Estimation and Control of the Operate Time of Relays

Part II — Design of Optimum Windings

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For each relay structure, there exists a best winding, once the circuit power has been specified. The best winding is that one which operates the relay in the least time. Methods for determining the best winding are developed.

On the basis of the operating time, the relay behavior is classed as mass or load controlled. The design method chosen depends upon this classification.

The design of windings for series connected relays is based on a method of determining equivalent single relays, the behavior of each corresponding to one of the series relays. This method is generalized to allow for different magnetic structures for the several relays. Each relay winding is then designed in turn, using the design data for its own type of structure.

The best winding design is not given directly by an explicit formula. Rather, methods are developed for determining the operate time for any winding. Then by choosing a range of windings, the best one is selected by interpolation.

INTRODUCTION

The selection of a relay for a circuit application, particularly so in common control systems, involves in part a determination of its operating and releasing time. In Part I of this article expressions are derived for the relations between these times, the design parameters of a relay, and the conditions of operation. These expressions are approximate, and have been developed with primary reference to the selection of favorable characteristics in design. The timing estimates they provide are of sufficient accuracy for the comparison of design alternatives. Once a basic design has been selected, the choice of windings and the prediction of the limiting times occurring in specific applications requires a more detailed and exact procedure than can be obtained entirely from the application of these approximate relations. The information furthermore has to be quite versatile, to permit all new type variations to be included, such as winding, number of contacts and armature travel. It must, moreover, be in a form that permits determination of the upper and lower limits to the time observed in each specific type of relay as actually used, as in applications, interest attaches not only to the nominal conditions, but to all variations that may arise in actual manufacture and use.

Relay operation is complex, and in principle the nonlinear differential equations which describe it can be solved exactly only by computers. Even if this is done, the results are subject to any uncertainty that exists as to the exact form of the relations that apply. The representation of non-linear magnetic material properties, the discontinuous load-travel characteristics, the eddy current effects, etc., do not make such an approach attractive.

The relay, however, is a perfect analog of itself. With the magnetic structure set, controlled models can be built to include dimensional and magnetic material variations. With these, exact solutions to a variety of conditions can be determined. With these data available, approximate solutions can be used for interpolation and extrapolation, determining the effect of small variations from the tested conditions.

This part of the article will exhibit the form of data presentation in two classes, mass and load controlled operation. It includes the theory used in selection of the forms, and the correction methods used for estimation of variations from the standards. The initial part will be concerned with a single relay operating in a local circuit. The latter part will consider series and series-parallel operation of similar structures but not necessarily identical windings. This latter problem is solved by determination of an equivalent relay in a local circuit for each of the several relays. The earlier analysis then can be applied to each in turn.

The order of analysis can be reversed. That is, given required operating times, windings can be determined which will provide these times.

In this article, a best winding is (1) that winding which, for the specified applied power, results in the minimum operate time or (2) that winding which, for a specified operating time, requires the minimum power. A unique solution exists.

Existence of Best Winding

Fig. 1 shows measured operate time of a relay for two different dc power conditions, versus number of turns in the winding. That a best winding, in the definition of this article exists, as exhibited by the minima on Fig. 1, was shown in Part I to be a consequence of equation (12). This best winding and its determination is the basic subject of this article.

OPERATE TIME — SINGLE RELAYS

As in Part I, the operating time of a relay is considered as made up of three stages: (1) the waiting time while the armature remains at the backstop and the pull builds up to equality with the back tension, (2) the motion time beginning at the end of the waiting time and continuing while the armature moves from the backstop to the position of the earliest contact, and (3) the stagger time during which the armature actuates all of the remaining contacts.

The mass controlled case, as a practical matter, simplifies to a determination of only the first two without regard for the spring load, with the displacement of the armature taken to the latest contact in the array. The armature pull builds up to values in excess of the load during its early motion and the velocity is so high during the stagger time that this small interval can be included as part of the motion time. This treatment is essentially that of the three stage approximation of Part I.

The load controlled case simplifies to a determination of the time required for the pull to build up to the maximum load, including the back tension, with most of the relatively slow motion taking place during this pull buildup. The remainder of the motion time is accounted for as an empirically determined correction factor in the expression for time of pull buildup. This treatment corresponds to the single stage approximation of Part I, with a correction term to account for the additional motion time.

Fig. 1 shows graphically the basic characteristics of the two types of operation. This chart displays the operating time of the same relay under the conditions of 1.0 or 5.7 watts, and three contact load conditions: 12, 18, and 24 contact pairs. For each of the six conditions, a curve is drawn showing the effect of the number of winding turns. It is clear that (1) there is a best number of turns for each case, (2) for the 5.7 watts the best number of turns is the same for all three loads, (3) for the 5.7 watts case the difference in operating time is only 0.1 millisec in 5.6 millisec, between 12 and 24 contact pairs, (4) for the 1.0 watt case the best number of turns increases with the number of contact pairs, and (5) for the 1.0 watt case the operate time is double for 24 compared to 12 contact pairs. These strikingly different behaviors form the basis for division of relays

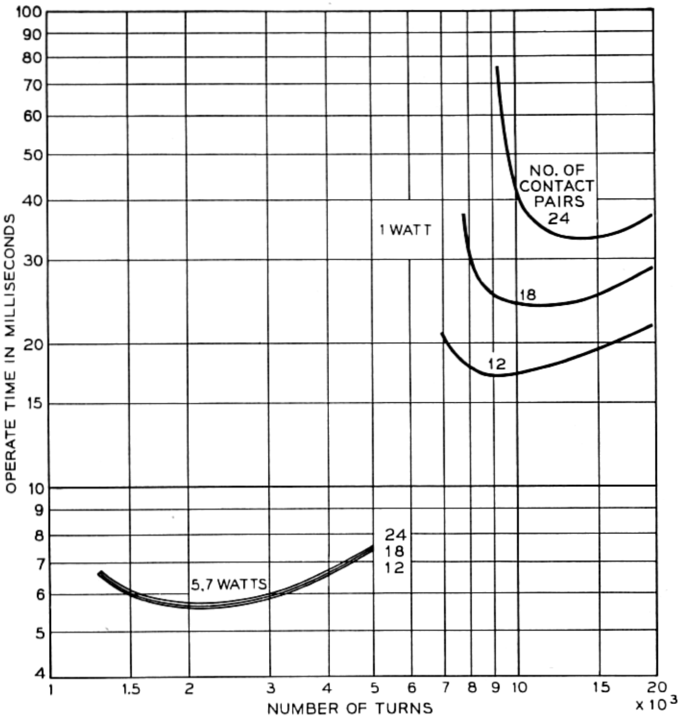


Fig. 1 — Chart showing typical mass and load controlled operation.

into two classes called mass and load controlled, and furthermore empirically establish which of the three stages of operate time predominates. This distinction corresponds to that made in Part I in the discussion of equation (12). For the lower power cases the term involving the spring load predominates, for the higher power cases the mass term predominates. As this equation shows, there is necessarily a transition range where the operation time merges from one class into the other. Both types of charts are extended to include this range. A rule of thumb for an estimate of whether a time under discussion is in the mass or load controlled case is that if the time is less than

$$t_M = \sqrt{\frac{27m(x_1 - x_3)}{F_3}} \tag{1}$$

it is mass controlled; otherwise it is load controlled. The derivation of this bound will be discussed when mass controlled operation is considered.

The single relay design data are general enough to include resistors external to the relay winding. Wherever a resistor or power term appears, the sum of all resistances or powers is indicated.

Maximum Versus Average Operate Time Presentation

In what follows, load controlled operation data are given in terms of maximum times, whereas mass controlled data are average. Either choice could be used for both. The data for converting to average or maximum respectively will be described. However, the choices made here are consistent with the normal use of the data.

Mass controlled, sometimes called speed, relays operate with relatively large power and ordinarily are used in common control circuits where many events occur in succession. The total number of similar common equipments necessary in an office is related to the control circuit holding time. For a system design, the cost of power is balanced against the cost of equipments to arrive at a minimum office cost.[1] Now the maximum holding time per call is never the sum of the maximum possible times of each of the several relays operating in succession. It rather is more nearly the sum of the averages, increased by considerably less than the common maximum to average ratio of one of them. For example, if n relays are assumed to have a distribution approaching normal, an analytical expression for the probable maximum is:

$$\delta t_{\text{av}} + \left(1 + \frac{\dfrac{t_{\max}}{t_{\text{av}}} - 1}{\sqrt{n}}\right) \sum t_{\text{av}}\,, \tag{2}$$

where δt_{av} is an allowance for short time deviations of the manufactured product from the long time average. Thus for mass controlled relays the most directly applicable type of data is in the form of average time, and maximum to average time ratio.

For load controlled relays just the opposite is true. Here the operating times are relatively long, either because the power drain is to be kept to a minimum, as in a long holding time circuit, or it is an event which takes place while several successive mass controlled events occur. For either case, it is a single event and only its maximum duration is desired. Here then the most directly applicable type of data is maximum. In addition to these, other data for minimum times are needed for studies of timing when two parallel circuit paths occur.

Because how far the relay armature has to move is the outstanding variable in mass controlled relay timing, these charts are prepared for each nominal distance which can be chosen. When there is no concern as to relative actuation time of the several contacts on one relay, the smallest armature travel is ordinarily provided. When a definite sequence is needed for circuit reasons, the actuating means is arranged to guarantee operation of certain contacts before the others. This necessitates the provision of a greater armature motion.

The principles used in preparing these other types of charts are identical to those used for the two types which are developed specifically in this article.

Local Circuit Load Controlled Operation

Power Given

The best winding for a load controlled relay is not here given explicitly by a formula, but rather is found indirectly by developing a method for determining the operate time for any winding. Then by assuming a range of these, the best one can be selected by interpolation of these derived data. In Part I, the waiting line for a linear system was shown as equation (9) to be:

$$t_1 = \frac{4\pi}{\mathcal{R}_3} (G_C + G_E + G_S) \ln \frac{1}{1 - I_0/I}, \tag{3}$$

with I_0/I substituted for ν. Without motion time, saturation, eddy currents, or non-linear effects, Part I of this article also shows that the best winding is that for which the turns are selected so that $NI_0/NI = 0.715$.

Taking the other variables explicitly into account complicates the determination and is at best only approximate. Instead they are included through this interpolation approach. In a previous article[2] a better representation for the core eddy current constant was shown to be:

$$G_E' = G_E e^{-G_E/(G_C+G_S)} = \text{effective core eddy current constant.}$$

Substituting this and explicitly indicating a correction term for the small motion time, the form best suited for the present discussion becomes:

$$t_0 = (1 + t_2/t_1) L_1 (G_C + G_S + G_E') \ln \frac{1}{1 - q}. \tag{4}$$

Except for the correction term for the motion time, t_2/t_1, this is still a linear equation with all factors known. Omitting the motion time correction, it can be solved either by numerical substitution or by a nomogram, once a value for L_1 has been chosen. A conservative value is the average one turn inductance, for the late contact critical load point where $x = x_3$. A more accurate value is determined through use of both the inductance at the open gap and at the critical load gap. These are weighted in proportion to the ampere turns developed at each location. The ampere turns necessary to overcome the back tension is one factor, and the other is the difference between this first value, and the total required for operation at the critical load gap. This weighting yields an effective inductance value intermediate between the two extremes for each problem, but a new chart does not have to be prepared. An operate time using the chart is first determined. This time is then adjusted by the ratio of the effective inductance applying and the inductance used for making up the chart. This is exact, as the inductance term appears only as a direct multiplier.

A typical nomogram is shown in Fig. 2. It already includes the motion time correction, whose determination will be discussed. The dashed line

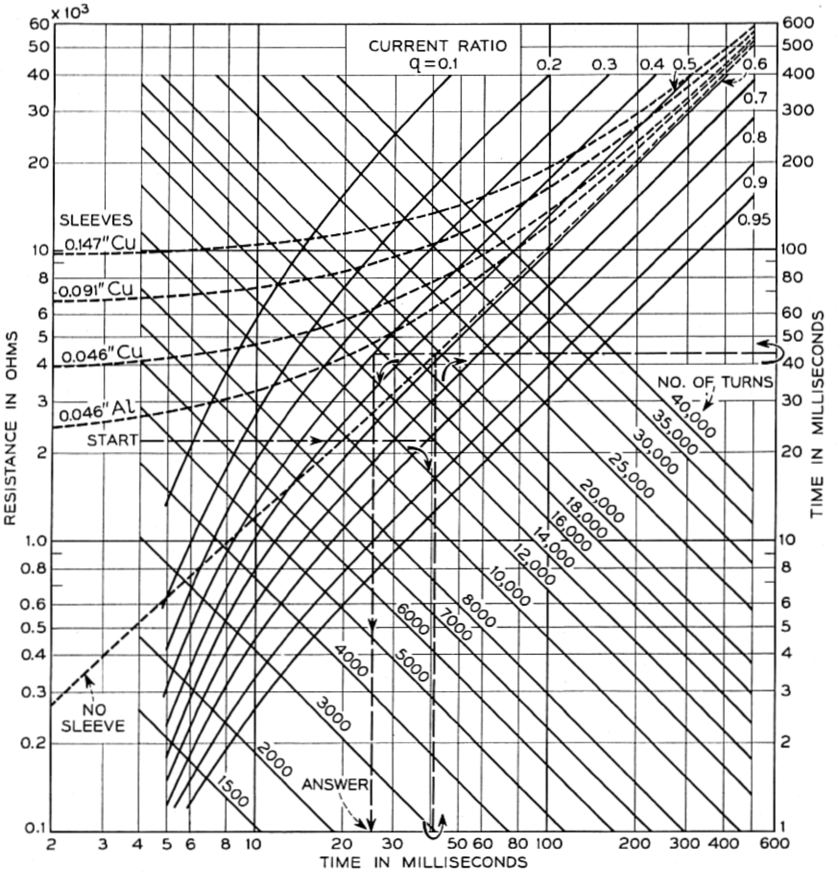


Fig. 2. — Nomogram for solution of load controlled operate time.

shows the successive steps taken in determining the numerical value for a specific case.

The dc circuit resistance is determined by the known circuit voltage and specified power. Entering the chart on the left ordinate at this resistance and proceeding horizontally to the right, intersections with increasing number of turns lines, sloping downward to the right, are found, and the appropriate one is chosen. Dropping vertically to the abscissa, the winding time constant $t_C = L_1G_C$ is determined.

To this is next numerically added the known effective core eddy current and a sleeve (if any) time constant by returning vertically to an intersection with the appropriate core indicated as "no sleeve," or core plus sleeve, curve. Proceeding horizontally to the right hand ordinate scale from this intersection, the time constant multiplier of the ln term in equation (4) is determined.

The multiplication of these two factors is accomplished by proceeding to the left along this same horizontal line, to an intersection with the proper q line, sloping downward to the left. Vertically below this last intersection is the operate time.

Initially, tentative q lines are drawn, omitting the motion time correction. These lines are straight with a positive 45° slope. Then with an actual relay whose just operate current has been measured, operate time measurements are made, keeping the final current, and hence q, fixed at several values in turn. This is done by adjusting an external resistance and battery voltage over a wide range, effectively changing N^2/R. These measured data are plotted, following the same steps through the nomogram, except the last intersection is with the measured time vertical, rather than the known q. This provides several empirically determined q lines. These are used as templates, to progressively alter the shapes of the tentative straight q lines drawn earlier and shift them to the right. This adjustment then introduces the motion time correction.

It has been found empirically that for large operating times, the motion time correction factor has a value of about 0.1 There is no definite division between mass and load controlled operation, but as the total time decreases, travel time becomes more important. The correction factor increases to a value of about 0.5, in the transition range. For completely mass controlled operation, there is no q effect, so the q lines must all eventually converge.

As stipulated above, this chart applies to the current, flux and pull range where the first approximation magnetic constants of the structure are applicable. For this reason, the tests for the motion time corrections are made under conditions meeting these restrictions.

These curves should be corrected for magnetic saturation if there is a

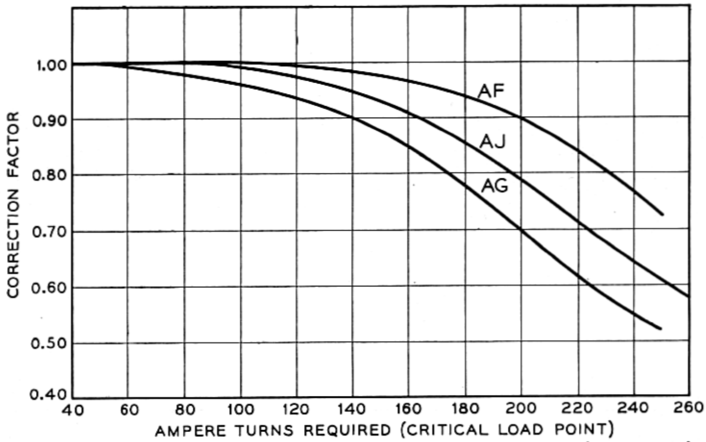


Fig. 3 — Correction of computed operate time for magnetic saturation.

large load at the critical load point and the ampere turns needed are in the saturation region for the core or armature. In the saturation region the current builds up faster than the linear time constants indicate, and, therefore, the indicated operate times are too large. Correction factors are determined by graphical integration of the integral form of the flux rise equation expressed as a ratio to the linear relationship:

$$\text{Saturation Correction Factor} = \frac{\displaystyle\int_0^{\varphi_3} \frac{d\varphi}{NI - Ni}}{-L_1 \ln (1 - NI_0/NI)}. \tag{5}$$

These corrections are plotted as a function of the just operate ampere turns NI_0, with the final NI as a parameter for each curve. Actually, because the correction factor is only of the order of 20 per cent maximum, assuming the final winding ampere turns are well into the saturation region, it is found that a single curve for any one type of relay fits all the computed points to an accuracy of a few per cent, and generally is used. Such composite curves are shown in Fig. 3 for the three types of wire spring relays.

A method has now been established for determination of the operate time of a relay with two restrictions (1) that the final ampere turns will operate the relay and (2) that the relay is in the load controlled class. An indication of the latter is whether the operate time determined is in the time region where the q curves are decreasing in curvature. For Fig. 2, 10 milliseconds is taken as the lower bound for load controlled relays.

Now the determination of an optimum winding for a particular problem can be finished. Starting with the power given and choosing an arbitrary number of winding turns, the operate time is determined as described above. If it falls into the load controlled class, then a different number of turns is next assumed and a second time determined. This procedure is repeated until a time curve versus number of turns similar to Fig. 1 can be plotted, including a minimum. The best winding is this minimum. A different curve and optimum number of turns will apply to each contact load assumed. Three computed curves corresponding to the measured 1-watt curves are shown on Fig. 4. These were determined using effective inductance values described earlier.

Finite wire sizes permit only certain number of turns to be physically realizable when the resistance has been specified, without splicing two gauges of wire. The nearest gauge on the coarser wire size side is chosen, resulting in slightly too many turns. Note that the curves rise less steeply on the high turn side and the time penalty therefore is less than if too few turns were supplied.

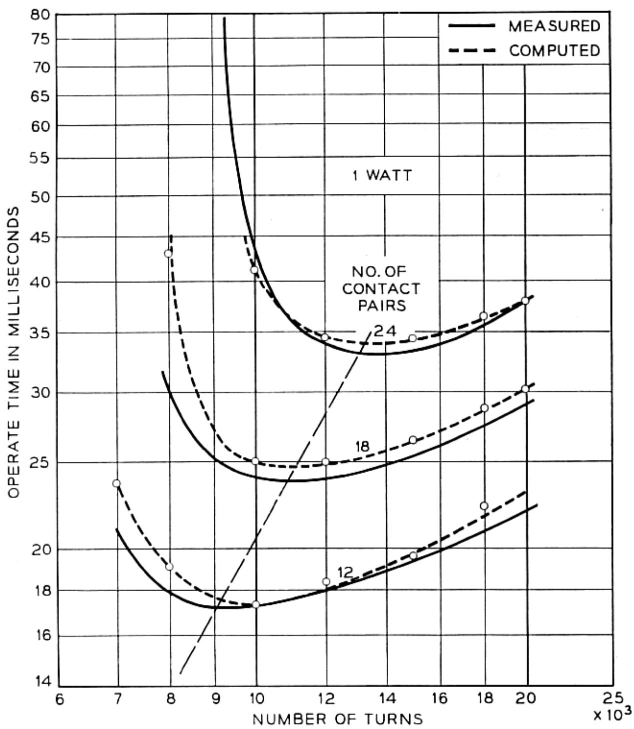


Fig. 4 — Typical computed maximum operate time curves for load controlled relays.

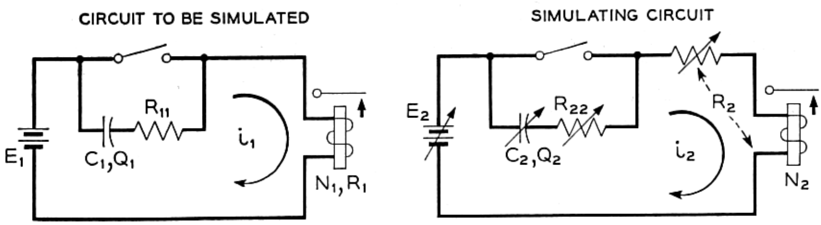
Maximum Time Given

For a specified maximum time, the above process is repeated for several assumed circuit powers until the specified time is bracketed. Then by interpolation of the optimum times indicated, the minimum power, maximum resistance and optimum turns are determined. As an example, Fig. 4 can be used to demonstrate the method. Assume that the three curves were computed for different circuit powers, rather than for different contact spring loads. A line is drawn through the minima. The intersection of this line with the required operate time determines the number of turns. For instance, if 30 millisec were required, the turns would be 12,500. The circuit power at this same intersection can be interpolated for, using the known circuit powers associated with the three curves.

It is not economical to have a different winding for each spring combination. For this reason, a winding is designed for the maximum spring load and then used for smaller loads. The operate time will always be less with the smaller loads.

Measurements of Time Curves

Before considering mass controlled operation, the simulation of windings will be discussed. In the above description for establishing the q curves, it was pointed out that by adjusting an external series resistor and the battery voltage to maintain constant final current, the coil constant N^2/R was altered without changing q. This can be further extended to permit simulation of any winding for test purposes providing only that the experimental coil fills the winding volume as much or more than the coil to be simulated. For this purpose, a special test winding



$$E_2 = \frac{N_2}{N_1} E_1 \qquad R_{22} = \left(\frac{N_2}{N_1}\right)^2 R_{11}$$

$$R_2 = \left(\frac{N_2}{N_1}\right)^2 R_1 \qquad C_2 = \left(\frac{N_1}{N_2}\right)^2 C_1$$

Fig. 5 — Simulation of winding circuit for timing tests.

always is used which completely fills the winding volume. Hence any winding which can be designed also can be simulated.

The conditions which must be fulfilled for perfect simulation derive from Lenz's Law. It is essentially an impedance transformation technique keeping the magnetic flux invariant, with the assumption that a winding can be considered as a lumped rather than a distributed network. This is equivalent to stating that at any instant the current flow is the same in every turn and there is no propagation time involved. This is true for times involved in electro-magnets.

Fig. 5 shows a circuit to be simulated, in which all the components with subscripts 1 have been given. The simulating circuit has only the number of turns N_2, of the test winding, given. The other four elements must be determined. After switch closure, the exact differential equations applying are

$$\begin{aligned} N_1 \frac{d\varphi_1}{dt} &= E_1 - i_1R_1, \\ & \qquad\qquad t = 0; \quad i_1 = i_2 = 0. \qquad (6) \\ N_2 \frac{d\varphi_2}{dt} &= E_2 - i_2R_2. \end{aligned}$$

Now for equality of magnetic flux, the two rates of flux change must be identical at all times including the first instant. Inserting the initial boundary conditions and equating the two rates, we have

$$\frac{E_1}{N_1} = \frac{E_2}{N_2}. \qquad (7)$$

At infinite time, the same magnetomotive force must apply to both circuits for equality of final flux. Equating these, and cancelling the 4π factor,

$$N_1I_1 = N_2I_2. \qquad (8)$$

Noting that

$$I_1 = \frac{E_1}{R_1},$$

and (9)

$$I_2 = \frac{E_2}{R_2},$$

we have, after using (7) and rearranging,

$$\frac{N_1^2}{R_1} = \frac{N_2^2}{R_2}, \qquad (10)$$

which states that the coil constants must be equal. After steady state has been reached and the switches opened, the two differential equations are:

$$E_1 = i_1(R_{11} + R_1) + N_1 \frac{d\varphi_1}{dt} + \frac{1}{C_1} \int_0^t i_1 \, dt,$$

$$E_2 = i_2(R_{22} + R_2) + N_2 \frac{d\varphi_2}{dt} + \frac{1}{C_2} \int_0^t i_2 \, dt, \tag{11}$$

at $t = 0, \quad i_1 = \frac{E_1}{R_1}, \quad i_2 = \frac{E_2}{R_2}, \quad Q_1 = Q_2 = 0.$

Multiplying the first equation by N_1C_1 and the second by N_2C_2, and equating term by term for equality at all times and equal magnetomotive forces, two additional equations result:

$$\begin{aligned} (R_{11} + R_1)\, C_1 &= (R_{22} + R_2)\, C_2\,, \\ N_1^2 C_1 &= N_2^2 C_2\,. \end{aligned} \tag{12}$$

The four defining equations, with some further rearrangements using (10), are shown in Fig. 5 as the relations applying for equality of magnetic flux in the two circuits. As the mechanical behavior of a magnetic structure is completely determined by the magnetic flux, it follows that the mechanical performances will be identical and timing measurements made with the simulating circuit will represent the actual circuit. This is true whether the relay is mass or load controlled. It includes eddy current effects and whether or not there is motion of the armature.

Local Circuit Mass Controlled Operation

Typical mass controlled operate time curves are shown in Fig. 6. These are for two different armature travels, indicated as short and intermediate. The curves are characterized by the circuit power used for each, with the coil constant plotted along the abscissa and the corresponding operate time as the ordinate. As mentioned earlier, these curves are substantially independent of contact spring load, and are plotted for average conditions, including an averaging of the time for the first and the last contact to be actuated.

It will be noted that the best coil constant is not independent of the circuit power, decreasing continuously as the power is increased. Also by increasing the circuit power from 2.3 to 23 watts, the operate time is decreased by a factor of a little less than 3, which is nearer the square

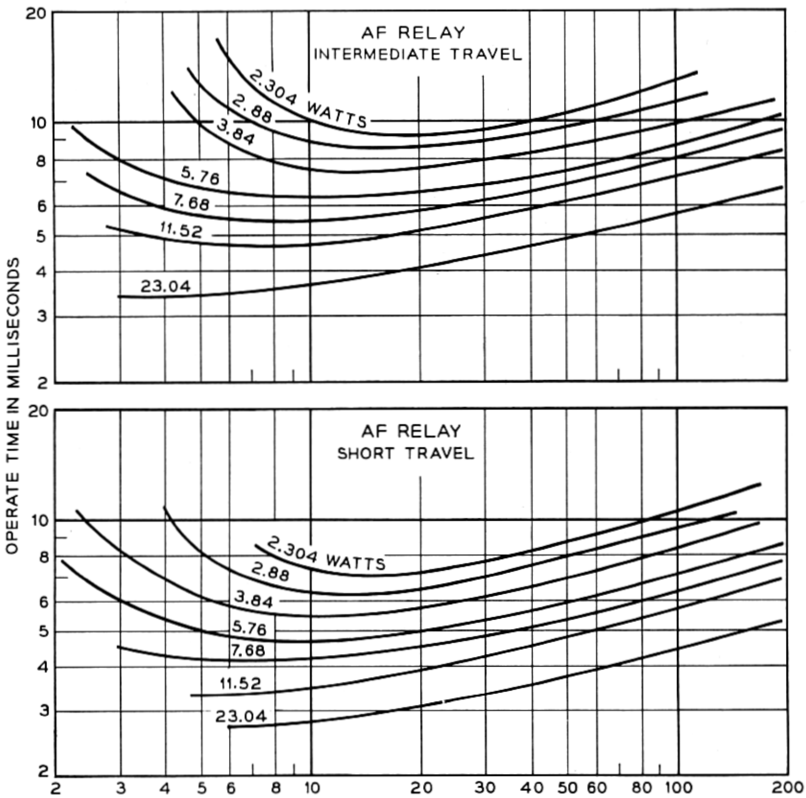


Fig. 6 — Mass controlled operate time tests.

root than the cube root of the power ratio, developed in Part I, for the mass controlled case. This is partly attributable to the fact that waiting time is included, and partly to the fact that the 2.3 watts case is in the transition range between mass and load controlled operation.

For the operating region where coil constants are larger than the best, the time curves are parallel and increasing. Considering any one vertical line representing a particular winding, increasing the power by increasing the battery voltage will always decrease the operate time. For curves plotted as in Fig. 1, where the battery voltage is kept constant and the time curves are plotted with winding turns as the independent variable, parallel curves again obtain. Thus an increase in circuit power obtained by keeping the voltage constant and reducing the circuit resistance always will reduce the operate time. Conversely, adding any series impedance, not including a capacitance, will always increase the operate time. This will be made more evident when the operate times of relays with their windings in series are considered.

The curves can be used in either of two ways (1) given the relay circuit for which the applied power and coil constant can be computed, the average operate time can be found from the chart, or (2) given a required average operate time, the necessary circuit power and coil constant can be found from the chart.

These curves can be plotted in this form to exhibit the best winding directly because of the independence to contact spring load. For the load controlled case, the contact spring load was an essential parameter. For the mass controlled case, the armature travel becomes the outstanding parameter to be considered, but there are only two or three of these. All the other factors except contact load also enter and need to be evaluated for two reasons, (1) to provide an estimate of the range in operate time to be expected and (2) to adjust experimental measured time data to average. For any experimental setup, it is seldom possible to provide a structure which is average in every respect. For any one structure, however, all the factors known to affect its performance can be measured. Comparison of these to the manufacturing specifications locates the experimental setup in the universe of all relays as regards each of the factors. It is thus necessary to develop representations relating each of these factors to the operate time, which will suffice for the two uses named above. The development of these relationships will be the subject of the following sections.

Waiting Time

The waiting time, whether an electromagnet is mass or load controlled, is given by the same form of equation used for the total operate time of a load controlled relay:

$$t_1 = L_1(G_C + G_E') \ln \frac{1}{1 - q_1}. \tag{13}$$

where now q_1 is determined by the armature back tension F_1; L_1 applies to the open gap; and no sleeve conductance is present. For the present purpose, it is desirable to rewrite this equation in terms of the fundamental parameters of the relay. For the open gap case the magnetic material is operated in its linear region and the open pole face gap provides additional linearity. For these reasons the expression is quite accurate. The sketches of Fig. 7 show the factors to be used. The value of

L_1,[4] the inductance for one winding turn is:

$$L_1 = 4\pi \left(\frac{1}{\mathcal{R}_L} + \frac{1}{\mathcal{R}_0 + x_1/A} \right). \tag{14}$$

Also

$$NI = \sqrt{G_C W}. \tag{15}$$

From the network of Fig. 7,

$$\varphi_G = \frac{4\pi N i}{\mathcal{R}_0 + x_1/A}. \tag{16}$$

The armature force developed is

$$-F = \frac{\varphi_G^2}{8\pi A}. \tag{17}$$

When the magnetic pull has reached F_1, the waiting time is over, determining φ_G, which in turn determines NI_0, and hence q.

Substituting (14), (15), (16), and (17) in (13) we have

$$t_1 = 4\pi \left[\frac{1}{\mathcal{R}_L} + \frac{1}{\mathcal{R}_0 + x_1/A} \right] (G_C + G_E') \ln \frac{1}{1 - (A\mathcal{R}_0 + x_1)\sqrt{\dfrac{F_1}{2\pi A G_C W}}} \tag{18}$$

as the desired expression for the waiting time in terms of the fundamental constants. Use will be made of this later when an expression for motion time has been developed.

Motion Time

The motion time immediately follows the waiting time and is the time required for the armature to move from the backstop position x_1, to the location of the last contact x_3. From Fig. 1, it is permissible to omit any consideration of contact spring load, and consider the motion to be controlled entirely by the armature mass and magnetic pull developed after the waiting time is over. The determination of motion time is simplified because the initial velocity and net force on the armature are both zero, the latter following from the definition of the end of t_1. In any problem involving motion, there must be established the initial velocity, position, and a suitable expression for the ensuing force. With these factors, the differential equation of motion can be solved, and the time for a given travel determined.

Thus there is now needed an expression for the armature force developed by the pole face gap flux as a function of time. Briefly, the basis for the method to be described is the assumption developed in Part I, that the winding flux continues to rise during the initial motion interval in the same way it would have risen if the armature had not moved. With this assumption, an expression for the motion can easily be derived. With this expression it then can be shown that the armature does spend most of its time in the close vicinity of the backstop, justifying the initial assumption.

The present approach differs from the more general one of Part I. It uses the results there developed regarding the behavior of mass controlled operation, to simplify the initial motion equation and obtain an expression for motion time in a form better suited for the present purpose.

Armature Force Rise and the $\dot{F}$ Concept

As an introduction to the method which will be used, consider the general character of flux build-up in a relay. It will have a shape somewhat similar to an exponential curve. Now with the armature at the backstop, from the first approximation magnetic network of Fig. 7, a fixed portion of this winding flux will pass through the pole face gap, with the result that the pole face gap flux curve will also have the same general character. Such a measured curve is shown in Fig. 8, as well as the curve when the armature is free to move.

Now the armature force developed is proportional to the square of

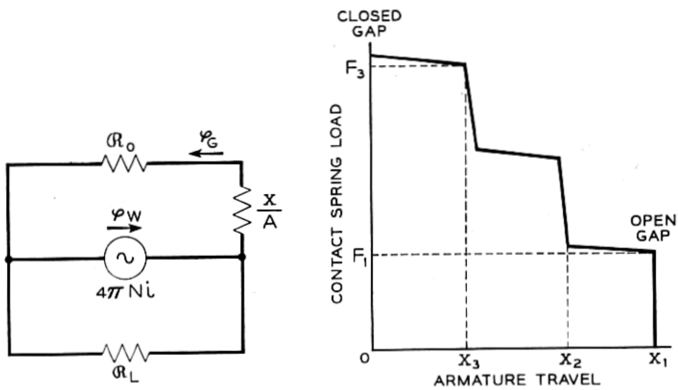


Fig. 7 — Schematics of nomenclature applying to operate time.

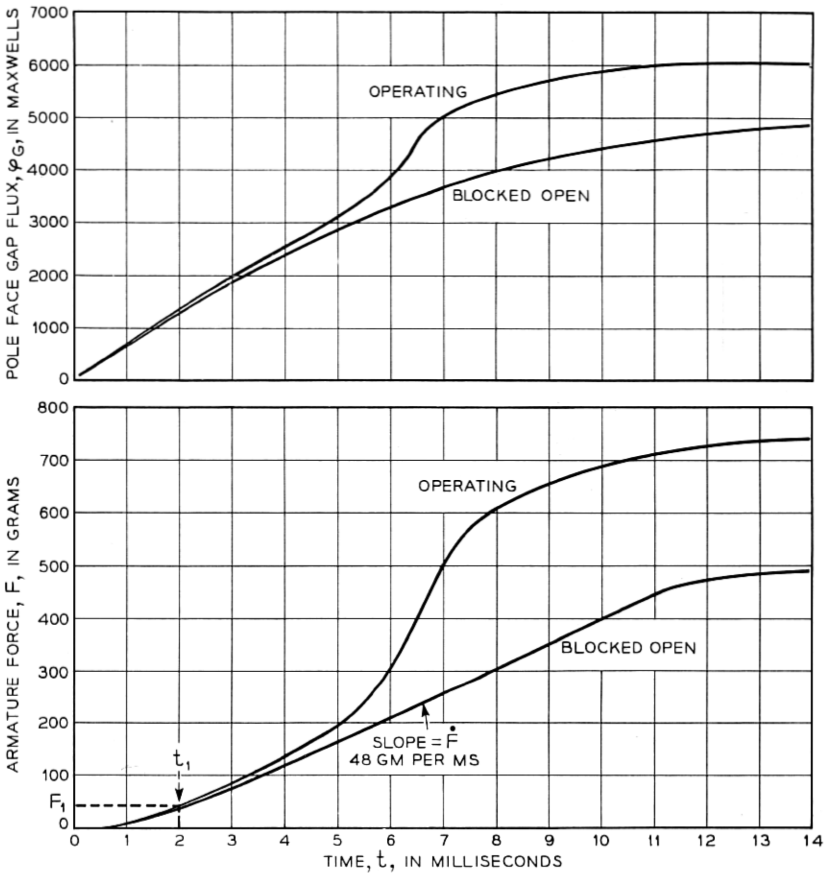


Fig. 8 — General character of winding flux and armature force rise.

the pole face gap flux as given by (17). Hence, the initial force rise will be parabolic, as shown by the lower portion of the curves of Fig. 8. After a long enough time however, the flux will have reached its maximum and the finally developed force will be constant with time. This is shown by the force curves approaching horizontal lines to the right in the same force diagram. As the force curve is continuous, there also must be an inflection point in the curve, and the entire curve has the general shape as shown.

The waiting time t_1 occurs while the magnetic force builds up to F_1, as indicated, amounting to 2 millisec for the example shown. This time generally includes all of the parabolic part of the curve. Following the waiting time, the initial force build-up necessarily is almost linear because of the inflection of the curve. Two other factors, caused by the ensuing motion, act to provide even more force than the open gap curve indicated by Fig. 8. The first is that a smaller gap takes a larger portion of the winding flux than assumed in the diagram, and the second is that the winding flux rises more rapidly and to a higher value than the open gap curve assumed. The operating force curve shown includes these effects. The relay operates in about 7 millisec and for 6 of the 7 millisec, the two force curves differ very little.

Hence, for a relatively long interval after the start of the motion time, a very simple force relationship holds, namely that the force is directly proportional to time, and the proportionality factor is the slope of the straight portion of the curve, designated as $\dot{F}$. For the example given, this amounts to about 48 grams per millisecond. By using Newton's Law, an expression for the motion time can be written.

$$m \frac{d^2(x_1 - x)}{dt^2} = \dot{F}t, \tag{19}$$

from which, with the initial velocity zero

$$m(x_1 - x) = \frac{\dot{F}t^3}{6}. \tag{20}$$

Rearranging, the expression for motion time becomes:

$$t_2 = \sqrt[3]{\frac{6m(x_1 - x_3)}{\dot{F}}}. \tag{21}$$

The factor $\dot{F}$ will now be considered.

Derivation of Expression for $\dot{F}$

Experimental—The factor $\dot{F}$ can be determined graphically. This brings in the second order effects such as quality of iron, cross section, residual magnetism, fit of parts, etc. For this purpose three working curves characteristic of the structure, all at the open gap, are first prepared for the particular winding:

(a) The dynamic flux rise curve,[2]

(b) The static flux curve versus ampere turns, and

(c) The static pull curve versus ampere turns.

Choosing a time on curve (a), the corresponding instantaneous flux transferred to curve (b), determines the equivalent magnetomotive force. This transferred to curve (c) yields the instantaneous magnetic force. Repeating this for other times in the range of interest, an armature force curve like Fig. 8 is established. The waiting time is read directly, corresponding to F_1. The slope of the ensuing linear force range determines $\dot{F}_1$. This, with equation (21) completes the determination of mass controlled motion time. Of course, the final pull developed has to exceed the operated load. This check is made from another pull curve taken at the armature gap x_3, using the known final ampere turns and the load.

Analytical—For our present purposes, an analytical relation, expressed in the fundamental constants, similar to that for the waiting time is needed. Its derivation follows:

The solution which will be developed is based on linear circuit theory. This necessarily implies exponential flux rise, which is not exactly true. However, the relation is dimensionally correct and accurate to better than first order. Then the use which will be made is to determine the motion time of an electromagnet as the parameters are varied one at a time. These are plotted as ratios to one of them, chosen as a reference. By this means the ratio curves become accurate to better than second order and provide excellent correction factors for actual measured data.

For a linear circuit, the pole face gap flux will increase, after the winding circuit is closed to a battery, with the same time constant as the winding:

$$\varphi_G = \frac{4\pi NI}{\mathcal{R}_0 + \dfrac{x}{A}} (1 - e^{-t/T}), \tag{22}$$

where $T = L_1 (G_C + G'_E)$ and $I = E/R$. Then the pull:

$$-F = \frac{\varphi_G^2}{8\pi A} = \frac{1}{8\pi A} \frac{(4\pi NI)^2}{\left(\mathcal{R}_0 + \dfrac{x}{A}\right)^2} (1 - e^{-t/T})^2, \tag{23}$$

$$\frac{dF}{dt} = \frac{4\pi(NI)^2}{AT\left(\mathcal{R}_0 + \dfrac{x}{A}\right)^2} [e^{-t/T}(1 - e^{-t/T})]. \tag{24}$$

The bracket term is of the form $a(1 - a)$, $0 < a < 1$, which has a maximum value of 0.25, and from $0.2 < t/T < 1.5$ is between 0.15 and 0.25. This range corresponds to the maximum slope of the armature force versus time plot in Fig. 8. Arbitrarily 0.2 is chosen and the expression for the maximum rate of force rise becomes:

$$\dot{F} = \frac{4\pi(NI)^2}{5AT\left(\mathcal{R}_0 + \dfrac{x}{A}\right)^2}. \tag{25}$$

Substituting for T and rearranging:

$$\dot{F} = \frac{(NI)^2}{5(G_C + G'_E)(A\mathcal{R}_0 + x_1)\left(1 + \frac{\mathcal{R}_0}{\mathcal{R}_L} + \frac{x}{A\mathcal{R}_L}\right)}. \qquad (26)$$

Substituting (26) and (15) into (21), the final expression for motion time becomes:

$$t_2 = \sqrt[3]{\frac{30m}{W}(x_1 - x_3)\left(1 + \frac{G'_E}{G_C}\right)(A\mathcal{R}_0 + x_1)\left(1 + \frac{\mathcal{R}_0}{\mathcal{R}_L} + \frac{x_1}{A\mathcal{R}_L}\right)}. \qquad (27)$$

This, with equation (18) for the waiting time, forms the basis for the variation effects which now will be determined. Its region of accuracy is for windings with turns exceeding the best, as it presupposes the relay will always operate. It thus is applicable in the region where the time curves are parallel. For studies of lower numbers of turns, the three stage approximation of Part I does not have this limitation.

Operate Time Variations

For a particular electromagnetic structure the factors in equations (18) and (27) can be measured. These measured factors can then be compared to the manufacturing specification and estimates made of average values for each. With these average values, the waiting time, the motion time and then the sum, which is the operate time, can be computed. This is the reference condition.

Each factor is then varied over its appropriate range, one at a time, and the computation repeated. The ratio of these variation times to the reference condition can then be plotted. Fig. 9 shows the chart for the wire spring type relay. For this chart, the range for each factor was taken as 3 without regard to the actual range. Similar charts have been prepared for other types of electromagnets, including quite a size difference. These ratio charts all agree remarkably well when plotted in this way. For this reason, for early estimates of any structure in the mass controlled class, this chart is entirely adequate for estimates of variations.

The basic assumption made in this method of determination is that for small variations, the interactions are negligible and a separated solution of products, one for each factor, is applicable. Checks made by varying two at a time confirm that essentially the presumption is fulfilled.

The G_C curve does not exhibit a minimum, as do the measured curves of Fig. 6. This results from the simplifying stipulation made that the

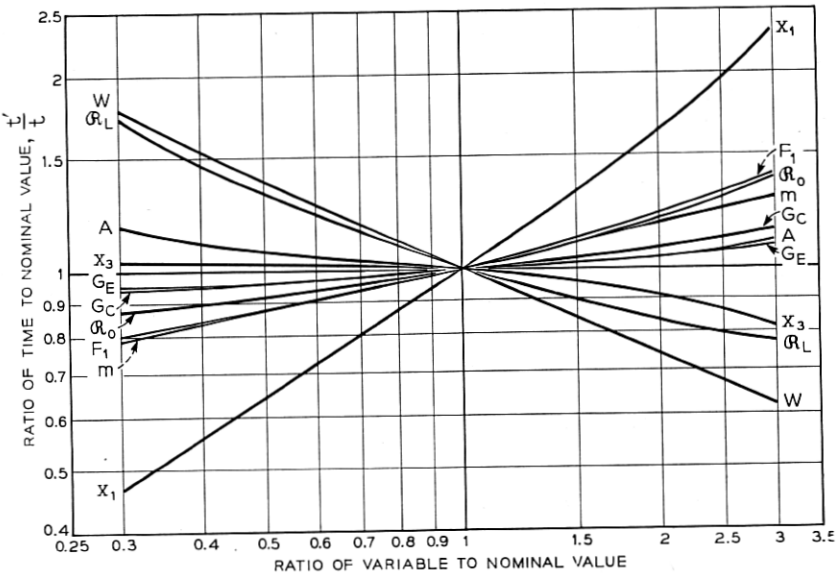


Fig. 9 — Mass controlled operate time variations.

relay always must operate. However, the purpose of these variation curves is to adjust measured data. The coil constant is the independent variable in any measurement and can always be set exactly. Hence it does not require an adjustment.

The chart shows clearly that the single most important factor is the armature travel. Following are power W, and leakage reluctance $\mathcal{R}_L$. The least sensitive is the pole face area A, because it has been optimized in the design. Taking the slopes at the (1, 1) point, the operate time, expressed in a separated variable form, becomes:

$$t_0 \approx \frac{x_1^{0.66} F_1^{0.24} \mathcal{R}_0^{0.2} m^{0.21} G_C^{.084} G_E^{.056} A^0}{W^{0.45} \mathcal{R}_L^{0.33} x_3^{.084}}. \qquad (28)$$

The exponents are a measure of sensitivity — the nearer they approach zero, the less the sensitivity. In designing electromagnets for speed, every effort should be made to keep the armature travel to a minimum as it is outstanding in its effect on time.

For our present purpose, the chart is used to adjust measured operate time data to the average value. The factors to be corrected are F_1, m, x_1, x_3, A, $\mathcal{R}_0$, $\mathcal{R}_L$, i.e., all the pertinent geometric and load factors shown in Fig. 7. This completes the description of the method used for establishing the average mass controlled operate time curves.

Maximum and Minimum Values

The next to last part of this phase of the article concerns estimates of the range of the mass controlled maximum operate time of a particular relay code. This involves determining the range of each of the variables from the manufacturing specifications. Then with the variation chart the range in the time, due to each component, is evaluated, as a ratio. The actual range of course is not the product of these, as all variables never will be adverse simultaneously, but it is greater than the largest single individual contribution. For general purposes, a root sum square addition is used. For the wire spring relay this amounts to a range of about ± 30 per cent.

Preliminary Estimate of Type of Operation

Earlier, a rule of thumb expression for an estimate of whether an electromagnet is in the mass or load controlled class was given. It is easily derived by use of the $\dot{F}$ concept, and the observation from computing the variation charts, that the waiting time generally is about one-half of the motion time when the electromagnet is mass controlled.

When the motion time begins, the force starts to build up according to equation (19). However, multiplying this initial rate by the motion time need not result in a force value equal to the load for operation to be complete. Four factors act to this end: (1) during the waiting time the back tension F_1 already has been overcome; (2) when the armature gap closes, the armature takes a greater portion of the winding flux; (3) as the armature moves in the winding flux builds up more rapidly; and (4) the kinetic energy can cause the armature to crash through a small distance where the load projects through the rising dynamic pull curve.

Calling t'_M the longest average motion time which is mass controlled, and arbitrarily setting

$$\dot{F}_1 t'_M = \frac{F_3}{2}$$

in view of the foregoing, an estimation of the limiting average $\dot{F}_1$ is determined. Substituting this in (20), the maximum average mass controlled motion time is:

$$t'_M = \sqrt{\frac{12m(x_3 - x_1)}{F_3}},$$

which, when multiplied by $3/2$ to allow for waiting time, and dropping the prime to indicate total time, becomes

$$t_M = \sqrt{\frac{27m(x_3 - x_1)}{F_3}}\,. \tag{29}$$

This equation was given as equation (1) in the early part of this article.

For the wire spring relay, this expression has a value of 7.5 millisec. Increasing this by 30 per cent for a maximum value, an estimate of 10 millisec results, agreeing with the earlier lower estimate of maximum time for load controlled operation.

Selection of Winding for Mass Controlled Operation

One more group of factors needs consideration before a winding is selected. These are (1) the range in dc resistance of the windings, (2) the winding temperature as determined by the duty cycle, and (3) the range in the battery voltage. The number of turns of the winding is ordinarily not considered as a variable once it is chosen because of the automatic machine method of winding. An examination of Fig. 6 shows that if the turns are too few, a greater time penalty obtains than if there are too many. Also, decreasing the circuit power, increases the best coil constant. These two considerations indicate that the best coil constant should be chosen under worst circuit conditions. For any other condition the operate time will be reduced. Further, the range between worst circuit and average time will be a minimum.

The procedure for choosing a winding is to determine the dc resistance of a maximum resistance winding at the operating temperature set by minimum battery voltage and the maximum duty cycle. This sets the worst circuit power, and by use of Fig. 6, the best number of turns. In no case is a winding specified with fewer turns than will supply sufficient ampere turns to operate the worst relay with the maximum load. In some cases of low power, this sets the number of turns. For some cases of intermediate power, heating requires the maximum winding surface area, also resulting in excess turns. The average resistance with its variation, all at a standardized temperature at 68°F, completes the design.

Summary of Single Relay Local Circuit Operation

An analytical determination of the operate time of a single relay cannot be obtained in closed form because it requires the solution of two simultaneous, non-linear, non-homogenous, differential equations, without adding the complications of representing the magnetic saturation and eddy current effects in some convenient form. Approximations for solutions have been developed which predict with good accuracy the order of operate time attainable for a design, but not of sufficient accuracy to exactly determine the best winding for a particular case. Thus once a magnetic structure has been established, the operate time data for a single relay necessarily have to be determined empirically by measurements using controlled samples.

Actually this procedure is quicker and easier, and besides it gives the correct answer, including all the non-linear effects, such as eddy currents, saturation and motion, no matter how complicated. Complete data for the range of all relays and windings using the given structure can be determined using one such relay with a full winding, through the use of impedance transformation techniques and estimation of small variation effects, using the approximate solutions. By using the approximate solutions only for corrections, the errors become of second or smaller order. Thus single relay operate time data can be determined accurately and presented in a form permitting either analysis or synthesis of performance.

The design of a best winding for load controlled operation, is accomplished by a variation of the method of successive approximations. Appropriate battery voltage, winding temperature and resistance of course are considered as part of the solution of the problem. A range of windings is chosen and operate time data established for each winding. If necessary, the range is extended in the appropriate direction until a minimum operate time is included. This minimum determines the best winding.

For mass controlled operation, the contact spring load is immaterial, and the data can be presented directly. As part of this type of study, the relative importance of the several parameters affecting the operate time has been determined. These show at a glance whether (1) a change will have a significant effect or (2) what change or changes are necessary to effect a necessary reduction in operate time.

OPERATE TIME — SERIES RELAYS

Series relays are two or more relays whose windings all are connected in series, and energized by the same current, controlled by a single contact. The impedance of each one enters into the manner in which the common current will increase, after contact closure. If the procedure used for single relays were followed, there would be a double infinity of combinations to portray, or else experimentally study each combination when proposed. This can be avoided, with no loss in generality, by transforming each relay into an equivalent single relay in a local circuit. Then the foregoing methods for single relays can be applied to each in turn.

This procedure is the common device of breaking up a complicated problem into parts, each of which can be solved by familiar methods. Two assumptions are made. The first is that each winding is a lumped two terminal network. At any instant the same current is in every turn of each relay and there is no propagation time involved. This holds for the times involved in electromagnets. The second assumption is that, when the relays have different operate times, the current reduction caused by the motional impedance of the first one to operate does not significantly extend the operate times of the later ones. In the following transformations, the winding turns, currents, and hence magnetomotive forces, are kept constant.

Identical Relays

If the two relays are identical they have some impedance $Z(p)$ which is the same for both. Part of this is the dc resistance and the other part is the ac effect, proportional to the turns squared. By the extension of Ohm's Law to ac circuits, when a potential source is applied to the two identical devices in series, exactly one-half the source appears across either device at any time after application, forming a virtual constant potential point. Thus if a battery of E_0 volts is applied, exactly one-half the battery appears across each, including the effect of eddy currents. Now if the voltage across a coil is known, then the response is uniquely determined, knowing just the relay characteristics and the voltage, disregarding the mechanism of how the voltage is applied. For this situation the voltage is in a most convenient form, represented by exactly one-half the battery. The operate time can easily be determined for either relay with this information, as the effects of eddy currents and motion are included in the data. The coil constant is already known and the power is one-half the total power.

General Case

This procedure can be generalized to include any division of dc resistance, different turns, and different magnetic structures providing, for the latter case, that eddy currents can be ignored. The justification for this will be considered later.

The basic problem is: given the battery voltage E_0, relay No. 1 of N_1 turns, a 1 turn inductance L_1, and resistance R_1, in series with relay No.

TABLE I — EQUATION 30

Relay	*Turns N*	*Resistance R_E*	*Voltage E_E*	*Power W_E*	*Coil Constant*
No. 1	N_1	$\dfrac{R_1 + R_2}{1 + \frac{L_2}{L_1}\left(\frac{N_2}{N_1}\right)^2}$	$\dfrac{E_0}{1 + \frac{L_2}{L_1}\left(\frac{N_2}{N_1}\right)^2}$	$\dfrac{E_0^2}{(R_1 + R_2)\left[1 + \frac{L_2}{L_1}\left(\frac{N_2}{N_1}\right)^2\right]}$	$\dfrac{N_1^2}{R_1 + R_2}\left[1 + \frac{L_2}{L_1}\left(\frac{N_2}{N_1}\right)^2\right]$
No. 2	N_2	$\dfrac{R_1 + R_2}{1 + \frac{L_1}{L_2}\left(\frac{N_1}{N_2}\right)^2}$	$\dfrac{E_0}{1 + \frac{L_1}{L_2}\left(\frac{N_1}{N_2}\right)^2}$	$\dfrac{E_0^2}{(R_1 + R_2)\left[1 + \frac{L_1}{L_2}\left(\frac{N_1}{N_2}\right)^2\right]}$	$\dfrac{N_2^2}{R_1 + R_2}\left[1 + \frac{L_1}{L_2}\left(\frac{N_1}{N_2}\right)^2\right]$

2 of N_2 turns, a 1 turn inductance L_2 and a resistance R_2. What are the two equivalent relays, each in a local circuit and what are their virtual applied voltages? The first step is to reassign the total resistance to obtain two equivalent windings having the same time constant N^2L_1/R. Then determine the two virtual voltages by division of the total in proportion to the impedances. These and their dependent power and coil constant relationships for the case of two relays are tabulated in Table I. The procedure can be extended to any number of dissimilar series structures.

For the case of all identical magnetic structures, the 1 turn inductances L_1, L_2, etc., are all equal and their ratios become unity, simplifying the expressions. Note that the coil constants are not equal unless the structures are the same magnetically, but that the time constants always are equal. Further, the total power is constant and equal to that of the original circuit.

After determining the effective powers and coil constants, the operate time for each can be read from the applicable single relay charts such as Fig. 6.

Two Like Parallel Relays in Series with a Third Relay, Identical Structures

The equivalent relay method can be extended to include the case of two like parallel relays in series with a third relay, as shown in Fig. 10. The first observation to make is that, because of symmetry, the current flow in the two parallel relays is identical. No winding current change would be made if, for instance, all the dc resistance of the two parallel relays were removed and half of either were connected in series with the single relay. Thus again, the resistances can be assigned as necessary to result in equal winding constants and total power. The same net dc resistance gives a second condition:

$$\frac{N_1^2}{R_{1E}} = \frac{N_2^2}{R_{2E}},$$
$$R_1 + \frac{R_2}{2} = R_{1E} + \frac{R_{2E}}{2}. \tag{31}$$

Solving

$$R_{1E} = \frac{2R_1 + R_2}{2 + \left(\frac{N_2}{N_1}\right)^2},$$

$$R_{2E} = 2\left(R_1 + \frac{R_2}{2} - R_{1E}\right). \tag{32}$$

Because the equivalent coil constants have the same ratio as the equivalent resistances, the voltage division will be:

$$E_{1E} = \frac{R_{1E}E_0}{R_1 + \frac{R_2}{2}},$$

$$E_{2E} = E_0 - E_{1E}. \tag{33}$$

With these, the equivalent coil constants and powers can be computed and charts such as Fig. 6 used as before.

Selection of Optimum Coils, Identical Structures

Neither Winding Known, Operate Times Given

In the above discussions it was assumed that the windings were known and that the operate time data were sought. The procedure can be reversed, starting with a desired operate time, or times, and then choosing optimum coils. From time curves such as Fig. 6, the watts corresponding to the desired times can be obtained provided they are read from the same coil constant vertical because the same current flows through both and hence the effective coil constants necessarily must be equal.

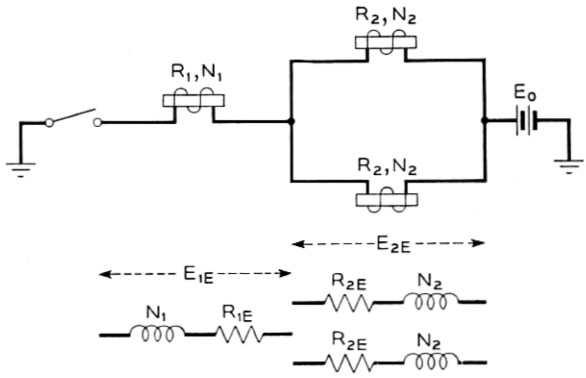


Fig. 10 — Transformation of series-parallel relay circuit.

The particular vertical N^2/R to be used is the one passing through the higher of the two given times, which lies at the minimum point on a watts curve. Satisfying the smaller wattage relay assures sufficient ampere-turns for both windings, and also gives a smaller time loss compared to optimum for the other higher speed relay, as the higher power curves are flatter for coil constants above the optimum. Thus the specified times determine equivalent watts for each coil, and the actual total circuit watts are exactly the sum of the equivalent watts. Hence:

$$W_{\text{total}} = \frac{E_0^2}{R_1 + R_2}. \tag{34}$$

This sets the total resistance $(R_1 + R_2)$. Also, knowing the current is the same through the two coils, and that the actual division of the dc resistance has no effect on time, the ratio of the two resistances can initially be chosen the same as the ratio of the two effective powers:

$$\frac{W_1}{W_2} = \frac{R_1}{R_2}. \tag{35}$$

Finally, knowing that the effective coil constants for the two relays are equal, the desired turns are determined using the already selected coil constant, G_C. Solving these three equations, the individual relays are:

$$R_1 = \frac{W_1 E_0^2}{W_{\text{total}}^2}, \quad R_2 = \frac{W_2 E_0^2}{W_{\text{total}}^2}, \quad N_1 = \sqrt{R_1 G_C}, \quad N_2 = \sqrt{R_2 G_C}. \tag{36}$$

The resistance values R_1 and R_2 can be used as shown above or divided in any other way as long as the total is unchanged. The numbers of turns, however have to be kept fixed.

A particularly simple relationship exists when it is desired to have equal times. In this case the turns on the two coils are equal.

One Winding Known

As is often the case in actual use, a winding must be chosen to be used in series with another known winding and have best operate times. The method described here can be applied to any relay structure, but the numerical values in the analysis are applicable to the wire spring relays only. The first step is to choose the desired resistance for the coil. This is usually set by the heating and power limits, knowing that the higher the total power is, the less the operate time. Then the choice of turns for the coil depends on which of the two coils needs speed the most.

Assume first that we want speed on the coil to be designed, say relay No. 1, rather than the known relay in the circuit, say relay No. 2. Then we want the effective power for this relay as high as possible provided that the coil constant is not too far from optimum. From Table I it has already been noted that the two effective coil constants are always equal when the structures are identical. Also the sum of the two effective powers is exactly equal to the total power; that is, as the effective power to the first relay increases by increasing winding turns, that to the other correspondingly decreases. We see that for relay No. 1, the effective power increases as N_1 increases, but also that the effective coil constant increases. Thus an optimum turns value can be found, where on one side the low effective power slows up the operate time, and on the other the high coil constant does. Fig. 11 shows these optimum values. The solid curves for relay No. 1 are plots of time versus the turns ratio N_2/N_1 with total power, $E^2/(R_1 + R_2)$, and the "series coil constant" of relay No. 2, $N_2^2/(R_1 + R_2)$, as parameters. The optimum turns values for relay No. 1 show up clearly on this curve, and are seen to vary with both parameters.

This relation of optimum turns to the two parameters is shown on Fig. 12, where optimum N_2/N_1 values are plotted against total power with the relay No. 2 series coil constant as the parameter. The added series relay always has the most turns when it is designed for least time. Where speed on relay No. 1 is the only concern, the optimum turns can be chosen directly and easily from Fig. 12. Fig. 11, however is of more general use since it actually gives the times and also shows the time values for the second relay (the dotted curves). Thus the turns can be chosen to approach optimum speed on either relay or to choose a compromise value.

Now for relay No. 2 with total power $E^2/(R_1 + R_2)$, and the second relay series coil constant $N_2^2/(R_1 + R_2)$ as parameters, the effective power decreases and the effective G increases when N_1 is increased, both increasing the time. In other words, the given relay will always be slowed down by any added series relay winding.

The minimum N_1 value is limited by sufficient ampere-turns to operate the first relay. As shown by the dotted curves of Fig. 11, which are the time versus N_2/N_1 curves for relay No. 2, the gain in speed is slight as N_2/N_1 is increased beyond about 2. Thus, although a compromise must always be chosen for speed on relay No. 2, the loss in speed for relay No. 2 is not necessarily great if N_2/N_1 can be chosen near 2. For the case of equal operate times, in every case equality of turns of course applies.

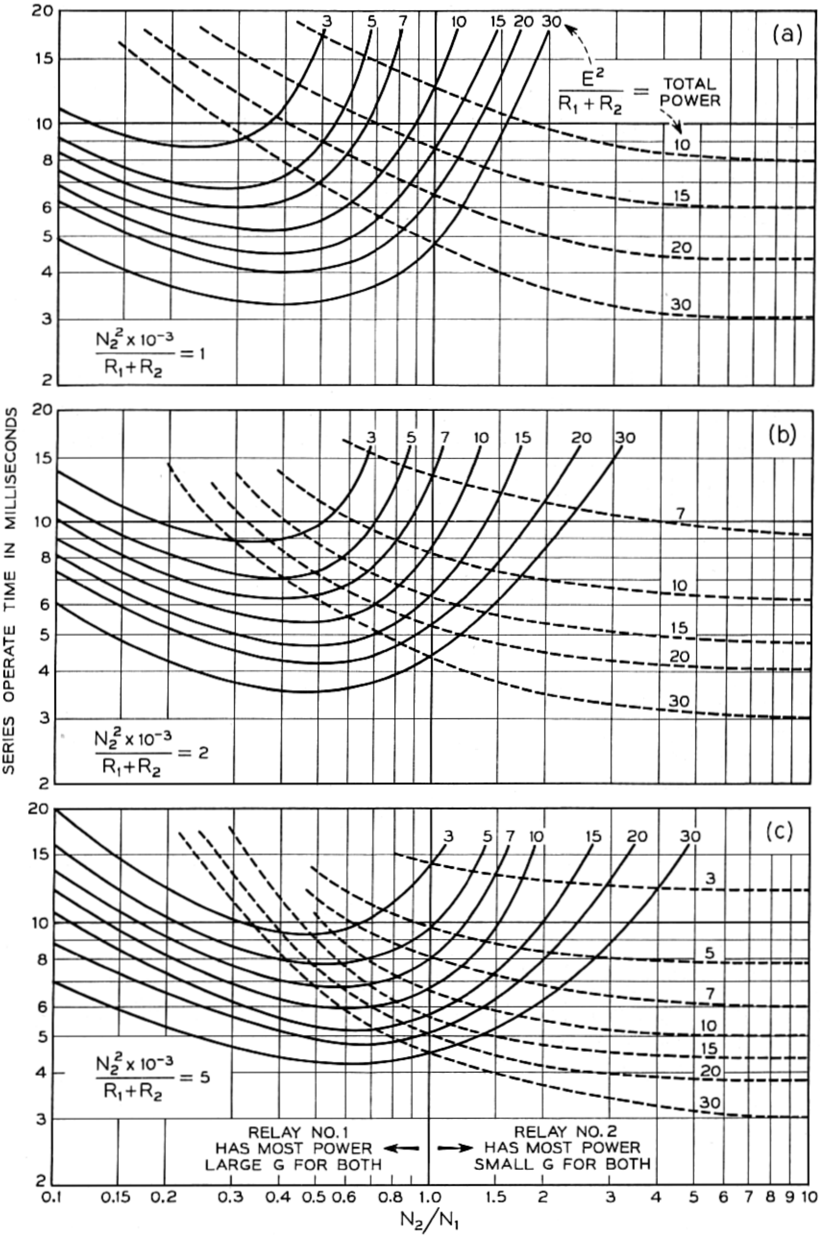
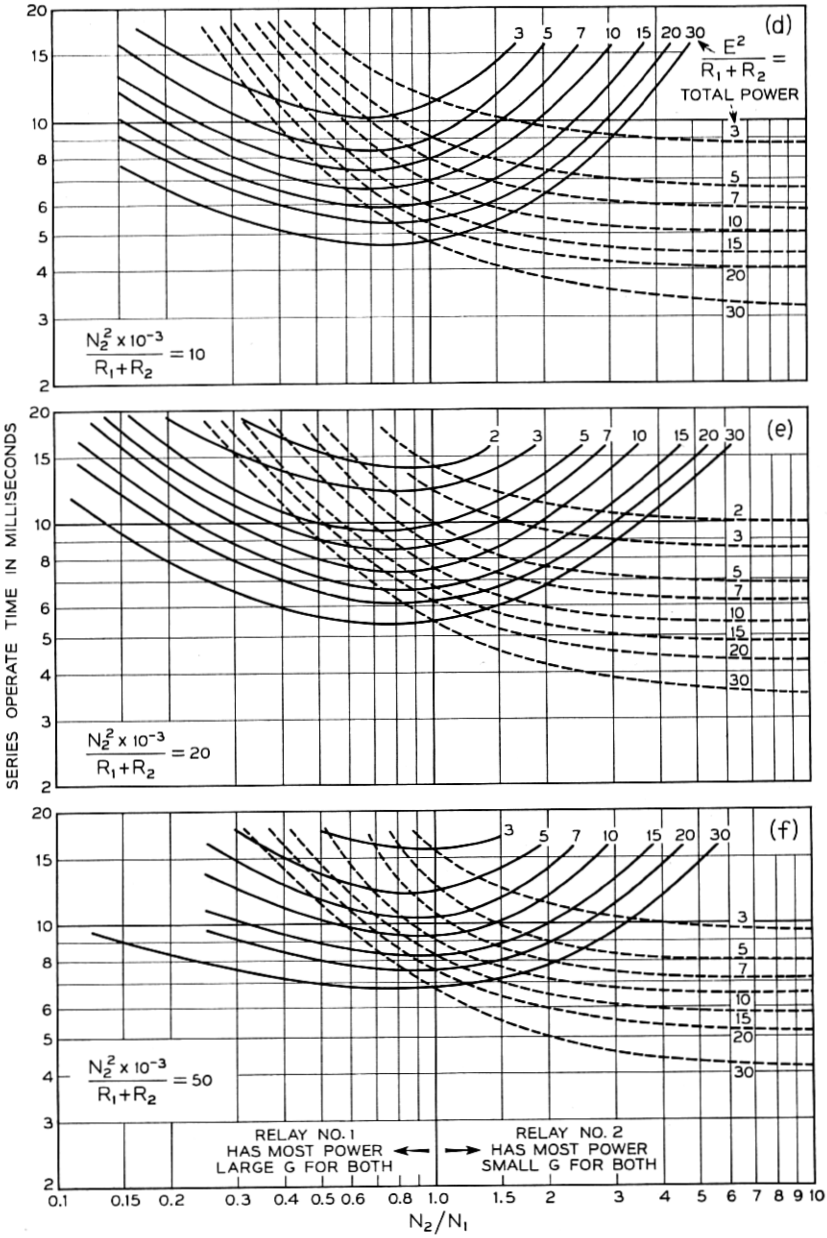


Fig. 11 — Operate time of



series relays versus turns ratio.

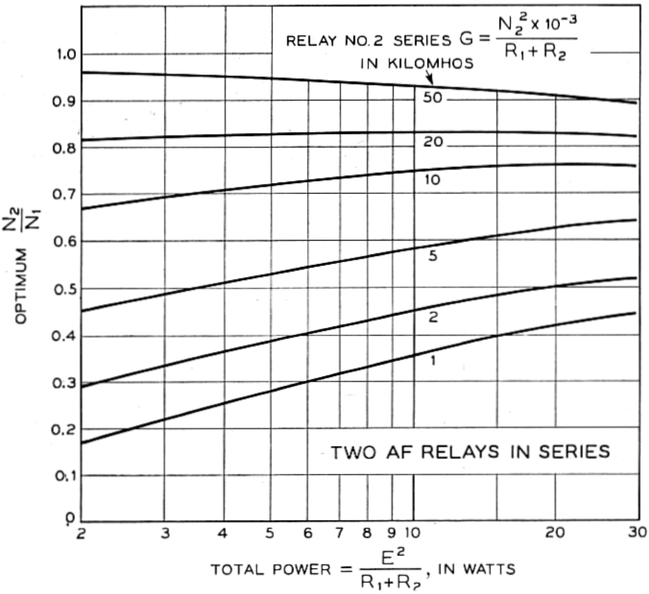


Fig. 12 — Optimum turns ratio for speed on relay No. 1, No. 2 known.

THE OMISSION OF EDDY CURRENT CORRECTIONS

The foregoing discussion of series connected relays developed transformation relationships ignoring eddy current effects. The following discussion will qualitatively show that ignoring the eddy current effects introduces second or smaller order errors.

For very large coil constants and different magnetic structures, the core constants can be ignored. With large coils, the current and flux rise are influenced very little by the core constant.[2] As the coil constant is reduced, the core effect becomes more important, but not significantly so until the speed class is reached. Unfortunately, we are generally only interested in the speed case. In the following, the most affected high speed applications are considered. It is found that the virtual constant potential point is only slightly affected by the eddy currents, and that equivalent single relays for each of the series relays determined using only the winding resistance and turns, with the relay reluctance correction as described above, are sufficiently accurate. The operate time for each relay itself, is affected by the core. This effect of course is included in the measured timing data which are used after equivalents are determined. The present discussion is directed toward the effect of the core eddy currents on the voltage division.

The linear equations used for developing the first approximation operate time equations also represent the behavior of the three element, two terminal network of Fig. 13, shown referred to a one turn admittance form. This exhibits the core eddy current effect as a resistor shunting an ideal inductance, rather than as an infinite line. Whatever value the shunting resistor may have, it affects only the transient response of the network, the part with which we are now concerned.

In what follows, it will be assumed that the transformation relations have already been applied, and the G_C values applying to Fig. 13 are effective values related to the winding turns in accordance with equations (30).

For two such networks in series, the voltage division would be independent of time if the ratios of the shunting to series conductances for each structure were equal. The method developed for similar structures then would apply with no error.

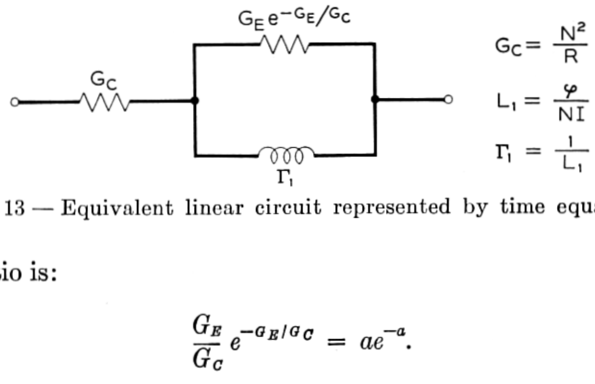


Fig. 13 — Equivalent linear circuit represented by time equation.

The ratio is:

$$\frac{G_E}{G_C} e^{-G_E/G_C} = ae^{-a}.$$

This function has a maximum value of $1/e$; that is, the shunting resistor is at least e times larger than the series resistor. An analysis of the range of core conductances and speed windings in use, shows that the actual resistance ratios are somewhat greater than this and hence the ratios are not quite independent of the structure. However, because the maximum is broad, it reduces the actual range to about 10 per cent, including all windings plus a 2 to 1 G_E change. In turn, this signifies that for a suddenly applied voltage, the initial linear network voltage division would differ from the final by less than 10 per cent. This error decreases with time.

The above discussion applies to linear networks such as Fig. 13. In an actual magnetic structure the initial voltage division is not affected by eddy currents as they have not had time to build up. The voltage division starts and ends exactly in the ratio of the effective series resistances by virtue of the method used in determining them. For times after circuit closure, a transient voltage error does develop, but it will not approach in magnitude the initial transient error of the linear network. The operate time error will be still smaller. The operate time varies as the two-thirds power of the voltage (power varies as the voltage squared).

These considerations lead to the conclusion that ignoring the eddy currents in setting up the equivalent relays, results at the most in errors of operate time estimates for series connected relays of the order of a few per cent.

RELEASE TIME

The relase time of a relay is not as directly affected by the winding as is the operate time. For instance, if it is opened by the controlling contact without an *RC* network, the winding current, ignoring arcing at the contacts, abruptly drops to zero. The winding subsequently plays no part in the flux decay. If the winding has a shunting resistor or *RC* circuit, then winding current does flow and some effect is present. A resistor always increases the release time. A favorable *RC* choice can cause a slight decrease in release time. Because of this minor effect, the winding almost always is designed from operate time or sensitivity considerations.

Differing slightly from operate time, the release time is divided into two parts, (1) waiting time plus motion time until actuation of the nearest contact and (2) stagger time. For simplicity the combination (1) above is merely called release waiting time.

The release time of a relay is a more complicated function than is the operate time. The primary cause of this is the closed gap situation, with little stabilizing effect from an air gap. For the earlier operate time studies the air gap largely contributed to the simple exponential relationships. A second effect in release is that the magnetic material is almost always in the non-linear saturated portion of its characteristic. Hence an approximately linear relationship between steady state flux and current cannot be assumed. Finally, for release without an *RC* network, the only current flows in the core, so it completely controls the flux decay. Even with an *RC* network, only a minor decrease in release time is possible. For these reasons, except for rough preliminary estimates of release times during preliminary design, all release data are based on measurements.

Conductance Shunt

In a companion article,[3] a hyperbolic relationship between the flux and current is used to represent the portion of the hysteresis loop of concern in release. This, with a conducting sleeve or shunted winding, gives an excellent representation of the release waiting time. It also provides an understanding of the controlling parameters, even with only eddy currents controlling the release. The form most useful for release consideration is:

$$t = \frac{(\varphi'' - \varphi_0)(G_S + G_C + G_E)}{NI_0} \left(\frac{\ln z}{z - 1} - \frac{1}{z} \right), \tag{37}$$

where

$$z = \frac{\varphi'' - \varphi_0}{\varphi - \varphi_0}.$$

In these experessions, φ is the flux corresponding to the ampere turns NI_0 at which the relay just releases, φ_0 is the residual flux, and φ'' is the asymptotic saturation flux. The conductance terms are the same as for the operate case except G_C now is computed using the dc winding resistance plus any winding shunting resistance. If the winding has no shunt then $G_C = 0$. The operated contact spring load enters through φ and NI_0.

The function of z in brackets has a broad maximum in the region of relay release. It therefore is appropriate to consider the releasing ampere turns as the independent variable and plot the measured release time with the total conductance as a parameter covering the range of interest. As G_E is a characteristic of the structure and not subject to adjustment, the curves are actually labelled in terms of just the sleeve, if any, plus the shunted winding. For the wire spring relay, Fig. 14 shows data in this form. If the function of z were truly constant, the curves would all have a slope of -1 in this plot.

Strictly speaking, the above equation applies only to the time until the magnetic pull has decayed to equality with the operated spring load. Following this is the motion time, during which the pull decays further. For convenience, however, the armature motion time through the distance to the nearest contact is included in the chart for release time as (a) the contact actuation is the means used to measure the time and (b) this much motion always takes place before any contact is actuated.

The further displacement of the armature continues at almost constant velocity. This is because the decreasing magnetic pull (called drag for the release case) has become small and the contact spring load is dropped by the motion. The further forces acting on the armature then are only the back tension and the difference between the rapidly decreasing contact spring load and magnetic drag.

Returning now to equation (37) above, the release time is directly proportional to the sum of the conductances, and to the difference between the saturation and residual fluxes. The conductance variations can be determined from temperature, winding data, sleeve dimensions, and material. The saturation flux is directly proportional to the core

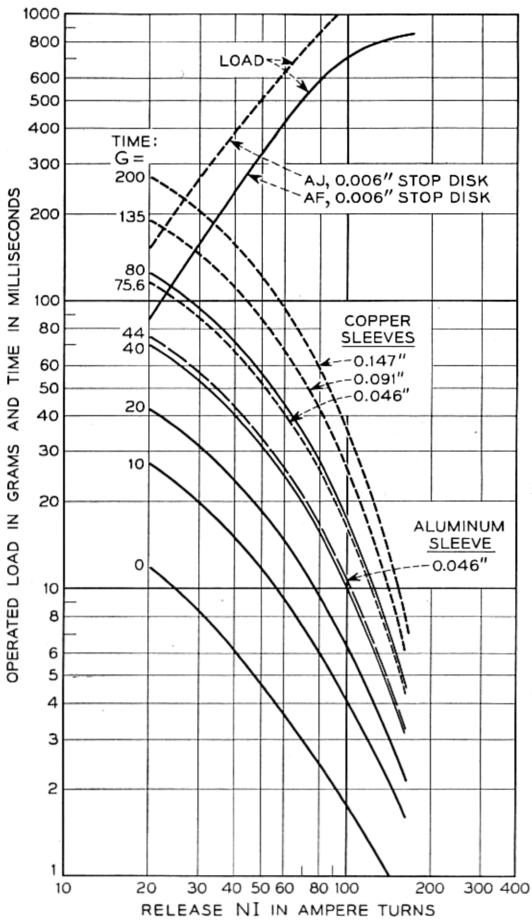


Fig. 14 — Release time with a shunted winding.

cross-section. The residual flux is directly proportional to the coercive force times the length of the magnetic material, and inversely propotional to the closed gap reluctance. The two largest factors affecting the closed gap reluctance are magnetic permeability, and fit of the joints, including variations in stop pin height. By measuring these factors on the test relay and estimating the corresponding values for the desired reference condition, the measured data can be corrected to the reference condition. Then having the release pull curves after magnetic soak for the same reference condition, the release ampere turns for any load under consideration and hence its release waiting time can be determined. These release pull curves are also shown as part of Fig. 14 for the wire spring relay. The ordinate scale is marked for both contact spring load in grams and releasing time in milliseconds. The particular chart shown is for a constant initial number of ampere turns. For other initial values, a correction chart is provided. If not as desired, the time can be adjusted either upward or downward by changing to a different sleeve, shunting resistor, or both. Of course, in no shunting conductance case can the release time be less than the open circuit time.

RC Shunt

The above considerations all related to shunting conductances. These serve the purpose of increasing the release time. The shunting resistor also greatly reduces the transient peak voltage developed when the winding circuit contact is opened. For contact protection reasons, *RC* networks are frequently used across the winding or contact for this same purpose. Such a network also has no power drain when the relay winding is energized. By a suitable choice of capacitance the network can reduce the release time to a value less than the open circuit time. It does this by developing a heavily damped oscillation of winding current. The frequency must be of the order of the reciprocal of the unprotected release time for a release time reduction. This fixes the choice of capacitance to a small range i.e., there is a best capacitor, one for each winding. Smaller values cause the time to increase toward the unshunted case. Larger values also cause an increase but for this case, if too large, an increased time beyond the unshunted case can result.

For speed windings, where timing is important, a network is designed for each. For the slower higher resistance windings, because time is not as important, the closest to the best of the available networks is chosen. This results in a time penalty but furthers standardization.[1]

To evaluate a network, the transformations developed and shown in

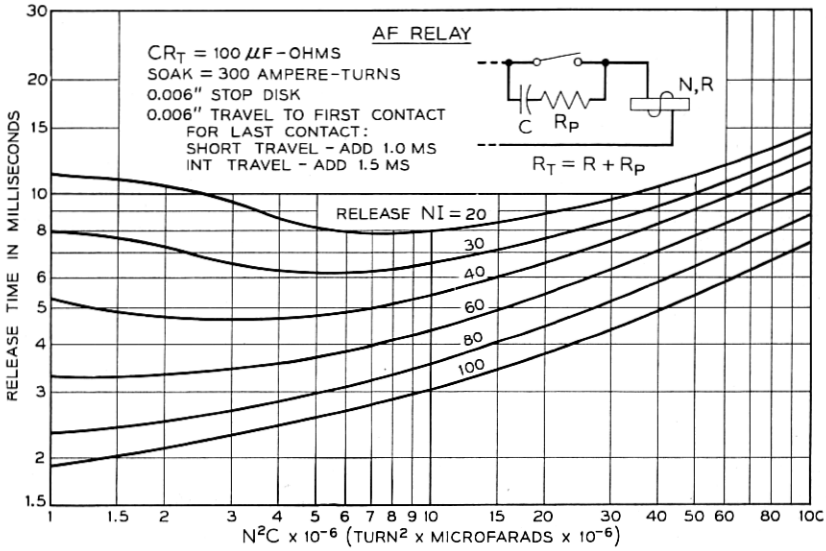


Fig. 15 — Release time as a function of N^2C.

Fig. 5 are used. This permits exact simulation for any winding and network from a time standpoint. For presentation of data, an arbitrary separated product form for the more important variables is used. The particular factors chosen are:

(1) N^2C, a measure of frequency, with NI_0 (RELEASE NI) as a parameter.

(2) RC, a damping term, with NI_0 as a parameter.

(3) Soak NI with N^2/R as a parameter.

Essentially what is assumed is that the release time can be represented well enough as a function of the form:

$$t_1 = f_1(N^2C) \times f_2(RC) \times f_3(NI). \tag{38}$$

Then by holding two factors fixed and varying the third, each factor in turn is evaluated. Generally the accuracy is better than 10 per cent even with this elementary approach. For a few cases the error approaches 15 per cent. Typical charts for these three factors are shown in Figs. 15, 16 and 17.

Release Time for Series Relays and an RC Shunt

With two series relay windings it is usual to provide a single RC network, as shown in Fig. 18. This results in a double infinity of possible combinations if handled directly. However, it can be broken into two

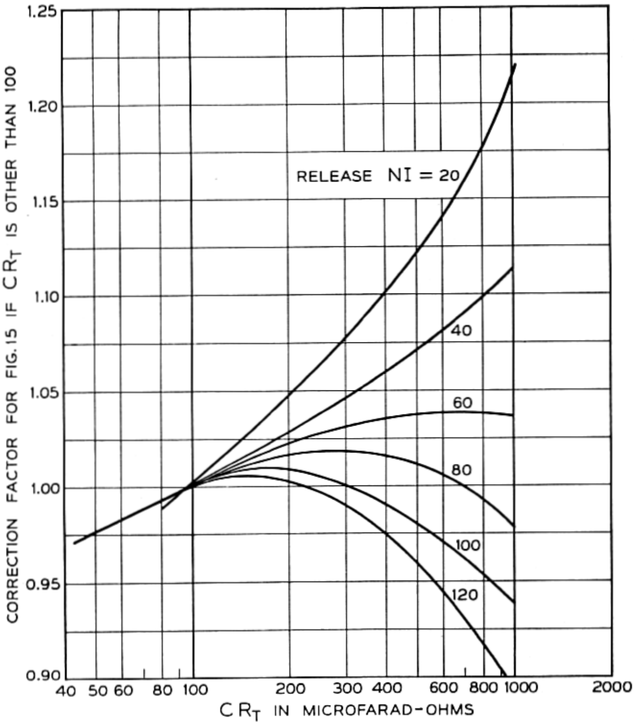


Fig. 16 — Release time correction for *RC* damping.

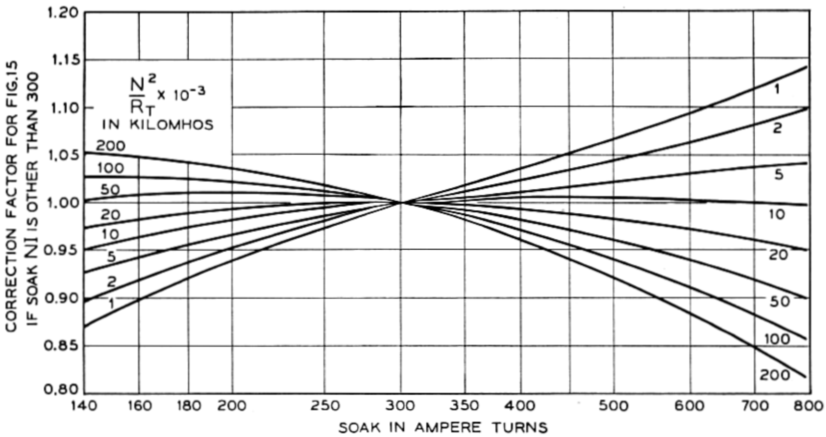


Fig. 17 — Release time correction for magnetic soak.

parts, each of which exactly represents, as individual relays, the two series relays. This is done by first assigning the total of the winding resistances for equal N^2/R values. The two equivalent relays then form a voltage divider, independent of frequency. The RC network next is drawn as two series RC networks as shown in Fig. 19. The procedure now is to determine the component RC values to have the identical voltage divider effect as the relays. Then the two equal voltage points can be connected as shown by the dotted line and no circulating current will flow. The resultant is a three node circuit and each branch can be considered independently, using the method of the preceding section.

There are four unknowns and hence four equations are needed. They are

$$R_{11} + R_{22} = R, \qquad \frac{N_1^2}{R_{11}} = \frac{N_2^2}{R_{22}},$$
$$\frac{C_1 C_2}{C_1 + C_2} = C, \qquad N_1^2 C_1 = N_2^2 C_2. \tag{39}$$

The solutions for the four unknowns are shown on Fig. 19.

For experimental measurements, each is transformed again by means of Fig. 5. Note that the time constants all are equal, as they must be, because the same current flows through all elements. However, the initial ampere turns NI are different by virtue of $N_1 \neq N_2$. Thus only one circuit like Fig. 5 need be set up. Measurements then are made with two different voltages to provide the two different ampere turns and the two release times. By choosing the subscript 1 network to represent the actual circuit, then no subscript confusion results in arriving at the simulating circuit of Fig. 5.

Release Time for Similar Parallel Relays and an RC Shunt

Similar parallel relays are split as shown in Fig. 20. Two equal parallel RC networks are first drawn. One is then assigned to each, and then the pairs are divided. The release times are each equal to that of one of the separated circuits.

Summary

The release time of fast electromagnets is influenced much more than the operate time by the fit of the magnetic parts. For release, the small non-magnetic stop disc introduces a relatively small stabilizing air gap compared to the open gap of the operate case. Secondly, the release always starts after an applied magnetomotive force which differs as between circuits, whereas operation invariably follows an open circuit. Thirdly, with *RC* protection networks connected, a more complicated winding current is present. Finally, the transmission line behavior of the magnetic core material is less masked by the winding, and in fact controls completely for the open circuit case. For these reasons, the

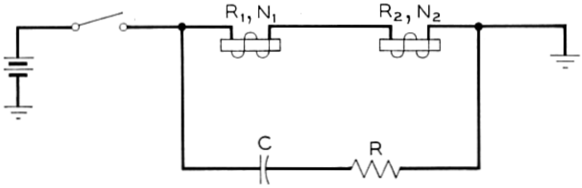
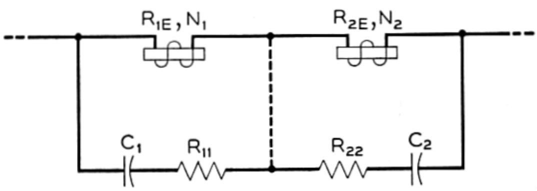


Fig. 18 — Series relays with one *RC* network.



$$R_{11} = \frac{R}{1 + \left(\frac{N_1}{N_2}\right)^2} \qquad C_1 = C\left[1 + \left(\frac{N_2}{N_1}\right)^2\right]$$

$$R_{22} = \frac{R}{1 + \left(\frac{N_2}{N_1}\right)^2} \qquad C_2 = C\left[1 + \left(\frac{N_1}{N_2}\right)^2\right]$$

Fig. 19 — Transformation to series *RC* networks.

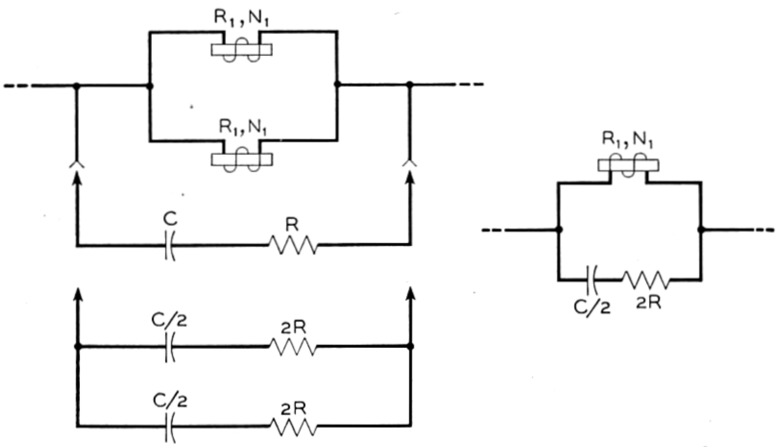


Fig. 20 — Transformation to parallel *RC* networks.

analytical presentation, as noted in Part I, of fast release time data is not now as advanced as the operate time data.

The material presented here describes the present state of the art. For slow releasing relays, the performance can be predicted with accuracy. For fast relays, while the general pattern is known, accurate means for estimating variations have not been developed. The present engineering of releasing relay circuits therefore, depends upon specific measured data in chart form, for each condition.

ACKNOWLEDGMENTS

The analyses leading to the forms of data presentation for the several types of relay timing information are the results of contributions from many people. In particular, the early nomograms for load controlled operation were developed by P. W. Swenson. The node method for separation of series releasing relays shunted by an *RC* network was developed by R. H. Gumley. The graphical method for design of optimum series windings was developed by Mrs. K. R. Randall. To her, I am also indebted for preparation of all the charts of measurements.

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