A Governor for Telephone Dials

Principles of Design

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This is a report on the development of a new type of governor for regulating the speed of rotary dials. The paper includes derivation of the equations of motion which determine the theoretical speed of the governor during dial run-down and an analysis of the operating characteristics of the governor as influenced by varying friction and input torque. Experimental verification of the relations is presented. A theoretical analysis which explains "governor chatter" or positional instability for friction-centrifugal governors is also given.

INTRODUCTION

Machine switching telephone systems depend on the telephone dial for originating information used in completing a call. During run-down, the dial originates current pulses which operate step-by-step switching equipment or are registered for use in common-control panel or crossbar systems. For reliable functioning of dial pulse controlled switching equipment, the pulses must be closely controlled in frequency and form. Since the pulses are produced during run-down of the dial after release by the customer, the run-down speed most be constant. Friction-centrifugal governors are commonly used to provide this required control of speed.

If the pulses reaching the central office were exactly like those generated by the dial, the designers of dials and central office switching apparatus and circuits would find themselves far less restricted. Unfortunately the dial pulses are distorted by the electrical characteristics of the customer's loop. To compensate for this distortion and insure accurate registration of the pulses at the central office, the dial and central office equipment must be designed to operate to close limits of performance. The designs must also be such that there is negligible change of adjustment resulting from use, time or weather conditions, which might affect the ability to accurately send or receive dial pulses.

To achieve the accuracy of timing required of dial governors, it was recognized that a general theoretical analysis which defined speed of governors would be beneficial. The late C. R. Moore investigated this problem in the thirties, and derived from theoretical considerations, general equations of motion relating to governors. The relationships derived by the Moore analysis are extremely useful in that they can be used to indicate the influence of various design factors on the performance of a governor. This theory was applied in developing the new governor used in the 7-type dial of the 500-type telephone set* and will be presented in this paper. To better demonstrate the operating characteristics of the new governor, it will be compared with a previous governor which was used in an older type dial. Photographs of the new governor as it is assembled in a dial are shown in Figs. 1A and 1B.

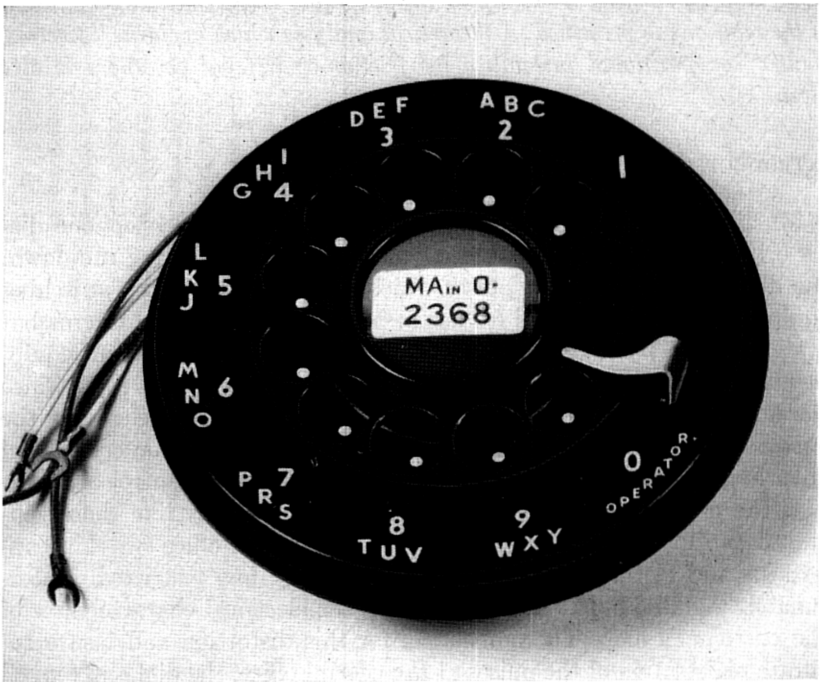

FIG. 1A — Front view of 7-type dial.

* Inglis, A. H., and Tuffnell, W. L., An Improved Telephone Set, B.S.T.J., **30,** pp. 239–270, April, 1951.

DIAL AND GOVERNOR OPERATION

In dialing, the fingerwheel is rotated through an angle proportional to the number being dialed and then released. Energy stored in the motor spring, Fig. 2, causes the mechanism to return to the start position. For each 30° rotation of the fingerwheel during run-down, the intermediate gear rotates one-half revolution and the cam pinion and pulsing cam rotate one full revolution. Once the pulsing pawl is in position, each revolution of the pulsing cam in the run-down direction results in a pulse being placed on the telephone loop. The intermediate gear also meshes with a governor pinion which is coupled to the governor shaft and governor through a spring clutch.* This clutch decouples the governor from the fingerwheel on windup to reduce the windup torque. On run-down the governor rotates two full revolutions for each 30° rotation of the fingerwheel.

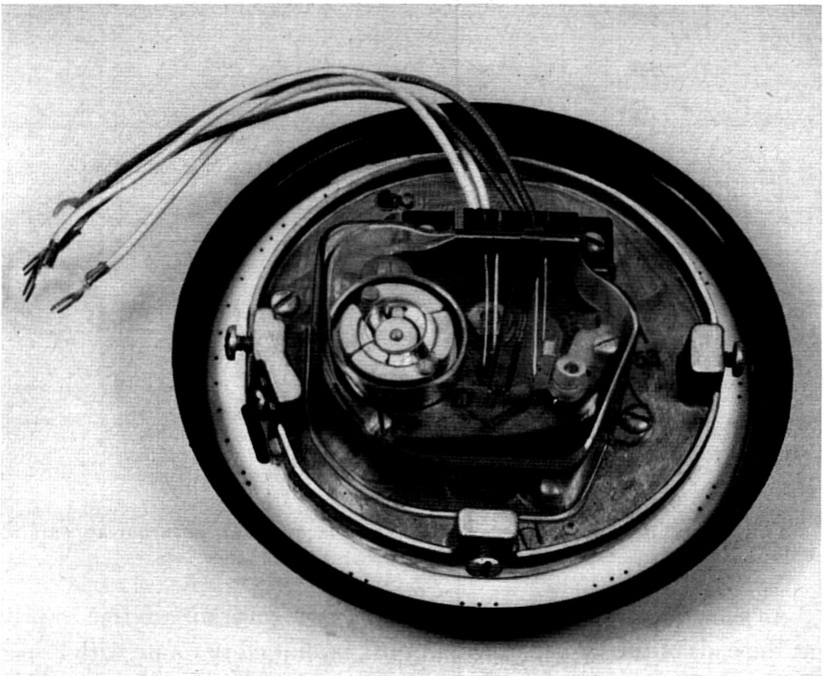

Fig. 1B — Rear view of 7-type dial. Left, speed governor, center, off-normal contact and right, pulsing mechanism.

* Wiebusch, C. F., The Spring Clutch, J. Appl. Mech., Sept., 1939.

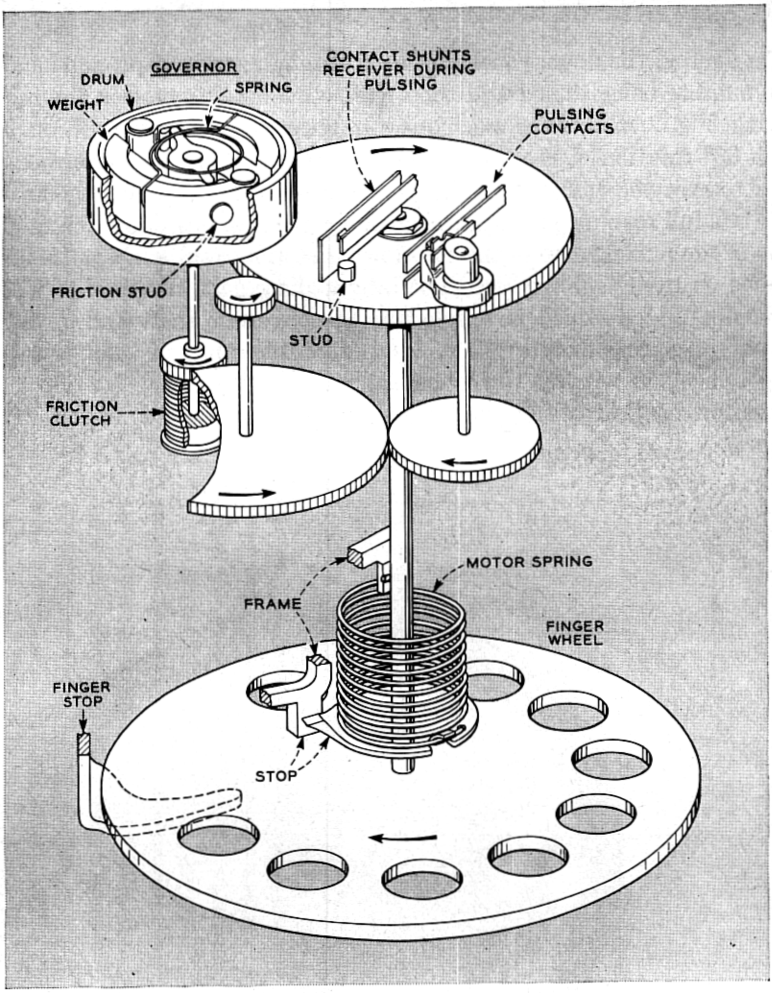

Fig. 2 — Simplified diagram of 7-type dial mechanism; governor appears top left.

As shown in Fig. 3, the weights of the new governor are free to pivot at the ends of the fly-bar which, in turn, is allowed to rotate with respect to the shaft. Rotation is imparted to the system by the drive-bar which presses against each weight at a specific point. As the mechanism begins to rotate during dial run-down, the two weights are caused by centrifugal force to move outward against the tension of a spring. Movement of the weights about their pivots continues until the friction studs touch the case. At this instant governing begins, and controls the dial speed until the end of run-down. The speed attained by the governor will be dependent on the friction between the studs and case, the magnitude of the driving torque, and the tension to which the spring of the governor is adjusted.

In the schematic of the new governor, Fig. 4, the driving force is designated as F. By applying this driving force between the weight pivot and the center of the governor, the mechanism behaves during operation as a true friction-centrifugal governor and also as a brake. This configuration results in a gain in the ability of the governor to resist the increase in speed which normally results from an increase in the applied rotational force. The drive-bar force, F, and the torque produced by the stud-to-case force about the pivot assists the centrifugal force in pressing the friction studs against the case.

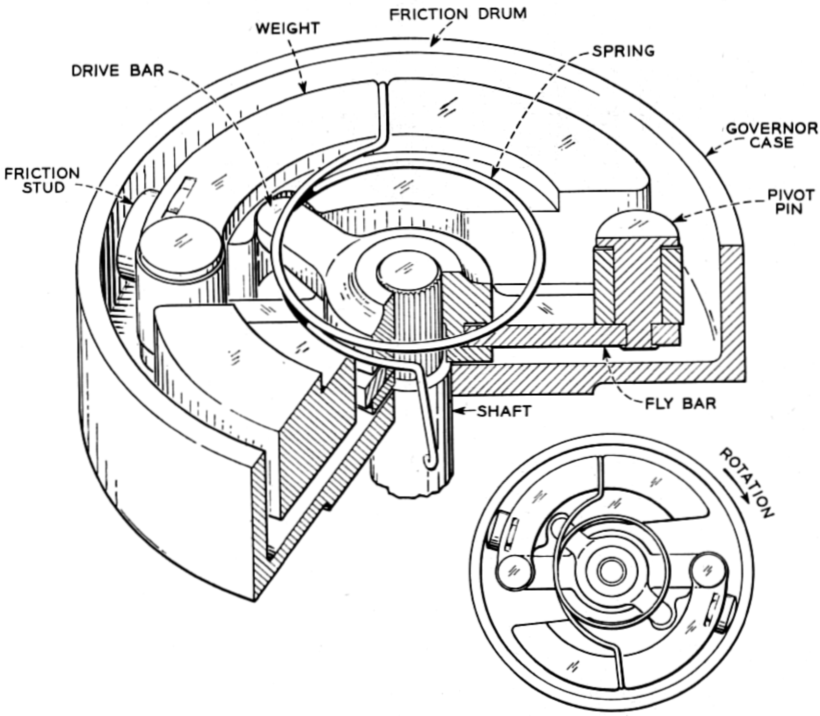

FIG. 3 — The 7-type dial drive-bar governor.

The comparative design shown in Fig. 5 is a conventional fly-bar type governor consisting of two weights each pivoted at the end of a fly-bar which is fixed to a shaft. As the shaft accelerates during run-down, the weights move outward under the influence of centrifugal force and are restrained in their motion by the tension of the governor spring. At a certain speed the friction studs contact the inner surface of the governor case. The governor gradually decelerates until the input torque to the governor is balanced by the stud-to-case frictional loss and the governor shaft and dial theoretically rotate at constant speed. It will be noted in this configuration that only the torque produced by the stud-to-case

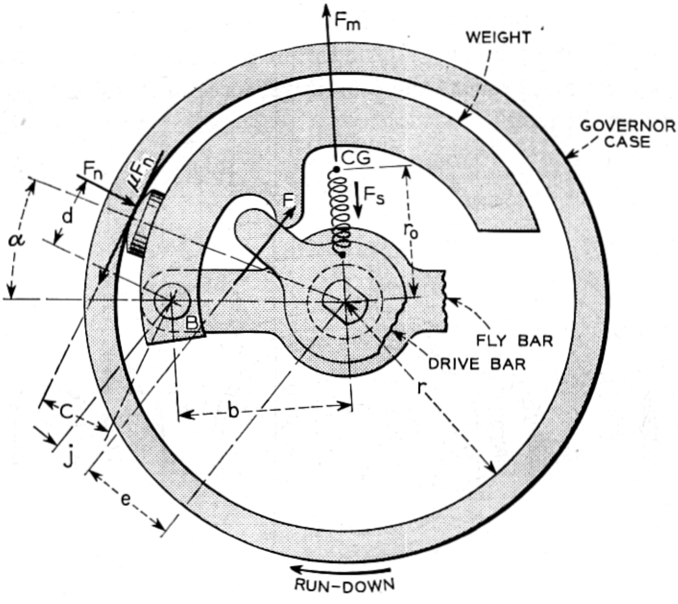

Fig. 4 — Schematic of drive-bar governor.

F — Force applied by torque on governor weights
F_n — Normal force of case acting on studs
F_s — Force exerted by spring when studs touch case
F_m — Centrifugal force acting at center of gravity of weight
μ — Coefficient of friction
I_0 — Moment of inertia of the governor about center shaft
ω — Angular velocity of governor
ω_0 — Critical angular velocity at which studs just touch the case
m — Mass of each weight
r_0 — Radius to the center of gravity of each weight
r — Radius of governor case
α — Stud angle
Neg. Rotation — Run down of governor

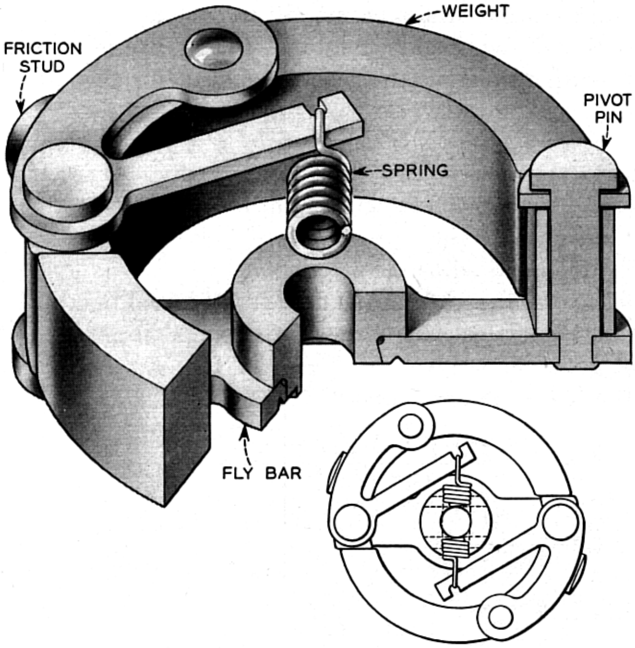

Fig. 5 — Fly-bar type governor.

friction about the pivots aids the centrifugal force in pressing the studs against the case.

EQUATIONS OF MOTION

In deriving the general equations of motion for a governor, two assumptions are made concerning the action. During the interval of time that the governor is approaching the critical velocity when contact of the friction surfaces first occurs, the motion is assumed to be that of a simple fly-wheel, constantly accelerating. The angular velocity of the governor during the time from rest to the critical velocity, ω_0, is then given by

$$\omega = \frac{Gt'}{I} \tag{1}$$

where G = applied torque

t' = time from start of motion

I = moment of inertia of the governor assembly about the shaft

After stud-to-case contact occurs, it is assumed that there is no further pivoting of the weights or fly-bar, and the assembly rotates as a rigid body. During this time the general equation for angular velocity is

$$\omega = q \tanh \left(\frac{ht}{q} + \ln \sqrt{A} \right) \tag{2}$$

where q = regulated or final angular velocity
h = design constant
A = design and adjustment constant
t = time measured from the moment of initial braking

The derivation of this general speed equation as it applies to the new 7-type dial governor is given in Appendix I. For this drive-bar type governor the following terms apply:

$$h = \frac{G(d - \mu c) + M\mu\omega_0^2 - \mu rjG/e}{I_0(d - \mu c)} \tag{3}$$

$$g = \frac{M\mu}{I_0(d - \mu c)} \tag{4}$$

$$A = \frac{q + \omega_0}{q - \omega_0} \tag{5}$$

$$q = \sqrt{\frac{h}{g}} = \sqrt{\frac{G(d - \mu c) + M\mu\omega_0^2 - \mu rjG/e}{M\mu}} \tag{6}$$

The derivation of the theoretical equation for speeds in excess of the critical speed for the comparative fly-bar design results in the following relationships:

$$\omega = q \tanh \left(\frac{ht}{q} + \ln \sqrt{A} \right)$$

where

$$h = \frac{G(d - \mu c) + M\mu\omega_0^2}{I_0(d - \mu c)} \tag{7}$$

$$g = \frac{M\mu}{I_0(d - \mu c)} \tag{8}$$

$$q = \sqrt{\frac{h}{g}} = \sqrt{\frac{G(d - \mu c) + M\mu\omega_0^2}{M\mu}} \tag{9}$$

The form of the general speed equation is the same for all types of friction-centrifugal governors but each particular governor will have different terms in the values for g, h and q. The theoretical equation of motion can be used to calculate the speed of the dial at any time, t, after the critical velocity is reached or the time required to reach any given speed once the governor studs touch the case. The equation shows that for large values of t, ω approaches q, so that steady state speed is given by the value of q for each type governor, and is in terms of the operating values and design constants of the mechanism.

For the drive-bar governor the steady state speed equation is

$$\omega = q = \sqrt{\frac{G(d - \mu c) + M\mu\omega_0^2 - \mu rjG/e}{M\mu}} \tag{10}$$

THEORETICAL SPEED-TIME CURVES

Drive-Bar Governor

The design constants and physical data given in Table I apply to the drive-bar governor, and were used to calculate the theoretical speed

TABLE I — REFER TO FIGS. 3 AND 4

d = 0.390 cm	I_0 = 7.40 gm cm² (experimental)
c = 0.361 cm	G = 7,500 dyne-cm (steady-state)*
r = 1.180 cm	13,500 dyne-cm (initial)
r_0 = 0.635 cm	μ = 0.25 (assumed)
b = 0.920 cm	m = 3.9 gms
e = 0.498 cm	$K = r_0/r$ = 0.538
j = 0.236 cm	$M = 2mr^2bk$ = 5.38

* Appendix II — Governor Input Torque.

versus time curve shown in Fig. 6. The dial governor is initially adjusted so that signaling is at the rate of 10.0 pulses per second, requiring a steady state governor shaft velocity of 125.6 radians per second. This steady state velocity was used to determine the critical velocity ω_0 by substituting the values in Table I in equation (10):

$$\omega = q = \sqrt{\frac{G(d - \mu c) + M\mu\omega_0^2 - \mu rjG/e}{M\mu}}$$

$$\omega_0 = 121.8 \text{ radians/second}$$

and from equations (3), (4), (5), and (6)

$$g = 0.606 \qquad h = 9{,}520 \qquad h/q = 75.9 \qquad A = 72.7.$$

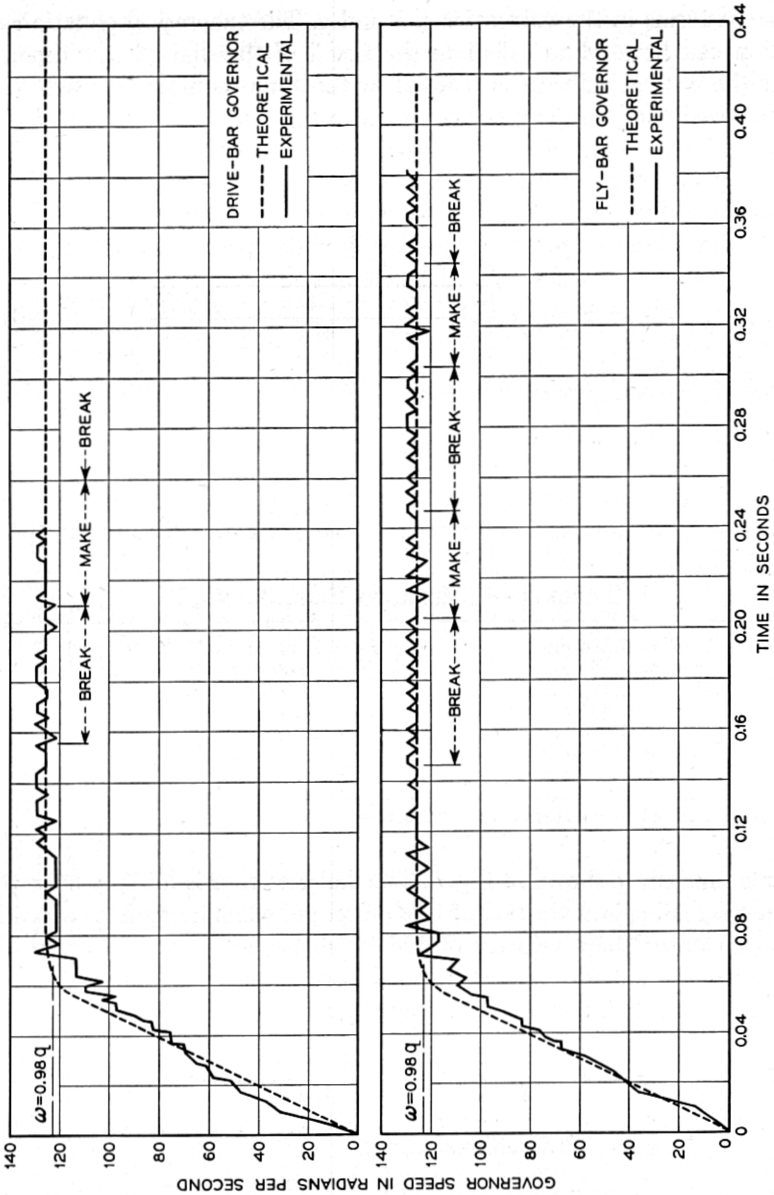

Fig. 6 — Speed curves for the drive-bar and fly-bar type governors.

TABLE II

t	$75.9t + 2.142$	$\omega = 125.6 \tanh (75.9t + 2.142)$
0.000	2.142	121.8
0.004	2.446	123.7
0.008	2.750	124.6
0.012	3.054	125.1
0.016	3.358	125.3
0.020	3.662	125.5

Substituting these values in the general speed equation,

$$\omega = q \tanh \left(\frac{ht}{q} + ln \sqrt{A} \right)$$

gives the velocity of the drive bar governor at any time t measured from the moment the governor reaches the critical velocity, i.e., when the friction studs first touch the inside of the governor case. (Table II.)

For governor speeds from start of rotation up to the critical velocity, it is assumed that the system rotates as a simple fly-wheel, therefore from equation (1)

$$t' = \frac{\omega_0 I_0}{G_I} = \frac{121.8(7.4)}{13{,}500} = 0.0668 \text{ seconds}$$

This time of 0.0668 seconds determines the slope of the straight line portion of the theoretical speed-time curve shown on Fig. 6.

Fly-Bar Governor

The data given in Table III applies to the fly-bar governor shown schematically in Fig. 7. Substituting the values given in this table in the steady state speed equation for the fly-bar governor,

$$\omega = q = \sqrt{\frac{G(d - \mu c) + M\mu\omega_0^2}{M\mu}}$$

$$\omega_0 = 118.5 \text{ radians/sec.}$$

and from equations (7), (8) and (9):

$$g = 0.609 \qquad h = 9{,}560 \qquad h/q = 76.4 \qquad A = 36.35$$

Substituting these values in the general speed equation gives the velocity of the fly-bar governor at any time (t) measured from the moment braking begins, Table IV. For this particular fly-bar design, the time required to reach the critical velocity is, from equation (1),

$$t' = \frac{\omega_0 I_0}{G_I} = \frac{118.5(7.36)}{13{,}500} = 0.0646 \text{ seconds}$$

EXPERIMENTAL SPEED-TIME CURVES — NORMAL TORQUE

Experimental velocity versus time curves were obtained for the fly-bar and drive-bar governors constructed to the specifications listed in Tables I and III. Data used in determining the true speed versus time picture for the experimental governors was taken from photographic traces exposed on a recording oscillograph. A thin disc having 36 radial slots spaced every 10° was fastened to the end of the governor shaft. Light detected through the slots of the moving disc by the element of a photo tube was used to deflect one of the strings of the oscillograph. The trace

Table III — Refer to Figs. 5 and 7

d = 0.390 cm	G = 7,500 (steady state)
c = 0.361 cm	13,500 (initial)
r = 1.18 cm	μ = 0.25 (assumed)
r_0 = 0.635 cm	m = 3.9 gm
b = 0.920 cm	$K = r_0/r$ = 0.538
I_0 = 7.36 gm cm^2	$M = 2mr^2bK$ = 5.38

of this string appeared on the photographic paper as a distorted sine wave. The distance between two successive wave peaks represented 10° of rotation of the governor. By noting on the trace the time between peaks, it was possible to determine the average velocity of the governor at 10° intervals after release of the finger-wheel, or start of rotation of the governor mechanism. The experimental speed curves for the drive-bar and fly-bar governors are plotted on Fig. 6 along with the theoretical speed curves.

It was assumed in the theoretical analysis that while accelerating up to the critical velocity, the governor assembly rotates as a simple fly-wheel. This requires that the velocity increase linearly. The theoretical and experimental velocity curves for both type governors during the initial accelerating period show the fly-wheel assumption to be justified. The slope of the velocity curve, or rate of acceleration is generally constant.

For that portion of the theoretical and experimental curves which show speeds from the critical velocity to 98 per cent of rated speed, agreement is not too clearly defined. The theoretical curve is naturally smooth in shape. An oscillating type characteristic appears in the experimental speed curves of both types of governors. This probably results from the shock and grabbing when the friction studs first touch the case and continues until the forces tending to move the weights outward increase to a value sufficient to hold them against the case for governing.

That part of the experimental curve, during which governing actually occurs at rated speed is relatively smooth through full run-down. Both the fly-bar and new drive-bar governors exhibit excellent speed regulation. The waves present on the trace do not necessarily indicate hunting or vibration since the variations in speed which appear are actually smaller in magnitude than the degree of accuracy present in measuring the experimental photographic trace.

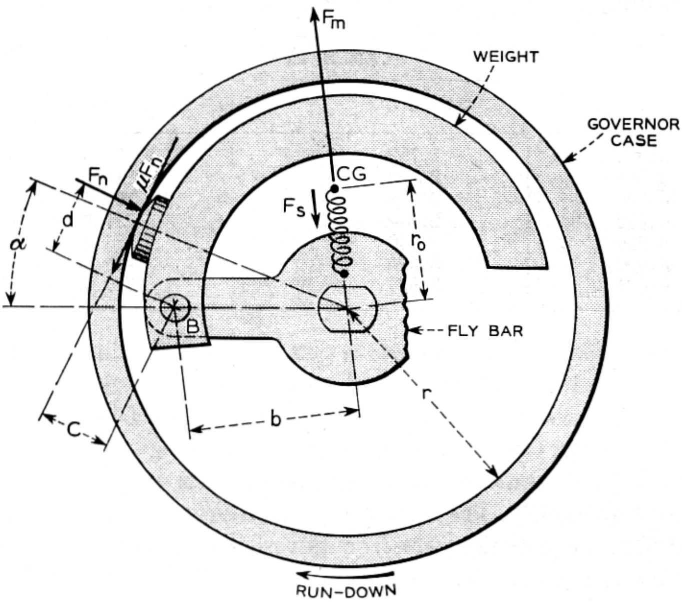

Fig. 7 — Schematic of fly-bar governor.

F_n — Normal force of case acting on studs
F_s — Force exerted by spring when studs touch case
F_m — Centrifugal force acting at center of gravity of each weight
μ — Coefficient of friction
I_0 — Moment of enertia of the governor about center shaft
ω — Angular velocity of governor
ω_0 — Critical angular velocity at which studs just touch the case
m — Mass of each weight
r_0 — Radius to the center of gravity of each weight
r — Radius of governor case
α — Stud angle
Neg. Rotation — Run down of governor

THEORETICAL OPERATING CHARACTERISTICS

The solution of the equation of motion for any governor is based on specific design constants and certain assumed and estimated values. Such dimensions as the governor case inside diameter r, and distance from the shaft center to the weight pivot b, are two examples of design constants. These constants establish the arrangement of the various component parts of the mechanisms and are, therefore, subject to practical manufacturing and space considerations as well as considerations from a speed regulation standpoint. The design constants are in effect static considerations. They are not subject to appreciable variation once established, and on any particular governor do not vary significantly over the life of the governor.

TABLE IV

t	$76.4t + 1.798$	$\omega = 125.6 \tanh (76.4t + 1.798)$
0.000	1.798	118.5
0.004	2.103	121.9
0.008	2.408	123.6
0.012	2.714	124.5
0.016	3.010	125.0
0.020	3.325	125.3
0.024	3.631	125.4

During actual operation there are two factors which directly affect the degree of speed control afforded by a governor; i.e., the coefficient of friction which exists between the governor case and friction studs, and the value of input torque to the governor shaft. Design control over these factors is present, but to a lesser degree than for the design constants mentioned previously. These factors are considered fixed in arriving at a given design but actually vary from governor to governor and over the life of a governor. It is therefore necessary to consider carefully the effect of variations in friction and torque if close regulation of speed is required.

The input torque to the governor shaft will vary because of the dimensional variations of the motor springs, the tolerance permitted for the driving torque at full windup of the dial, dial friction and the variation in pulsing spring forces. These variables appear at the governor as a range of input torques during run-down of the mechanism. For the motor spring used in the 7-type dial, input torque referred to the governor shaft decreases during run-down on the average from 20,000 dyne-cm to 13,000 dyne-cm. Torque required to overcome bearing and gear friction and the loads imposed by the pulsing mechanism result in an average torque of 7,500 dyne-cm at the governor. Over the life of a dial this input torque at the governor will vary as the dial efficiency varies. Initially the dial mechanism is lubricated and the bearings and gears turn freely. With time and continued operation, the accumulation of dirt and wear products affect the dial so that more torque is needed to drive the moving parts. This causes a decrease in the remainder torque going to the governor.

Another aspect of torque requiring consideration is that resulting from forcing of the finger-wheel during run-down. This action can produce torque values at the governor of the order of 110,000 dyne-cm or a torque of approximately fifteen times that which appears at the governor during normal operation.

The second factor, which can vary during dial life, is the value of the stud-to-case coefficient of friction, μ. Both the drive-bar and fly-bar governors have studs of Ebonite compounded with 40 per cent by weight of hard rubber dust and cases of ASTM B16 brass. Actual service tests show the satisfactory wearing ability of these materials.

Each governor is initially adjusted for speed by changing the tension of the governor spring. At the time this adjustment is made, a particular friction condition exists between the governor studs and case. With time or continued operation there is always the possibility of a change occurring in this friction value. Such factors as very high humidity, lubrication products traveling to the stud operating region, or the accumulation of foreign-material or wear particles may produce different values of friction and hence result in variation in governor and dial speed from the initially adjusted value.

The range of coefficient of friction values μ expected for rubber on brass is from 0.05 to 0.35. These are the extreme conditions produced by oil in the governor case for the 0.05 value and very low unit pressure on a scored brass surface for the 0.35 value. For this study a representative figure for the average stud-to-case friction value was taken to be 0.25.

The problem of variation in steady state governor speed with changes in the coefficient of friction and input torque can be analyzed by considering the derivatives of speed with respect to these values. This is done by operating on the equations for terminal speed.

Speed with Respect to Coefficient of Friction

For the drive-bar governor, the partial derivative of speed, with respect to coefficient of friction, is as follows:

$$\omega = q = \sqrt{\frac{G(d - \mu c) + M\mu\omega_0^2 - \mu rjG/e}{M\mu}}$$

Taking the derivative of ω with respect to μ gives

$$\frac{\partial\omega}{\partial\mu} = -\frac{Gd}{2\omega M\mu^2} \tag{11}$$

where $M = 2mrr_0b$. For the fly-bar governor, $\partial\omega/\partial\mu$, is as follows from

$$\omega = q = \sqrt{\frac{G(d - \mu c) + M\mu\omega_0^2}{M\mu}}$$
$$\frac{\partial\omega}{\partial\mu} = -\frac{Gd}{2\omega M\mu^2} \tag{12}$$

Since the equations are identical for both governors the same considerations exist in holding to a minimum the change in speed caused by a change in the coefficient of friction.

For optimum speed regulation, the partial derivative, $\partial\omega/\partial\mu$, should be a minimum. Small values of $\partial\omega/\partial\mu$ can be obtained by operating on the design constants, controlling the value of μ, or having high governor speeds. Specifically, the design constants m, r_0, r and b should all be large. Inspection of the drive-bar governor schematic, Fig. 4, shows that there are physical limitations to the arbitrary enlargement of these values. Space, manufacturability, and cost of materials must be considered in establishing these terms. In selecting materials for fixing the coefficient of friction value the wearing quality of the materials to be used must be of first consideration. A high governor speed is advantageous but must be weighed against the primary disadvantage of high inertia loads for the entire mechanism.

The terms G and d in the numerator of equations (11) and (12) indicate that low input torque and a small value for d would be desirable. For G, one must consider the anticipated change in dial efficiency and variation in torque produced by the pulsing mechanism plus the torque necessary for the governor to maintain adequate speed control.

As shown in the drive-bar governor schematic, Fig. 4, d is the distance from the weight pivot to the normal of the point of contact between the rubber stud and governor case. Its magnitude is controlled by the stud angle α, and the distance between the stud and case when the weights are in the closed position.

As indicated, d should be as small as possible for best regulation with friction change. This requirement imposes a difficult design problem because as d decreases, the stud and bearing hole in the weight approach each other. A minimum d is therefore fixed by interference of the parts themselves. Inspection of the governor mechanism also shows that even if the interference problem were not limiting, the weight turning angle imposes a further restriction on the d value. The life of a governor is considered to end when the friction material is worn to the point of allowing the weight to touch the inside of the case. Because of this initially large weight motion, little material is provided for wearing and hence the possible life of the mechanism is reduced. In the design of the drive-bar governor, the stud angle, α, was made as small as manufacturing techniques would permit. The stud angle is 22° from the weight pivot with a corresponding $d = 0.390$ cm.

Since both governors under study were designed to operate in the case of the same dial the terms shown in Tables I and III which apply in the friction equations (11 and 12) are identical. This identity of terms indicates that both type governors should exhibit identical frictional characteristics. Fig. 8 represents a plot of the $\partial\omega/\partial\mu$ for various governor input torque values. One set of curves applies. The range of torque values covered by the curves is from zero to the forcing condition at fifteen times normal motor spring torque. Curves for coefficient of friction values, 0.05, 0.10, 0.20, 0.35, are shown as encompassing the extremes in operating range.

Speed with Respect to Input Torque

For the drive-bar governor, the partial derivative of speed with respect to torque is determined as follows from the steady state speed equation.

$$\omega = q = \sqrt{\frac{G(d - \mu c) + M\mu\omega_0^2 - \mu rjG/e}{M\mu}}$$

Taking the derivative of ω with respect to G gives:

$$\frac{\partial\omega}{\partial G} = \frac{1}{2}\left[\frac{G(d - \mu c) + M\mu\omega_0^2 - \mu rjG/e}{M\mu}\right]^{-1/2} \frac{(d - \mu c - \mu rjG/e)}{M\mu}$$

and

$$\frac{\partial\omega}{\partial G} = \frac{1}{2}\frac{(d - \mu c - \mu rj/e)}{M\mu\omega} \qquad (13)$$

where

$$M = 2\,mrr_0b$$

For the fly-bar governor the derivative of ω with respect to G can be developed similarly to give

$$\frac{\partial \omega}{\partial G} = \frac{1}{2} \frac{(d - \mu c)}{M \mu \omega} \tag{14}$$

Here also the design constants, m, r_0, r and b, the coefficient of friction μ, and the governor speed ω, should all be high values. Minimum change in dial speed would then occur for a given change in the input torque.

Inspection of the equations for $\partial\omega/\partial G$, indicates that it is possible to have perfect torque regulation for at least one value of μ. For this limiting condition, $\partial\omega/\partial G$, would equal zero and equation (13), for the drive-bar governor, equates to

$$d - \mu c - \mu rj/e = 0 \tag{15}$$

and equation (14) for the fly-bar type governor

$$d - \mu c = 0 \tag{16}$$

If these equations were satisfied, there would be zero change in speed for a given change in input torque to the governor. As stated previously μ is predetermined and has in this design a maximum known value of 0.35. A margin of safety is considered by taking $\mu = 0.425$ for the limiting case and equation (15) becomes for the new drive-bar governor

$$d - 0.425\ (c + rj/e) = 0 \tag{17}$$

and equation (16) for the fly-bar governor

$$d - 0.425\ c = 0 \tag{18}$$

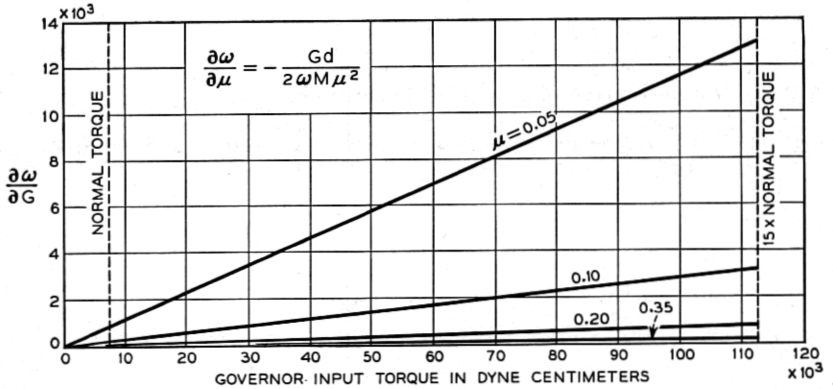

Fig. 8 — Derivative of governor speed with respect to coefficient of friction versus input torque to the governor.

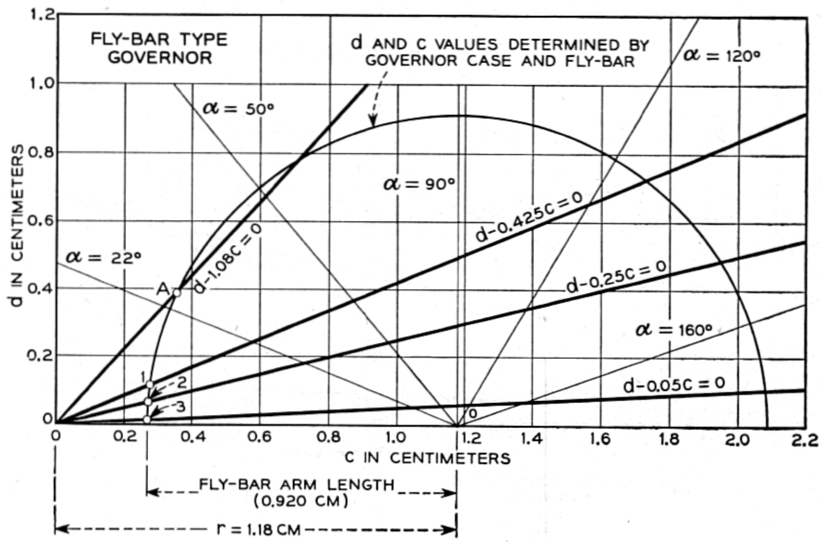

Fig. 9 — Design diagram for fly-bar type governor.

Comparison of these two equations shows a very important difference in the term multiplied by $\mu = 0.425$. For the drive-bar governor, there are four variables which can be operated on to satisfy the equation; i.e., c, r, j and e. For the fly-bar governor only c is available. The importance of these additional terms can be realized when one considers that the value for c results from our choice of d in making $\partial\omega/\partial\mu$ a minimum. For both type governors c is equal to 0.361 cm. Substitution of the d and c values in the limiting equation, (18), for the fly-bar governor does not lead to a solution when $\mu = 0.425$. Solving for this limiting μ in the fly-bar governor gives

$$(0.390) - (0.361)\,\mu = 0$$

$$\mu = 1.08$$

This value of μ is far beyond that encountered in actual governor operation and, in effect, represents useless margin. This is graphically shown in Fig. 9 where all d and c values which conform to the geometry of the fly-bar governor mechanism are plotted as a design diagram. The three straight lines radiating from the origin represent plots of the limiting equation for $\mu = 0.425$, 0.25 and 0.05. The intersection of these lines with the d and c semicircle, noted at points 1, 2 and 3, give the particular angle at which the stud should be located for optimum regula-

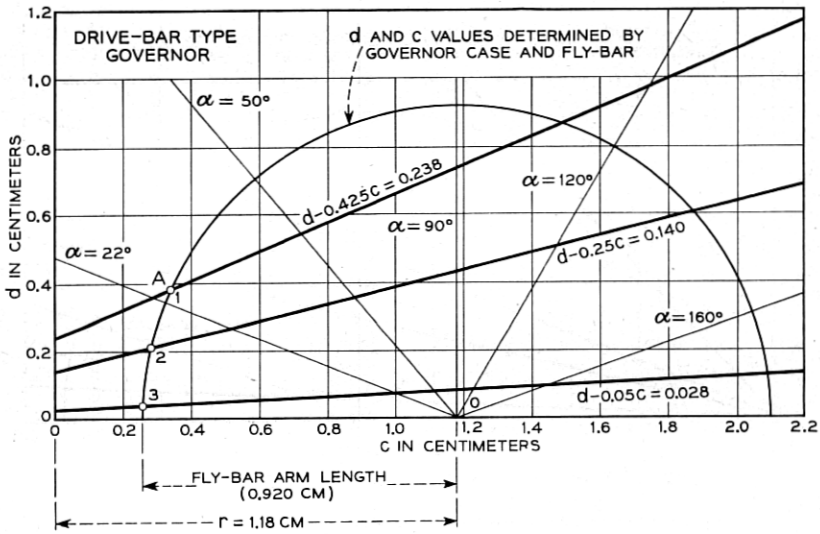

Fig. 10 — Design diagram for drive-bar type governor.

tion for these particular friction values. The point A on the d and c circle denotes the d and c values which result from the stud being at $\alpha = 22°$. The off-set appears because stud-to-case contact is not made on the center line of the stud. The points 1, 2 and 3 are all below the point, A, established by the minimum permissible angle, $\alpha = 22°$. This indicates that the fly-bar governor has its $\partial\omega/\partial G$ equal to zero at some coefficient of friction value higher than $\mu = 0.425$. As previously determined in this value is $\mu = 1.08$.

Fig. 10 represents the design diagram for the new drive-bar governor. Here again, the large semi-circle is a plot of all d and c values which conform to the geometry of this governor mechanism. Point A is determined by the stud angle $\alpha = 22°$. The straight lines represent the limiting equations for $\mu = 0.425$, 0.25 and 0.05 and are shown intersecting the d and c circle at 1, 2 and 3 respectively. For this particular governor, the line representing $\mu = 0.425$,

$$d - 0.425\ (c + rj/e) = 0$$

intersects the d and c circle at the point A. This is possible by making the term $(\mu rj/e)$ equal to 0.238. The intersection of this curve at A indicates that there will be zero change in speed for a given change in input torque to the governor when the stud-to-case coefficient of friction value is 0.425.

The additional terms, r, j and e in the limiting equation make it possible to design the governor for optimum regulation for any particular value of μ desired. Since the term r, the case inside radius, is controlled by space requirements, the terms j and e assume added importance. They are determined by the point at which the drive-bar arms act against the weights and can be made any value required to meet the design objective.

Figure 11 is a plot of $\partial\omega/\partial G$ at various values of μ for the drive-bar and fly-bar governors specified in Tables I and III. It indicates that for any coefficient of friction the new drive-bar governor should exhibit less change for a change in input torque than is possible with the fly-bar governor.

EXPERIMENTAL DATA

To substantiate the theoretical conclusions drawn from the analysis of the steady state speed equations, experimental data were compiled from a number of models of each type of governor. Drive-bar and fly-bar governors, made to the specifications listed in Tables I and III were investigated to determine their response to changes in input torque and changes in the stud-to-case coefficient of friction value, μ. The tests were conducted on 7-type dials manufactured by the Western Electric Company as standard product.

Dial Speed Versus Coefficient of Friction

The theoretical analysis of the equation

$$\frac{\partial\omega}{\partial\mu} = -\frac{Gd}{2\omega M\mu^2}$$

indicates that the two types of governors should exhibit identical frictional characteristics. Fig. 12 represents the theoretical plot of dial speed in pulses per second versus coefficient of friction. The single curve satisfies both the fly-bar and drive-bar governors as specified in Tables I and III. This curve shows the change in speed of a dial initially adjusted to 10.0 pulses per second, operating at normal torque, as the coefficient of friction varies. It indicates that if there is a decrease in the value of μ from that which existed at the time of initial adjustment, there will be a corresponding increase in the dial speed. The curve is drawn with $\mu = 0.25$ as representing the normal stud-to-case condition and a normal governor input torque of 7,500 dyne-cm.

The experimental data were compiled for the following operating con-

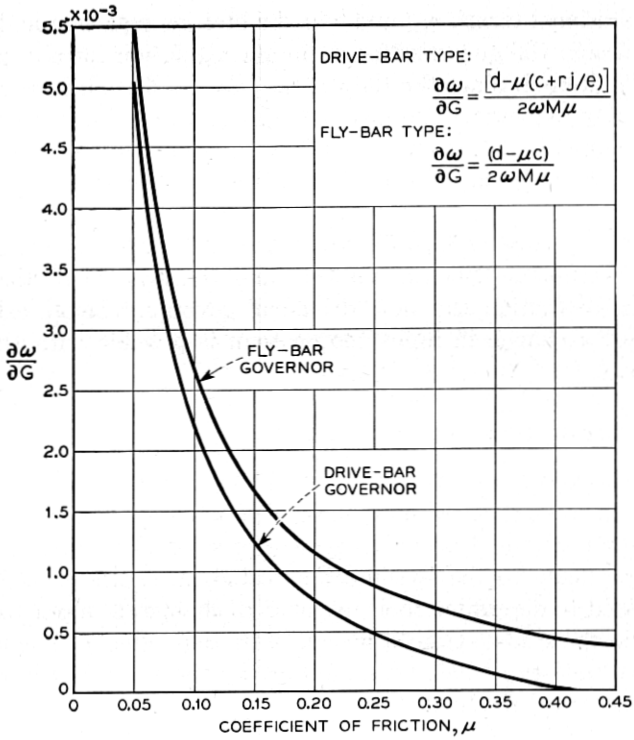

Fig. 11 — Derivative of governor speed with respect to governor input torque versus coefficient of friction.

ditions: as received, governor case cleaned with acetone, damp atmosphere, and SAE 10 petroleum base oil in the case. Each of the governors were initially adjusted to 10 pulses per second with the governor cases in the "as received" condition. The governors were then removed and the cases and governor studs were cleaned with acetone. The governors were then reassembled and the new speed recorded. During this procedure extreme care was exercised so as not to disturb the governor spring adjustment. Speed was next recorded for the damp atmosphere condition, and finally for the condition with one drop of SAE 10 oil in the governor case. For these last two conditions the governors were not removed from the dials.

The average speeds recorded for the four conditions are plotted on the theoretical speed curve of Fig. 12. The points were arbitrarily placed on the theoretical curve. No attempt was made to determine the exact coefficient of friction values corresponding to the four conditions. In this respect, the speed attained with oil in the case shows a value of $\mu = 0.06$ which is very close to the $\mu = 0.05$ taken as the lower limit.

To produce a decrease in governor speed one must increase the value of μ. This is difficult to do on a controlled basis, since it is brought about by the progressive action of wear particles and foreign material scoring the surface of the brass case during the life of the governor. The theoretical analysis indicates that when the coefficient of friction increases to $\mu = 0.35$ the speed of the governor will decrease from 10.00 to 9.65 pulses per second. This assumes that there would be no speed change due to wear in any part of the governor. In practice of course the governor studs wear as the case is scored and, therefore, are made progressively shorter. For this condition the increased outward motion of the weight required for stud-to-case contact produces an increase in the spring force, F_s, acting on the weights. The speed change which results from increasing the spring force is in the direction to compensate for the loss in speed due to increase in coefficient of friction. Therefore, considering only stud friction and wear as effective in causing change in speed, generally the governor and dial speed should increase from its initially adjusted value during life.

It can be concluded from the experimental and theoretical evidence that there is the possibility of a speed change due to varying coefficient

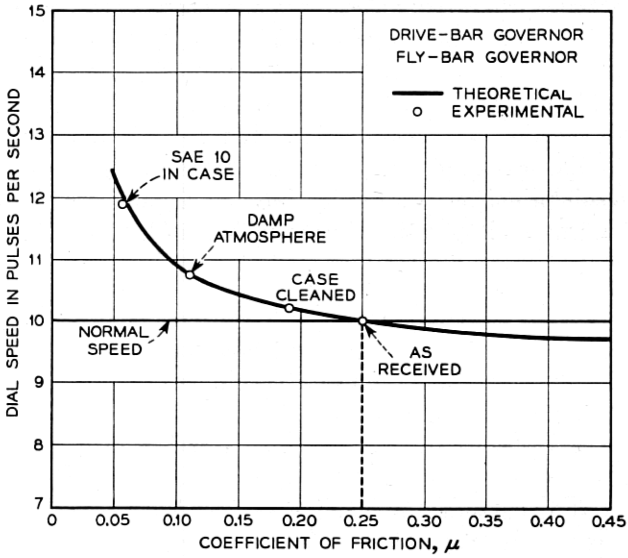

Fig. 12 — Dial speed versus coefficient of friction.

of friction to values between 9.65 and 12.0 pulses per second during life for dials initially adjusted to 10.00 pulses per second. An increase in speed may result from a decrease in stud friction due to the existence of lubricant or high humidity in the stud operating region or, the speed may decrease due to high friction.

The above changes represent the extremes in dial speed determined solely by reaction of the governor to change in stud friction. In practice it is anticipated that dials adjusted to 10 pulses per second initially can vary from 9 to 11 pulses per second during normal usage. A reduction in speed will occur as more torque is required to compensate for the increase in bearing friction caused by the accumulation of dirt and wear products during ordinary life. This additional bearing drag will cause a decrease in the torque available for governoring and therefore the dial will be regulated at reduced speed. For extreme cases of wear and contamination, it is of course possible that the dial will stop altogether during run-down. Such cases are not controllable by the governor. They result from the expected attrition during extended life or unfavorable environment.

To guard against excessive increase in dial speed from the value at time of initial adjustment, precaution is taken during manufacture. As stated previously, lubricant traveling into the governor case after initial adjustment will cause a marked increase in dial speed. To avoid this sort of contamination, the governor case is washed after machining and swabbed with clean chamois prior to insertion of the governor onto the shaft. Care is also taken to see that no lubricant is placed in the governor case during lubrication of the shaft bearings. These practices assure that initially the friction surfaces are relatively free from contamination. The increase in dial speed up to the 11 pulses per second possible during dial life will result primarily as a result of stud wear, increase in efficiency of the mechanism, and operation during periods of high humidity.

Dial Speed Versus Governor Input Torque

To substantiate the theoretical conclusion that the drive-bar governor should exhibit better regulation due to changes in input torque, experimental data were compiled on the dials equipped with the two type governors. The results of this test are plotted on Fig. 13, along with theoretical forcing curves for both governors. A coefficient of friction value of 0.25 was assumed in arriving at the theoretical curves.

The dials were initially adjusted to 10.00 pulses per second by bending the governor spring to have proper tension. Loads of 1, 3 and 5 lb were

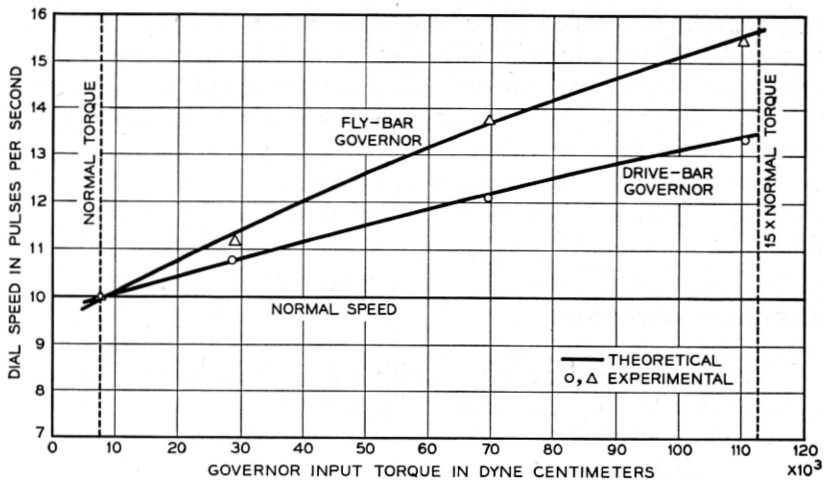

Fig. 13 — Dial speed versus governor input torque.

suspended from the fingerwheel at ⅝″ radius and released. Average experimental speeds were recorded for the three forcing conditions and are noted in Fig. 13 for both the fly-bar and the new drive-bar governors. Good agreement, between the theoretical and experimental values, is evident. For a forcing condition of fifteen times normal motor spring torque, an average speed of 15.6 pulses per second is shown for the fly-bar governor while an average speed of 13.4 pulses per second is noted for the drive-bar type. Theoretically the speeds should be 15.3, and 13.2, respectively. This type agreement is also present for the 1 and 3 lb forcing conditions, and therefore, it may be concluded that for any input torque resulting from forcing the fingerwheel a dial equipped with a drive-bar governor will exhibit less speed increase than one having the fly-bar governor.

The theoretical analysis indicates that for the torque available during normal rundown, drive-bar governors as specified in Table I will decrease in speed from 10.00 to 9.80 pulses per second and fly-bar governors as specified in Table III will decrease in speed to 9.70 pulses per second. These theoretical speed changes were checked experimentally by recording on a rapid record oscillograph a trace of the make and break times of the pulsing contacts during rundown of the dial from digit zero. This information was used to determine the average dial speed in pulses per second for each sequence of make and break times. Actual loss in speed from the first to the ninth pulse for dials equipped with drive-bar governors averaged 0.24 pulses per second while dials with fly-bar governors decreased 0.33 pulses per second. The slight additional loss in speed noted experimentally was probably due to friction in the gear mechanism which is not considered by the theoretical analysis. However, since the differences recorded are quite small, one can conclude that the theoretical and experimental results are in good agreement even when concerned with small changes in torque experienced during normal rundown of a dial.

CHATTER IN GOVERNORS

It is not uncommon for governors with fine regulating ability to produce an objectionable chattering noise when operated near or at the vertical position. This chattering, while in most cases not particularly adverse from a regulation or wear point of view creates in the mind of the listener grave doubts as to the correctness of the design. In severe cases a sharp noise is heard during every half revolution of the governor shaft as each weight alternately leaves the case and strikes against the end of the other governor weight. During every revolution of the governor shaft, each weight is alternately supported as shown in the schematic, Fig. 4. At this instant, the gravity moment about B is a maximum, and along with the spring moment, opposes the centrifugal moment. If the gravity component, or effective mass of the weight, is sufficiently large, a new system is produced which has a critical velocity in excess of the regulated speed. Since the governor speed is continually regulated by the bottom weight at a speed lower than the new critical velocity, the top weight falls from contact with the case.

The magnitude of the gravity component is a function of the angle at which the governor operates and is a maximum when the dial is in the vertical position. As the operating plane of the dial decreases to the horizontal, the gravity effect decreases to zero. Chatter will not occur when the operating angle produces a gravity component smaller than the difference between the centrifugal force and the spring force.

Since the chattering effect is the result of a balance of forces on the governor, it is apparent that a relationship can be derived which will express the effect in terms of governor constants. This derivation is given in Appendix III and shows the chatter equation for a conventional fly-bar governor to be

$$m \leqq \frac{G(d - \mu c)}{2r\mu\ell \sin \beta} \tag{19}$$

This expression must be satisfied if the governor is to operate free of chatter.

By substituting the constants for the fly-bar governor, in Table III, we have for this governor operating in the vertical plane

$$3.9(980) \leqq \frac{7{,}500(0.390 - (0.25)(0.361))}{2(118)(0.25)(1.092)}$$

or

$$3{,}820 \text{ dyne-cm} \leqq 2{,}790 \text{ dyne-cm}.$$

Since the equation is not satisfied instability should be present and governors of this fly-bar design do chatter loudly when operating in the vertical plane.

The chatter equation for the fly-bar governor indicates that by adjusting the design constants, one can eliminate the instability effect. This is true. A set of values could be used which would result in a fly-bar governor which operates free of chatter. Unfortunately such a governor would also have reduced ability to govern. The relationship between chatter and governing is explained as follows.

The equations which define changes in governor speed with respect to changes in friction and torque for the fly-bar governor

$$\frac{\partial \omega}{\partial \mu} = -\frac{Gd}{2\omega M \mu^2}$$

and

$$\frac{\partial \omega}{\partial G} = \frac{1}{2} \frac{(d - \mu c)}{M \mu \omega}$$

and the chatter equation, (19), show that operation without chatter and good speed regulation are totally incompatible. Those terms in the equations which should be small for good speed regulation; i.e., torque G and stud location d and c, should be large to avoid chattering of the governor. Those terms which should be large for good speed regulation; i.e., case radius r, friction μ, and the distance from the pivot to the center of gravity l, must be small for no chatter. As the theoretical analysis indicates there is no term in the fly-bar governor chatter equation which can be operated on to eliminate chatter without impairing regulation of speed.

A similar analysis, given in Appendix IV, shows the chatter equation for the new drive-bar governor to be

$$m \leqq \frac{G(d - \mu c - \mu r j/e)}{2 r \mu \ell \sin \beta} \qquad (20)$$

This expression must be satisfied if there is to be no chattering during operation. A comparison of this equation with that determined for the fly-bar governor, equation (19), shows an additional term $(\mu rj/e)$. Substitution of the design constants for the drive-bar governor given in Table I leads to the following.

$$3.9(980) \leqq \frac{7{,}500(0.390 - (0.25)(0.361) - (0.25)(1.18)(0.236)/(0.498))}{2(1.18)(0.25)(1.092)}$$

or

$$3{,}820 \text{ dyne-cm} \leqq 1{,}862 \text{ dyne-cm}.$$

This indicates that instability should be present in this governor, and that the additional term $(\mu rj/e)$ causes a greater difference between the mass term and the torque term, than for the conventional fly-bar governor. This is to be expected, since for the drive-bar governor also, adequate speed regulation, and a design which has no chatter, are totally incompatible. The sensitivity of speed to torque change and changes in coefficient of friction.

$$\frac{\partial \omega}{\partial G} = \frac{1}{2} \frac{(d - \mu c - \mu rj/e)}{M\omega\mu}$$

and

$$\frac{\partial \omega}{\partial \mu} = -\frac{Gd}{2M\omega\mu}$$

were made as small as possible for best regulation and this results in a small value to oppose chatter.

This chatter analysis indicates that a new approach in design is needed in order to provide a governor which will operate without excessive noise and still regulate speed as required for use in the 7-type dial. This is found in a design which prohibits rapid movement of the governor weights during the unstable period.

Referring to the assembly drawing of the drive-bar governor, Fig. 3, which shows the governor in the rest position, one can see that each governor weigh rests on the end of one of the arms of the drive-bar. By supporting the weights in this manner the following two beneficial effects are achieved. One, during operation in new assemblies, the weights move outward to touch the case only a nominal distance of 0.007″. This small allowable motion in the drive-bar governor results in a low velocity of the weight at closure and hence, less impact noise. Two, because the drive-bar presses to rotate the governor weights, impact occurs as the unstable weight skids against its arm of the drive-bar. This introduces friction damping to still further reduce the noise on closure of the weight. Some additional damping is provided by making the drive-bar and weight of powdered metal rather than wrought material. These features, which result from the particular physical arrangements of the component parts make possible the relatively quiet operation of the drive-bar governor. Experience with dials equipped with drive-bar governors indicates that they are effective since the noise due to chattering has been satisfactorily reduced so as to not be objectionable.

SUMMARY

This study has carried forward the work of C. R. Moore by presenting the derivation of theoretical equations which define speed for the drive-bar type governor. Design considerations necessary for optimum speed regulation indicated by the theory were applied in establishing the shape and working relationship of various components in the drive-bar governor. Governors constructed to these dimensions have operated as forecast by the theory. The excellent agreement between theory and practice indicates that it is both desirable and practicable to apply the Moore theory in the design of governors.

The initial requirements imposed on the design of a governor for the 7-type dial were two-fold. The new governor had to provide speed regulation at least equal to the conventional fly-bar type and no objectionable noise could be created by the governor during operation. To better understand the reasons for noise in governors, suitable theory was developed for investigating this phenomenon. Application of this theory to any type governor results in an equation which defines chatter in terms of the constants of the governor. This equation and the equations determined by the Moore theory for speed regulation indicate the existence of an interrelationship between speed regulation and noise in governors. The theory indicates that noise free operation and good regulation are totally at variance. The fly-bar governor supports this conclusion since this governor having fine control of speed also produces a chattering noise during operation. To satisfy both requirements, good regulation and quiet operation, it is first necessary to design a governor which will regulate properly and secondly, if the constants selected indicate that chattering will occur, prohibit excessive noise by providing means for restraining the system during the unstable period.

This method of attack was taken in the design of the new drive-bar governor. By applying the Moore theory, a governor was developed for

use in telephone dials which is an improvement over the conventional fly-bar type governor. Fine regulation is provided by the drive-bar governor for a given change in the coefficient of friction between the studs and case. This is achieved by locating the studs as close to the weight pivot as manufacturing techniques will permit. Improved speed regulation is provided for varying input torque in the new governor as compared with the fly-bar governor. The experimental data shows the new design able to control speed approximately twice as well. It is an effective non-forcing governor, prohibiting excessive increase in dial speed as a result of forcing the fingerwheel during rundown. This nonforcing feature is achieved by applying the driving torque to the weights at a point to develop a moment about the weight pivot. This drive-bar moment assists the centrifugal force in maintaining pressure of the friction stud against the case for friction governoring.

Having established a design which provided the degree of dial speed regulation considered necessary, it was then possible to investigate the second requirement of noise free operation. Application of the chatter theory to the drive-bar governor indicated the design to be unstable. This situation was controlled by using the ends of the drive-bar to limit the fall of the governor weights. This configuration of parts allows only small movement of the weights during the unstable period and provides damping as the weights close on the arms. The small motion and friction damping in the assembly results in a governor which is relatively free from noise during operation. Experience with dials equipped with drive-bar governors indicate that the chattering effect has been controlled.

ACKNOWLEDGMENT

The unpublished work of C. R. Moore covering the theory of fly-ball, fly-bar and band type governors has served as a foundation for this paper and the experimental work of R. E. Prescott aided considerably in determining the final drive-bar governor design. The writer also wishes to express appreciation to Mr. Prescott and H. F. Hopkins for their helpful comments and suggestions during preparation of this paper.

Appendix I

DERIVATION OF THE DRIVE-BAR TYPE GOVERNOR SPEED EQUATION

Referring to Fig. 4, as the governor mechanism rotates in a clockwise direction each weight tends to move outward under the influence of centrifugal force and the torque force, F. The centrifugal force, F_m, acts through the center of gravity of the weights radially from the turning center of the governor shaft. The torque force, F, is applied on the weights by the drive-bar arms. These forces are opposed by the spring force, F_s. A stud-to-case force, F_n, and a frictional component of this force, μF_n, act on the weights when the friction studs are in contact with the case. In deriving the equation of motion for speeds in excess of the critical velocity the following symbols will be used as noted on Fig. 4.

F — Force applied by torque on governor weights
F_n — Normal force of case acting on studs
F_s — Force exerted by spring when studs touch case
F_m — Centrifugal force acting at center of gravity of each weight
μ — Coefficient of friction
I_0 — Moment of inertia of the governor about center shaft
ω — Angular velocity of governor
ω_0 — Critical angular velocity at which studs just touch the case
m — Mass of each weight
r_0 — Radius to the center of gravity of each weight
r — Radius of governor case
α — Stud angle
Neg. Rotation — Rundown of governor

From the schematic, Fig. 4, taking moments about B we have

$$F_m b - F_s b + \mu F_n c - F_n d + F = 0 \tag{1}$$

collecting terms

$$b(F_m - F_s) - F_n(d - \mu c) + Fj = 0$$

$$F_n = \frac{b(F_m - F_s) + Fj}{(d - \mu c)} \tag{2}$$

The driving torque on the governor is $G = 2Fe$; the retarding torque, $2\mu F_n r$. The difference between the driving torque and the retarding torque is as follows:

$$I_0 \dot{\omega} = G - 2\mu F_n r \tag{3}$$

where I_0 = Moment of inertia of the governor about the shaft center
$\dot{\omega}$ = Angular acceleration about shaft center

equating

$$F_n = \frac{G - I_0 \dot{\omega}}{2\mu r} \tag{4}$$

Combining equations (2) and (4) and solving for the acceleration, $\dot{\omega}$,

$$\frac{b(F_m - F_s) + Fj}{(d - \mu c)} = \frac{G - I_0\dot{\omega}}{2\mu r}$$

$$\frac{2F_m b\mu r}{(d - \mu c)} - \frac{2(F_s b - Fj)\mu r}{(d - \mu c)} = G - I_0\dot{\omega}$$

or

$$\dot{\omega} = \frac{G}{I_0} - \frac{2F_m b\mu r}{I_0(d - \mu c)} + \frac{2(F_s b - Fj)\mu r}{I_0(d - \mu c)}$$

Substituting the following values for F_m, F_s, F

$$F_m = m\omega^2 r_0$$

$$F_s = m\omega_0^2 r_0$$

$$F = \frac{G}{2e}$$

then

$$\dot{\omega} = \frac{G}{I_0} - \frac{2m\omega^2 r_0 b\mu r}{I_0(d - \mu c)} + \frac{2m\omega_0^2 r_0 b\mu r}{I_0(d - \mu c)} - \frac{j\mu r G}{I_0(d - \mu c)e}$$

Substituting for the design constants

$$K = r_0/r$$

$$M = 2mr^2 bK$$

$$\dot{\omega} = \frac{G}{I_0} - \frac{M\mu\omega^2}{I_0(d - \mu c)} + \frac{M\mu\omega_0^2}{I_0(d - \mu c)} - \frac{\mu r j G}{I_0(d - \mu c)e}$$

or

$$\dot{\omega} + \frac{M\mu}{I_0(d - \mu c)}\,\omega^2 = \frac{G}{I_0} + \frac{M\mu}{I_0(d - \mu c)}\,\omega_0^2 - \frac{\mu r j G}{I_0(d - \mu c)e} \tag{5}$$

This is of the form

$$\frac{d\omega}{dt} + g\omega^2 = h$$

or

$$dt = \frac{d\omega}{h - g\omega^2}.$$

Separating variables and integrating*

$$t = \frac{q}{2h} \operatorname{Ln} \frac{q + \omega}{q - \omega} + c \qquad \text{where} \qquad q^2 = \frac{h}{g} \tag{6}$$

Applying the initial conditions to solve for the constant c

$$\omega = \omega_0 \qquad \text{at} \qquad t = 0$$

$$c = -\frac{q}{2h} \operatorname{Ln} \frac{q + \omega_0}{q - \omega_0}$$

Substituting in equation (6)

$$t = \frac{q}{2h} \operatorname{Ln} \frac{q + \omega}{q - \omega} - \frac{q}{2h} \operatorname{Ln} \frac{q + \omega_0}{q - \omega_0}$$

Letting

$$A = \frac{q + \omega_0}{q - \omega_0}$$

Then

$$t = \frac{q}{2h} \operatorname{Ln} \frac{1}{A} \frac{(q + \omega)}{(q - \omega)} \tag{7}$$

and

$$\omega = q \tanh \left(\frac{ht}{q} + \operatorname{Ln} \sqrt{A} \right) \tag{8}$$

Equation (8) applies as the equation of motion for the drive-bar governor for speeds in excess of the critical speed when

$$g = \frac{M\mu}{I_0(d - \mu c)} \tag{9}$$

$$h = \frac{G(d - \mu c) + M\mu\omega_0^2 - \mu rjG/e}{I_0(d - \mu c)} \tag{10}$$

$$q = \sqrt{\frac{h}{g}} = \sqrt{\frac{G(d - \mu c) + M\mu_0^2 - \mu rjG/e}{M\mu}} \tag{11}$$

Appendix II

GOVERNOR INPUT TORQUE

In order to apply the theoretical speed equations and the chatter equations developed for governors one must determine suitable values

* Short Table of Integrals, Pierce, B. O., pp. 8, No. 50.

for the stud-to-case coefficient of friction, μ, and governor input torque, G. Experimental evidence indicates a μ of 0.25 exists during normal operating conditions for hard rubber on brass. The values for governor input torque given in Tables I and III and used in the theoretical analysis for the governors were determined as follows.

The initial torque applied to the governor for the period up to the critical velocity was calculated from oscillograph string traces. These traces were obtained by mounting on the end of the governor shaft a thin disc having 36 radial slots spaced uniformly about the circumference. Light, detected through the slots of the rotating disc on the element of a photo tube, appeared as a distorted sine wave on the photographic paper. The distance between two successive wave peaks represented 10° of rotation of the governor. By noting on the trace the time between peaks, it was possible to determine the average velocity of the governor at 10° intervals after release of the fingerwheel, or start of rotation of the governor mechanism. The complete plot of these velocities appear as the experimental speed curve on Fig. 6. Inspection of the experimental curve for the drive-bar governor shows constant acceleration immediately after release. This appears as a straight line in the velocity time curve. Using the slope of this line and the moment of inertia, $I_0 = 7.4$ gm cm^2, the initial governor torque was calculated as follows:

$$G_I = I\dot{\omega} = \frac{I_0}{t'} = \frac{100(7.4)}{55} = 13{,}480 \text{ dyne-cm.}$$

The governor torque value during normal rundown was found by first determining the governor stud-to-case force, F_n, and using this value in the equation

$$G = 2\mu F_n r$$

The fly-bar governor mechanism was used to determine the F_n force since the moment equation for this type governor contains measurable values. Referring to Fig. 7, the schematic of the fly-bar governor, the moment equation about B is as follows:

$$F_m b - F_s b - F_n d + \mu F_n c = 0$$

To solve this equation for F_n one must determine the centrifugal force, F_m, and the spring force, F_s. Using electronic flash equipment with an exposure time of 1/10,000 of a second, it was possible to take distortion free photographs of the governor mechanism at the middle of the rundown. These photographs were taken with the governor in the horizontal

TABLE V — TYPE DIAL GOVERNOR INPUT TORQUE AT PULSE NO. 5

Dial No.	P.P.S.	F_m	F_s	F_n	G
1	9.89	37,820	32,820	15,320	9,050
	10.05	39,050	35,950	12,580	7,420
	9.92	38,100	34,300	11,650	6,880
2	10.02	38,920	35,230	11,340	6,700
	10.01	38,600	34,880	11,400	6,730
	9.96	38,400	35,000	10,420	6,160
3	10.03	38,950	34,100	14,850	8,770
	10.22	40,600	35,420	15,880	9,360
	10.08	39,400	35,000	13,522	7,980

Average 7,500 dyne-cm, approximately.

position, thus eliminating the gravity effect. The deflection of the governor spring measured on the photograph was used to determine the F_s force, and the governor speed, necessary to determine F_m, was taken from the oscillograph string trace. Three dials were tested, each having the same maximum motor spring torque, 490,000 dyne-cm, and clean governor cases and studs to produce an assumed coefficient of friction value, $u = 0.25$. For three 7-type dials with fly-bar governors, the experimental data given in Table V applies.

As determined, this 7,500 dyne-cm torque at the governor exists at the middle of rundown of the dial. In order to compare it with the 13,500 dyne-cm initial torque previously determined, it is first necessary to consider the effect of the motor spring. As stated previously, the torque provided by the motor spring during dial rundown decreases approximately 35 per cent from the initial value of 490,000 dyne-cm. Logically, the torque at the governor would decrease by the same percentage. Applying this factor to the torque value for the middle of rundown gives a value of 10,500 dyne-cm which can be compared with the 13,500 dyne-cm torque. A difference of 3,000 dyne-cm exists for the value of initial torque at the governor as determined by the two test methods. This remaining difference can be explained by considering the frictional losses in the dial mechanism during rundown. This analysis follows:

During rundown of the dial mechanism, a pair of pulsing springs tensioned against components on the pulsing shaft are alternately raised and lowered. This action allows contacts on the springs to open and close for pulsing in the telephone line. A portion of the input torque provided by the motor spring is required for performing this function. On Fig. 14 are plotted the torque curves for these pulsing springs as the pulsing

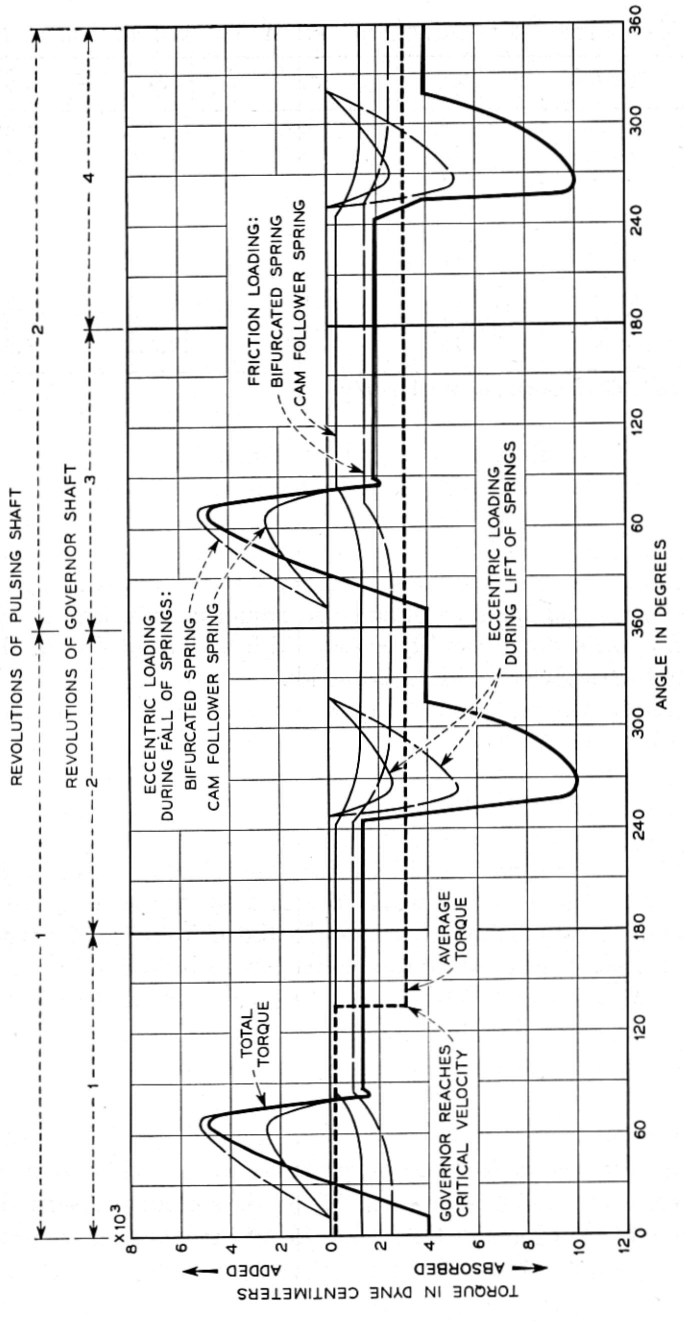

Fig. 14 — Torque analysis for the 7-type dial pulsing shaft.

shaft rotates during rundown. Fig. 15 is a schematic of the pulsing springs as they appear when the contacts are closed and when open.

For a short period during each revolution of the pulsing shaft, the pulsing springs aid the motor spring in driving the gear system. This occurs when the springs are being lowered by the cam just prior to opening of the contacts. For the remaining portion of each revolution the motor spring must provide energy to overcome frictional losses and lift the springs. These changes in energy required for moving the springs have been combined to give the total instantaneous torque curve also shown on Fig. 14.

As indicated by the pulsing spring torque analysis, the pulsing mechanism absorbs an average of only 200 dyne-cm during the period when accelerating up to the critical velocity. For rotation after the critical velocity, the average torque needed to drive the pulsing mechanism is approximately 3,000 dyne-cm. The difference between these average torque values appear at the governor shaft as a 1,500 dyne-cm torque difference. That is, 1,500 dyne-cm more torque is available for driving the governor prior to the time the critical velocity is reached as compared to that available after this time. This accounts for one half of the 3,000 dyne-cm difference in initial governor torques as calculated by the

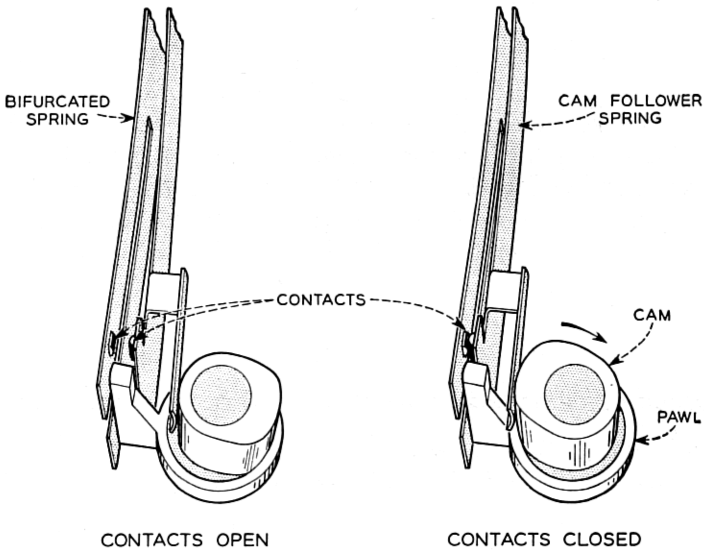

Fig. 15 — Pulsing springs of 7-type dial.

TABLE VI

Average torque at governor — Test No. 1, up to the critical velocity		13,500
Average torque at governor — Test No. 2, after reaching the critical velocity	10,500	
Torque at pulsing mechanism	1,500	
Torque required to overcome friction in the bearings	1,500	
Total	13,500 dyne-cm	13,500 dyne-cm

two test methods. The remaining 1,500 dyne-cm difference can be accounted for by considering the friction in the mechanism before and after the critical velocity.

Initially the system is accelerating as a simple fly wheel under the influence of the motor spring. At the critical velocity the governor studs engage the case and the dial rotates at virtually constant speed for the rest of the rundown period. This implies that the average speed of the moving parts in the mechanism will be twice as great for the period after the critical velocity as before. By considering the friction which exists in the dial bearings,* one can see the effect of this change in speed on the torque required to drive the mechanism. In sliding bearings with film lubrication the coefficient of friction is a function of speed. Specifically, as the speed of rotation of the journals increases, the coefficient of friction in the bearings will increase. Since the regulated dial speed is greater than the average speed while accelerating, friction in the system will also be greater. One can, therefore, justify the remaining 1,500 dyne-cm torque difference which exists before and after the critical velocity by considering it to result from the increased friction at the higher speed.

Therefore, by considering the effect of the pulsing mechanism and friction in the system, it has been possible to account for the difference in torque determined by the two test methods. Table VI shows the disposition of the torque.

APPENDIX III

DERIVATION OF CHATTER EQUATION FOR FLY-BAR TYPE GOVERNOR

Consider the governor rotating in the vertical plane at constant speed ω. Referring to the schematic Fig. 7 and taking moments about B

$$F_m b - F_s b - F_n d + \mu F_n c - ml \sin \beta = 0$$

* Design of Machine Members, Valence and Doughtie, p. 255.

where m = mass of weight
l = distance from C.G. of the weight to the pivot (B)
β = operating angle of governor weights with respect to a horizontal plane

For chattering to occur we know that the weight must leave the governor case. Therefore, at some angle β the gravity component will equal the forces tending to move the weight outward. For this condition pressure of the friction stud against the case will be equal to zero and

$$F_n d = 0$$

$$\mu F_n C = 0$$

and the moment equation becomes for this equilibrium condition

$$F_m b = F_s b + ml \sin \beta \tag{1}$$

Centrifugal Moment = Spring Moment + Gravity Moment

By applying the steady-state speed equation and the equation for centrifugal force we can transpose equation (1) to contain only design constants of the governor. From the steady-state speed equation

$$\omega = \sqrt{\frac{G(d - \mu c) + M\mu\omega_0^2}{M\mu}}$$

Substituting $F_s = m\omega_0^2\, r_0$ and $M = 2mrr_0 b$

$$mrr_0 b\mu\omega^2 = \frac{G}{2}(d - \mu c) + \frac{mrr_0 b\mu F_s}{mr_0}$$

or

$$F_s = mr_0\omega^2 - \frac{G}{2}\frac{(d - \mu c)}{rb\mu}$$

also, the centrifugal force is

$$F_m = m\omega^2 r_0$$

Substituting in equation (1) the values for (F_s) and (F_m)

$$m\omega^2 r_0 b = mr_0\omega^2 b - \frac{G(d - \mu c)}{2r\mu} + ml \sin \beta$$

or

$$ml \sin \beta = \frac{G(d - \mu c)}{2r\mu}$$

where the gravity moment ($ml \sin \beta$) must be just equal to or smaller than $[G(d - \mu c)]/2r\mu$ to have no chatter occur in the governor. Expressing this in terms of the mass of the weight we have the chatter equation for the fly-bar governor as

$$m \leqq \frac{G(d - \mu c)}{2rul \sin \beta} \tag{2}$$

Appendix IV

DERIVATION OF CHATTER EQUATION FOR DRIVE-BAR TYPE GOVERNOR

Referring to the schematic, Fig. 4, consider the governor rotating at constant speed ω in the vertical plane. Taking moments about B

$$F_m b - F_s b + F_j + \mu F_n c - F_n d - ml \sin \beta = 0$$

where m = mass of weight
l = distance from C.G. of weight to pivot B
β = operating angle of governor weights

Assume that at some angle β the gravity component will be large enough to make the F_n force equal to zero and a condition of equilibrium exists. For this condition

$$F_n d = 0$$

$$\mu F_n C = 0$$

and therefore, F_j, the torque component on the weight, must also equal zero. The moment equation becomes

$$F_m b = F_s b + ml \sin \beta \tag{1}$$

Using the equation for centrifugal force

$$F_m = m\omega^2 r_0$$

and steady-state speed for the drive-bar governor

$$\omega = \sqrt{\frac{G(d - \mu c) + M\mu\omega_0^2 - \mu rjG/e}{M\mu}}$$

where

$$F_s = m\omega_0^2 r_0 \text{ and } M = 2mrr_0 b$$

Solving for (F_s)

$$F_s = mr_0\omega^2 - \frac{G}{2r\mu b}(d - \mu c - \mu rj/e)$$

Substituting in equation (1), the values for F_m and F_s

$$m\omega^2 r_0 b = mr_0\omega^2 b - \frac{G}{2r\mu}(d - \mu c - \mu rj/e) + ml \sin\beta$$

or

$$ml \sin\beta = \frac{G}{2r\mu}(d - \mu c - \mu rj/e)$$

Expressing the equation in terms of mass of the weight we have the chatter equation for the drive-bar governor.

$$m \leqq \frac{G(d - \mu c - \mu rj/e)}{2r\mu l \sin\beta} \qquad (2)$$