optimum pole face area for fast operation. This is a smaller pole face area than that for maximum sensitivity, for which x_1/A should equal $\mathcal{R}_0$, as shown in the companion article[1] cited above. If the pole face area satisfies the preceding condition, the expression for A_1 reduces to:

$$A_1 = \frac{\sqrt{C_L} + 1}{\sqrt{C_L} - 1} x_1,$$

and the term $A_1\mathcal{R}_1$ in (24) reduces to $x_1(x_1 - x_2)$ multiplied by the factor $(\sqrt{C_L} + 1)/(\sqrt{C_L} - 1)$. This leakage factor, which increases with the ratio of leakage to useful flux, is then the only term in (24) which varies with the design of the electromagnet.

If $f_2(v_1, z)$ were strictly proportional to z^3, equation (24) would be nearly independent of the coil constant N^2/R, to which t_C is proportional. As $f_2(v_1, z)$ departs from this cube law relation, and also varies with v_1, t_2 is not independent of the coil constant. The optimum coil constant is that which minimizes $t_1 + t_2$. In any specific case, a succession of values may be assumed for G_C, and t_1 and t_2 evaluated by the relations given above. The resulting relation between $t_1 + t_2$ and G_C is similar in character to that described above in connection with the two stage approximation.

6 Design for Fast Operation

The approximations discussed above provide a means for estimating the operate time attainable in specific cases, and for selecting certain design variables to obtain maximum speed. In particular, it has been shown that for a given relay, with a specified load and power input, a winding can be selected to minimize the operate time. Aside from this, the relations show that for a given relay, the operate time depends only upon the load and travel and the power input, varying inversely as the latter at low levels of power input, and inversely as its cube root at high levels.

Further conclusions can be drawn from these relations with reference to development studies of relays and other electromagnets. The spring load and travel may be considered as fixed requirements, so far as the magnet design is concerned. The available power may be fixed by circuit considerations, or it may be related to the design by a requirement that the winding dissipate this power in the holding interval, a condition that imposes a minimum size on the coil and the magnet structure. Subject to this and some other limitations, there is a design choice of the dimensions and configuration which determine the magnetic circuit constants, the mass of the armature, and the eddy current conductance.

The preceding discussion has shown that, with an optimum choice of pole face area, the magnetic characteristics affect the time only with respect to the leakage factor, the ratio of leakage to useful flux. This factor may be reduced by using a "tight" magnetic circuit, but if this is done the factor tends to vary directly as the length of the magnetic path and inversely as the separation of the core and return members in relation to their cross sections. The leakage may be minimized by using a square outline for the magnetic path. The optimum speed magnet then has a specific configuration in which all dimensions are fixed in relation to the cross section of the magnetic members. This dimension then determines the mass of the armature.

For this optimum configuration, the power, the spring load, the mass, and the travel determine a level of pull which minimizes the operate time. This pull requires a certain armature cross section if saturation is to be avoided, as the effective pole face area is fixed in relation to the cross section. If the resulting armature mass is minor compared with the mass of the load, the operate time attainable varies with the cube root of the applied power, as in the cases discussed above.

With increased power, however, the optimum pull and armature cross section increase, and must eventually reach a level where the armature mass becomes the dominant portion of the mass term. As this condition is approached, any increase in power is offset by an increase in armature mass, such that a lower limit is imposed on the operate time proportional to the travel, corresponding to an upper limit to the average armature velocity attainable with a neutral electromagnet. This upper limit is of the order of 100 cm/sec. Thus, for example, an operate time less than 0.25 millisec cannot be attained with an armature travel of 10 mil-in, no matter how large the steady state power applied.

7 Release Waiting Time

Like operation, release is made up of an initial stage of flux change with the armature at rest, followed by a stage of armature motion, and the total time is the sum of the waiting time and the motion time. In release, the waiting time is usually larger than the motion time.

There are three distinct circuit conditions under which release occurs. These are:

Normal, or unprotected release, in which the coil circuit is open, and the only magnetomotive force maintaining the field is that of the eddy currents.

Protected release, in which the coil circuit is closed through a protective shunt, usually comprising a condenser and a resistance in series. The magnetomotive force comprises that of the eddy currents and that of the coil circuit transient.

Slow release, in which a sleeve or short circuited winding is used to maintain the field and delay release. The magnetomotive force is predominantly that of the sleeve or winding current.

The slow release case is the only one of the three for which the flux decay relation is accurately represented by equation (2). In protected release, the coil current transient is controlled by the condenser, as discussed below. In normal release, the variation in the field linked by different eddy current paths results in the changes in the field pattern discussed in Section 1. As noted there, equation (2) applies to this case only as a very crude first approximation.

In all three cases, the relation between φ and $\mathfrak{F}$ applying is that for decreasing magnetization, as illustrated by the curve for $x = 0$ in Fig. 7. Unlike the linear relation applying in operate, corresponding to the constant reluctance given by (4), the decreasing magnetization curve has a hyperbolic character, and is asymptotic to the saturation flux φ''. Residual magnetism results in a residual flux φ_0, the intercept of the decreasing magnetization curve on the φ axis. An analytical treatment of the decreasing magnetization curve is given in a companion article,[5] where it is shown to provide a satisfactory basis for predicting the release time of slow release relays, the third case above. The other two cases, to which the following discussion is confined, involve both non-linear mag-

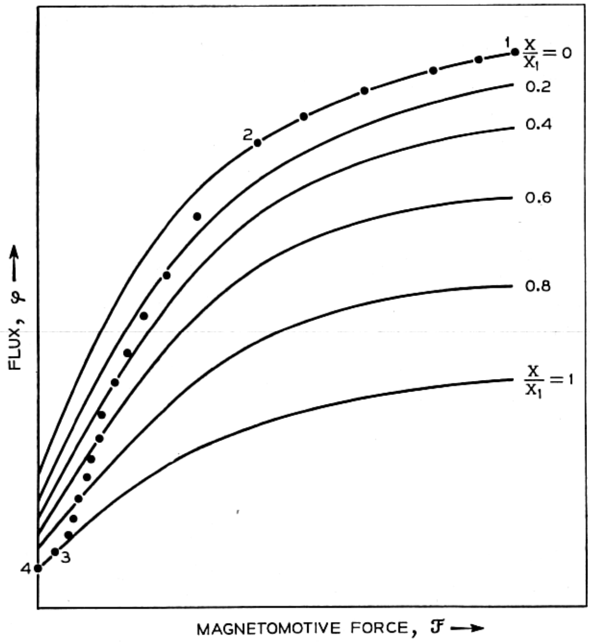

Fig. 7 — Field energy relations in release.

netization and a variable magnetic field pattern. In an approximation neglecting the latter effect, no advantage is derived from an accurate formulation of the former, and a linear approximation to the demagnetization curve may be employed.

NORMAL UNPROTECTED RELEASE

As indicated above, a rough approximation to the decay of the gap flux, and hence of the pull, may be obtained by assuming that this flux decays in conformity with equation (2), with $G = G_E$, where the eddy current conductance G_E has the same value as in operate. As a linear approximation to the demagnetization curve, the reluctance $\mathcal{R}$ can be taken as the closed gap reluctance $\mathcal{R}(0)$. Allowance may be made for residual magnetism by postulating a steady state magnetomotive force $\mathfrak{F}_0 = \mathcal{R}(0)\varphi_0$. Then integration of (2) gives the following expression for the time t at which the flux has decayed to the value φ from its initial value φ_1:

$$t = t_E \ln \frac{\varphi_1 - \varphi_0}{\varphi - \varphi_0}, \tag{25}$$

where $t_E = 4\pi G_E/\mathcal{R}(0)$. Unless φ/φ_0 is near unity, φ_0 may be omitted from this expression. If this is done, the expression may be re-written in terms of pull F in the form:

$$t = \frac{t_E}{2} \ln \frac{F_1}{F}, \tag{26}$$

where F is the pull at time t, and F_1 the initial pull. This expression follows from (25), for $\varphi_0 = 0$, because of the proportionality between F and φ^2. If F is the spring load or tension for the operated position, and F_1 the steady state pull there, (26) is an approximate expression for the release time.

PROTECTED RELEASE

The commonly employed method of contact protection is to use a condenser-resistance shunt connected either across the coil, as in Fig. 8, or across the contact. Except for the steady state voltage of the condenser, the circuit relations are the same for the two cases. As a first approximation, eddy currents may be represented by the current in a short circuited secondary, as in Fig. 8. The effect of such a secondary is determined by the value of t_E representing the ratio of its inductance to its resistance, as this determines the ratio of its contribution to the flux to the time rate of change of the total field.

If perfect coupling and linear magnetization are assumed to apply, the circuit equations for Fig. 8 can be solved and expressions obtained for the flux, current and condenser voltage as functions of time after the opening of the contact. These expressions are similar to those for the simple C-R-L circuit without a secondary, except that they involve the time constant t_E as well as CR and L/R. The discharge may be either exponential or a damped oscillation, but while the discharge of the simple C-R-L circuit is always oscillatory for small values of C, the discharge in the circuit of Fig. 8 is only oscillatory for an intermediate

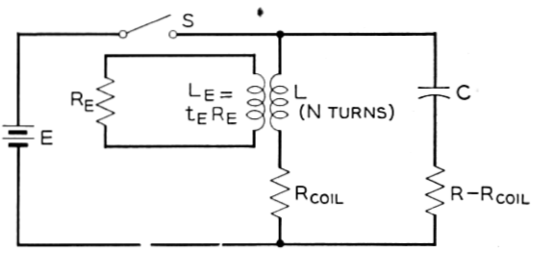

Fig. 8 — Coil circuit with capacitative contact protection.

range of capacity values. For large values of C, the discharge is damped by the coil resistance, and for small values of C it is damped by the currents in the secondary.

The latter case is essentially the condition applying for the small capacity values used for contact protection. Initially, the flux decay is opposed by the magnetomotive force resulting from both the eddy currents and the coil current. The latter discharges the condenser and develops a charge of opposite polarity. The initial rate of flux decay is therefore similar to that for exponential decay with a conductance equal to $G_E + N^2/R$, where R includes the external resistor as well as the coil resistance. The effect of the condenser charge is equivalent to a continuously increasing coil resistance, so that the decay approaches the rate that would apply for exponential decay with $G = G_E$, and the decay rate is then further increased by the reverse magnetomotive force resulting from the coil current reversal in the subsequent discharge of the condenser.

If CR is less than t_E, the effect of the coil circuit is to change the character of the flux versus time relation, without materially changing the time scale of the discharge. The initial decay is delayed, while the later decay is accelerated. Hence the release time is increased for a heavy load, corresponding to a high value of φ for release, while it may be slightly decreased for a light load, corresponding to a small value of φ. The utility of the coil-condenser circuit for contact protection results from the low initial rate of flux decay, which holds the induced voltage across the contact to a relatively low value in the initial stage of contact opening, when the contact separation is small.

Except initially, the predominant magnetomotive force is that of the eddy currents, and this results in a changing distribution of field intensity, as in the case of simple release. Hence the analysis of Fig. 8, as described above, is not applicable quantitatively, and is therefore not given here in detail. Qualitatively, the relations are similar, the protected release has a flux time relation similar in time scale to that for simple release, with a lower initial rate of decay, and a higher rate for the later stage. An illustration of this effect is included in the article by M. A. Logan[4] cited above. No analytical treatment is available for determining the differences in release time between simple and protected release, at least for values of CR of the same order at t_E, as in the protection networks commonly employed.

8 Release Motion

It was stated in Section 7 that the waiting time in release is usually larger than the motion time. This, however, is not necessarily or invariably the case, and interest attaches to the conditions under which the motion time may become relatively large. Another important aspect of the release motion is the final velocity attained when the armature strikes the backstop. This impact results in a rebound which may, in the relay case, result in re-actuation of the contacts: rebound chatter. Rebound may be reduced by appropriate mechanical design, as discussed in an article by E. E. Sumner,[8] but its amplitude is always, other things being equal, proportional to the kinetic energy of the armature in impact on the backstop.

Fig. 9 shows the force travel relations in release, corresponding to the operate relations of Fig. 4. The solid line, as in Fig. 4, represents the spring load, which has an operated value F_0 at the point marked 2. This corresponds to the similarly marked point in Fig. 7, which shows the corresponding φ versus $\mathfrak{F}$ relations. The field decays along the demagnetization curve for $x = 0$ to the point 2, where load and pull are in equilibrium. Further decay results in a pull less than the spring load, and hence in a net accelerating force producing armature motion. The flux continues to decrease as indicated in Fig. 7, following the path 2–3–4. The pull follows the similarly marked curve in Fig. 9, related to the flux-travel path by equation (5).

The net accelerating energy is therefore represented by the area marked T, lying between the pull curve and the spring load. Thus only this portion of the energy V stored in the spring load appears as kinetic energy of the armature. The remainder of V is represented by the area

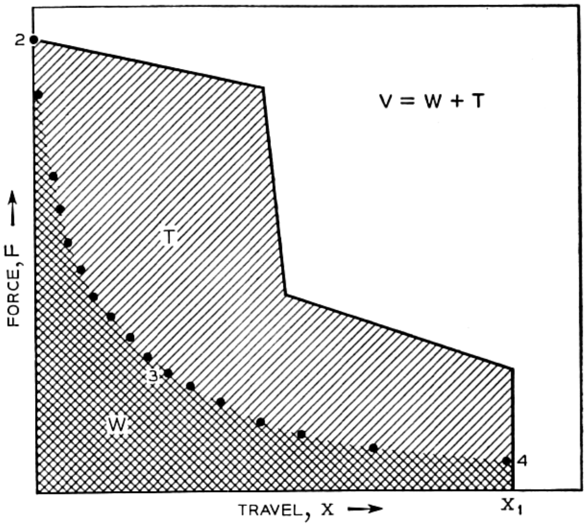

Fig. 9 — Load and pull relations in release.

marked W, lying below the release pull curve. This energy may be termed the magnetic drag: it is restored to the magnetic field and dissipated in the eddy current paths or in other circuits linking the field. Experimental studies have shown that the kinetic energy T of the armature in backstop impact may vary from 20 per cent to 90 per cent of V. Such a wide variation in armature energy has a proportional effect on rebound amplitude. Within certain limitations, the following analysis establishes the relations controlling the ratio W/V, and also serves for the evaluation of the release motion time.

EQUATIONS APPLYING

As the motion starts after an initial period of field decay, the relations applying are given approximately by (2) and (3), with $\mathcal{R}$ and F given by (4A) and (5A), and $G = G_E$. The use of (4A) and (5A) is justified by the fact that the lower portion of the demagnetization curve, which applies to the later stages of flux decay, is approximately linear. The assumption that (2) applies, with $G = G_E$, is justified by the fact, discussed in Section 1, that the initial rapid decay of the field in the outer layers of the core is followed by an exponential decay of the greater part (about 70 per cent) of the initial field. Thus if t_E is written for $4\pi G_E/\mathcal{R}(0)$ in (2), this expression applies in the latter stages of flux decay when the armature is at rest at $x = 0$. In this case, the value of t_E is a constant, even if the values of G_E and $\mathcal{R}(0)$ applying differ somewhat from those applying in the operate case.

Taking these equations to apply, it is desired to determine the solution for the initial conditions applying to the release case. This cannot be obtained by the approximations used for the operate case, as the simplifying condition of a nearly constant reluctance does not apply: the change of reluctance with x is a maximum for $x = 0$. The procedure that has been employed is the use of the analogue computer described in articles by A. A. Currie[6] and E. Lakatos[7] to obtain solutions for a restricted category of cases. The equations may be reduced to a form adapted to such solution as follows:

An expression for $\mathcal{R}/\mathcal{R}(0)$ obtained from (4A) is substituted in (2). Writing t_E for $4\pi G_E/\mathcal{R}(0)$, there is obtained:

$$\frac{C_L(1 + u)}{C_L u} \varphi^2 + \frac{t_E}{2} \frac{d\varphi^2}{dt} = 0,$$

where u is written for x/x_0, or $x/(A\mathcal{R}_0)$. By substituting in (3) the expression for F given by (5A) there is obtained an expression for φ^2.

On substituting this in the preceding equation, there is obtained:

$$(1 + u)(C_L + u)\left(\frac{dV}{dx} - m\frac{d^2x}{dt^2}\right) + \frac{t_E}{2C_L}\frac{d}{dt}\left((C_L + u)^2\left(\frac{dV}{dx} - m\frac{d^2x}{dt^2}\right)\right) = 0.$$

To reduce this expression to dimensionless form, the following substitutions are made:

f is written for $\dfrac{dV}{dx}\Big/F_0$, where F_0 is the operated load, so that $f = 1$ at the start of the release motion.

τ is written for $2C_L\,t/t_E$, expressing the time t as a multiple of $t_E/(2C_L)$.

t_M^2 is written for $2mA\mathcal{R}_0/F_0$. Thus t_M is the time for travel of the mass through the distance $A\mathcal{R}_0$ for a constant accelerating force F_0.

K is written for $2C_L\; t_M^2/t_E^2$.

The preceding equation then becomes:

$$(1 + u)(C_L + u)\left(f - K\frac{d^2u}{d\tau^2}\right) + \frac{d}{d\tau}\left((C_L + u)^2\left(f - K\frac{d^2u}{d\tau^2}\right)\right) = 0. \qquad (27)$$

This is the form of the release motion equation to which analogue computer solutions were obtained. It is a third order differential equation giving the travel, expressed as a multiple u of $A\mathcal{R}_0$, as a function of the time expressed as a multiple τ of $t_E/(2C_L)$. The boundary conditions correspond to zero initial values of travel, velocity, and acceleration, so that for $\tau = 0$, $u = du/d\tau = d^2u/d\tau^2 = 0$.

ANALOGUE COMPUTER SOLUTION

The analogue computer gives solutions to specific cases, and a specific case of (27) is defined by the values of C_L and t_M/t_E applying, and by the form of the relation between f and u, defining the shape of the spring load. To confine the cases considered to a manageable category, they were limited to those for which $f = 1$ for all values of u: the case of a constant spring load.

The constant C_L, the ratio $(\mathcal{R}_0 + \mathcal{R}_L)/\mathcal{R}_0$, is an inverse measure of the relative magnitude of the leakage field. As noted in Section 2, its value is usually in excess of 4. Solutions were obtained for two cases: $C_L = 3$ and $C_L = 5$. For $f = 1$, and a fixed value of C_L, the remaining parameter defining a specific case is t_M/t_E, or the corresponding value of K $(2C_L t_M^2/t_E^2)$. For the two selected values of C_L, solutions were obtained for the following values of K: 0.05, 0.1, 0.3, 1.0 and 10.

The computer set-up for this problem is indicated in Fig. 9 of the article by E. Lakatos cited above.[7] Solutions were obtained to (27) for $f = 1$ and the values of C_L and K cited above, for the given boundary conditions. The solutions were obtained in the form of machine drawn curves giving, u versus τ and $du/d\tau$ versus τ, over the range $0 < u < 3$. In accordance with the character of the net accelerating force, all these curves were smooth and monotonic.

For the conditions covered by the solution, these results give the motion time for a given travel directly in the form of the relation between τ and u. It is convenient to express the values of τ in terms of the corresponding values of t/t_M. The results for $C_L = 5$ are shown in Fig. 10 as curves of t/t_M versus u for various values of t_M/t_E.

The other result of interest is the ratio of the magnetic drag W to the spring load energy V, or of $(V - T)/V$, where T is the kinetic energy at the end of a given travel: the total travel in an actual case. T equals $m(dx/dt)^2/2$, or $2m(C_L A\mathcal{R}_0 \cdot du/d\tau)^2/t_E^2$. For $f = 1$, V equals $F_0 x$, or $F_0 u A\mathcal{R}_0$. It follows that T/V is given by:

$$\frac{T}{V} = \frac{mA\mathcal{R}_0}{2F_0 u}\left(\frac{2C_L}{t_E}\right)^2\left(\frac{du}{d\tau}\right)^2,$$

and therefore by:

$$\frac{T}{V} = \frac{K}{2u}\left(\frac{du}{d\tau}\right)^2. \tag{28}$$

Corresponding values of u and $du/d\tau$ were read from the computer results and substituted in (28) to determine T/V and thus W/V. The resulting values of W/V for the case $C_L = 5$ are shown in Fig. 11 plotted against t_M/t_E for various values of u.

DISCUSSION OF COMPUTER RESULTS

The significance of the results shown in Figs. 10 and 11 can be more readily grasped by reference to representative values of the parameters involved. For relay electromagnets, representative values of $\mathcal{R}_0$ and A are 0.04 cm^{-1} and 1 cm^2, respectively, for which $A\mathcal{R}_0 = 0.04$ cm, or 16 mil-in. This distance is, for this case, the travel for which $u = 1$. For this value of $A\mathcal{R}_0$, an effective mass m of 10 gm, and an operated load F_0 of 2×10^5 dynes (200 gm wt), $t_M = 2$ millisec. If the load were

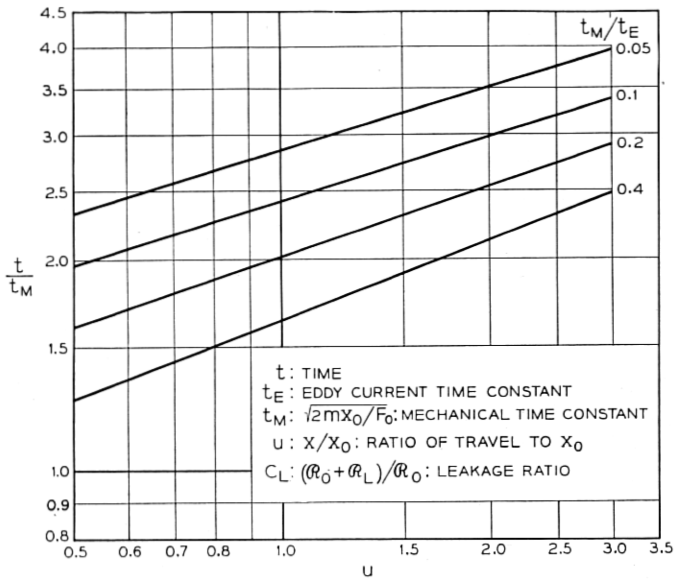

Fig. 10 — Release motion time relations.

doubled and the mass halved, t_M would be halved. These values of t_M are of the same order but smaller than the values of t_E generally applying, corresponding to the range in t_M/t_E covered by the figures. Similarly, the total travel usually lies in the range covered by the values of u, corresponding to travels up to 48 mils for the above value of $A\mathcal{R}_0$.

The curves of Fig. 11 show that the fraction W/V of the spring load energy absorbed by magnetic drag varies from 15 to 80 per cent over the range covered, and consequently that the kinetic energy at the end of the stroke, which determines the rebound amplitude, varies from 85 per cent to 20 per cent of V, or by a factor of 4 to 1.

The way in which W/V varies with u and with t_M/t_E is readily understood from physical considerations. Referring to Fig. 9, it is apparent that for a given time rate of field decay, the faster the rate of armature motion, the higher is the restraining pull and the less the kinetic energy T. For a short travel, or low value of u, the restraining pull will be larger than for a high value of u: hence W/V decreases as u is increased. As t_M measures the time for a travel of $A\mathcal{R}_0$, or $u = 1$, for a force F_0, t_M/t_E is a measure of the motion time relative to the time of field decay. Hence a large value of t_M/t_E corresponds to a condition where the motion is slow relative to the field decay, giving little restraining pull. Thus W/V decreases as t_M/t_E increases.

The effect of the magnetic drag on the motion time is apparent in the curves of Fig. 10. The motion time for a given value of u increases as t_M/t_E decreases, corresponding to increased drag. Thus a reduction in rebound by increased magnetic drag is, of course, accompanied by a longer motion time. It is of interest to note that the increasing acceleration results in an armature displacement which varies as the cube of the time, as in the relation applying in operate.

The results shown are for the case $C_L = 5$. For $C_L = 3$, corresponding to higher leakage, the results are similar, but the drag ratio W/V values are from 5 to 10 per cent lower than those for $C_L = 5$, and the values of t/t_M are correspondingly smaller. Thus the value of C_L applying has only a secondary effect on the release motion.

While these results are formally limited to the special case of a constant retractile force, the conclusions are of broader application. A load curve that decreases as the gap opens modifies the solution for the same operated load F_0 only in reducing the net accelerating force, increasing the drag ratio and the motion time. The residual magnetism, neglected in this discussion, has a similar, and minor, effect. The results show C_L to have little effect on the motion, which is governed primarily by the values of u and t_M/t_E applying.

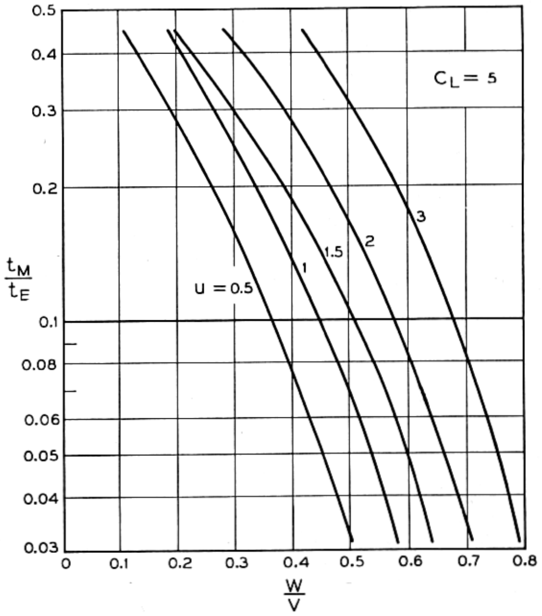

Fig. 11 — Energy absorbed by magnetic drag in release motion.

As minimizing rebound is of major importance in relay design, the relations controlling the kinetic energy to which it is proportional are of design interest. Most of the quantities appearing in the above relations, such as the spring load and the travel, are fixed by operating requirements or other design considerations. There is, however, considerable freedom in the design relations fixing the effective mass of the armature, particularly in selecting its axis of rotation. The preceding relations show that the magnetic drag ratio can be increased, and rebound reduced, by the decrease in t_M resulting from a decrease in effective mass. While such a decrease increases the ratio t/t_M, it results in a net decrease in the value of t, the motion time. A low effective armature mass is therefore advantageous both for fast release and for a high drag ratio for the reduction of rebound.

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