The Application of Designed Experiments to the Card Translator

By C. B. BROWN and M. E. TERRY

(Manuscript received October 27, 1953)

In the course of development of the card translator for use in the No. 4A toll crossbar system it was necessary to evaluate in detail the performance characteristics of all phases of the translator operation. One of the most important phases was that of the action of the translator cards. Since the action of these cards is controlled by the simultaneous influence of many independent variables a study of the card action was made using statistically designed experiments. This study made use of Graeco-Latin Square and Factorial Designs; herewith is presented a detailed description of their application to the problem. Also the method of analysis of the data and resulting conclusions are described in detail.

I. INTRODUCTION

In order to permit circuit designers to use the full capabilities of card translators[1] it has been necessary to conduct tests to determine (1) the time intervals required to drop the cards into reading positions and (2) the maximum number of cards which a translator can operate reliably. It was also desired to establish the maximum and minimum number of cards that could be operated efficiently in a card translator. Early estimates indicated a machine capacity of 1,000 cards but if a higher capacity could be demonstrated, a considerable cost saving in the 4A system would result. Since some differences in card drop time known to exist are due to card position and machine loading this study also was to consider whether or not any special loading instructions were necessary or desirable for field use of card translators. As the study commenced the Western Electric Company Installation Department requested information as to the need for leveling card translators on installation and it was felt that such information could readily be obtained in the course of this study. In investigating the effect of the many variables related to card drop time, a consideration was given to the possibility of the need for any design changes.

All previous tests of card drop time were made on Laboratories' built models of the translators and since tests made after any changes in model design have produced considerable differences in card drop times, it was felt that this study should be made on a sample of the Western Electric product. In addition, previous tests did not encompass all of the known variables over their extreme ranges.

For this study, a new translator was obtained from Western Electric. This translator was one of the regular production and was selected to be a representative unit. Fourteen hundred new 200A blanks were also obtained for use in this translator. Of these, 10 were coded for use as test cards to be observed. One hundred were coded to fill some bins with all coded cards. The balance of the 1,400 cards was left uncoded. This translator was connected to a simplified test set which would cycle the machine through the operation of the 10 coded test cards. Although this test set was simplified in its operation all of the pertinent time relationships involving card dropping were the same as are used in the standard translator circuits.

This study considered all of the variables, with the machine working in a normal cycle of operation as is currently used in the 4A system. These variables are:

1. *Bin* — Some differences in card dropping time had been observed depending on the bin in which the operating card was located.

2. *Position of Card Within Bin* — Earlier tests indicated that the location of the card within its bin made a considerable effect on dropping time.

3. *Code of Card* — Since both three and six digit cards will be used in the translator the code becomes a variable which may affect dropping time.

4. *Coded or Uncoded Cards in Working Bin* — Previous tests were made using only a few coded cards in the bin under observation and it was felt necessary to learn whether or not having all coded cards in a bin made any difference in the dropping time.

5. *Load in Working Bin* — This relates to the number of cards in the bin under observation. A range of from 15 to 105 cards per bin was used.

6. *Load in Machine* — This has to do with the possibility of any effect into the bin under observation from the cards in the other bins of the translator.

7. *Consistency of Data* — This has to do with repeated measurements to observe if the card dropping time is consistent over short periods of time.

8. *Balanced versus Unbalanced Loading* — Since the earlier installations will have machines that are less than full, it was necessary to learn whether or not any special loading considerations with regard to the position of the cards should be specified.

9. *Tilt* — (Machine) This has to do with the accuracy with which the translators should be leveled on installation.

10. *Life* — This variable considers the effect of repeated operations on cards and what effect, if any, these repeated operations have on the dropping time.

The tests on the first nine of the variables have been completed and a discussion of their effect on card dropping time is possible at this time. The tests on the tenth variable, life, are still in progress and have not as yet proceeded far enough to draw any conclusions. Therefore no consideration will be given to the effects of usage in this article. The effect of most of these variables was considered when the translator was operated both with and without the card support bars being used. The card support bars operate at the same time the card is dropped. Their operation is required for every card in the machine. Since their operation is slower than the free fall time of the operating card the card may ride down into the operated position on these bars and the observed drop time will simply be the operate time of the card support bars. This masks the effect of the various friction, gravity, and magnetic forces on the cards. Operating without the card support bars gives information on the free dropping time of the cards uninfluenced by these bars. Thus the masking effect of the card support bars can be removed and the tests made more sensitive to any physical effects which may be taking place at the cards themselves.

II. DESIGN OF EXPERIMENTS

Theory of Designed Experiments

In this problem it was required to evaluate the effect of each of the nine variables independently. It might have been convenient to take one variable at a time and vary it over its range while holding the associated variables at sets of constant values. Then a second independent variable could be chosen, the first variable placed in the group of associated variables, and a new set of runs made. This could be repeated until all the variables had been tested. This is obviously a logical, straightforward but unwieldy procedure.

A simpler procedure would be to vary the chosen variable over its range and at the same time let the associated variables vary over their assigned ranges in such a way that each associated variable is set an equal number of times at each of its possible values. This will cause any effect of the associated variable to be balanced out with respect to the chosen variable. That is, the average response of the system to the chosen variable can be evaluated because the contributions to the response from the associated variables are balanced out with respect to their variations.

Mathematically stated, the purpose of the latter procedure is to allow the independent evaluation of each variable over the observed range of its variation in the presence of the other variables.

Until recent years the standard experimental procedure was to make measurements for several values of one variable while taking great precautions to hold everything else constant. It was early recognized that such a technique was expensive but until the recent development of experimental designs in the field of Statistics there was no known alternative. In the class of designs treated here,[2] the set of numbers representing the respective sums of the measurements taken at the selected values of a given variable has the following properties:

1. If there are K discrete values of the given variables and N measurements made, then each sum contains N/K measurements.

2. If there are K_i values of the i^{th} remaining variable then N/K must be divisible by K_i, for all i.

3. Within each sum, for any given variable, each of the discrete values of the remaining variables must occur the same number of times.

4. The set of values of any of the remaining variables occurring in any sum for a given variable must be identical.

These properties imply that the logical subgroups for the discrete values of any given variable all contain the same number of measurements balanced with respect to the values of all the remaining variables.

The joint simultaneous evaluation of the specified variables can thus be made since it can be shown that the logical subgroups with respect to a given variable are independent of the logical subgroups of any other variable.

One of the difficulties in the use of this new technique is that both an engineer and statistician are required, and only rarely are both these professions found in an individual. It therefore becomes the responsibility of the engineer to outline the major and minor variables of the experiment, the description of the measuring devices to be employed, the accuracies desired in the results and his budget. The statistician must then propose the types of designs that will be appropriate together with their costs, methods of analysis, and salient features. The engineer and statistician must then select the design which best fits the situation.

The design selected for the first experiment of this study combined two basic design types, the Latin Square,[2] and the Factorial Design.[2] A brief discussion of each is in order at this point.

The Latin Square is peculiarly well suited to engineering research problems where many variables exist but the number of discrete levels necessary to describe the variation of any one is small, where relatively great precision is possible in measurements, and where the interaction of the variables is not a factor of the experiment. A key design[2] for a 5x5 Latin Square is as follows:

	Column				
Row	1	2	3	4	5
1	*A*	*B*	*C*	*D*	*E*
2	*B*	*C*	*D*	*E*	*A*
3	*C*	*D*	*E*	*A*	*B*
4	*D*	*E*	*A*	*B*	*C*
5	*E*	*A*	*B*	*C*	*D*

It will be observed that each Latin letter falls once and only once in every row and column. It is also true that if the logical subgroups *A*, *B*, *C*, *D*, *E* are considered, each of these sums has each row and column included once. If to the five row values, column values and letters, the five values of the first, second and third variables are associated respectively, the variables represented by Row, Column, and Letter can be evaluated independently of each other.

On the Key Latin Square, another properly chosen square represented by Greek Letters can be superimposed, and the five values of the fourth variable assigned to these letters at random:

	1	*2*	*3*	*4*	5
1	α	β	γ	δ	ϵ
2	γ	δ	ϵ	α	β
3	ϵ	α	β	γ	δ
4	β	γ	δ	ϵ	α
5	δ	ϵ	α	β	γ

The Graeco-Latin Square below is then produced:

	1	*2*	*3*	*4*	5
1	$A\alpha$	$B\beta$	$C\gamma$	$D\delta$	$E\epsilon$
2	$B\gamma$	$C\delta$	$D\epsilon$	$E\alpha$	$A\beta$
3	$C\epsilon$	$D\alpha$	$E\beta$	$A\gamma$	$B\delta$
4	$D\beta$	$E\gamma$	$A\delta$	$B\epsilon$	$C\alpha$
5	$E\delta$	$A\epsilon$	$B\alpha$	$C\beta$	$D\gamma$

Note that now each of the variables associated with Row, Column, Latin letter, Greek letter has the property that each element of the four categories contains one and only one element of the remaining three categories. For example, consider the five Latin-Greek letter combinations, or sample cells, containing ϵ:

Row 1 Col 5, *E*; Row 2 Col 3, *D*; Row 3 Col 1, C; Row 4 Col 4, B; Row 5 Col 2, *A*.

Then the sum of these five cells will contain the contribution of the 5 Rows, 5 Columns, and 5 Latin letters.

It should be noted that the rows and columns can be permuted without affecting the properties of the square. Indeed to protect against systematic effects which may be detrimental, it is usual to assign at random the row and column number, as well as the Latin and Greek letters to the values of variables represented by them. A notable exception to randomization occurs when time is a variable and those measurements made under essentially the same conditions within the same unit of time become the experimental unit. In the first experiment, all measurements made on one Run become the experimental unit with respect to time.

The Factorial Design serves a different purpose. In this discussion only two independent variables X, Z will be predicated but the extension to more variables follows directly. The XZ plane is the plane of the independent variables and we seek the point (X_0, Z_0) which gives a value of y (the dependent variable), $Y = \varphi(X, Z)$, which is optimum in some sense. That is, min Y, max Y may be sought, or the surface $f(X, Y, Z)$ shown to be a plane.

Generally only the region in the neighborhood of y (optimum) is of interest to the experimenter. Hence it is imperative (1) to bracket this point with respect to each independent variable and (2) to have a method of estimating y (optimum). If a factorial experiment has l_x different values of X and l_z different values of Z, then each replication of the experiment will require $l_x \cdot l_z$ units or points (X, Z). The first repetition of an experiment is called the second replication in the same way that the first overtone in music is called the second harmonic by engineers. Since the number of units available for test is usually limited, this places a practical ceiling on the magnitude of l_x and l_z. As a practical limit in general l should be 7 or less and the values 2, 3, or 4 are far more common. It is generally better to use the smaller values of l and repeat the experiment, than to conduct an experiment involving only a single replication. In addition to evaluating one variable averaged over the second, we are interested in evaluating the interaction of the variables on each other, when such interaction exists and is of interest. In a sense this interaction measures the departure of the system y, X, Z from linearity.

Once the basic designs have been selected and appropriately combined to fit most efficiently the requirements of the proposed experiment, and the values of the variables randomly assigned to the schematic layout, a detailed experimental layout must be drawn up. This layout must show concisely and clearly each experimental unit and the makeup of every basic element giving its assigned value of each variable. Explicit directions must be drawn up as to the order of selection of the elements of the unit. It is generally advisable for simplicity to assign the elements at random to the M possible consecutive order integers of the experimental unit.

Performance Study

The first seven variables listed in the Introduction are:

1. The Bin in use, *Bins*.
2. Position of Test card pair within the bin, *Position*.
3. The use of 3 digit or 6 digit Cards, *Code*.
4. Arrangement of Coded and Uncoded Cards, *Runs*.
5. Load of Bins containing Test Cards, *Load*.
6. Load of Bins not containing Test Cards, *Idlers*.
7. Order of repeated measurement, *Look*.

These constitute a system — that is, any or all can be varied at will and hence a design involving all of them simultaneously can be sought.

The test set can operate ten test cards; 5 with a 3 digit code and 5 with a 6 digit code. This immediately suggests 5 packages of two coded card pairs, each pair containing a 3 and a 6 digit card. Five pairs can also be handled neatly in 5 bins. The combination of the standard load of 85 cards, with two overloads and two underloads would give a fair evaluation of load criterion. Budget restrictions force the use of only a limited number of coded cards, with blanks used to fill out the experiment. Hence the type of card making up the load must be varied over the loads. It was further found that 5 positions of test cards within the bin covers the range of positions adequately.

The pairs of 3 and 6 digit cards now are associated with the Graeco-Latin Square design with Columns identified with Bins; Rows with distribution of coded cards; or Runs; Latin letters with Load; and Greek letters with position within the bin. Now if the position of the 3 digit card is randomly assigned in the pairs, the design absorbs the first five

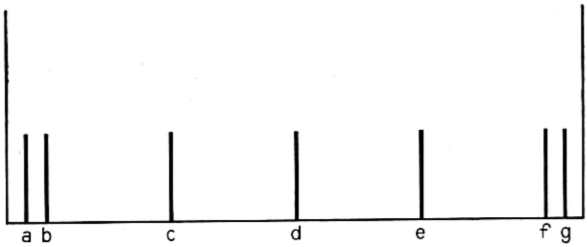
TABLE I — DESIGN OF GRAECO-LATIN SQUARE EXPERIMENT

Each run made with the "x" bins loaded with 0, 50, 85 and 100 cards and consists of four operations (A, B, C, D) of each coded card.

BIN NO.

Photocell Mounting	I	II	III	X	IV	V		X	X	X	X	X	X	Lamp

POSITION IN BIN



		Bin I	Bin II	Bin III	Bin IV	Bin V
	Card →	1 6	2 7	3 8	4 9	5 10
Run #1	Pos. in Bin	*d g*	*f g*	*a b*	*a d*	*c e*
	No. of Coded Cards	2	41	50	15	2
	No. of Blank Cards	98	44	0	0	103
Run #2	Pos. in Bin	*c e*	*d g*	*f g*	*a b*	*a d*
	No. of Coded Cards	85	9	15	2	2
	No. of Blank Cards	0	41	0	103	98
Run #3	Pos. in Bin	*f g*	*a b*	*a d*	*c e*	*d g*
	No. of Coded Cards	2	100	2	2	7
	No. of Blank Cards	103	0	83	48	8
Run #4	Pos. in Bin	*a d*	*c e*	*d g*	*f g*	*a b*
	No. of Coded Cards	2	2	105	2	2
	No. of Blank Cards	48	13	0	98	83
Run #5	Pos. in Bin	*a b*	*a d*	*c e*	*d g*	*f g*
	No. of Coded Cards	2	2	2	85	22
	No. of Blank Cards	13	103	98	0	28

variables of the list. The layout of these variables in the Graeco-Latin Square design is given in Table I.

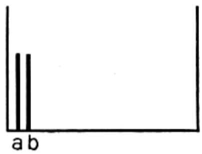
The remaining two variables are functions of different portions of the machine. The rows of the Square, involving distribution of coded and uncoded cards, represent distinct machine set-ups, and hence if for every set-up the load of the seven not-measured bins is varied, and for every variation in this load 4 observations of the 5 pairs are taken, consideration of the remaining two variables is achieved. Yet the tedious part of handling the test cards has been reduced to a very reasonable amount. These last two variables, considered by themselves, form a factorial design and where 4 loads and 4 measurements per load are used, 16 measurements being taken for each machine set-up. Similarly any of the other five variables considered pairwise with one of the latter two forms another factorial design. This resulting complex overall pattern is shown in Table I.

TABLE II — TRANSLATOR TILT TEST — DESIGN OF EXPERIMENT

TEST CARDS LOCATION

Bin	I	II	III		IV	V
Cards	6, 1	2, 7	8, 3		4, 9	10, 5

POSITION OF TEST CARDS IN BINS



Total number of cards in each bin: 100
Bin # I contains all coded cards.

	Description of Tilt
Tilt #0	Lamp end lower than photocell end by 1/16″ in 3 feet. Translator resting on table at all four points. North Lamp end higher than South Lamp end by 1/16″ in 22″. North Photocell end higher than South Photocell end by 1/16″ in 22″
Tilt #1	Lamp end higher than photocell end by 9/16″ in 3″ ft. Lamp end supports blocked up 1/4″ North Lamp end higher than South Lamp end by 1/8″ in 22″ North Photocell end higher than South Photocell end by 1/8″ in 22″
Tilt #2	Lamp end higher than photocell end by 15/16″ in 3 ft. Lamp end supports blocked up 1/2″ North Lamp end higher than South Lamp end by 3/16″ in 22″ North Photocell end higher than South Photocell end by 3/16″ in 22″
Tilt #3	Lamp end higher than photocell end by 1 1/8″ in 3 ft. Lamp end supports blocked up 5/8″ North Lamp end higher than South Lamp end by 3/16″ in 22″ North Photocell end higher than Souh Photocell end by 3/16″ in 22″

Tilt Study

Routine field practice for the installation of a card translator calls for leveling the table before positioning of the translator. After a translator was placed it had been found that the level was not maintained, and the question of final level requirements was raised. Accordingly the test machine was run at 0 in., ⅝ in., 1 in., and 1$\frac{3}{16}$ in. tilt, Table II. Two test cards were placed in each of five bins. Since the two end card positions on the low side of each bin were suspected as being critical, the two test cards were placed in these positions in the five bins. The experiment is shown schematically below:

	Bins				
	I	II	III	IV	V
Tilt 0	a_6b_1	a_2b_7	a_8b_3	a_4b_9	$a_{10}b_5$
1	a_6b_1	a_2b_7	a_8b_3	a_4b_9	$a_{10}b_5$
2	a_6b_1	a_2b_7	a_8b_3	a_4b_9	$a_{10}b_5$
3	a_6b_1	a_2b_7	a_8b_3	a_4b_9	$a_{10}b_5$

where a is the end position, b is the next-but-end position, and the subscripts identify the actual card number. Considering any bin, the two card positions and the four values of tilt may be considered as a factorial design. Similarly, the five bins combine with the four values of tilt as a factorial design.

Balanced Loading

At the onset of general usage many translators will not be fully loaded. It was desirable to investigate the effects of various patterns of loading at three representative low loads of 200, 400 and 600 cards respectively. Four logical patterns were studied (see Tables III and IV) all of which are shown below for a load of approximately 400 cards. (Similar patterns follow with loads of 200 and 600 cards.):

Bin	1	2	3	4	5	6	7	8	9	10	11	12
W	x	x	x	x								
X	x	x					x	x				
Y	x	x									x	x
Z	0	0	0	0	0	0	0	0	0	0	0	0

x = 100 cards, 0 = $\frac{1}{12}$ load (approximately)

The ten test cards were all placed in Bin One.

This experiment involves only two factors; the four loading patterns,

TABLE III — LOADING PATTERNS IN PARTIALLY LOADED CARD TRANSLATOR — BALANCED VERSUS UNBALANCED LOAD TEST

Treatments	Load	No. of Cards in Bins											
		Bin I	Bin II	Bin III	Bin IV	Bin V	Bin VI	Bin VII	Bin VIII	Bin IX	Bin X	Bin XI	Bin XII
W	600	100	100	100	100	100	100	—	—	—	—	—	—
	400	100	100	100	100	—	—	—	—	—	—	—	—
	200	100	100	—	—	—	—	—	—	—	—	—	—
X	600	100	100	100	—	—	—	100	100	100	—	—	—
	400	100	100	—	—	—	—	100	100	—	—	—	—
	200	100	—	—	—	—	—	100	—	—	—	—	—
Y	600	100	100	100	—	—	—	—	—	—	100	100	100
	400	100	100	—	—	—	—	—	—	—	—	100	100
	200	100	—	—	—	—	—	—	—	—	—	—	100
Z	600	50	50	50	50	50	50	50	50	50	50	50	50
	400	33	33	33	33	33	33	33	33	33	33	33	37
	200	16	16	16	16	16	16	16	16	16	16	16	24

Note: All cards in Bin I are coded, see Table IV. Cards in Bins II–XII not coded.

and the three loads of cards. A suitable design is the factorial design with one factor, the loading pattern, at four levels (W, X, Y, Z) and the other factor, the loads of cards at three levels (200, 400, 600) with ten test card measurements taken at each of the twelve points.

III. DATA

The data on card dropping time were obtained by means of shadowgrams of the light output from two of the light channels of the translator. Each shadowgram comprised the operation of the 10 test cards in sequence. Samples of these shadowgrams are shown on Figs. 1 and 2. For the purpose of these experiments the card dropping time is defined as the time from the release of the pull-up magnets until the full closure of the light channels of the translator exclusive of any card rebound. The data from the various experiments were tabulated and are given in Tables V to XI, inclusive.

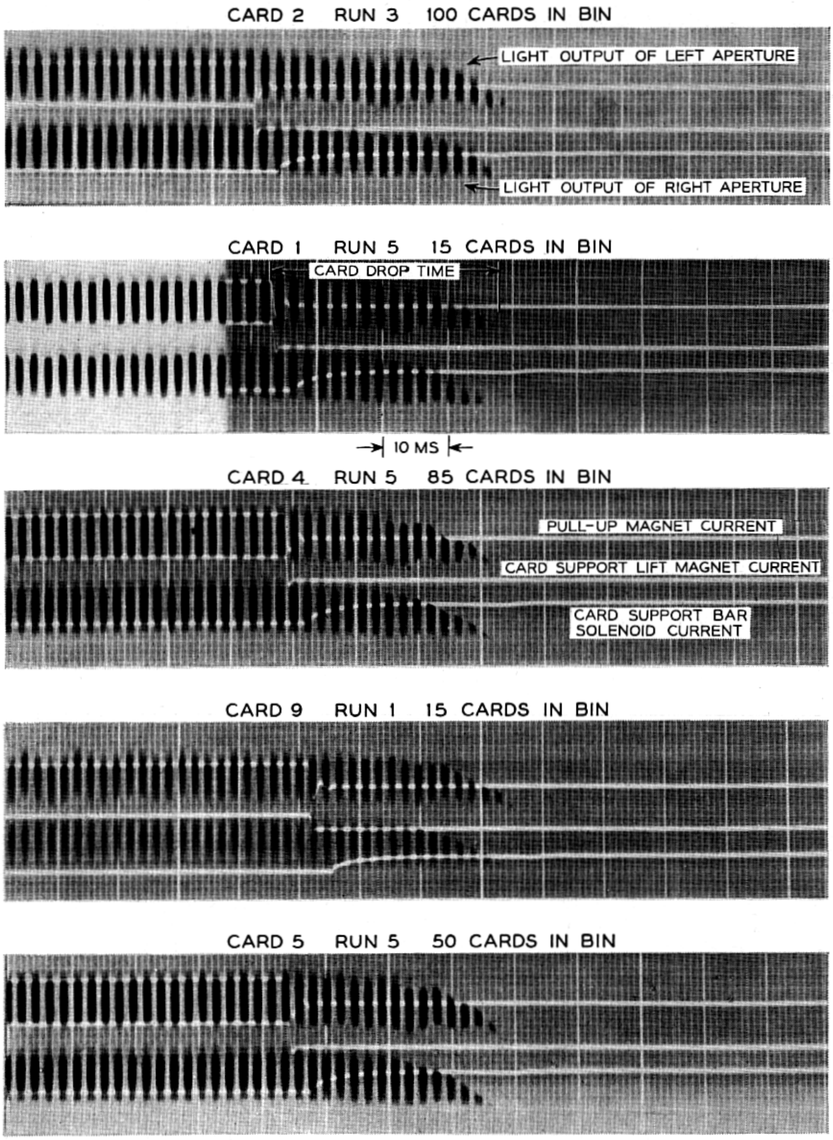


FIG. 1 — Card motion shadowgraphs, card support bars working.

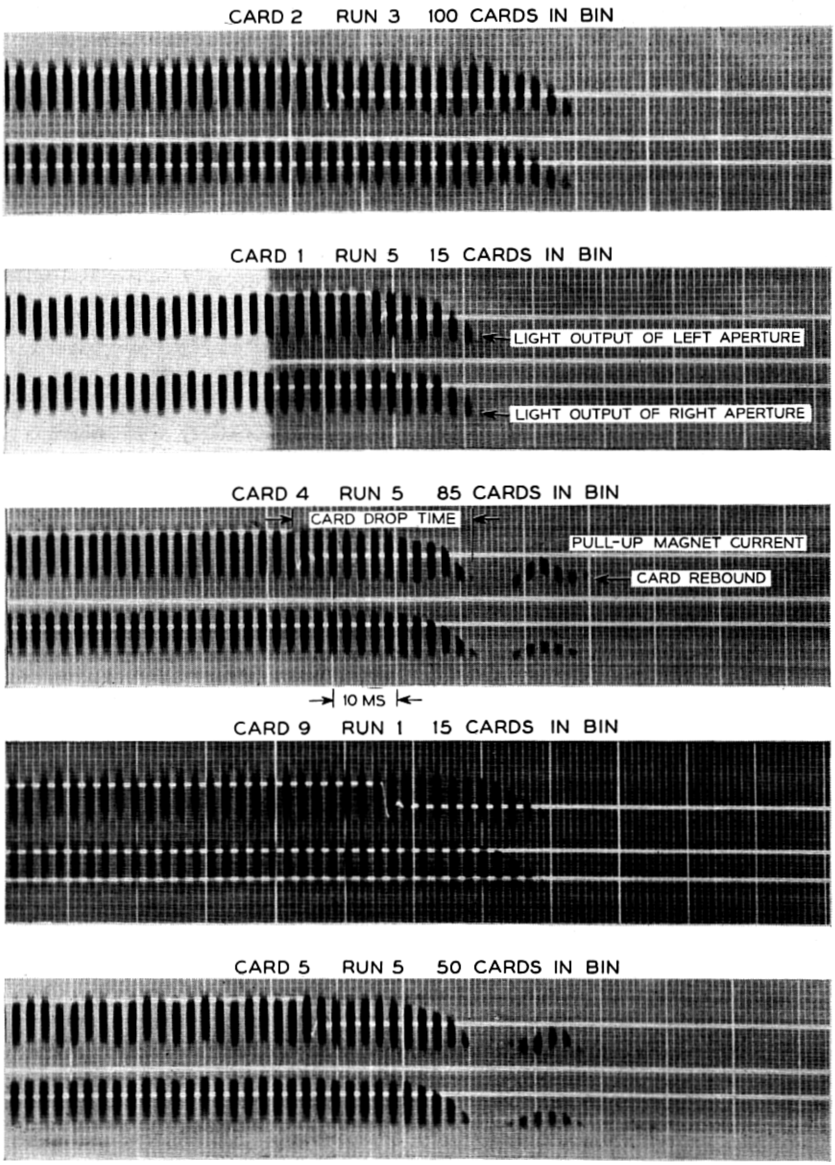


Fig. 2 — Card motion shadowgraphs, card support bars out.

Table IV — Card Translator Balanced versus Unbalanced Load Test — Arrangement of Coded Cards in Bin I

No. of Cards	Arrangement of Coded Cards in Bin I	
	← Left end of Bin	Right end of Bin →
100	#1 #2 *10* #3 *15* #4 *15* #5 *10* #6 *15* #7 *15* #8 *10* #9 #10	
50	#1 #2 *5* #3 6 #4 *7* #5 *4* #6 *7* #7 6 #8 *5* #9 #10	
33	#1 #2 *3* #3 *4* #4 *3* #5 *3* #6 *3* #7 *4* #8 *3* #9 #10	
16	#1 #2 *1* #3 *1* #4 *1* #5 *0* #6 *1* #7 *1* #8 *1* #9 #10	

Note: Numbers preceded by # represent the test cards. Numbers in italics represent quantity of non-test coded cards placed between two consecutive test cards.

IV. ANALYSIS OF THE DATA

After checking the raw data for recording errors, they were analyzed and reduced in three distinct steps as follows:

1. Using the techniques of the analysis of variance* each variable and each measured interaction of several variables was tested as a possible assignable cause of variation.

2. Using the components of variance analysis† on those variables and combinations of variables found to be assignable causes in Step 1 their contributions to the overall variation was estimated, and

3. After tabulating the arithmetic mean for each value of the variables an upper and lower bounds were determined which estimates the allowance, or probable range of means, for similar experiments and operations on card translators having the same residual σ. The technique used was proposed and formulated by J. W. Tukey.[3]

The analysis of variance tables were reduced to the summary in Tables XII and XIII for the seven variable studied.

Proceeding to the tabulation of the means and the calculation of the allowances appropriate to these means, the data was reduced as in Table XIV. The magnitude of the several effects can now be noted. The effect of Idler load is quite meaningful — as the load on the machine is increased to the normal load of 85 we see a decrease in mean dropping time from 36.5 to 34.4 milliseconds. Also the slight increase for the overload of 100 is not significant statistically or engineeringwise. At first glance the results of Idler loads of 0, 50, 85 and 100 might seem to be inconsistent with those of Operating Loads of 15, 50, 85, 100, and 105. The mean dropping time of the Load of 15 (say) is the average dropping time of

* See Appendix for Discussion and References 2, 4 and 5.
† See Appendix for Discussion and Reference 6.

Table V — Card Drop Time in Milliseconds — No Cards in X Bins — Card Support Bars Operating

		Bin # I		Bin # II		Bin # III		Bin # IV		Bin # V	
	Card # →	1	6	2	7	3	8	4	9	5	10
Run #1	A	36	38	36	35.5	37	36.5	37	37.5	37.5	40
	B	36.5	38	36.25	37.5	35	37	36.5	37	37	39.5
	C	35.25	37.5	35	36	37	36	36	36	37.5	39
	D	35.5	38	37	36.25	36.5	36.5	36.5	36.5	37	39
	$\bar{X}$	35.8	37.8	36.06	36.32	36.37	36.5	36.5	36.75	37.25	39.37
	R	1.25	0.5	2.0	2.0	2.0	1.0	1.0	1.5	0.5	1.0
Run #2	A	34	35	34	33.5	34	34	36	35	37	39
	B	33.5	34.5	34	34	34	34	35.5	33.5	37	38
	C	34	34.5	33.5	34	35	34	36	35	36.5	38.5
	D	35.5	33	33.5	34.5	34.5	34	35.5	34.5	36.5	38
Run #2	X	34.25	34.25	33.75	34.0	34.37	34	35.75	34.5	36.75	38.37
	R	2.0	2.0	0.5	1.0	1.0	0	0.5	1.5	0.5	1.0
Run #3	A	36.5	37.5	38	36.5	37	37	37	38	37.5	39.5
	B	36	37	36.5	35	36	37	36.5	36.5	38	39
	C	36	36.5	37.5	35.5	36	35	37	36.5	38	39.5
	D	35.5	36.5	37.5	35.5	36	35.5	35.5	36.5	37.5	38.5
Run #3	X	36.0	36.37	37.37	35.62	36.25	36.12	36.5	36.37	37.75	39.12
	R	1.0	1.0	1.5	1.5	1.0	2.0	1.5	1.5	0.5	1.0
Run #4	A	37	37	35.5	37	36	37	37	37.5	38	38.5
	B	36	37.5	35.5	36	36	37	36.5	37.5	38	38.5
	C	36	37.5	35	36	35.5	36	37	38	37.5	38
	D	35.5	37	36	36	36	36	37	37.5	37.5	37.5
Run #4	X	36.12	37.25	35.50	36.25	35.87	36.50	36.87	37.62	37.75	38.12
	R	1.5	0.5	1.0	1.0	0.5	1.0	0.5	0.5	0.5	1.0
Run #5	A	36	36	37	37	35.5	36.5	37	36.5	38.5	38.5
	B	36	35.5	37	37	37	37	37	37	38.5	39
	C	36	36	36	36	36	36.5	37	37	38.5	39
	D	36	35	35.5	36	35	36	36.5	37	38	39
	$\bar{X}$	36	35.62	36.37	36.50	35.87	36.50	36.87	36.87	38.37	38.87
	R	0	0.5	1.5	1.0	2.0	1.0	0.5	0.5	0.5	0.5

Cards in Bins loaded with 15 cards in the presence of 4 bins loaded with 50, 85, 100, and 105 cards respectively and of bins loaded as a group with 0, 50, 85, or 100 cards. On the other hand the mean dropping time attributable to an Idler load of 50 cards (say) is the average of all dropping cards in the operating bins when the 7 Idler loads are 50 cards. Hence the statistical conclusion that the effect of loads in the operating bins is slight over the range of loads of 15 to 105 cards is reached, and that the

TABLE VI — CARD DROP TIME IN MILLISECONDS — 50 CARDS IN X BINS — CARD SUPPORT BARS OPERATING

		Bin #I		Bin #II		Bin #III		Bin #IV		Bin #V	
	Card # →	1	6	2	7	3	8	4	9	5	10
Run #1	A	35	36	34	35	34.5	35	33	34	33.5	35
	B	33.5	35	34.25	34	33	33.5	32.5	33.5	33.5	34.5
	C	34.5	34	32.5	34	32.5	32.5	32	33.25	32	34
	D	34	32.5	32	34	33.5	33.5	33.25	32.5	33	33
	$\overline{X}$	34.25	34.37	33.19	34.25	33.37	33.62	32.69	33.31	33	34.12
	R	1.5	3.5	2.25	1.0	2.0	2.5	0.75	1.5	1.5	2.0
Run #2	A	35.5	35	34	34	32.5	35	35	35.5	35	35.5
	B	35.5	35.5	34	33.5	34.5	34.5	35.5	35	35.5	35
	C	35	35	34	35	34.5	34	36	35	34.5	34
	D	34	35	34	34	32.5	34	35.5	34	35	35
	$\overline{X}$	35.0	35.1	34	34.1	33.5	34.4	35.5	34.9	35.0	34.9
	R	1.5	0.5	0	1.5	2.0	1.0	1.0	1.0	1.0	1.5
Run #3	A	35	35	37.5	34	34.5	34.5	34	34	34	34.5
	B	36	36.5	37	36	35.5	35.5	34	34.5	34.5	36
	C	35	35.5	39	35	35.5	35	34.5	34.5	34.5	35.5
	D	35.5	37	39	35	35.5	34	34.5	34.5	34.5	35.5
	$\overline{X}$	35.37	36.0	38.12	35.0	35.25	34.75	34.25	33.37	33.37	35.37
	R	1.0	2.0	2.0	2.0	1.0	1.5	0.5	0.5	0.5	1.5
Run #4	A	37	37	35.5	35.5	36.5	35	35	35.5	36	37
	B	36.5	37	36	36.5	36.5	36	35.5	35.5	36	37
	C	37.5	38	35.5	36	36.5	35	36	35.5	35.5	36.5
	D	37	37.5	35.5	37.5	36.5	36.5	36.5	36	37	38
	$\overline{X}$	37	37.37	35.62	36.37	36.50	35.62	35.75	35.62	36.12	37.12
	R	1.0	1.0	0.5	2.0	0	1.5	1.5	0.5	1.5	1.5
Run #5	A	36	36	35.5	36	35.5	36	34.5	36	34.5	35
	B	35	35.5	35	36	35	35	34	34	34.5	35
	C	35.5	36	35.5	35.5	34.5	35	34	35	34.5	35
	D	36	35.5	35	35.5	34.5	35.5	34	34.5	34	34.5
	$\overline{X}$	35.62	35.75	35.25	35.75	34.87	35.37	34.12	34.87	34.37	34.89
	R	1.0	0.5	0.5	0.5	1.0	1.0	0.5	1.5	0.5	0.5

improvement in dropping time caused by increasing the overall load is generally consistent over the test bin loads.

One last estimate must be made — that of the σ' for a single measurement where*

$$\sigma'^2 = \sigma_s'^2 + \sigma_e'^2 + \sum \sigma'^2_{\text{assignable causes}} .$$

* See Appendix for meaning of symbols and discussion.

TABLE VII — CARD DROP TIME IN MILLISECONDS — 85 CARDS IN X BINS — CARD SUPPORT BARS OPERATING

		Bin #I		Bin #II		Bin #III		Bin #IV		Bin #V	
	Card # →	1	6	2	7	3	8	4	9	5	10
Run #1	A	34.5	34	32.5	32.5	33	34	32.5	34	33.5	33.5
	B	35	34	34	32	32	34	33	33	33.5	33
	C	33	34	33.5	33.5	32.5	33.5	33	33.5	33.5	33
	D	35	34	34	32	33	33.5	33	33.5	32.5	34
	$\bar{X}$	34.4	34	33.5	32.5	32.9	33.7	32.9	33.5	33.25	33.4
	R	2.0	0	1.5	1.0	1.5	0.5	0.5	1.0	1.0	1.0
Run #2	A	33	34.5	32	34	33	34.5	34	33.25	33.5	33.5
	B	32	35	34.5	34.5	32	33.5	35	34	33.75	34
	C	34	35	33.5	34.5	33	34.5	35.5	35	35	35
	D	34	34	34	34.5	33.5	35	35	34.5	35	33
	$\bar{X}$	33.25	34.62	33.5	33.4	32.9	34.4	34.19	33.55	34.31	33.79
	R	2.0	1.0	2.5	0.5	1.5	1.5	1.5	1.75	1.5	1.5
Run #3	A	35	35	36.5	32.5	34	34.5	33	34.5	34	34
	B	35	35.5	37	34.5	35.5	34	33	34.5	34.5	34
	C	35	34.5	36	34.5	35.5	33	33.5	34	34.75	35
	D	35	35	37	34.5	35	34.5	34	34.5	34	34.5
	$\bar{X}$	35	35	36.62	34	35	34	33.37	34.37	34.31	34.37
	R	0	1.0	1.0	2.0	1.5	1.5	1.0	0.5	0.75	1.0
Run #4	A	37	37	36.5	36.5	36	36	35	35	36.5	36.5
	B	38	37.5	37	36	36.5	37	36	35	36.5	36.5
	C	37	37	37	35	36	36.5	35.5	35	36	36.5
	D	36	35	36	36	36	36	35	35.5	35.5	36.5
	$\bar{X}$	37	36.62	36.62	35.87	36.12	38.37	35.37	35.12	36.12	36.25
	R	2.0	2.5	1.0	1.5	0.5	1.0	1.0	0.5	1.0	1.0
Run #5	A	35	35	34	35	33.5	34	34	33.5	33.5	34
	B	34.5	34.5	34	34	34	33.5	34	33	33.5	33.5
	C	34	34.5	34	34.5	34	33.5	33	32.5	33	34
	D	34	35	33.5	34.5	33.5	34	33	33.5	32.5	34
	$\bar{X}$	34.37	34.75	33.87	34.50	33.75	33.75	33.50	33.12	33.12	33.87
	R	1.0	0.5	0.5	1.0	0.5	0.5	1.0	1.0	1.0	0.5

If the assignable causes contributing to this estimate are not removed then this σ' is the well known parameter for control charts.

The variable Runs may be an experimental variable which will not occur in operational usage since coded cards only will be used in the machines. The joint effect, however, of this variable with all the others is slight, for

$$\sigma'_{w0} = 2.68 \text{ ms without Runs, and}$$
$$\sigma'_{w} = 2.98 \text{ ms with Runs included.}$$

TABLE VIII — CARD DROP TIME IN MILLISECONDS — 100 CARDS IN X BINS — CARD SUPPORT BARS OPERATING

		Bin #I		Bin #II		Bin #III		Bin #IV		Bin #V	
Card # →		1	6	2	7	3	8	4	9	5	10
Run #1	A	34	35.5	34.5	35	34	35	33.5	34	34.5	35
	B	34	35	35	35	34.5	34.5	34.5	35	35	34
	C	34	35	34	33	35	34.5	33.5	33.5	35	34
	D	35	33.5	34.5	35	35	34	35	33.5	33.5	34.5
	$\bar{X}$	34.75	34.75	34.5	34.5	34.62	34.5	34.12	34.0	34.5	34.37
	R	1.0	2.0	1.0	2.0	1.0	1.0	1.5	1.5	1.5	1.0
Run #2	A	34	36.5	34.5	35.5	33.5	34.5	35.5	36	35	35.5
	B	34.5	35	35	35.5	33.5	35	35.5	36.5	35.5	34
	C	34	34	34	33	34	33.5	34	34	33.5	33
	D	33	32	33.5	32.5	33.5	33.5	34.5	34	33	34
	$\bar{X}$	33.9	34.37	34.25	34.12	33.6	34.12	34.9	35.12	34.25	34.12
	R	1.5	4.5	1.5	3.0	0.5	1.5	1.5	2.5	2.5	2.5
Run #3	A	34.5	34.5	36.5	33.5	35.5	33.5	34	34	34	34
	B	35	34.5	38	34	34.5	33.5	34	33.5	33	34
	C	35	35.5	38.5	34	34.5	34.5	33.5	33.5	34.5	34.5
	D	34.5	35	39.5	34	35	35	34	34	34	35
	$\bar{X}$	34.75	34.87	38.12	33.87	34.87	34.12	38.87	33.75	33.87	34.37
	R	0.5	1.0	3.0	0.5	1.0	1.5	0.5	0.5	1.5	1.0
Run #4	A	37	37	35.5	35.5	36	35.5	35	35.5	36	37
	B	36	36	37	36	36	35.5	35	35.5	37	36
	C	36	36.5	35.5	37	36	36.5	35	36.5	36	37
	D	37	36	35.5	35.5	35.5	35.5	34.5	35	36	37
	$\bar{X}$	36.5	36.37	35.87	36	35.87	35.75	34.87	35.62	36.25	36.75
	R	1.0	1.0	1.5	1.5	0.5	1.0	0.5	1.5	1.0	1.0
Run #5	A	33.5	34	33.5	33.5	33.5	33.5	32.5	33.5	33.5	33
	B	33.5	33	34	34.5	34	34	32.5	33.5	33.5	33.5
	C	35	35	34	34.5	33.5	34	33	33.5	33.5	33
	D	34	34	34.5	33.5	34	34	33	33	33	34
	$\bar{X}$	34	34	34	34	33.75	33.87	32.75	33.37	33.37	33.37
	R	1.5	2.0	1.0	1.0	0.5	0.5	0.5	0.5	0.5	1.0

The overall estimate of Mean dropping time was found to be 35.1 ms. We can predict therefore that the dropping time of a card chosen at random from a normal translator at the beginning of its life will be 35.1 $\pm$ $3\sigma'$ (without Runs) or between 27 and 43 ms from the well known 3σ limits.

When the analysis of variance with the card support bars out is examined it is found that the sampling error σ_s^2 remains stationary but

TABLE IX — CARD DROP TIME IN MILLISECONDS — 85 CARDS IN X BINS — CARD SUPPORT BARS OUT

		Bin #I		Bin #II		Bin #III		Bin #IV		Bin #V	
	Card # →	1	6	2	7	3	8	4	9	5	10
Run #1	A	23.5	27	26.5	26	28	28	25	24	26	26.5
	B	23.5	28	25.5	26.5	28	27	25	24	25.5	26
	C	23	27	26	26	29	28	25	23	26	26.5
	D	24	26.5	26.5	27	29.5	27.5	25	24	26.25	26
	$\bar{X}$	23.5	27.1	26.1	26.4	28.6	27.6	25	23.75	25.94	26.25
	R	1.0	1.5	1.0	1.0	1.5	1.0	0	1.0	1.75	0.5
Run #2	A	24.5	26.5	26	23.5	22	21.5	34	31.5	32	27.5
	B	25	27	25	22	21.5	22	33.5	31	32	28
	C	24	27.5	26.5	23.5	21.5	23	34.5	31.5	31.5	27.5
	D	24.5	26.5	25	22.5	22	22	34	31.5	33	28
	$\bar{X}$	24.5	26.9	25.6	22.9	21.75	22.12	34.0	31.4	34.6	27.75
	R	1.0	1.0	1.5	1.5	0.5	1.5	1.0	0.5	1.5	0.5
Run #3	A	28	29	37.5	28	34	28	29	28.5	24	22
	B	27	29	37	28	33	28	30	28	24	21
	C	28.5	30	37.5	29	32.5	28	28.5	28	23.5	20
	D	28	30	38	29	33	28	29	27.5	24	21
	$\bar{X}$	27.87	29.5	37.5	28.5	33.12	28	34.12	28	23.87	21
	R	1.5	1.0	1.0	1.0	1.5	0	1.5	1.0	0.5	2.0
Run #4	A	16.5	23.5	22	24	28	29	28.5	32	34	34
	B	17	23	22	23.5	28	29	28	33	33.5	33.5
	C	17	23	21	22	28	29	27.5	32	30.5	33
	D	17	23.5	22	22.5	28	29	28	31	33.5	36.5
	$\bar{X}$	16.87	23.25	21.75	23	28	29	28	32	32.87	33.50
	R	0.5	0.5	1.0	2.0	0	0	1.0	2.0	3.5	3.0
Run #5	A	16.5	17	25	25.5	28.5	28	29	30	26	25.5
	B	16	17	26	26.5	28	28	29	29.5	26	25
	C	18	17.5	25	25.5	28	28	28.5	29.5	26	25
	D	16.5	17.5	25	25.5	28	28	29	30	26	25.5
	$\bar{X}$	16.75	17.25	25.25	25.75	28.12	28	28.87	29.75	26	25.25
	R	2.0	0.5	1.0	1.0	0.5	0	0.5	0.5	0	0.5

the experimental error is inflated tenfold. The assignable causes found in the previous experiment are found here, where measured, with one exception — the interaction of Codes on Positions. The estimate of σ' for a single reading with the card support bars out now becomes

$$\sigma' = 8.56 \text{ ms.}$$

Since the overall estimate of mean dropping time with the card support

TABLE X — TILT STUDY DATA — CARD DROP TIME IN MILLISECONDS — CARD SUPPORT BARS OUT

		Bin #I		Bin #II		Bin #III		Bin #IV		Bin #V	
	Card # →	6	1	2	7	8	3	4	9	10	5
Tilt #0	A	21.5	19.5	29	26	30.5	27.5	32.5	32.5	31	29
	B	21.5	19.5	31	25.5	32	28.5	31.5	35	30.5	28.5
	C	21.5	19.0	29.5	26	31	28	31	34	30.5	29
	D	21.5	19.0	29.5	25.5	30.5	28.5	32	34.5	30.5	29.5
	Σ	86	77	119	103	124	112.5	127	136	122.5	116
	$\bar{X}$	21.5	19.25	29.75	29.75	31	28.13	31.75	34	30.63	29
	R	0	0.5	2	0.5	1.5	1	1.5	2.5	0.5	1.0
Tilt #1	A	27.5	21	38	28	38.5	28.5	44	28.5	47	31.5
	B	25	21	37	27.5	38	29	44	29	49	31
	C	25.5	20.5	38	27.5	43	28.5	47	29.5	46	31
	D	25.5	21.5	39	28	42.5	28	43.5	29	46.5	32
	Σ	103.5	84	152	111	162	114	178.5	116	188.5	125.5
	$\bar{X}$	25.88	21	38	27.75	40.5	28.5	44.63	29	47.13	31.38
	R	2.5	1	2	0.5	5	1	3.5	1	3	1
Tilt #2	A	29	23.5	62.5	32	67	32	73	32.5	92.5	38.5
	B	38.5	23	57.5	31	63.5	31.5	72.5	32	62.5	37
	C	33	22.5	58.5	30.5	59	31	68.5	32	66	37.5
	D	33	23	59	31.5	59.5	31.5	75	33	67.5	37.5
	Σ	132.5	92	237.5	125	249	126	289	129.5	288.5	150.5
	$\bar{X}$	33.13	23	59.38	31.25	62.25	31.5	72.25	32.38	72.13	37.63
	R	9.5	1	5	1.5	8	1	6.5	1	30	1.5
Tilt #3	A	58	23	68	33.5	109.5	34	Did not oqerate	34.5	69.5	39.5
	B	41.5	23.5	69	33	95	33.5		34.5	91.5	38.5
	C	34	23	80	33.5	77	34		36	87	39.5
	D	31.5	24.5	80.5	34	86.5	35		36	95	39
	Σ	165	94	297.5	134	368	136.5		141	343	156.5
	$\bar{X}$	41.25	23.5	74.38	33.5	92	34.13		35.25	85.75	39.13
	R	26.5	1.5	12.5	1	32.5	1.5		1.5	25.5	1

bars out was 26.7 ms, it can be predicted that the limits for the dropping time for a single card at the onset of life of a card translator will be 1 and 52 ms. It is to be noted that while the upper limits in both these estimates are reasonably close, the lower limits are quite different. This is to be expected since the card support bars may delay the dropping of a card and thus restrict operations that might be fast.

Tilt Study Analysis

A fundamental assumption underlying any comparison is that the compared elements have been measured with the same precision. Scru-

TABLE XI — BALANCED VERSUS UNBALANCED LOAD — CARD DROP TIME IN MILLISECONDS

Treatment	Load	Look	Test Card 1	2	3	4	5	6	7	8	9	10
W	600	A	24.5	21.5	25	21.5	28.5	28	26.5	29	30.5	31.5
		B	24	22	26	27.5	28	27.5	27	29	29.5	31
	400	A	24	22	27.5	29.5	28.5	29	28	30	29	34
		B	24	22.5	28	28	28	28	28	30	30.5	33
	200	A	26	24.5	29.5	30	30	29	29	30	31	33
		B	25.5	24	30	30	30	29.5	28.5	29.5	30	33.5
X	600	A	25	22.5	27	28	27.5	28	27.5	26	30	32
		B	24.5	23	27	28.5	28	28	28	29	30	33
	400	A	25	24.5	29	29	29.5	28	28.5	30	30	33
		B	25	24.5	29.5	29	29.5	28	28	29	30.5	33
	200	A	28	26	29.5	29	29	28.5	27.5	27.5	24.5	26
		B	27	25.5	29.5	29	28.5	27.5	26.5	28	25	26
Y	600	A	24.5	22.5	27	28	28.5	28.5	28	29	30	34
		B	24	23	27	28	28	28	28.5	30	29	32.5
	400	A	25.5	23	29	29.5	29	28	28.5	28.5	28.5	33.5
		B	25	24	29.5	29.5	29	28	28.5	29	29.5	33
	200	A	27.5	26.5	28.5	28.5	27.5	27.5	26	27	25	26
		B	27.5	26	28.5	28.5	27.5	28	26	28	23.5	26
Z	600	A	21	20	23	23.5	25	25.5	26	25.5	26	26.5
		B	21.5	20	23	24.5	25.5	26.5	25.5	25.5	26.5	26.5
	400	A	18	18.5	23	26.5	27	25	24.5	24.5	24.5	27
		B	19	18.5	22	26	27	25.5	25.5	25	25	26
	200	A	19.5	18.5	20	21.5	22	22	22	22	22	22
		B	18	18.5	21	22	22.5	22	21.5	21.5	22.5	22

tiny of the data cast grave doubt on this assumption and each card position was analyzed separately as shown below.

CARD POSITION A

ANALYSIS OF VARIANCE

Source	Degrees of Freedom	Sum of Squares	Mean Square
Bins	4	8947	2238
Tilt	3	25653	8551
(Expt. Error) Bins x Tilt*	11	2882	262
Total	18	37482	

* One observation was lost, and Fisher's Missing Plot Technique was used (5).

CARD POSITION B

ANALYSIS OF VARIANCE

Source	Degrees of Freedom	Sum of Squares	Mean Square
Bins	4	1515	379
Tilt	3	492	164
(Expt. Error) Bins x Tilt	12	168	14
Total	19	2175	

The two estimates of experimental error being 262 and 14 $\overline{\text{ms}}^2$ respectively against a previous estimate (Table XIII) on 24 degrees of freedom of 10.9 $\overline{\text{ms}}^2$, we must accept Position A as coming from a different universe with respect to position B and the previous seven varible experiments. Further Position B on the same evidence has been measured with equivalent precision when compared to the previous study with the card support bars out.

The individual dropping times of Position B cards over the experiment range from 19 to 39.5 ms., well within the predicted dropping times. The mean dropping times for each tilt of card Position B are as follows:

Tilt	0	1	2	3
	27.2 ms	27.5 ms	31.2 ms	33.8 ms

By contrast the mean dropping times for Position A are:

Tilt	0	1	2	3
	28.9 ms	39.2 ms	59.9 ms	73.6 ms

with a range of 21 ms to 109.5 ms on the fourth tilt.

TABLE XII — ANALYSIS OF VARIANCE — SUMMARY — WITH CARD SUPPORT BARS

Source	Degrees of Freedom	Sum of Squares	Mean Squares	Mean Squares is an Estimate of*
1. Looks	3	5.65	1.88 N.S.(+)	
2. Code	1	4.46	4.46†	$\sigma_e^2 + 20(\sigma_{cb}^2 + \sigma_{cp}^2 + \sigma_{cl}^2 + \sigma_{cr}^2) + 100\sigma_c^2$
3. Idlers	3	520.48	173.49‡	$\sigma_e^2 + 10(\sigma_{ib}^2 + \sigma_{ir}^2) + 25\sigma_{ci}^2 + 50\sigma_i^2$
4. Bins	4	53.43	13.35‡	$\sigma_e^2 + 10\sigma_{ib}^2 + 20\sigma_{cb}^2 + 40\sigma_b^2$
5. Runs	4	359.64	89.91‡	$\sigma_e^2 + 20\sigma_{cr}^2 + 10\sigma_{ir}^2 + 40\sigma_r^2$
6. Loads	4	26.07	6.52‡	$\sigma_e^2 + 20\sigma_{cl}^2 + 40\sigma_l^2$
7. Positions	4	35.96	8.99‡	$\sigma_e^2 + 20\sigma_{cp}^2 + 40\sigma_p^2$
8. Codes x Idlers	3	6.58	2.19 N.S.	$\sigma_e^2 + 25\sigma_{ci}^2$
9. Codes x Bins	4	20.34	5.08‡	$\sigma_e^2 + 20\sigma_{cb}^2$
10. Codes x Runs	4	16.78	4.20‡	$\sigma_e^2 + 20\sigma_{cr}^2$
11. Codes x Loads	4	11.99	3.00†	$\sigma_e^2 + 20\sigma_{cl}^2$
12. Codes x Positions	4	25.47	6.37‡	$\sigma_e^2 + 20\sigma_{cp}^2$
13. Idlers x Bins	12	162.52	13.54‡	$\sigma_e^2 + 10\sigma_{ib}^2$
14. Idlers x Runs	12	203.99	17.00‡	$\sigma_e^2 + 10\sigma_{ir}^2$
15. Experimental Error	136	128.76	0.95	σ_e^2
16. Sampling Error	597	224.88	0.39	σ_s^2
	799	1807.00		

* See Appendix and Reference.
† Significant at 5 per cent level.
‡ Significant at 1 per cent level.
(+) N.S. = Not significant at 5 per cent level.

Clearly, Card Position A gives rise to an undesirable assignable cause which must be dealt with, and Card Position B while showing tilt as an assignable cause has a dropping time at the extreme tilt well within the allowable tolerances (see Conclusions).

Balanced Loading

The two main variables were total machine load in numbers of cards (Number) and distribution of cards over bins (Loading). The results of this experiment were so clear that little analysis was necessary. Although the mean dropping time of cards distributed uniformly over the twelve bins was statistically significantly different from the means of cards from the three nonuniform distributions the magnitude of the difference 4.8 ms was not sufficiently large to cause concern. Furthermore a decrease in mean dropping time was observed as the total load in-

TABLE XIII — ANALYSIS OF VARIANCE — SUMMARY — WITHOUT CARD SUPPORT BARS

Source	Degrees of Freedom	Sum of Squares	Mean Square	Mean Square is an Estimate of*
1. Looks	3	1.72	0.57 N.S.	
2. Code	1	3.43	3.43 N.S.	$\sigma_e^2 + 5\sigma_{cb}^2 + 5\sigma_{cr}^2 + 5\sigma_{cl}^2 + 5\sigma_{cp}^2 + 25\sigma_c^2$
3. Idlers				
4. Bins	4	714.69	178.67‡	$\sigma_e^2 + 5\sigma_{cb}^2 + 10\sigma_b^2$
5. Runs	4	275.61	68.90‡	$\sigma_e^2 + 5\sigma_{cr}^2 + 10\sigma_r^2$
6. Loads	4	1695.52	423.88‡	$\sigma_e^2 + 5\sigma_{cl}^2 + 10\sigma_l^2$
7. Position	4	239.42	59.86‡	$\sigma_e^2 + 5\sigma_{cp}^2 + 10\sigma_p^2$
8. Codes x Idlers				
9. Codes x Bins	4	205.39	51.35‡	$\sigma_e^2 + 5\sigma_{cb}^2$
10. Codes x Runs				
11. Codes x Loads				
12. Codes x Positions	4	144.09	36.02‡	$\sigma_e^2 + 5\sigma_{cp}^2$
13. Idlers x Bins				
14. Idlers x Runs				
15. Experimental Error	24	261.28	10.89	σ_e^2
16. Sampling Error	147	48.86	0.33	σ_s^2
	199	3689.01		

* See Appendix and Reference 6.
† Significant at 5% level.
‡ Significant at 1% level.
(+) N.S. = Not significant at 5% level.

creased regardless of the distribution of cards, confirming a conclusion from the first experiment.

V. CONCLUSIONS OF OVER-ALL STUDY

When considered in the normal cycle of translator operation with the card support bars functioning, the range of card drop times was observed to be from 32 to 40 ms with a mean of 35.1 ms. Based on the results obtained in this study it is predicted the dropping time of a card chosen at random in a new translator will be between 27 and 43 ms. This includes all the known variables except the life of the cards. The test on this variable is continuing and all conclusions reached in this report may be modified somewhat by the results of this life study which will be reported at a later date. The relative effect of the several variables is tabulated in Table XIV. The only two variables found to have any sig-

TABLE XIV — MEAN CARD DROPPING TIMES WITH ALLOWANCES SUMMARIZED WITH REGARD TO VARIABLES

Variable	No. Readings in each Mean	Mean Dropping Time Milliseconds					Range of Means	Allowance
Looks		A	B	C	D			
C.S. In.	200	35.2	35.2	35.1	34.9		Negligible	
C.S. Out	50	26.8	26.6	26.6	26.8		Negligible	
Codes		3 digit	6 digit					
C.S. In.	400	35.0	35.2				Negligible	
C.S. Out	100	26.6	26.8				Negligible	
Idlers		0 cards	50	85	100			
C.S. In.	200	36.5	35.0	34.4	34.6		2.1	±0.4
Graeco-Latin Square								
Runs		1	2	3	4	5		
C.S. In.	160	34.6	34.5	35.3	36.3	34.8	1.8	±0.5
C.S. Out	40	26.0	26.9	28.6	26.9	25.1	3.5	±3.0
Bins		I	II	III	IV	V		
C.S. In.	160	35.3	35.1	34.8	34.8	35.5	0.7	±0.5
C.S. Out	40	23.4	26.3	27.4	29.0	27.5	5.6	±3.0
Positions		ab	ad	ce	dg	fg		
C.S. In.	160	35.5	35.3	34.9	35.0	34.9	0.6	±0.5
C.S. Out	40	28.9	26.1	26.2	26.0	26.5	2.9	±3.0
Loads		15 cards	50	85	100	105		
C.S. In.	160	34.9	35.0	35.0	35.4	35.3	0.5	±0.5
C.S. Out	40	21.6	25.3	29.1	29.3	28.3	7.7	±3.0
Graeco-Latin Sq. & Codes		Min		Max.				
C.S. In.	16	33.6		37.6			4.0	±2.5
C.S. Out	4	16.8		37.5			20.7	±8.4
Idlers & Bins								
C.S. In.	40	34.0		38.2			4.2	±1.6
Idlers & Runs								
C.S. In.	40	33.4		36.8			3.4	±1.6

nificance when operating in the normal manner were the load in the machine and the number of coded and uncoded cards in the working bin. In a lightly loaded machine the flux from the pull-up magnet is concentrated in the few cards and therefore in each card there is a greater amount of flux to decay before the card is free to drop. When a bin is loaded with all coded cards the dropping time may be increased because the cards are slightly deformed by the coding process which in turn increases the effective thickness of the cards thus reducing their freedom in the bin.

Although these two variables were significantly large they still are included in the range of dropping times mentioned above and are not of sufficient magnitude to warrant special consideration in loading the

Table XV — Card Translator — Summary of Data on Card Rebound — Card Support Bars Out

No. of operations	200	100%
No. of operations with no rebound	83	41.5%
No. of operations with one rebound	116	58%
No. of operations with two rebounds	1	0.5%
Maximum duration of one rebound	17 ms.	
Minimum duration of one rebound	6 ms.	
Duration of two rebounds	28 ms.	
Maximum magnitude of a rebound	⅓ reopening	

Bins	No. of Operations	No. of Operations with Rebound	Per cent of Operations with Rebound
1	40	28	70
2	40	24	60
3	40	25	62.5
4	40	15	37.5
5	40	25	62.5

Load	No. of Operations	No. of Operations with Rebound	Per cent of Operations with Rebound
15	40	15	37.5
50	40	28	70
85	40	23	57.5
100	40	23	57.5
105	40	28	70

translators. The effect of the load in the machine was further investigated in the balanced vs. unbalanced loading test and although it was found that a balanced load produced a faster drop time, the amount of improvement obtained by balancing the load is not enough to warrant special loading instructions.

The card dropping time as observed with the card support bars out of the circuit ranged from 16 to 40 ms with a mean of 26.7 ms and the predicted total range is from 1 to 52 ms. Although the minimum dropping time is considerably less with the card support bars out, the translator cannot be operated satisfactorily in this manner. The cards rebound on a majority of the operations; in fact, 58 per cent of the operations with the card support bars out had measurable rebound. When operated in the normal manner no card rebound was observed at any time. A summary of the amount of card rebound observed is given in Table XV.

As a result of this study, it was concluded that the requirement for minimum 65 cards per bin should be removed and that the field be permitted to place as few cards as they find convenient in any bin. A second conclusion was that the maximum number of cards per bin be set at 100. This will allow a total of 1,200 cards per machine, an increase of

20 per cent in the capacity of a translator over the design objectives. The requirement that an uncoded card be placed next to each separator should remain until the life test which is currently in progress is concluded. These uncoded cards are to be included in the 100 cards per bin figure. A preliminary analysis of the data indicates that it is probably necessary that this requirement be retained for field use.

With regard to the leveling of the machine, it was found that if the translator table is leveled consistent with the normal practice of the installation department no further leveling is necessary when the translator is installed on the table provided an uncoded card is next to each separator. When only cards that were at least one removed from the separator were considered it was found that the Translator could be tilted 1 inch in 3 feet without seriously affecting the card drop time.

Considering the ranges of the several variables considered and the results of the analysis of the data, it appears that there is no major unknown variable having an effect on the card dropping time. It is also believed that the results of the work on this machine can be considered representative of the results that will be obtained on another new production model translator.

Appendices

I. ANALYSIS OF VARIANCE

The general theory of the analysis of variance has been formulated and discussed at length by several authors.[2, 4, 5] Basically it reduces to the concept that in any set of data obtained from a statistically designed experiment the total sum of squares of deviations from the mean can be partitioned into orthogonal components, and that under certain restrictions the distribution of each component falls into known patterns. Hence data taken from designed experiments can be examined for conformance to the known pattern, and a lack of conformity indicates an assignable cause of variation. Further, the distribution of the ratio of mean square deviations under specified conditions has been tabulated as the table of the F ratio. It has also been shown that when a treatment variable is not a parameter or assignable cause of variation in the experiment, the partitioned component for that variable must contain only residual variation. Thus, the analysis of variance tests the hypothesis that the treatment means for a given variable are all equal (i.e., the variable is not a parameter) by testing the ratio of the mean squares of mean deviations for the variable to the residual mean square, i.e., the F ratio. When this F ratio is larger than the critical value at the α^{th} level, the variable is said to be *significant at the* α^{th} *level*. That is, let us assume a null hypothesis that the variable in question is not a parameter or assignable cause, and select a critical value, F^*, such that the probability of observing an F ratio greater than F^* (when the null hypothesis is true) is small (say 0.01). Then if the F ratio is computed from our experimental observations and the null hypothesis rejected when this ratio is larger than F^*, on the average incorrect decisions will be made not more than 1 per cent of the time. This method of evaluation is not trivial and in complex situations reference should be made to the literature or to experts.

II. COMPONENTS OF VARIANCE

A basic difference between the estimation of the Components of Variance[6] and the Analysis of Variance above is the concept of the underlying model or law. The Analysis of Variance tests the hypothesis that the treatment variable is not a parameter. In the estimation of Components of Variance we assume that the observed effect of the several values of a given variable is a random sample from a normal population of effects from these values. If u_{iv} is the true effect of the i^{th} value of the v^{th} variable, then the component of variance due to the v^{th} variable, σ_v^2, is found from

$$\sigma_u^2 = \frac{\displaystyle\sum_{i=1}^{k} \mu_{iv}^2 - \frac{\left(\sum_{i=1} \mu_{iv}\right)^2}{k}}{k - 1}.$$

If $u_{1v} = u_{2v} = \cdots = u_{kv}$, then $\sigma_v^2 = 0$, and the mean square for variable v contains only residual variation. If the variable v is a parameter, $\sigma_v^2 > 0$; and the mean square for variable v contains $\sigma_e^2 + M\sigma_v^2$ (M measurements being made at each of the k levels of the variable). It is desirable to estimate the component of variability of each variable, in order to be able to estimate the variability of a measurement which is affected by these variables. That is, if there are ρ variables whose components of variance are σ_i^2, $i = 1, \cdots, \rho$ respectively and if the measurement, x, is influenced by all of these variables, then

$$\sigma_x^2 = \sum_{i=1}^{\rho} \sigma_i^2 + \sigma_e^2, \qquad \text{and} \qquad \sigma_x = \sqrt{\sum_{i=1}^{\rho} \sigma_i^2 + \sigma_{\text{error}}^2}.$$

Referring to Table XII and using the column of mean squares, we make the following inferences:

Since the ratio of mean square for variables to mean square for experimental error is "significantly" large for all the main variables, excluding Looks, these main variables are considered to be assignable causes of variation. It is also evident that the three digit cards are causing an effect quite different from that of the six digit cards when the four variables, Bins, Runs, Loads, and Position are allowed to vary. Since the ratio of the mean square of variables of lines 9, 10, 11, and 12 of Table XII to mean square for experimental error is significantly large, in the same way lines 13 and 14 show that the effect of Idler loads is not independent of the Bins and Runs effects. Statistically then, we have isolated many significant effects, but note that our experimental error, σ^2, for samples of four readings is 0.95 $\overline{\text{ms}}^2$ and σ is then 0.975 ms.

If we compare two means $\bar{x}_1$ and $\bar{x}_2$ each based on N samples of four observations each we will detect as significant, differences as small as

$$3\sigma_{\bar{x}_1-\bar{x}_2} = \frac{3\sqrt{2}\sigma}{\sqrt{N}} \frac{4.095}{\sqrt{N}}.$$

When N is large we will, therefore detect as significant, differences which may be of no interest engineering-wise. Thus not only is the significance of the effects of interest but also the magnitude of the effect.

When the component of variance attributable to each of the significant effects is estimated only four are so large as to be of interest to the engineer. The four variables are Idlers, Runs, interaction of Idlers on Runs and Bins. The estimates of the components of variance, $\hat{\sigma}^2_{\text{effect}}$, are obtained by equating the linear combinations of the components of variance shown in the right hand column of Table XII to the mean squares which estimate them and solving.

The component estimates are:

	σ^2	
Idlers	3.	$\overline{ms}^2$
Runs	1.55	$\overline{ms}^2$
Idlers x Runs	1.60	$\overline{ms}^2$
Idlers x Runs	1.26	$\overline{ms}^2$

III. SOURCES AND MEASURES OF ERROR

In any experiment a decision must be made as to the number of experimental units to be measured and the number of repeated measurements to be made on each unit.[7] It is important to note that measurements made on the same unit and a measurement made on each of several units give rise to two distinct sources of variation, and that both of these should be estimated. Consider making n measurements on each of k units, where the k units are a random sample of units belonging to a normal universe with mean u and variance σ_b^2. Further a set of measurements on the i^{th} unit is a random sample of measurements from a normal universe of measurements with mean u_i and variance σ_w^2. Clearly, if the set contains only one measurement ($n = 1$), we cannot estimate σ_w^2, and if there is only one unit ($k = 1$) we cannot estimate σ_b^2. When both $n, k > 1$ we can estimate simultaneously both σ_b^2 and σ_w^2. Let Xij be the j^{th} measurement on the i^{th} unit,

$$j = 1, \cdots, n; i = 1, \ldots, k.$$

and $\bar{X}_i$ be the mean of the i^{th} unit,

$\bar{\bar{X}}$ be the mean of all the units.

We can estimate σ_w^2 directly by computing

$$\sigma_w^2 = \frac{\sum_{i=1}^{k} \sum_{j=1}^{n} (X_{ij} - \bar{X}_i)^2}{k(n - 1)}, \qquad \text{but}$$

σ_b^2 can only be estimated indirectly by first estimating $\sigma_w^2 + n\sigma_b^2$ from

$$\frac{n \sum (\bar{X}_i - \bar{\bar{X}})^2}{k - 1}$$

Then the estimate of σ_b^2 is

$$\frac{1}{n}\left(\frac{n \sum_{i=1}^{k} \frac{(\bar{X}_i - \bar{\bar{X}})^2}{k - 1}}{k - 1} - \frac{\sum_{i=1}^{k} \sum_{j=1}^{n} (X_{ij} - \bar{X}_i)^2}{(n - 1)k}\right).$$

Since $\sigma_{\bar{x}}^2 = \frac{1}{k}\left(\sigma_b^2 + \frac{\sigma_w^2}{n}\right)$, it is clear that if σ_b^2 is large relative to σ_w^2, then $\bar{X}$, for fixed $M = nk$, will have greater precision if k is large and n is small. The estimate of σ_w^2 is called the sampling variance or sampling attributable to repeated measurements. The estimate of σ_b^2 is called the component of variance due to experimental variation free of sampling error. For a given experiment the estimate of $\sigma_w^2 + n\sigma_b^2$ is called the experimental error term and measures the precision of measurement of a unit. In the experiment $\sigma_w^2 = \sigma'^2_s$, and $\sigma_b^2 = \sigma'^2_e$.

BIBLIOGRAPHY

1. L. N. Hampton and J. B. Newsom, The Card Translator for Nationwide Dialing, B.S.T.J., **32,** pp. 1037–1098, Sept., 1953.
2. W. G. Cochran and G. M. Cox, "Experimental Designs", J. Wiley and Sons, New Yor,, 1950.
3. J. W. Tukey, "Comparing Individual Means in the Analysis of Variance", Biometrics, Vol. 5, No. 2, 1949.
4. W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis", McGraw Hill, New York, 1951.
5. R. A. Fisher, "Design of Experiments", Oliver and Boyd, London, 1950.
6. S. Lee Crump, "The Estimation of Variance Components in the Analysis of Variance", Biometrics, Vol. 2, No. 1, 1946.
7. C. Eisenhart, "Assumptions Underlying the Analysis of Variance", Biometrics, Vol. 3, No. 1, 1947.