Economics of Telephone Relay Applications

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Today's telephone central office is largely built around the telephone relay. As each office may use some 60,000 of them, their performance characteristics, as well as their first cost, have a very important influence on the cost of the office. By properly balancing such factors economically, the lowest cost office can be planned.

This paper shows how performance features and design factors may all be expressed as "equivalent first cost," and so be related to manufacturing cost itself. The influence of lot-size on manufacturing cost is considered including a determination of optimum lot-sizes, to aid the relay designer in deciding on standardized models.

With a basic performance language formulated — equivalent first cost — optimum relations among the variables may be established. Such results are given for (a) optimum coil-plus-power costs, (b) optimum power-plus-speed costs, (c) optimum number of relay codes, and other related problems. Methods are also given to evaluate how serious the effect may be when optimum conditions are not satisfied.

Were it not for the application of these methods, central office costs would be much higher.

INTRODUCTION

The telephone central office of today is in part an enormous computing machine which, upon instructions from the customer, given by his dial, calculates how to find the called party; then the remaining mechanism completes the call. This machine is composed largely of interconnected relays, used in enormous quantities. Every dial call in a large city involves around 1000 relays. A typical large office contains more than 60,000 of them; in fact, they are used so extensively that the Western Electric Company manufactures about ten of them for every new subscriber, and their output is figured in tens of millions per year. It is not surprising that relay use has an enormous influence on the cost of the central office — not just because of relay purchase cost but also the relay's influence on other office factors. For example, the size of the power plant and the total number of equipments for common control, which depend on the functioning time of the relays, are decided by the relay characteristics. In the application of relays, then, it may well prove that the largest economies can be realized by spending a little more for each relay in the beginning in order to save still more in the cost of the power plant, common control equipment, size of the building, and so forth. It has been found that attention to ways of optimizing the application costs for relays can lead to an appreciably lower cost central office, and this paper will illustrate a few cases of how such a problem is approached. Because the telephone system is so large, the influence of each relay, taken in total, is also large. Fortunately, at Bell Telephone Laboratories, it has been possible to take the over-all view of the subject through familiarity with all phases of the relay application problem; and large economies which will eventually benefit the customer are resulting.

The basic problem to be discussed is how best to realize maximum economy of the central office so far as relays and their uses are concerned. As in most engineering problems, it is necessary to evaluate and compromise between oppositely varying cost effects of such things as efficiency, manufacturing cost, amount of equipment, ease of mounting and wiring, maintenance in all its aspects, and the like. The end result of each of these variables is its economic effect on the office as a whole, and they can only be compared if their values can be stated on a comparable basis. It has been found that each such effect may be considered as an incremental cost over and above a reference cost for its particular ideal condition. If properly chosen so as to be independent, then all such incremental costs may be added. They then represent the net "cost penalty," compared to the design with all ideal conditions taken together, and describe the merit of the design. They can also be used to find the optimum design. The methods apply generally to many other similar problems.

BREAKDOWN OF THE PROBLEM

Consider first how relays are applied in the telephone switching system. To the greatest extent possible a basic relay structure is chosen, carefully planned for low maintenance effort, all of whose basic parts can be made by mass production methods. On this basic framework one

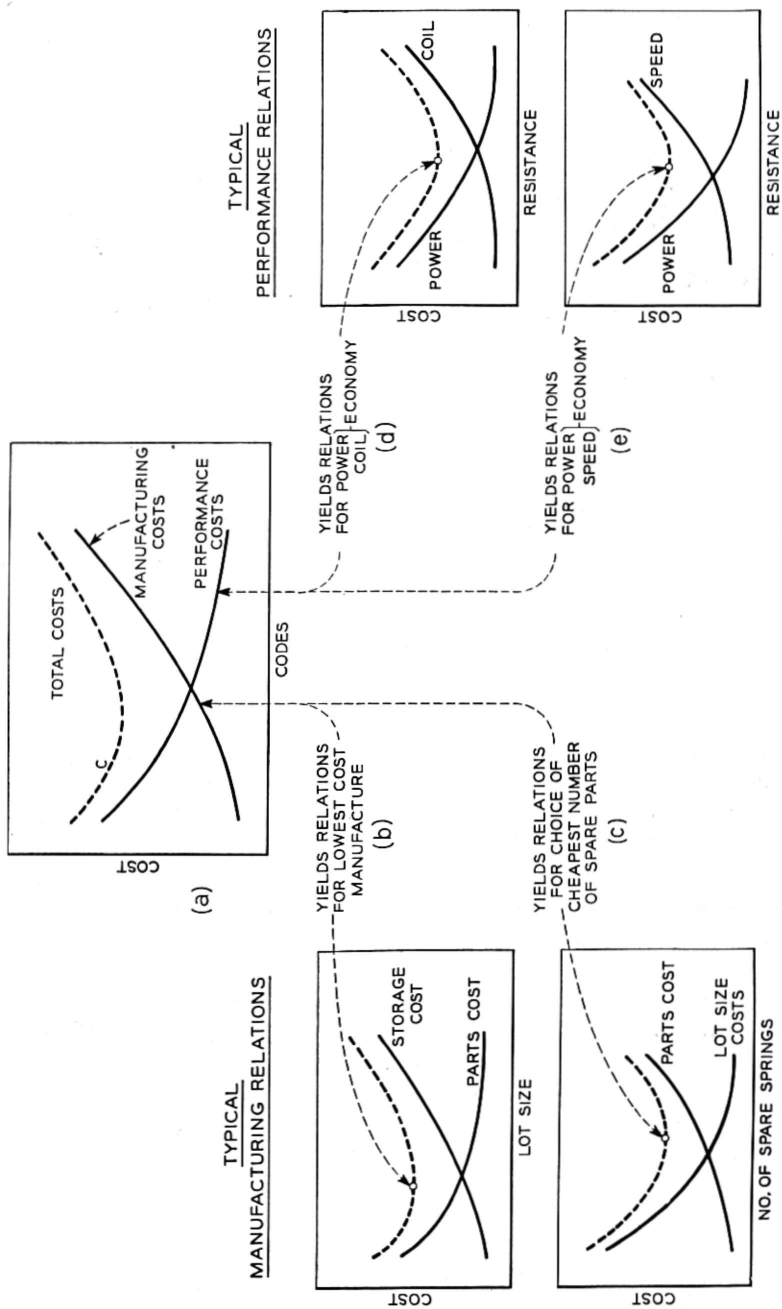


Fig. 1 — Simplified view of procedure for gaining maximum system economy.

may apply (a) suitable windings to satisfy the various conditions imposed by a need for sensitivity, speed, lowest possible first cost, or some other of the many specialized requirements to be encountered; and (b) particular arrangements of contact springs which will provide for the desired functions in the circuit ranging from a single "make" up to complicated sequences such as 12 sets of "transfers." Every different combination of the basic parts is termed a "code," meaning another *kind* of relay built up from the parts common to the basic *type* in question. If an increasingly large number of such codes is encouraged, each tailored to some special circuit function, then the advantages of mass production begin to be lost as the total demand is split into more and more subdivisions, each with comparatively low demand. Each additional *kind* of relay is a problem in "coding economics," and can lead to excessive telephone office costs unless it is properly considered.

Satisfactory solutions to this coding problem in turn depend on a detailed knowledge of all the factors governing either the first cost of the relays, or the first cost of other features of the system, as controlled by the relays. When all such effects are properly stated, they can be compared and brought into economic balance. Though such studies soon branch out into many fields, the enormous dollar savings are strong incentives to do whatever work is needed.

Best economic balance in the system is formed through a series of optimizing steps, generally illustrated by Fig. 1. Graph (a) shows how the total cost of an office may vary with number of codes. The performance part of the cost diminishes with increasing codes because each code will more nearly satisfy the precise circuit needs. But the manufacturing costs of the relays will rise with increasing codes because the lot-sizes grow ever smaller and hence more costly. The lowest point on the summation curve represents a desirable goal. The two base curves of (a) in turn are built from detailed facts about parts costs, coil costs, power plant and equipment costs, and many other similar data. The curve of manufacturing cost, for instance, is built from graphs like (b) and (c) which tell how factory costs will vary. The curve of performance cost is built from graphs like (d) and (e) which tell how office costs will vary for things like power and speed. In each such case, important individual economies result by following similar optimizing steps. Procedures for the practical application of these ideas have been of particular help in recent redesigns of the No. 5 crossbar system to use the newly developed wire spring relay family.[1]

[1] A. C. Keller, A New General Purpose Relay for Telephone Switching Systems, B. S. T. J., **31**, pp. 1023–1067, Nov., 1952.

In carrying out an actual study, a point of reference is needed first. This involves (a) fixing a standard against which all costs will be compared, and (b) expressing the value of all features in a common language. Once this has been done, every factor may be evaluated as an incremental "cost penalty," over and above the reference standard. Finally, in making comparisons, the reference standard will always subtract out, so that one needs only to add all incremental cost penalties to get the total cost of the design changes in mind.

The evaluation of all variables on a common basis is covered in Part I, which considers various manufacturing costs, power costs, and the cost of functioning time.

The remaining parts of this paper then develop relations for maximum economy in the switching system, for some important cases that arise in practice:

Coil design for maximum power economy.

Economical number of occasionally unused extra parts.

Economical adjustment for speed relay.

Coil design for maximum combined power and speed economy.

So far as possible, results are given in general form, permitting one to work from charts when considering specific application problems. Of further benefit are charts which permit the designer to decide the economic disadvantage of a design which may depart from optimum.

Design for systems economy through relay design includes many other important topics not considered here, as for example: how to make best use of molding, welding and other manufacturing processes; or how to design for long life, reliability, and other maintenance concerns. This article covers only some typical problems for which optimizing methods are readily applied.

Part I — Expressing Systems Performance and Manufacturing Variables in a Common Language

In the over-all switching system, one must evaluate various effects whose costs seem quite different, and a decision must be made as to what design course to follow, no matter how varied the conditions may be. Many of the most important such cases can be handled quite easily by a process of converting actual cost in the telephone plant back to an equivalent cost in terms of the factory production cost of each individual relay. Once all such costs are so stated, it is possible to examine the incremental effect of each change, and confidently draw conclusions. For example, if it can be stated that a change in a relay coil will save an amount of power that is worth 50 cents per relay equivalent first cost, then there is no doubt about designing such a new coil if its manufacturing cost will increase by only 20 cents. So the relay designer needs to understand how various factors, from original manufacture to final application, may be stated on a cost basis that is comparable throughout. Among the most important special cases are the following:

1. Manufacturing cost of winding a coil,
2. Manufacturing cost as a function of annual demand,
3. Equivalent manufacturing cost of power consumed by relays,
4. Equivalent manufacturing cost of functioning time of a relay.

The method of evaluating each on the same basis will now be briefly outlined. In each case, it should be noted that since only comparisons between variable portions of the system are considered, only incremental values need be considered.

1.1 THE COST OF A WINDING

In comparisons between coils, certain common operations, such as soldering the leads, using and cementing spoolheads, etc., will always subtract out, leaving only the difference due to varying amount of copper, which is paid for by the pound, (or per ohm), and the number of turns, which depends on the speed of the winding machine and number of coils wound at one time. Thus, winding costs, which depend only on the electrical design, nay be considered as varying only with the cost per ohm and cost per turn, as given in equation (1):

$$C_W = c_R R + c_N N, \tag{1}$$

where C_W = total cost of the variable part of the winding,
c_R = cost per ohm, which may be tabulated by wire size,
c_N = cost per turn of winding, also given in tables by wire size,
R = resistance of coil, ohms,
N = number of turns in winding.

Typical values of c_R and c_N for a particular kind of wire and coil are shown in Fig. 2.

1.2 COST AS A FUNCTION OF ANNUAL DEMAND

Various components of relays, as well as their assembly routines, occur in several variants of a general basic pattern. In the course of their manufacture it is necessary to stop the manufacture of one unit, reset the machine, and proceed with manufacture of another. Whenever this happens, production stops; and work must be done to reset the machine, all of which may be evaluated as a set-up cost (designated S). As more and more variants are required in the over-all general structure, the number of set-ups will increase to the point where the economical manufacture of the part is seriously penalized. Thus a point will be reached where it is advantageous to make more parts than are immediately needed, and store the remainder, so as to avoid excessive cost of shutdowns. When the variable effects of (a) set-up costs and (b) inventory costs are fully considered, the ideal quantity to manufacture in one lot may then be found by comparing the results of (a) and (b), as will now be shown. The costs which correspond to these ideal quantities will immediately follow.

1.21 *Lot-Size Costs*

In manufacturing practice there are two outstanding expenses which may affect the cost of any particular item which is classed as a code, or kind, of the basic family of articles. These are the administration costs, and the set-up costs.

1.211 *Administration Costs*

For certain kinds of operations there is a considerable amount of paper-work, drafting, checking, etc. to maintain in normal up-to-date condition. Occasionally, this cost is enough to influence the cost of the

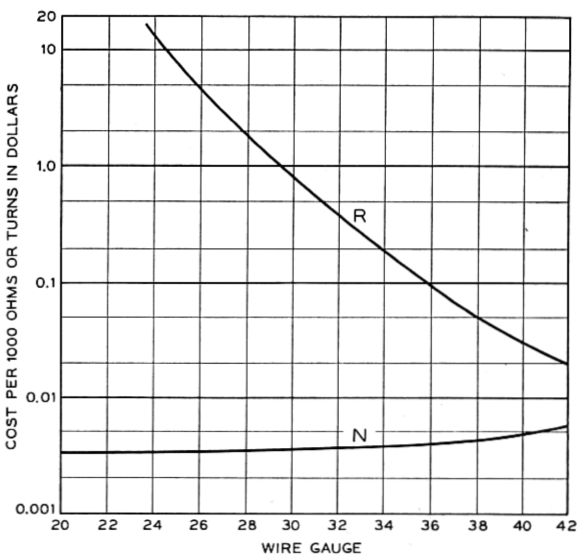


Fig. 2 — Typical winding costs.

item. The resulting individual effect may then be stated as

$$C_A = \frac{A}{n}, \tag{2}$$

where C_A = cost penalty per unit due to administration,
A = annual administration cost for maintaining one code in good standing,
n = annual demand for a given code.

1.212 *Set-up Costs*

If only one kind of part or assembly were needed, it could be continuously built in the same way, with no time lost for changeover to other parts, no particular bookkeeping necessary to control the proper flow of differing parts, and with more mechanized action. Usually this condition is far from realized in practice; nevertheless it represents the peak of manufacturing economy, and may be taken as a standard of reference for comparing all other less favorable conditions. The cost incurred, per item, due to its lot size, as against its cost if there were always but one lot, will be called the "lot-size cost penalty." The manufacturing lot-size cost penalty per part for any particular process, then, may be stated as follows:

$$C_L = \frac{S}{L}, \tag{3}$$

$$= \frac{S\ell}{n}, \tag{4}$$

where C_L = lot size cost per item,
S = cost of one "set-up",
L = size of lot for which one set-up is made,
= n/ℓ,
n = annual demand for a given code,
ℓ = number of lots per year, or lot-frequency.

Equation (4) gives the cost over and above single-kind manufacture, so far as lot-size is concerned. Hence, when values for each of these variables can be established, the lot-size cost penalty for any particular process can be determined.

1.213 *Over-all Lot-Size Costs, Related to Annual Demand*

Combining the results of equations (2) and (4), the total cost penalty per part due to lot-size is

$$C_L = \frac{A}{n} + \frac{S\ell}{n}. \tag{5}$$

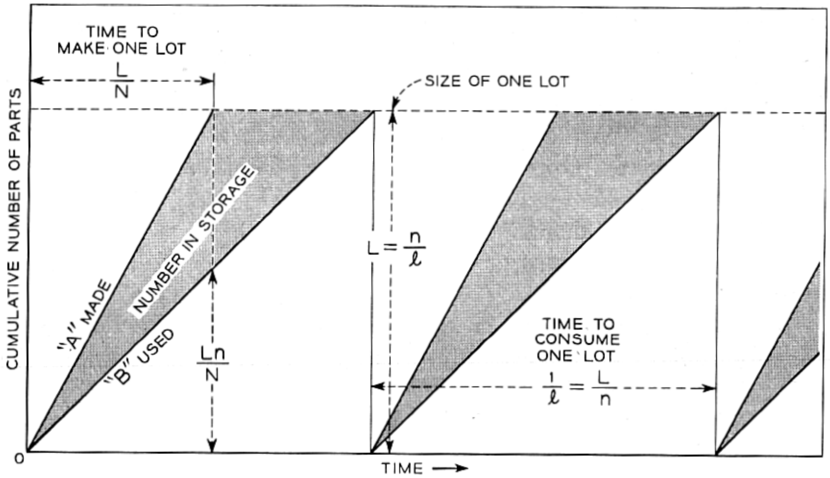


Fig. 3 — Lot-size cost penalty relationships.

1.22 *Inventory Costs*

When more parts are made than are immediately needed, it becomes necessary to store them until they are all used, when the process will be repeated. The resulting cost penalty, compared to no storage at all, may be stated analytically with the help of Fig. 3, which is a plot of production as a function of time. Line A represents the number of parts *made* in a lot as a function of time, while line B represents the parts *used* in the next manufacturing stage, as a function of time. The difference between the two lines at any time represents the number in storage, from which their value and hence the interest charges may be found.

The cost penalty per part due to storage is:

$$C = \frac{\text{Annual interest factor} \times \text{value of one part} \times \text{av. no. of parts in storage}}{\text{Annual production of this part}}. \qquad (6)$$

The following steps can be taken to supply values for this equation. From the figure, the maximum quantity of parts stored equals

$$L - \frac{Ln}{N},$$

or

$$\frac{n}{\ell}\left(1 - \frac{n}{N}\right).$$

The average number of parts stored is one-half this, or

$$\frac{n}{2\ell}\left(1 - \frac{n}{N}\right), \tag{7}$$

where N = rate of output of machine per year.

The penalty per part due to lot-size and administration was given in (5), and when added to its base cost C_1 gives the total value of one part as

$$C_1 + \frac{S\ell}{n} + \frac{A}{n}. \tag{8}$$

The penalty per part, due to storage, equation (6), is then the product of k/n times (7) times (8), where k = interest charges on stored parts expressed as a ratio. Then the storage cost penalty, C_S, is

$$C_S = \frac{k}{n}\left(C_1 + \frac{S\ell}{n} + \frac{A}{n}\right)\frac{n}{2\ell}\left(1 - \frac{n}{N}\right). \tag{9}$$

1.23 *Total Cost Penalty*

The entire penalty resulting from lot size charges and inventory charges may now be stated as a function of annual demand by summing equations (5) and (9). Writing q for $(1 - n/N)$, and rearranging terms, the total cost penalty in terms of annual demand is

$$C_P = \frac{kq}{2\ell}\left(C_1 + \frac{A}{n}\right) + \frac{kqS}{2n} + \frac{1}{n}(S\ell + A). \tag{10}$$

Equation (10) is the basic relation between costs and annual demand, and is of the general shape shown in graph (b) of Fig. 1.

It is of greatest interest to determine conditions where this curve has its lowest value, which occurs when $dC/d\ell = 0$. The result is

$$\ell_0 = \sqrt{\frac{kqn}{2S}\left(C_1 + \frac{A}{n}\right)}. \tag{11}$$

which gives the lot-frequency, ℓ_0, for which total cost penalty is a minimum. When ℓ_0 is substituted for ℓ in equation (10), the resulting optimum cost, as a function of annual demand, C_0, is given:

$$C_0 = \sqrt{\frac{2kqS(C_1 + A/n)}{n}} + \frac{kqS}{2n} + \frac{A}{n}. \tag{12}$$

Equations (11) and (12) furnish the means for deciding how to plan manufacture under differing conditions of annual demand, and how much the product is penalized even after planning is carried out as efficiently as possible. Two illustrations will be given: for a comparatively low-cost part involving expensive set-up charges and no administrative charges, and for a more costly operation with comparatively inexpensive set-up costs but high administrative costs. For Case I assume a part for which the following constants apply:

$$k = 0.15 \qquad C_1 = \$0.10$$
$$N = 2{,}000{,}000 \qquad A = 0$$
$$S = \$10.00$$

In Cast II assume that

$$k = 0.15 \qquad C_1 = \$1.00$$
$$N = 2{,}000{,}000 \qquad A = \$100$$
$$S = \$2.00$$

The results for these two cases are given in Fig. 4.

Results such as these are of the utmost value both in manufacturing planning for lowest cost, and in applications planning to show when the lot-size penalties can be tolerated. However, no case is to be expected in practice where optimum conditions can be met more than a part of the time. What with unexpected rush orders, fluctuation of annual demands, rescheduling, possible moves, or breakdown of machinery, it is usually impossible to plan production to remain at all times at the optimum value. The variation from optimum cost that results from a departure from optimum lot size is thus of interest; it is readily stated

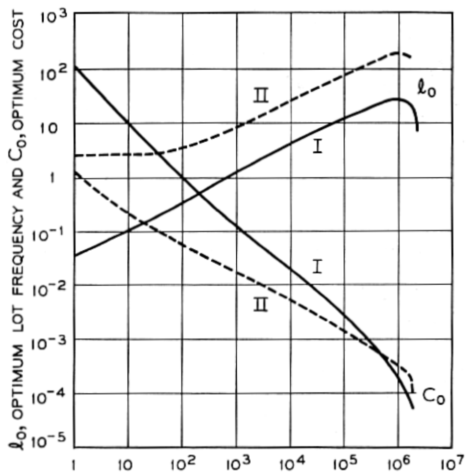


Fig. 4 — Optimum lot sizes and costs.

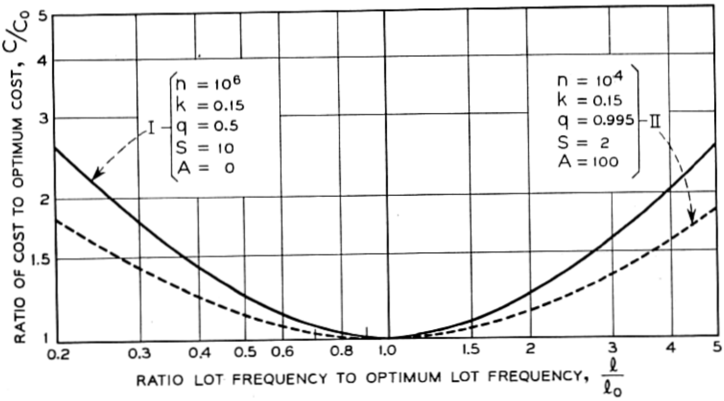


Fig. 5 — Cost variations with variations from optimum.

by rewriting equation (10) in terms of the values of ℓ_0 and C_0 just found in equations (11) and (12). The result is

$$C/C_0 = \frac{\ell_0/\ell + \ell/\ell_0 + D/\ell_0}{2 + D/\ell_0}, \tag{13}$$

where

$$D = \frac{kq}{2} + \frac{A}{S}.$$

This relationship is seen to depend on annual demands, and also administrative and set-up costs, which will be peculiar to each case. Therefore, it will be necessary to use the equation with appropriate values of these constants for each case that arises. Typical values for the two previous illustrations are shown in Fig. 5. Inspection of these curves shows that the cost penalty of departing from optimum lot-frequency is somewhat variable depending on the ideal lot-frequency for the particular case. However, the higher values of lot-frequency, which produce the highest penalties, are those most easily under control of the factory, and variations averaging in excess of 2 or 3 to one from ideal are not at all likely. When the ideal lot-frequency is low, manufacturing control is not so easy, but the cost penalty ratio of departing from the ideal is far less. From inspection of these curves, then, the value of C/C_0 must be judged by the conditions of each problem; a value of cost penalty amounting to about 1.25 C_0 is often found to be a quite satisfactory value to assume.

Thus, (a) a method is available to the manufacturing engineer to help plan optimum lot size, and (b) a method is available to the relay and circuit designer for deciding how much his designs may be penalized by varying the quantities used. It consists of using the costs determined from equation (12), and modified by an amount indicated by (13), or in ordinary cases by arbitrarily increasing (12) by the factor 1.25.

1.3 THE EQUIVALENT MANUFACTURING COST OF POWER CONSUMED BY A RELAY

The largest part of the power plant used in telephone central offices is the 50-volt equipment needed for the switching apparatus. Because of its special voltage range and the requirements on its stability and reliability, it must be provided as a part of every central office to convert the normal power company voltages into the desired telephone values. This equipment involves generators, banks of storage batteries, and switchboard, bus-bar and control equipment; it is an expensive portion of every central office which, of course, it is desirable to minimize as far as possible. For every relay required in the central office, one must associate a small "chunk" of this power plant; so that each relay needed implies an associated investment in the power plant. In a later section the means for minimizing the power and relay costs are described, and since they will involve comparisons on the same basis, it is first necessary to state the value of the power plant in terms related to the amount of power consumed by any individual unit. The method of evaluating the power plant is given in this section.

The problem may be broken into two parts: (a) the equivalent first costs assignable to the power plant equipment, and (b) the equivalent first costs of the charges per kilowatt-hour paid to the power company. Then the figures are combined.

1.31 *The Power Plant*

Power plants furnished for central offices vary over a wide range of size and cost, depending on the size of the office and its estimated activity. This plant must of course be planned so as to carry the load during the period of peak activity, even though this occurs for only a small part of the time, so that in the long run the cost of the power plant is decided by the share of busy-hour power taken by each relay. To get such costs one must first find the cost of the plant as a function of total power requirements.

Present practice in power plant planning results in the purchase of basic power units which may be combined to cover certain ranges of power supplied over roughly a two-to-one range. Within this range, by adding generators and batteries in parallel the needed capacity can be attained. Beyond the range, basically heavier machines are needed. The resulting installed price of such power plants has the character shown in Fig. 6. Since prices and installation charges change from year to year, the values shown here are given in qualitative form as illustrations — they must of course be evaluated as concisely as possible when considering new central office designs. As seen in the figure, the power plant price varies in two ways: in fairly big jumps as basic plant size is changed, and in smaller but rather uniform manner as the basic size varies within its lower and upper limits.

From Fig. 6 it is seen that for each basic size of plant the rate of increase in price with small increases in capacity is about the same for all plants. The average rate for this incremental power capacity, corresponding to the expense if relatively small power changes are made in any one basic type of power plant will be called a. As the power requirements of the office change beyond a certain point, it is necessary to install basically larger or smaller equipment with correspondingly different base prices. At the points of maximum capacity, there is an abrupt change in the basic price of the plant, amounting often to many thousands of dollars. These abrupt increments, designated A, represent a lump sum of money that can be saved whenever the power plant size can be re-

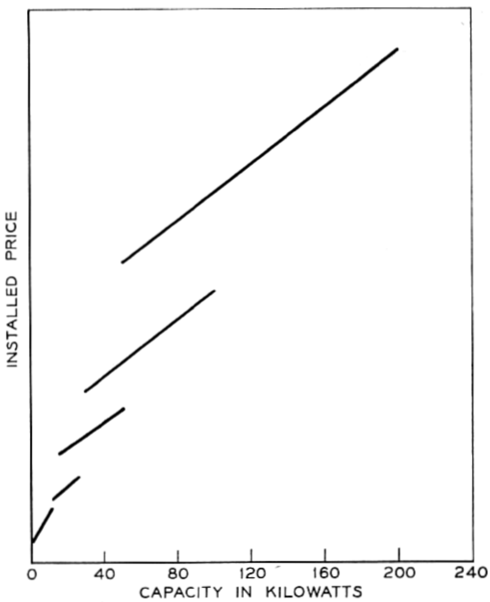


Fig. 6 — Installed value of power plants.

duced to the next lower-range unit. Such a saving can be realized only part of the time, but it is desirable to attempt to weight its value so that over the telephone system as a whole there is adequate encouragement to strive for power savings.

The method of weighting is complicated by such factors as the fraction of power that may be saved in an office, by the likelihood of certain size offices being representative, and by the extent to which offices are designed with safety factors for growth and overload. Nevertheless a reasonable estimate can be made by assuming differing amounts of the office power can be saved, and by assuming that all sizes of office are equally likely. This gives the result that the over-all incremental value of power is

$$k = \frac{C}{W} = a + \frac{2AW_0}{(1 - p)(W_2^2 - W_0^2)}, \qquad \text{(dollars per kw)}, \qquad (14)$$

the average value of power plant per kilowatt saved over the range between W_0 and W_2, taking into account the proportionate number of plants that can be converted to the next cheaper size, where

C = total price saving,
W = total power saving,
p = fraction of power saved, not to exceed about .5, (above which point two steps of base price saving might be realized),
W_0 = lower limit of range,
W_2 = upper limit of range,
a = incremental price of one size power plant, assumed to be about constant for all plants,
A = difference in base price to next lower plant, as may be tabulated for any particular case.

By these methods, incremental values of installed power plant may be found; it is even possible in many cases to assume an average slope for the family of curves in Fig. 6, without greatly changing the results in practice.

It is now desired to convert these figures of installed expense to figures comparable to manufacturing cost, in the factory, of the relays which will consume this power. This may be done through standard economic-study practice by which first cost is converted to annual charges through a knowledge of charges for interest, taxes, depreciation, etc. As these factors are not necessarily alike for power equipment and relays, the factors used will differ to suit the problem. For the present study, with recognition of the accounting practices then in effect, it was found that on a basis comparable to relay manufacturing cost, savable power was worth between $500 and $2,500 per kilowatt, depending on size, savable power, etc.

Now these figures must be associated with the power needed by any particular relay. The total switching power during the busy part of the day, which is the period determining the size of the power plant, is the sum of all the individual relay power drains at any particular moment. This in turn may be taken as the sum of the power required by each relay times the probability that it will be energized at any particular time, which is taken as the fraction of the busy hour that the relay is expected to be energized. For every relay, this ratio can be determined with some certainty, by consultation with the circuit designers.

Summarizing, the switching power capacity required by the office may be stated as the sum, for all relays, of power w for each relay, times m, the fraction of the busy hour that it is energized (i.e., hrs. per busy hr.). The cost of the power plant capacity required for each relay is then

$$C = kmw \text{ dollars,} \tag{15}$$

where k is the equivalent value of power.

Very often it is desirable to state this cost on the basis of the annual power drain. In this way, the plant investment can be easily correlated with the costs paid to the power companies. In telephone offices, the annual power has been found to correspond to 3,000 times the power consumed in a busy hour, and as a matter of convenience power drain in the central office is often stated in terms of the energy per year based on a 3,000-busy-hour year. Then the ratio of use during the busy hour may also be stated as

$$m = \frac{h}{3000},$$

where h = hours per year energized.

Writing $E^2/1000R$ for the power in kilowatts drawn by any relay, the equivalent first cost of the power plant assignable to the relay is then

$$\begin{aligned} C_{\text{power plant}} &= \frac{E^2}{1000R} \times \frac{kh}{3000}, \\ &= c_{P_1} \frac{E^2 h}{R}. \end{aligned} \tag{16}$$

For this case, in other words, the equivalent first cost of power plant has a value c_{P_1} dollars for each watt-hour-per-year of switching power required.

1.32 *Cost of Power Purchased from Power Companies*

The power used by the switching apparatus corresponds to a larger amount of power furnished by the power company, since the power machines are not 100 per cent efficient. Its cost may be stated as

$$c_P = \frac{c_0}{e},$$

where c_0 = average marginal cost of power furnished,
e = conversion efficiency.

The annual power bill will then be

$$\frac{c_0}{e} \frac{E^2 h}{1000R} \text{ dollars.}$$

Converting from annual charges to first cost, through the same steps as above, gives

$$C_{\text{power}} = c_{P_2} \frac{E^2 h}{R}, \tag{17}$$

or c_{P_2} dollars per watt-hour-per-year.

1.33 *Total Power Cost*

With both power plant costs and purchased power costs now stated on the same basis, they may be combined to give the total power cost, C_P:

$$C_P = (c_{P_1} + c_{P_2}) \frac{E^2 h}{1000R} = \frac{c_P E^2 h}{R}, \tag{18}$$

where c_P represents the dollars per watt-hour-per-year for power consumed by a particular relay, and is the equivalent first-cost value of power.

Thus a method is available to find the value of the power that magnet coils may need — to be balanced against the first cost of the coils (Section 1.1). Procedures for best results are given later.

1.4 THE EQUIVALENT MANUFACTURING COST OF THE FUNCTIONING TIME OF A RELAY

Another large part of the investment in every crossbar-type central office is the common-control equipment, most important of which is the "marker." This equipment has the sole duty of selecting the proper path for each call, and sending it on its way. After each such function, the equipment restores to normal, ready to serve the next call. It takes a marker about a half-second to do its job; nevertheless many markers may be needed to handle the traffic during each day's heaviest calling period. Since the cost of each marker is measured in the tens of thousands of dollars, it becomes a matter of great importance to shorten the marker work time — in other words, to shorten the time of each relay in the chain of marker events. Since faster relays are usually more expensive it is important to find the dollar value of speed so as to guide an intelligent design of each speed relay.

As in the case of power, it has been found possible to state the value of speed in terms equivalent to the manufacturing costs of relays. This may be done through a knowledge of the number of markers and associated equipment needed in relation to their work time, and their cost. Such relationships are shown in Fig. 7, which gives a typical curve for the markers per line needed to handle the traffic. Then, when the value of the marker is known, the value per line per millisecond, c_t, follows. Corresponding to this, a value per millisecond per marker, c_M, can then be found, which varies inversely as the number of markers needed:

$$c_M = \frac{c_t}{p},$$

where p = number of markers per line that are needed. This figure is the value of a millisecond for any complete event or series of events

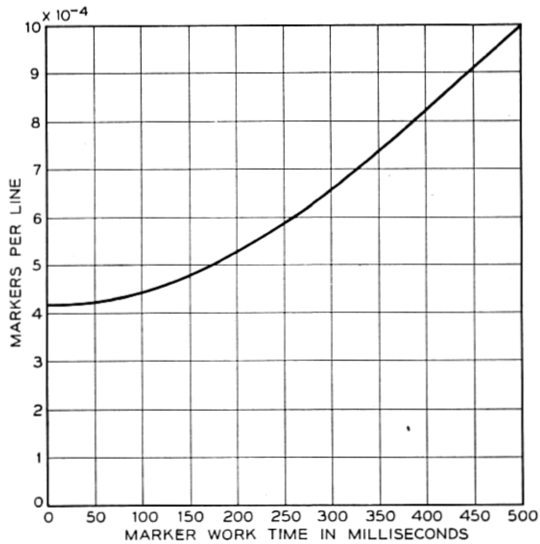


Fig. 7 — Markers as a function of work time.

within the marker which directly affects its working time, referred to as a "major event."

1.41 *The Value of Speed per Event*

Within each major event there may be a whole cycle of operations happening partly in parallel and partly in sequence. Each separate action is called an "event." In the case of events happening in parallel, the circuit designer will usually know whenever one path is in outstanding control of the work time as a whole. Whenever such is the case, it is a "major event," and the other paths may be ignored. Often, however, the times for each of several paths will be about the same, and special consideration is needed to determine their value. In breaking down these operations so as to arrive at the cost per relay, one must evaluate the events within any major event as in the region marked in Fig. 8. Supposing that the number of subevents within a major event are labelled a, b, c, etc. for each of the several parallel paths, then the total number of subevents is $a + b + c + d + \cdots$. The total time for the major event is t, which has the dollar value, c_t/p, per millisecond, as just previously given.

Along any path, A, of events, the total time for the events is the time taken by the group as a whole. If this total time is t, then the effective time per event along any path is

$$\begin{aligned} t_A &= t/a \text{ for path } A \text{ (seconds)}, \\ t_B &= t/b \text{ for path } B \text{ (seconds)}, \\ &= \text{etc.} \end{aligned}$$

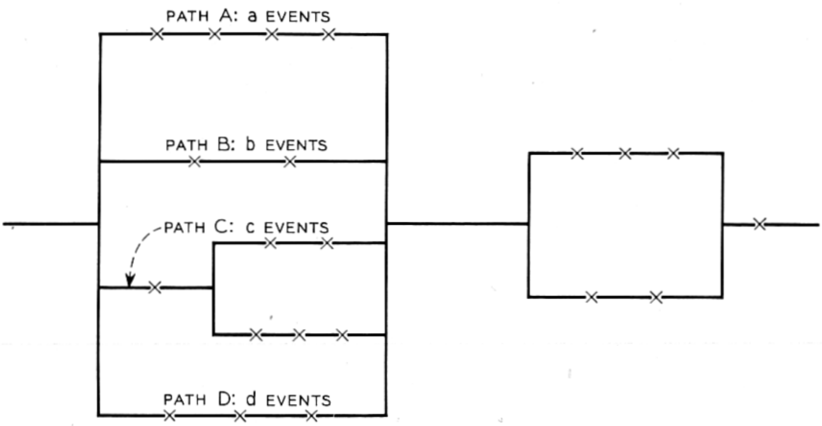


Fig. 8 — Typical chain of events in controller operation.

The worth of the time taken for the group as a whole is then

$$C_M = c_t t/p = c_t a t_A/p = c_t b t_B/p = \text{etc., dollars.}$$

The value per event is usually of principal interest. Since there are $(a + b + c + \cdots)$ events in any one "major event," then the total value per event C_E is

$$C_E = \frac{C_M}{(a + b + c + \cdots)} \text{ dollars per event,}$$

$$= \frac{a}{(a + b + c + \cdots)p} c_t t_A, \qquad \text{for path } A,$$

$$= \frac{b}{(a + b + c + \cdots)p} c_t t_B, \qquad \text{for path } B,$$

$$= \text{etc.}$$

Then the value of an event, per millisecond, is

$$c_E = q_A c_t/p = q_B c_t/p = \text{etc., dollars per event per millisecond.}$$

where $q_A = \dfrac{a}{a + b + c + \cdots}, \qquad q_B = \dfrac{b}{a + b + c + \cdots}, \qquad$ etc.

In general, then, the value of an event is

$$C_E = c_E t_E \text{ dollars,}$$

where $t_E =$ the duration of the event,

$$c_E = \left(\frac{q}{p}\right) c_t \text{ dollars per millisecond.} \tag{19}$$

The quantity q will be called the "path factor." It is seen to simplify to a value of unity when a major event is involved, ($q = a/a$), and in all other cases to have a value less than this.

1.42 *The Value of Speed, per Relay Operation*

Each event may consist of the action of one relay out of several possible alternates. This means that several relays must be purchased for each such event, though only one determines the time. Calling w the number of relays provided per marker to perform one function, each function being designated by subscript corresponding to path and sequence, then the value per relay of one millisecond is

$$
\begin{aligned}
c_r &= \frac{q_A c_t}{w_{A_1} p}, \quad \frac{q_A c_t}{w_{A_2} p} \quad \text{etc., for path } A, \\
&= \frac{q_B c_t}{w_{B_1} p}, \quad \frac{q_B c_t}{w_{B_2} p} \quad \text{etc., for path } B, \\
&= \frac{q c_t}{w p}, \quad \text{in general, (dollars per millisecond).}
\end{aligned} \tag{20}
$$

These equations give the general expression for the value of functioning time in terms of cost. They permit the direct comparison with cost of a relay design, including its coding cost, manufacturing cost, or power drain cost, in deciding on how much investment is warranted in switching apparatus in order to shorten the marker work time.

1.43 *Simple Examples of Relay Design for Speed*

The above relations make it possible to rapidly evaluate design changes for speeding up relays, or to state the value of seeking a design change that will make speed possible. Several examples will be given.

(a) Relay Involved in a Major Event

Suppose the following typical conditions are given:

$$
\begin{aligned}
\text{No. of markers per line} &= p = 0.0006. \\
\text{Path factor} &= q = 1. \\
\text{Relays per marker function} &= w = 1. \\
\text{Value per millisecond per line} &= c_t = \$0.04.
\end{aligned}
$$

Then value per millisecond per relay is

$$c_r = \frac{0.04 \times 1}{0.0006 \times 1} = \$67.$$

In this case a very large investment per relay could be justified in order to gain small savings in time.

(b) Relay Involved in a Subevent

Suppose the following typical conditions are given:

$$
\begin{aligned}
\text{No. of markers per line} &= p = 0.0006. \\
\text{Path factor} &= q = 2/8 = 0.25.
\end{aligned}
$$

$$\text{Relays per marker function} = w = 2,$$
$$c_t = 0.04.$$

Then value per millisecond per relay is

$$c_r = \frac{0.04 \times 0.25}{0.0006 \times 2} = \$8.33.$$

In this case, too, it is worth a considerable manufacturing cost penalty to shorten the functioning time of the relay.

(c) *Relay Involved in a Subevent—Release*

A particularly common form of subevent is the release of numerous relays at the end of one phase of marker action. In this case, assume values of the constants are:

$$\text{No. of markers per line} = p = 0.0006.$$
$$\text{Path factor} = q = 0{,}02.$$
$$\text{Relays per marker function} = w = 5.$$
$$c_t = \$0.04.$$

Then value per millisecond per relay is

$$c_r = \frac{0.04 \times 0.02}{0.0006 \times 5} = \$0.266$$

Thus even in this case, an appreciable first cost value is associated with the releasing speed of each relay.

1.5 REVIEW OF EQUIVALENT FIRST COST EXPRESSIONS

The previous sections have shown how some of the variable portions of the relay design that affect its performance may be stated in terms of first cost. Likewise methods are given for evaluating sensitivity and speed performance in the same terms. These may be summarized as

Cost of a coil:

$$C_W = c_R R + c_N N. \tag{1}$$

Cost of an item as a function of demand:

$$C_o = \sqrt{\frac{2kqS(C_1 + A/n)}{n}} + \frac{kqS}{2n} + \frac{A}{n}. \tag{12}$$

Cost of power:

$$C_P = \frac{c_P E^2 h}{R}. \tag{18}$$

Cost of functioning time:

$$C_r = \frac{q c_t}{wp}. \tag{20}$$

Examination of these equations will show that, in nearly every case, a cheapening of the cost of the structure leads to more expensive (i.e., poorer) performance, and vice versa. However, the availability of these relations, together with accurate values for the constants, gives the information needed to vary the relay design to the point where maximum over-all economy (i.e., the cheapest central office) can be realized.

In the following sections, some examples of methods for finding this optimum point, and how much it is worth to find it, will be given.

Part II — Design for Maximum Power-Plus-Coil Economy

Among the more common problems facing the designer of switching circuits is the selection of the relay magnet which gives best economy in use. On the one hand, equation (1) has shown that the relay coil cost will be less if the resistance is less (but power consumed is more). On the other hand, equation (18) shows that the equivalent first cost of that part of the power plant furnished to operate the relay increases as the resistance is reduced (but coil is more expensive). These opposite effects are indicated in Fig. 9 where the curve labelled C_W shows how wind-

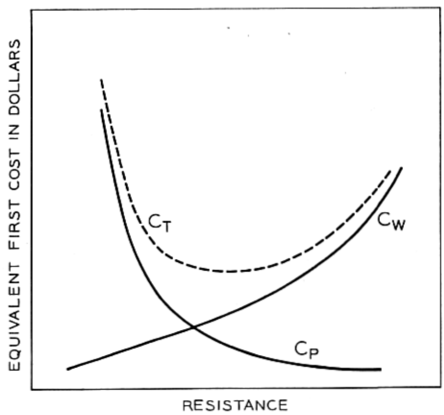


Fig. 9 — Power and winding costs.

ing cost for a certain wire gauge varies while the curve labelled C_P shows how power plant cost is changing. The desirable end result is a coil with resistance value to permit the sum of these to be as low as possible, as in the curve marked C_T. The following methods will give such a design.

The value of C_T, made up of power cost, C_P, and winding cost, C_W, represents the cost of all the parts of the central office which are influenced by a change in resistance of the coil. Thus we wish to minimize this equation:

$$C_T = C_P + C_W,$$

where, from the previous relations,

$$C_P = \frac{K_1}{R},$$

$$C_W = K_2 R,$$

$$K_1 = c_P h E^2,$$

$$K_2 = c_R \left(1 + \frac{c_N}{c_R} \frac{N}{R}\right).$$

The resulting expression for this total cost is

$$C_T = \frac{K_1}{R} + K_2 R. \tag{21}$$

In this equation K_1 is a constant, but K_2 varies both with resistance and wire size, as the latter affects the values of c_N, c_R, and the quantity N/R, which is determined by the coil design equation

$$R = AN\,(h + d),$$

in which, using the special notation common to coil design problems:

N = turns in winding = $h\ell/K$,

R = coil circuit resistance,

A = π times the resistivity of wire in ohms per inch length, given in tables for each size of wire,

h = depth of coil space occupied by wire,

ℓ = length of winding space,

d = inside diameter of coil,

K = effective cross-sectional area required by one turn of wire, given in tables for each size.

For a given relay, the dimensions ℓ and d are fixed, and only the wire size and the coil depth h can be varied. For a given wire size, therefore, the values of h and N are fixed by the resistance. As C_N and C_R are known quantities, the values of K_2 can be determined and plotted against R for each wire size, as illustrated in Fig. 10. Such a chart applies to a particular relay, as the curves depend on the fixed values of ℓ and d applying. Here it can be seen that for a given resistance the finest gauge wire is the cheapest. Because of this, and since power costs are minimized with the highest resistance, the finest gauge wire would always give the cheapest coil. However, this becomes a theoretical case. As finer and finer wires are used, $A = \pi\sigma$ becomes larger, and thus N/R or NI/E becomes smaller. In all relay design, an ampere-turns requirement must be met. Thus a practical solution to this problem of optimum costs is to choose the finest gauge wire which will meet the ampere-turns requirement.

If the same relay is assumed as for Fig. 10, the variations of ampere turns with resistance are as shown in Fig. 11. Starting with the NI required, the wire size can be determined from a plot like Figure 11. Then K_2 is determined from a plot like Fig. 10.

Now, with the gauge of wire chosen, K_2 is almost a constant and can be considered independent of R. Thus, upon differentiating (21), C_T will be found to be a minimum when R has a value designated by

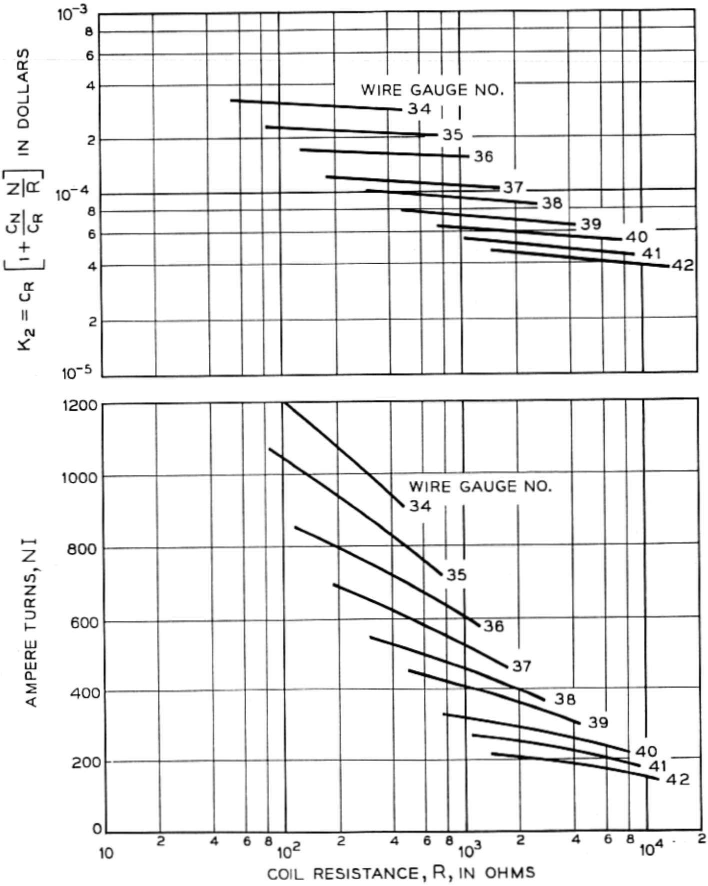
$$R_0 = \sqrt{K_1/K_2}\,. \tag{22}$$

The most economical coil, resulting from use of a resistance R_0, will have a system cost of

$$C_0 = 2\sqrt{K_1K_2}\,. \tag{23}$$

As can be seen by the slopes of the curves on Fig. 11, if the optimum resistance, R_0, is larger than the resistance value at the desired ampere turns, the NI requirement will not be met. Then either the selection of R_0 must be made for the next coarser wire, or the resistance value chosen at just the required NI, whichever is cheaper. Conversely, if there is a very large NI margin, a finer gauge wire could be tried. In all, two trials should give the optimum values.

A procedure has now been outlined for choosing the coil design for optimum power-plus-winding cost. Curves similar to Figs. 10 and 11 should be drawn for the applicable relay parameters. Then the K_2 value is obtained for the finest gauge meeting the ampere-turns requirement, and R_0 and C_0 found. Then, the NI value applying for R_0 must be checked and adjustments made as above, if necessary.



Figs. 10 and 11 — K_2 and NI values versus resistance for various wire gauges for a typical relay.

COST WHEN OPTIMUM RESISTANCE CONDITION IS NOT SATISFIED

There will, of course, be cases in practice where the optimum resistance is not used, for such reasons as standardizing of certain coil sizes, need for speed, or insufficient winding space. The penalty in cost from departing from the ideal winding size may be found by comparing the cost for any value of resistance, (equation 21), with the cost if resistance were optimum, (equation 23):

$$\frac{C}{C_0} = \frac{K_1/R + K_2R}{2\sqrt{K_1K_2}},$$

$$= \frac{1}{2}\left(\frac{R_0}{R} + \frac{R}{R_0}\right). \tag{24}$$

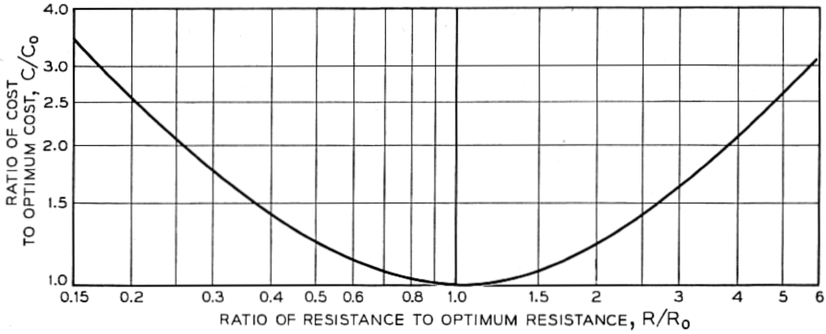


Fig. 12 — Cost penalty when resistance is not optimum.

Thus, the fractional change in cost caused by any departure from the best resistance is readily estimated, expressed in ratio form as R/R_0. This relation is shown in Fig. 12.

SUMMARY

In summary of Part II, methods have been given for deciding on the magnet coil design which will provide the cheapest net cost in the switching system. When the conditions of use are known, a series of charts may be prepared such as the one shown, from which the applications engineer may decide the proper coil design, and how much it may be worth to the system. As experience will show, the cost of departing from an optimum design may run well over $1.00 per individual relay, with the result that aggregate savings in the whole central office may run to thousands of dollars compared to the cost when this problem is ignored.

New relay development may be guided by the use of these same relations, since through a process of considering various hypothetical designs of various sizes, work outputs and magnetic qualities, the potential optimum costs may be found. Design of the recently adopted wire spring relay was very materially guided by considerations of this sort.

Part III—Choice of Contact Sets for Maximum Economy

One of the many problems involving lot-size costs of relays is the number of standardized sets of contacts that should be provided: i.e., how many kinds of contact arrangements give maximum economy for the product as a whole? By providing only a few choices of kinds of contacts, for example, the annual output is less divided and there are fewer setup charges, with the result that the initial manufacturing economy is large. However, because of the small selection of kinds of contact sets, there will result many cases where extra contacts are provided at greater than needed cost. The problem is how best to combine these opposing factors.

In illustration, consider a relay which is to be equipped with molded spring arrangements, capable of providing any number of "transfers" from 1 to 12, each of which is used in approximately equal numbers in the system. At the one extreme, only one molding might be provided, always equipped with 12 transfers. At the other extreme 12 different moldings might be provided, covering each individually needed quantity of "transfers." In the case of the single molding there is no lot-size penalty at all, but on the average a large penalty in surplus contacts; while for the twelve kinds of moldings, lot-size penalties corresponding to one-twelfth of the possible annual output are incurred, but no spare parts at all will be needed. Such a problem may be treated as given below.

Let the number of kinds of spring sets chosen be designated by v. Then the annual output of each kind is the total output N divided by v, or

$$n = \frac{N}{v}. \tag{25}$$

Now it was shown in Section 1.2 that the cost penalty for making things in lots is given by equation (12). For the present problem, this may be expressed in terms of v, by substituting equation (25) into equation (12). The resulting lot-size cost penalty is then

$$C_0 = \frac{v}{N}\left[\sqrt{\frac{2kSN}{v}\left(1 - \frac{1}{v}\right)\left(C_1 + \frac{Av}{N}\right)} + \frac{kS\left(1 - \frac{1}{v}\right)}{2} + A\right]. \tag{26}$$

The penalty due to providing unneeded springs is the dollar value of the average number of spare springs. The average number of spares is approximately

$$E = \frac{12 - v}{2v}.$$

Thus for only one kind furnished, always 12 transfers, the number of spares can vary between 0 and 11, an average of 5.5, as given by the equation. If there were eight kinds, consisting possibly of moldings of 1, 3, 5, 6, 7, 8, 10 and 12 sets of transfers, the average number of extras would be 0.25. The dollar penalty compared to no extras ever needed is

$$C_E = C_1\left(\frac{12 - v}{2v}\right). \tag{27}$$

and the sum of this equation and that for lot size represents the system cost for any one condition.

Fig. 13 shows this variation for an assumed case for which

$$N = 500{,}000 \text{ parts per year.}$$

$$k = 0.15.$$

$$S = \$100.$$

$$C_1 = \begin{cases} 0.20 \text{ for spring set cost.} \\ 0.005 \text{ for spare spring cost.} \end{cases}$$

$$A = 0.$$

Here it is found that the greatest benefit comes from using in the neighborhood of six kinds of spring sets. It is further clearly seen that the system as a whole will not be particularly penalized if the number of sets chosen is a few different from this quantity — but a penalty of almost two cents per relay, or about two thousand dolllars per year would be incurred by erring too far toward the extreme.

Part IV — Choice of Special Adjustment for Maximum Speed Economy

Speed can be an exceedingly valuable property of a relay, as shown in Section 1.4. There are cases for example where every saving of 1

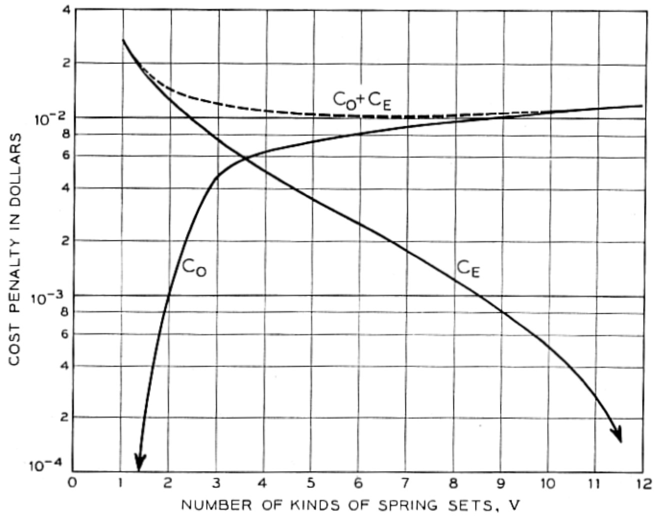


Fig. 13 — Optimum number of spring sets.

millisecond in working time will reduce the average quantity of equipment needed by an amount equivalent to \$35, or even more. For such cases a considerable sum of money can be spent on each relay in order to make it faster. The following approach helps guide the circuit and relay designer in the choice of relay they should make in order to gain the greatest over-all advantage.

It is well known to relay designers that one of the most important design parameters controlling operating time of a relay is the spacing between the contacts when they are at rest. This space cannot be less than a certain small distance of about 0.005″ to avoid contact failures due to vibration, buildups, sparkover, etc. However, because it is costly to hold adjustments accurately to a few thousandths of an inch, the maximum spacing is often quite large, sometimes as much as 0.050″. The spread in this distance is primarily a matter of economics; if it were clear that more money could be spent on the relay to narrow down this spacing while gaining material reductions in operating time as a result, then some closer adjustment would be specified. Such a problem is readily treated by summing (a) the cost penalties for differing values of this spacing and (b) the cost penalties of the corresponding functioning time, and finding the optimum condition that results. One such case is shown below.

The cost penalty due to changing spacing may be estimated by considering various actual values, and determining what procedures would be used in the factory in each case. Such a cost study can be made in the factory. In illustration, assume a particular relay type deemed to suffer no cost penalty if its armature stroke is allowed to take on any value up to 0.050″, but which cannot be less than 0.010″ without impairing performance. The actual cost per unit of various kinds of manufacturing procedures might be found according to the hypothetical conditions given in Table I. Thus a curve can be plotted of the cost penalties for each setting of gap.

The design may also be studied for the influence of the gap settings on speed. The operating time of the magnet as a function of spacing is readily determined by experiment, and may also be checked against known operating principles, to be sure the figures are reasonable. For a typical case, operating times corresponding to various spacings might be as given in Table II. At the same time, the value of this functioning time is given by equation (20) to be

$$C_r = \frac{qc_t}{wp}.$$

TABLE I — COST PER UNIT OF VARIOUS KINDS OF MANUFACTURING PROCEDURES

Assumed Max. Stroke	Manufacturing Procedure	Cost
0.050″	Acceptance of wide dimensional tolerances on all parts, and no adjustment permitted	$0
0.045″	As above, but crude adjustments added	0.05
0.040″	Closer dimensional tolerances, no adjustment	0.07
0.034″	As just above, with rough adjustment	0.10
0.025″	Very close tolerances on all parts, no adjustment	0.15
0.020″	As just above, with touch-up permitted	0.20
0.015″	Screw adjustment added, (more expensive parts, but simpler adjustment)	0.35
0.010″	Precision setting of all parts	0.65

Assuming for this hypothetical case that the values applying are

$$c_t = \$0.05,$$

$$w = 100,$$

$$q = 0.2, \text{ and}$$

$$p = 0.0006,$$

then the cost of the resulting time is found in the last column of Table II.

A summation of these two sets of cost penalties is given in Fig. 14, which clearly shows that the ideal value of stroke is about 0.020″, and that that it may be permitted to vary between the limits 0.015″ and 0.025″ without a serious economic penalty.

In summary, a method has been indicated for deciding how important it may be to build in, or omit, expensive design features which have an effect on functioning time. An illustration has been given for an assumed common control cirucit and an assumed change in a design feature of the relay that importantly affects the time. The method, however, is equally

TABLE II — OPERATING TIMES CORRESPONDING TO VARIOUS SPACINGS

Assumed Max. Stroke	Functioning Time (Milliseconds)	Equivalent First Cost Value of Time
0.050	6.1	$1.02
0.045	5.6	0.933
0.040	5.2	0.867
0.035	4.75	0.792
0.025	3.75	0.625
0.020	3.2	0.533
0.015	2.55	0.425
0.010	1.9	0.317

applicable to other speed problems, once the value of speed is known, and once the influence of the design change and its cost are known.

Part V — Choice of Winding for Maximum Combined Economy of Speed and Power

In the selection of magnet designs for use in circuits where speed is important, much can be gained by modifying the mass, the stroke, the force against the stop, etc., but when all these devices have been exhausted there remains the possibility of supplying more power to the magnet coil. If this is done in enough cases, an over-all penalty in central office cost is incurred through the added over-all power plant capacity that may be needed. As already seen in Section 1.3, this cost penalty is proportional to the base value of one watt-hour-per-year, the voltage squared, and the hours per year energized, but is inversely proportional to the resistance. In telephone usage, the voltage is usually fixed at 50 volts, and equation (18) may be rewritten as

$$C_P = \frac{k_P h}{R}, \tag{28}$$

where

$$k_P = c_P E^2.$$

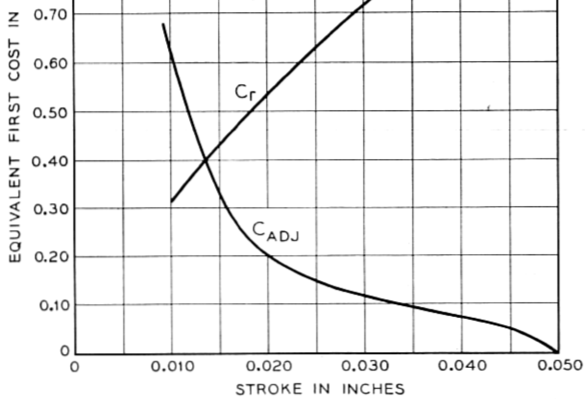


Fig. 14 — Optimizing a speed relay design.

The above expression gives the equivalent first cost of the power taken by one relay. For each relay operation, or event, however, there may be several relays energized. Then the total cost penalty per event, due to power, is

$$C_P = \frac{k_P h g}{R} \text{ dollars,} \tag{29}$$

where g = number of relays energized, per marker, for the particular function in question even though only one appears in the time sequence. For the power cost to be compared with the cost due to speed, this cost must be spread over the number of relays, w, involved in the function. Then

$$C_P = \frac{k_P h g}{wR} \tag{30}$$

This is the equivalent first cost penalty incurred against each relay, due to its power consumption. As R increases, it is seen to decrease.

However, as R increases, the operate time increases. For speed relays, the time t has been found to be a complicated function of the resistance, varying approximately as $R^{1/3}$. Thus when R increases, time increases, and the equivalent first cost of this action time increases, as already seen in Section 1.4.

As power is changed, then, there results two oppositely varying costs — one due to power, and one due to speed. The problem is graphically shown in Fig. 15, where Curve A shows the typical cost variation due to power drain and Curve B shows the variation in cost due to speed. The point where the sum of the two is a minimum is the ideal point to operate, assuming that the coil costs are about equal in each case. Curve C shows how the total cost will vary. Actually, it may be necessary to add the effects of a third variable: the cost of the coil itself as it is redesigned for different power consumption. This can be easily done by the methods shown above, but is ignored in the present treatment for the sake of simplicity. In many actual cases, coil cost has been found to be a much smaller factor than the other variables.

CHOICE OF OPTIMUM COIL

In practice the above result may be obtained by a knowledge of how a specific magnet design will vary in operate time as its coil resistance is changed. The values may be substituted in the following equation for total cost:

$$C = \frac{qc_t t}{wp} + \frac{k_P hg}{wR}, \\ = \frac{g}{w}\left(\frac{qc_t t}{pg} + \frac{k_P h}{R}\right). \tag{31}$$

The expression may be evaluated when a relation between t and circuit resistance R is established, usually a matter for experiment in any given case.

Illustration

For a particular case let us assume that $p = 0.0006$, so that equation (31) is

$$C = \frac{g}{w}\left(1670\,\frac{q}{g}\,c_t t + \frac{k_P h}{R}\right). \tag{32}$$

Typical values of operate time, t, for a speed relay in 50-volt telephone applications are given in Table III. From this information, one may readily evaluate equation (32) to cover the practical range of conditions that are met in service. This has been done, in Fig. 16, to give the total

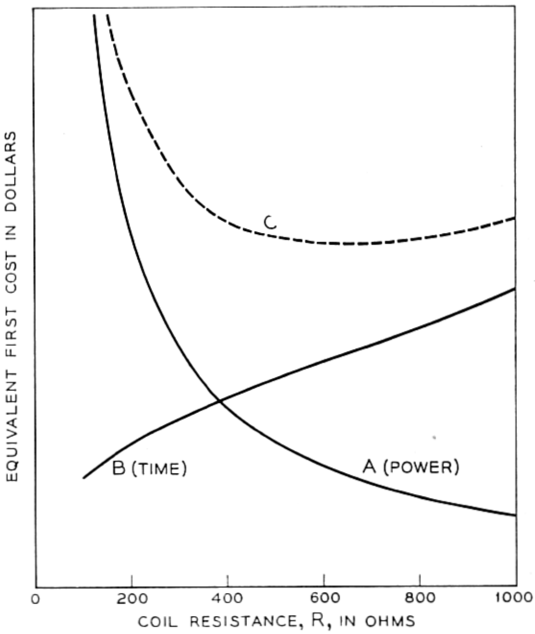


Fig. 15 — Optimizing costs of speed plus power by relay coil design.

TABLE III — TYPICAL VALUES OF OPERATE TIME

Coil Resistance (Ohms)	Optimum Time (Milliseconds)
100	2.45
200	3.1
400	4.15
600	5.0
800	5.8
1000	6.7

cost penalty for the case of very short holding time ($h = 100$ hr. per year) and for long holding time ($h = 2000$ hr. per year). The optimum coil resistances for these cases are seen to lie between 200 and 400 ohms for the long holding time condition, and to be as low as possible for the case of short holding time. As a result we see an appreciable economic incentive to plan relay designs which are capable of operating on low resistances. Further study of heat dissipation, operation with holding windings, contacts to withstand heavy currents, and other devices to gain fast action, therefore take on added importance to the relay designer.

PART VI — CHOICE OF THE ACTUAL CODE

With the background of the previous sections, the systems designer can now decide what codes he needs to use in his circuits so as to be

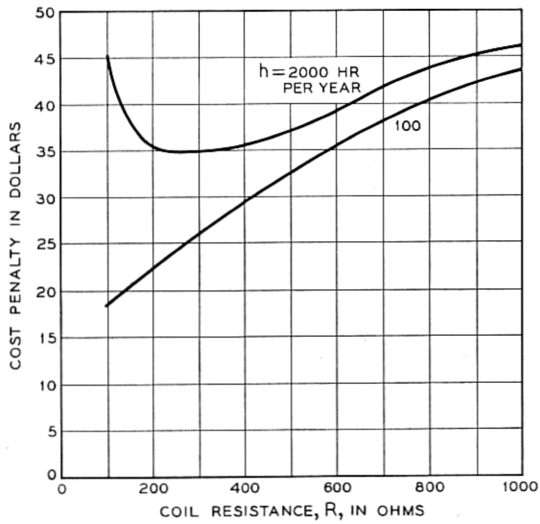


Fig. 16 — Best coil design for both speed and power economy.

most economical, over-all. The problem will first be discussed in a generalized form, and the means for getting the economy will then be reviewed.

6.1 GENERAL DISCUSSION OF CODING

If enough information were available to the circuit designer, he should be able to choose a relay for any particular application by considering

(a) The penalties in performance (expressed in terms of first cost) resulting from having only a certain limited number of available combinations of relays, as against complete flexibility, i.e., the penalties due to standardizing, and

(b) The penalties in first cost resulting from many variations of a basic type, as compared with one single standard type, i.e., the penalties due to *not* standardizing.

By weighing both the penalties and the advantages of standardization in each case, they may be maintained in approximate balance, and give the economical number of codes.

In the development of an entire new switching system using relays, there will be approximate ideas on the number of relays needed in the system, and on the number of kinds of relays required to fairly well satisfy the circuit functions. Based on past experience, the statistical distribution of kinds of relays for the various uses in relation to annual demand may also be approximated. The number of codes provided will determine the number of cells into which the statistical distribution is divided, and also the relative annual demand for each. In the extreme, only one code might be provided. It would have the advantage of very large lot-size, but also it would have to do the hardest as well as the easiest function. Such a requirement could lead to such absurdities as requiring that all relays have three windings, twelve transfers, best grade of contact metal, and the like; or to a greatly increased number of simpler relays. In addition, with but little flexibility in the design, there would be the added performance disadvantages due to imperfectly matching certain needs for power economy, speed, and the like. Hence, there is a large performance disadvantage, if but one kind of relay were provided.

At the other extreme would be cell divisions of kinds of relays to give the perfect match for every circuit use. Then, with no spare contacts, the best possible power economy, and every other condition at its ideal value, no performance penalty at all would be entailed. But every code increase would further subdivide the manufacture, till the lot-size penalties grew excessive.

These variations are shown by the curves in Fig. 1, and it is their sum, as seen in the curve marked *c*, that should decide where best to work. In planning a whole new central office which may use from 40,000 to 80,000 relays, and a new design of relay whose manufacturing costs are not yet well established, the data to give numerical values for Fig. 1 can only be quite tentative, and the results can only serve as guides. Yet such work has been most useful in recent effort on a redesigned No. 5 crossbar system to use the wire spring relay. Calculations could be made with enough assurance to determine the general shape of the total penalty curve (Curve c), and showed an optimum value for kinds of relays of about 200. Even more important was the indication that the curve was extremely flat in its optimum region, giving but a small change in the total penalty if an error, even as great as 2 to 1, were made in the number of codes to be used.

6.2 THE DETAILED PROCEDURES OF CODE SELECTION

Background information as gained above helps in guiding the early steps in picking codes of relays for the job, but there is a better method of picking codes when the circuit designer is actually down to cases. By following the steps below, the optimum number of codes will automatically result.

The first step in the coding program is to pick a list of basic codes. This can be done quite arbitrarily, initially, and should be based on general knowledge of types of circuit functions, together with specific knowledge of certain uses with very large demands. For example, certain coils will be needed to cover the range from very fast operation with no concern for current drain, to slow release and emphasis on economy of power; there will be single and multiple windings as dictated by circuit functions. Also, certain arrangements of contacts may be assumed to span the range of needs from a single make up to the capacity of the design — twelve transfers or twenty-four makes in the case of the new wire spring relay. Further, certain combinations of springs and coils will be evident at once as ideally suited to particular large-demand uses, and these should be on the original list. Soon the likely minimum demand for each will be quite evident. Such a list of tentative codes and demands is the first step in the coding scheme.

Now, as each circuit application arises, this basic list may be consulted. For some cases, an available code will be ideal, but for many others a new arrangement may be desired. In each such case a simple calculation will show the economical choice between a new code or the old code with more features than are really needed. Steps in the calculations are given below, where the extension of the old code is identified as Plan A, and the use of a new code as Plan B.

For Plan A, there will be a new demand, $(X + Y)$, which is greater than its previous demand by Y, the quantity of the new application. The demand cost penalty can then be read from charts such as Fig. 4. Performance penalties for pertinent features such as speed, power, extra contacts, etc. may be read from charts of the kind described in the earlier sections above. The sum of these values, multiplied by Y, is the cost penalty for the new application of an existing code.

For Plan B, there will be a lot-size cost penalty due to its demand. But there will also be a penalty imposed on the original code, (whose demand was X), because its demand was not enlarged to value $X + Y$. Thus the total lot-size cost penalty is made up of two parts:

(a) Penalty due to demand Y, multiplied by Y.

(b) [Penalty due to demand X − Penalty due to demand $(X + Y)$] multiplied by X.

The unit lot-size cost penalty for Plan B is the sum of these two factors, divided by the quantity Y. The design of relay for Plan B may be chosen by the methods previously outlined to be as nearly optimum as is feasible, and then performance penalties itemized as in Plan A above. The sum of these penalties, added to the lot-size penalties just mentioned, give a number for comparison with Plan A.

After the penalties due to either plan are compared, the least costly should be chosen. In cases where the costs are approximately equal, preference should probably be given to Plan A, as encouraging standardization.

In summary, the ideal number of codes in a newly designed system

TABLE IV — CHECK LIST FOR RELAY CODE SELECTION
AMOUNT OF EQUIVALENT COST PENALTY

Kind of Penalty	Plan A		Plan B
Lot-size (a)	Note (1)	Note (2)	Note (3)
(b)		$[(2) - (1)] \times \frac{X}{Y} =$	
Power			
Speed			
Extra parts			
Other			
Total			

Note (1): Fill in for demand $X + Y$.
Note (2): Fill in for demand X.
Note (3): Fill in for demand Y.

may be approximated by choosing the more economical alternative resulting from filling in the check list given in Table IV.

Part VII — Summary

In the telephone central office some 60,000 relays are needed to carry out the automatic switching functions. Since the entire office is built around them, they determine the cost of the office, not only by their first cost, and maintenance cost, but at least equally importantly by their influence on the power plant, and the amount of common control equipment that is needed. There is opportunity for great economy, overall, when manufacturing cost is put in proper balance with the cost of power, cost of functioning time, and similar performance variables.

The preceding pages have shown how each factor in the total system cost may be stated in a common language — the equivalent first cost value. When each variable, expressed as an incremental figure denoting the cost compared to an ideal standard, is so stated, all relay factors in the switching system may be cross-compared. This permits a large number of optimizing procedures to be carried out.

Optimizing methods for several important cases have been illustrated, and relations or procedures of value to the relay designer are presented. Particularly outstanding cases are the means for realizing (a) power-plus-coil economy and (b) power-plus-speed economy.

On the side of manufacturing cost, relay designers need some guidance as to how their applications problems will be influenced by the lot-size, or annual demand. Optimizing procedures are also given for this problem, to yield approximate values for optimum lot-sizes under various manufacturing conditions, and the corresponding cost penalty as a function of annual demand.

The steps to be taken in order to strike the best balance between standardizing for maximum manufacturing economy and diversifying for maximum performance economy are also given.

The problem as a whole is a striking example of how design for service can be applied on a very large scale. The economies realized by the approach used here have avoided the expenditure of many additional thousands of dollars yearly to the telephone companies, with corresponding economy to the customer.

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Symbols

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The following list gives the symbols used through the several articles appearing in this issue of the JOURNAL. The list does not include all the variant forms of the different symbols distinguished by subscripts, as these distinctions are indicated in the context where they are used.

MECHANICAL:

a Cross sectional area
ℓ Length
m Effective armature mass
T Kinetic energy
V Work done to overcome static load
x Armature displacement

ELECTRICAL:

e Copper efficiency or fraction of the copper volume occupied by conductor
E Applied battery voltage
G Total equivalent single turn conductance
G_C Equivalent single turn conductance of coil; N^2/R
G_E Equivalent single turn eddy current conductance
G'_E Effective single turn eddy current conductance; $G_E e^{-G_E/(G_C+G_S)}$
G_S Equivalent single turn conductance of sleeve
i Instantaneous current
I Steady state current
m Mean length of turn in winding
N Number of turns in winding
L_1 Single turn inductance; $\frac{4\pi}{\mathcal{R}'(x)}$
NI Ampere turns
NI_0 Just operate or just release ampere turn value
q Ratio of just operate to final ampere turns; $\frac{NI_0}{NI}$

ρ Resistivity of material
R Resistance
S Coil volume
v Ratio of flux attained at time t to steady state flux
W Power; I^2R

MAGNETIC:

A Equivalent pole face area
A_2 Effective pole face area (design value)
B Induction (flux density)
B' Value of B for maximum permeability
B'' Saturation density
B_M Density at $\mu = 1000$
B_R Remanence (density at $H = 0$)
C_L $\dfrac{\mathcal{R}_0 + \mathcal{R}_L}{\mathcal{R}_0}$
F Magnetic pull
$\mathfrak{F}$ Magnetomotive force; $4\pi NI$
$\mathfrak{F}_C$ Coercive magnetomotive force
H Field intensity
H_C Coercive force
L Inductance
μ Permeability
μ_A Permeability of air
μ' Maximum value of permeability
$\mathcal{R}$ Reluctance
$\mathcal{R}_C$ Reluctance of the core
$\mathcal{R}_C'$ Minimum value of core reluctance
$\mathcal{R}_1$ $\mathcal{R}(x)$ for $x = x_1$
$\mathcal{R}_i$ Initial incremental reluctance
$\mathcal{R}_0$ Equivalent closed gap reluctance
$\mathcal{R}_L$ Equivalent leakage reluctance
$\mathcal{R}_{L2}$ Effective leakage reluctance (design value)
$\mathcal{R}_{02}$ Effective closed gap reluctance (design value)
$\mathcal{R}(x)$ Equivalent relay reluctance $\dfrac{\mathcal{R}_L\left(\mathcal{R}_0 + \dfrac{x}{A}\right)}{\mathcal{R}_0 + \mathcal{R}_L + \dfrac{x}{A}}$

φ flux
Φ Steady state flux

φ_1 Initial equilibrium flux
φ' Flux for maximum permeability or minimum reluctance
φ'' Flux at saturation
φ_0 Residual flux
φ_G Gap flux
U Field energy
u $x/A\mathcal{R}_0$ or x/x_0
W Mechanical work done by magnet
x_0 $A\mathcal{R}_0$

TIME:

t Time
t_0 Operate or release time
t_1 Waiting time
t_2 Motion time
t_3 Stagger time
t_E Eddy current time constant; $L_1G'_E$
t_C Winding time constant; L_1G_C
t_S Sleeve time constant; L_1G_S