The Wave Picture of Microwave Tubes

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Many microwave tubes make use of a long electron beam. The radio frequency excitation on such a beam can be expressed in terms of two space-charge waves, one of which has negative energy and negative power flow. The electron beam may pass through resonators, through lossy surroundings, through slow-wave circuits. In this paper the low-level operation of klystrons, resistive-wall amplifiers, easitrons, space-charge-wave amplifiers, traveling-wave tubes and double-stream amplifiers is explained in terms of waves on electron beams and on circuits. Noise is discussed in terms of such waves.

INTRODUCTION

There are many different ways in which one can make a valid analysis of the low-level or small-signal behavior of the many types of microwave tubes which use long electron beams. Which way one should choose depends partly on one's purpose in making the analysis, and partly on the particular problem to be solved.

All of these analyses lead at some point to waves or modes of propagation: waves which travel along an electron stream, along a circuit, or along the two together; waves which are unattenuated or which increase or decrease with distance. Sometimes, the analysis starts out with electron current, electron velocity and circuit dimensions as the fundamental physical quantities, just as network analysis can start out with inductance, capacitance and resistance. However, an analysis can start out instead with waves, their propagation constants and their characteristic impedances as the fundamental physical bases of the analysis. We might argue that as we are to end with waves, we may well start with waves. As it turns out, the picture of the operation of various tubes in terms of waves is simple and pleasing.

It is the purpose of this paper to present a picture of the operation of microwave tubes in terms of waves. This may be of some interest to those outside of the tube field, in that it gives an account of many recent devices. For experts in the field it can serve as an introduction to a method of analysis which is fairly recent and which may be unfamiliar.

In this analysis, certain simplifications are made. One underlying simplification is that of linearity; it is assumed that at low signal levels the behavior of the electron stream, which is inherently non-linear, can be represented by that of a truly linear system.

As this paper purports to give an accurate and useful picture of the low-level operation of microwave beam devices rather than an exhaustive discussion, some details have been omitted or passed over lightly because they seemed to be of secondary importance. Material which may be unfamiliar to workers in other fields but which is important as background is presented in appendices A to C. Various points can be pursued further in the literature. References to publications and to those responsible for various advances are not given in the body of the paper or in the appendices; they are given for each topic in Appendix D.

SPACE-CHARGE WAVES

Many microwave tubes embody a long, narrow electron beam surrounded by a conducting tube and focused or confined by a longitudinal magnetic field. At low levels of operation, the radio-frequency disturbances on such an electron stream can be expressed in terms of space-charge waves.

In these waves, two forms of energy are of primary importance: electrostatic energy associated with the bunching together of electrons, and kinetic energy, associated with differences in the velocities of the electrons. Thus, the waves may be called electromechanical; the electric energy which we associate with waves in transmission lines and waveguides is present, but the magnetic energy is replaced by kinetic energy. In circuit terms, we have an electrical capacitive element, but the inductive element is inertial, not magnetic in nature. When the electron and wave velocities are slow compared with the velocity of light, the magnetic fields produced by the electron convection current are negligible.

There may be many space-charge modes or waves in an electron stream, some with complex radial and angular variations of amplitude over the electron stream. Two waves predominate in the operation of tubes, however, and one simplification we will make is to deal with these only, and to disregard other modes of propagation on the electron stream. Appendix A discusses such a pair of waves in a simplified physical system.

We can associate with these two waves an ac electron convection current i and an ac electron velocity v. Either we can assume that the electron beam is narrow and disregard the fact that these quantities vary across the beam, or we can deal with peak or effective values much as in the case of voltages and currents in waveguides.

These ac quantities are assumed to contain a factor

$$e^{j\omega t}\, e^{-j\beta z}$$

That is, they vary sinusoidally with time and with distance, and (assuming β to be positive) propagate in the $+z$ direction. The phase constants β of the two waves will be called β_1 and β_2. For beams of moderate charge they are very nearly

$$\beta_1 = \frac{\omega}{u_0} + \frac{\omega_q}{u_0}$$

$$\beta_2 = \frac{\omega}{u_0} - \frac{\omega_q}{u_0}$$

Here u_0 is the electron speed, ω is the operating radian frequency and ω_q is the effective plasma radian frequency.

The plasma frequency of the electron beam ω_p is given by

$$\omega_p^{\,2} = \frac{\dfrac{e}{m}\rho_0}{\epsilon}$$

Here e/m is the charge-to-mass ratio of the electron, ρ_0 is the charge density and ϵ is the dielectric constant of vacuum. In terms of ω_p, ω_q may be expressed

$$\omega_q = R\omega_p$$

Here R is a factor somewhat less than unity which depends on the geometry of the electron beam, on ω and ω_p, and on the velocity distribution of the electrons (see Appendices A and C).

Let us consider the simple case in which R is unity and the effective plasma frequency is equal to the plasma frequency. The phase velocities v_1 and v_2 of the two waves, which are ω divided by β, are

$$v_1 = \frac{u_0}{1 + \dfrac{\omega_p}{\omega}}$$

$$v_2 = \frac{u_0}{1 - \dfrac{\omega_p}{\omega}}$$

Thus, the first wave has a phase velocity less than that of the electrons; it is a slow wave, and the second wave is a fast wave.

Suppose we make up a radio-frequency pulse out of various frequency components of one wave. The pulse envelope generally travels with a different velocity from that of the rf sinusoids under the envelope. The velocity of the envelope is called the group velocity. The group velocity is the velocity with which a signal is transmitted. The direction of the group velocity is the direction in which causality acts (for some waves the phase velocity and the group velocity have opposite directions). The group velocity tells in which direction energy flows, and the power flow P is the stored energy per unit length, W, times the group velocity, v_g.

$$P = Wv_g$$

The group velocity is given by

$$v_g = \frac{1}{\partial\beta/\partial\omega}$$

We see that for our assumption ω_q is equal to ω_p, the group velocity for each wave or mode is u_0, the velocity of the electrons in the beam

$$v_g = u_0$$

Thus, of the two waves, the first has a phase velocity slower than that of the electrons, the second has a phase velocity faster than that of the electrons, and each has a group velocity equal to that of the electrons.

A simple discussion of power flow is given in Appendix B. In describing the excitation of the electron stream we can use the convection current i together with a quantity U which is analogous to voltage. In terms of the ac electron velocity v,

$$U = -\frac{u_0}{\frac{|e|}{m}} v$$

The real power flow P is given by

$$P = \tfrac{1}{2}(iU^* + i^*U)$$

This relation is justified in Appendix B.

For each of the two waves the voltage U bears a constant ratio to the current i; this ratio is the characteristic impedance K of the wave. We find that

$$K_1 = \frac{U_1}{i_1} = -2\frac{\omega_q}{\omega}\frac{V_0}{I_0}$$

$$K_2 = \frac{U_2}{i_2} = 2\frac{\omega_q}{\omega}\frac{V_0}{I_0}$$

Here V_0 is the accelerating voltage specifying the electron velocity u_0 and I_0 is the total beam current.

We see that the characteristic impedance K_1 of the slow wave is negative. This means that the power flow in the $+z$ direction is negative. We could also say that positive power flows in the $-z$ direction, but this may carry an unfortunate implication as to the direction in which causality acts. An example may be helpful.

Fig. 1 shows an electron beam acted on by the fields of two devices A and B. The fields in A are such as to set up the slow wave only. This travels between A and B. The fields of B are such as to just remove the slow wave entirely, so that the electron beam leaves B with no ac disturbance on it. The electron velocity u_0, phase velocity v, group velocity v_g and negative power flow $-P$ are all directed in the $+z$ direction, that is, to the right.

We must remove a power P from A to set up the slow wave. A power $-P$ flows from A to B. We must add a power P to B to remove the slow wave from the electron beam. Causality acts from A to B. To change the amplitude of the slow wave between A and B we must change the fields in A, not the fields in B.

The power flow is the group velocity times the stored energy per unit length. As the group velocity for the slow wave is positive and the power flow is negative, we see that the stored energy must be negative.

If we moved with the electrons and observed the waves, we would find that the average kinetic energy associated with the ac electron velocity was equal to the average potential energy of the electric field, and that both were positive; this is characteristic of waves in a stationary medium. The kinetic energy of the electrons relative to a fixed observer is proportional to the square of their total velocity, that is, the ac velocity plus the average velocity. The average velocity is larger than the ac velocity, so that energy terms involving the product of the average

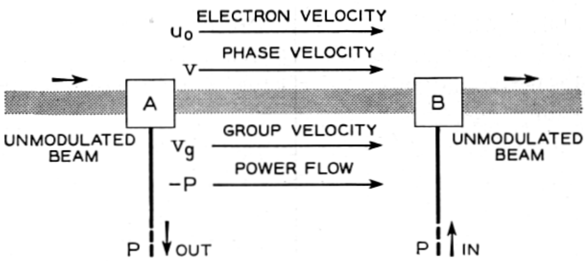

Fig. 1 — Device A sets up the slow space-charge wave only, and device B removes it. u_0, v, v_g and $-P$ are respectively the electron velocity, the phase velocity, the group velocity and the power flow between A and B.

velocity and the ac velocity are larger than terms involving the square of the ac velocity. The product terms may be negative or positive.

We can understand the negative energy of the slow wave qualitatively through a simple argument of a somewhat different sort. In the slow wave, the charge density is greatest in regions of less-than-average velocity and least in regions of more-than-average velocity, so that the electron beam has less total kinetic energy in the presence of the slow wave than it does in the absence of the slow wave. How does this come to be? Suppose that we move with the wave; we then see electrons moving in an electric field which is constant with time, and hence, as electrons move through the field their velocities vary as the square root of the potential. Relative to the wave, the electrons move slowest in the low-potential regions, and correspondingly, they are bunched together in regions of low potential. Now, for the slow wave the total electron velocity is the arithmetic sum of the wave velocity and the electron velocity relative to the wave, so if the electrons are bunched in regions of lowest velocity relative to the wave they are necessarily bunched in the regions of least total electron velocity, and the kinetic energy of the slow wave is thus negative.

In the case of the fast wave, the electrons travel backward relative to the wave. The total electron velocity is the arithmetic difference between the wave velocity and the electron velocity relative to the wave. Hence, the total electron velocity is greatest at the bunches, where the velocity relative to the wave is least, and the kinetic energy of the fast wave is positive.

THE KLYSTRON

We can explain the operation of a number of types of vacuum tubes in terms of space-charge waves. Consider the klystron, illustrated in Fig. 2. The voltage produced across the input resonator by the input signal sets up on the electron beam both the slow and the fast space-charge waves in equal magnitudes and so phased that the velocities v, or the voltages U add, while the currents cancel. Thus, just beyond the input resonator, the beam has an ac velocity; it is velocity modulated, but it has no ac convection current.

Because the two space-charge waves, one with negative power flow and one with positive power flow, are set up with equal magnitudes, the ac power flow in the beam between the input and the output resonators is zero. The input resonator neither adds power to nor subtracts power from the beam.

Because the two waves have different phase velocities, their relative phase changes as they travel along the beam. If we go along the beam a distance L such that

$$2 \frac{\omega_q}{u_0} L = \pi$$

we will find that the ac velocities of the two waves cancel and their currents add. If at this point we put an output resonator, the current will produce a voltage across the resonator which will act on the electron beam to set up new components of the slow and the fast waves.

If the resonator is on tune, so that it acts as a resistive impedance, the phase of the voltage is such with respect to the space-charge wave producing it that the new component of the fast space-charge wave

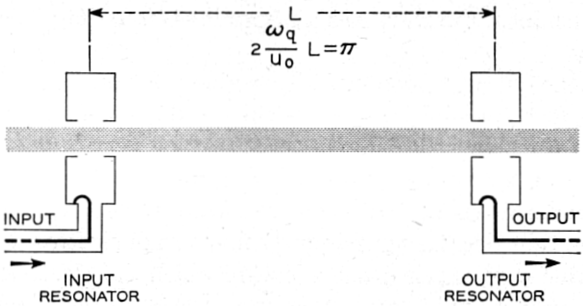

Fig. 2 — In a klystron the input resonator sets up slow and fast space-charge waves so phased that the velocities add and the currents cancel. At the output resonator the currents add and the velocities cancel. The voltage across the output resonator increases the amplitude of the slow, negative-power wave and decreases the amplitude of the fast, positive-power wave.

subtracts from the old component, while the new component of the slow space-charge wave adds to the old component. Thus, while to the left of the output resonator the two space-charge waves have equal magnitudes, so that the net power flow is zero, to the right of the output resonator the slow space-charge wave has a greater magnitude than the fast space-charge wave, so that the power flow in the beam is negative. The missing power appears as the output from the output resonator.

Of course, klystrons are frequently used in the nonlinear range of operation, and the distance L between resonators may be chosen differently from other considerations.

THE RESISTIVE-WALL AMPLIFIER

Consider a tube much like a klystron, but in which the electron beam is surrounded by a glass tube coated with lossy material, such as graphite, as shown in Fig. 3.

As in the klystron, the input resonator produces both the slow and the fast waves with equal magnitudes. As each wave travels, it induces currents in the resistive wall surrounding it and dissipates power in the wall. Thus, the power in each wave must decrease as the wave travels.

The fast wave has a positive power, and so for its power to decrease the amplitude must decrease. Thus, in the resistive-wall region the amplitude of the fast space-charge wave decreases exponentially with distance.

Because the slow space-charge wave has a negative power, its power can decrease only if the amplitude of the wave increases, so that the power flow becomes less (more negative). Thus, in the resistive-wall region the amplitude of the slow space-charge wave increases exponentially with distance; the wave has a negative attenuation; it is amplified as it travels.

If we put the output resonator far from the input resonator, the amplitude of the fast space-charge wave will be very small there, but the amplitude of the slow space-charge wave may be very large. Its current will produce a large voltage across the output resonator. As in the case of the klystron, this voltage will increase the amplitude of the slow space-charge wave, thus decreasing the power flow in the electron stream.

The resistive wall amplifier has a feature which the klystron lacks; the process of amplification involves an actual growing wave along the electron stream.

THE EASITRON; INCREASING WAVE IN A LOSSLESS SYSTEM

Consider a tube somewhat similar to the resistive wall amplifier, but in which the beam is surrounded, not by a lossy tube, but by a series of pill-box resonators, as shown in Fig. 4. Imagine that the resonators are so tuned that at the operating frequency they present a lossless negative susceptance to the electron beam.

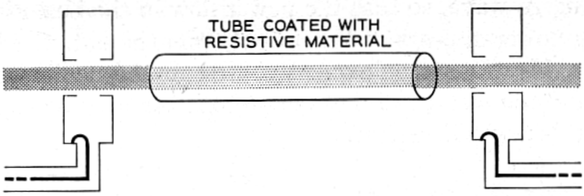

Fig. 3 — In a resistive-wall amplifier the currents excited in the lossy wall by the slow, negative-power wave decrease the power in the wave, so that the amplitude of the wave must increase.

The impedance an electron beam sees in traveling through free space or in a concentric lossless tube is capacitive. In section 1 the space-charge waves were described as involving the stored energy of the electric field, capacitive in nature, and the kinetic energy of the electrons, which has an inductive effect. We might liken the beam and its capacitive circuit to the ladder network of Fig. 5. We know that such a network supports waves.

When the charge of the beam sees a negative susceptance, the behavior is much as if the capacitances in the ladder network of Fig. 5 were negative.* In this case the waves characteristic of the circuit are not traveling waves, but are a pair of waves, one of which decays with distance and one of which increases with distance. Neither has any net stored energy.

We can express the propagation constants of the waves much as in the section on, "Space-Charge Waves," but the effective plasma frequency ω_q is now imaginary; we will call it $j\omega_q'$. The phase constants β_1 and β_2

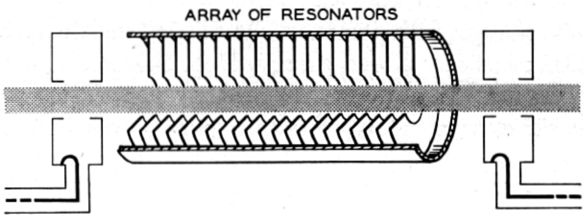

Fig. 4 — In the easitron, resonators surrounding the beam change the susceptance the electrons see from positive to negative. The system no longer supports two traveling waves, but rather, a growing and a decaying wave.

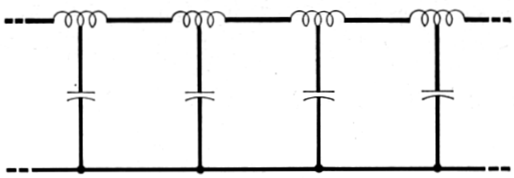

Fig. 5 — If the capacitances in this ladder network were negative it would support growing and decaying waves rather than traveling waves.

* Some care must be used in arriving at proper equivalent circuits. For instance, neither of the electric waves on a ladder network has negative energy if the network is set in motion, but we have seen that one of the longitudinal space-charge waves does have negative energy. If both the capacitances and the inductances of a ladder network are negative, the waves on the network will have negative energies.

become

$$\beta_1 = \frac{\omega}{u_0} + j\frac{\omega_q'}{u_0}$$

$$\beta_2 = \frac{\omega}{u_0} + j\frac{\omega_q'}{u_0}$$

The characteristic impedances become

$$K_1 = -j2\frac{\omega_q'}{\omega}\frac{V_0}{I_0}$$

$$K_2 = j2\frac{\omega_q'}{\omega}\frac{V_0}{I_0}$$

The fact that the characteristic impedances of the two waves are imaginary means that neither of the waves alone has any power flow. Neither of the waves can very well carry power. The amplitudes change with distance; hence for each wave Ui^* and iU^* increase or decrease with distance. But, the circuit and the electron beam are lossless, and the power cannot change with distance. As the waves do have a group velocity, neither has any stored energy. Does this mean that the beam cannot carry any power? The beam can carry power, just as a filter in its stop band can carry some power from a source at one end to a resistive load at the other end. The power flow is still given properly in terms of the total current i and the total voltage U by the same expression used in section 1. Suppose that the two waves have currents i_1 and i_2. Then the total power flow is

$$P = \tfrac{1}{2}[(i_1 + i_2)(K_1^*i_1^* + K_2^*i_2^*) + (i_1^* + i_2^*)(K_1i_1 + K_2i_2)]$$

$$P = \tfrac{1}{2}[(K_1 + K_1^*)(i_1i_1^*) + (K_2 + K_2^*)i_2i_2^* + i_1i_2^*(K_1 + K_2^*) + (i_1i_2^*(K_1 + K_2^*))^*]$$

First consider the case in which ω_q is real and for which the characteristis impedances are real and

$$K_1 = -K_2$$

In this case

$$P = K_1i_1i_1^* + K_2i_2i_2^*$$

This is the familiar case of unattenuated waves. The total power is the power of each wave calculated individually.

Let us now consider the case in which the effective plasma frequency is imaginary. In this case we can write

$$K_1 = -jK_0$$

$$K_2 = +jK_0$$

where K_0 is real. We have

$$P = [-ji_1i_2^*K_0 + (-ji_1i_2^*K_0)^*]$$

Either wave alone carries no power; there is power flow only when the two waves are present simultaneously. The two waves vary with distance as

$$e^{-j(\omega/u_0)z}e^{(\omega_q'/u_0)z}; \qquad e^{-j(\omega/u_0/z)}e^{-(\omega_q'/u_0)z}$$

so the $i_1i_2^*$ is constant with distance. If this were not so the power would change with distance, but as the resonators have been assumed to be lossless, neither taking power from the beam nor adding power to the beam, this is impossible. Thus, in a lossless system an increasing wave is always one of a pair, and the other member decreases with distance in such a way as to keep the product of the amplitudes of the two waves constant with distance. Neither the increasing wave alone nor the decreasing wave alone carries any power, but the two together can carry power.

We will note that in the easitron the direction of the group velocity, that is, the direction of causality, is the direction of electron flow. Thus, the waves are both set up at the input resonator; it is there that boundary conditions on both current and voltage must be satisfied.

COUPLING OF MODES OF PROPAGATION

We know that waves which increase and decrease exponentially with distance are characteristic of a ladder network in which the susceptances of the shunt and series arms have the same signs. They occur in other networks as well. Consider a smooth transmission line loaded periodically with shunt capacitances, as shown in Fig. 6. Each capacitance reflects

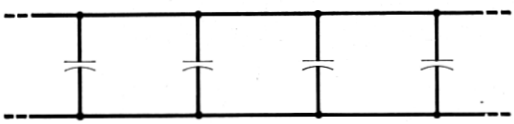

Fig. 6 — Capacitances connected across a smooth transmission line periodically couple the forward and backward waves and produce stop bands characterized by growing and decaying waves.

part of a wave approaching it. In other terms, each capacitance acts to couple one mode or wave (say the forward wave) to another (say, the backward wave). When the distance between the capacitances is such that the couplings reinforce, that is, near a half wavelength in this case, the system is a filter in its stop band; it does not transmit traveling waves, but supports rather a wave which increases exponentially with distance and a wave which decays exponentially with distance. Neither of these waves alone carries any power.

The space-charge waves of an electron stream can be coupled to one another, to a space-charge wave of another stream, or to an electromagnetic wave. In any of these cases we can have increasing waves.

THE SPACE-CHARGE-WAVE AMPLIFIER

Consider an electron beam surrounded by a series of metallic tubes $A, B, A, B \cdots$, alternately at different potentials with respect to the cathode from which the electrons come, as shown in Fig. 7. The impedances of the space-charge waves will be different in tubes A from what they are in tubes B. The behavior of this system is much like that for the transmission line system shown in Fig. 8, in which we have alternate line sections of different characteristic impedances K_A and K_B. We know that such a series of line sections forms a filter with stop bands.

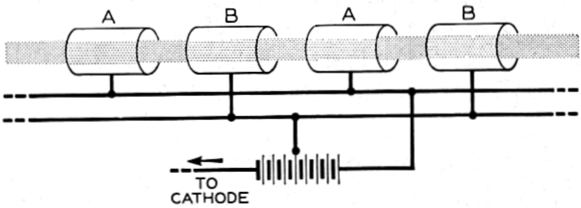

Fig. 7 — The impedances of waves in an electron beam passing through electrodes at alternately higher and lower potentials differ in regions of different potentials. This can result in stop bands characterized by growing and decaying waves. Such a device is a space-charge-wave amplifier.

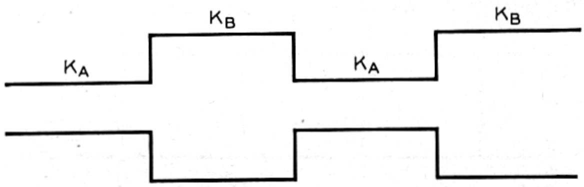

Fig. 8 — A transmission line with alternating sections of impedances K_A and K_B is somewhat analogous to the space-charge-wave amplifier.

In the case of the space-charge-wave structure of Fig. 7, the stop band occurs for conditions near that in which for both sections A and B the section lengths L_A and L_B are such that

$$2 \frac{\omega_{qA}}{u_0} L_A = \pi$$

$$2 \frac{\omega_{qB}}{u_0} L_B = \pi$$

Here ω_{qA} and ω_{qB} are the effective plasma frequencies for sections A and B.

A structure such as that of Fig. 7 can be interposed between input and output circuits, such as resonant cavities, to give a space-charge-wave amplifier dependent for its action on the growing wave of the pair.

THE TRAVELING-WAVE TUBE

In the space-charge-wave tube, the two waves which are coupled together have different velocities, just as the forward and backward waves on an electron stream have different velocities. Hence, they can be coupled strongly only through the use of some periodic structure in which the period is related to the difference in phase constants of the two waves.

In a traveling-wave tube we can have coupling between a space-charge wave and a wave traveling on a circuit, and both of the waves can have velocities which are nearly or exactly the same.

Here we must consider two different cases. If both of the coupled waves carry power in the same direction (that is, if the power is positive for both, or negative for both), coupling cannot result in a stop band, but only in transfer of power between one wave and the other. In order to have a stop band, power which we try to send in on one wave must come back to us on the other. Hence, to produce a stop band and gaining waves, the two coupled waves must carry powers with opposite signs.

A traveling-wave tube can consist of a helix of wire, which can support a slow electromagnetic wave, surrounding an electron beam, as shown in Fig. 9.

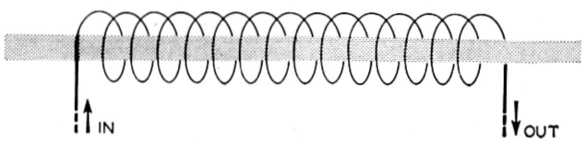

Fig. 9 — The vital elements of a traveling-wave amplifier are an electron stream and a slow-wave circuit which may be a helix surrounding the electron stream.

Traveling-wave tubes really involve at least four waves: two space-charge waves and two circuit waves. Usually, the backward circuit wave is so far out of synchronism with the space-charge waves that we can neglect its coupling with them. Further, if the space-charge waves are well separated in velocity, that is, when ω_q is large enough, then when one is coupled to the circuit wave the other isn't, and so we can get some idea of traveling-wave tube operation by considering waves in pairs. The simple mathematics of such coupling is given in Appendix D.

In Fig. 10, the behavior of various phase constants, plotted as a function of ω/u_0, is shown qualitatively. Here ω is radian frequency and u_0 is electron velocity. We may consider that ω/u_0 is varied by changing the electron velocity u_0 and keeping the frequency ω constant. The horizontal line β_c is the phase constant of the forward circuit wave in the absence of electrons, or when the coupling to the electrons is zero. β_c does not change with electron velocity. β_s and β_f are the phase constants of the slow and fast space-charge waves, respectively, with zero coupling to the circuit wave. For the slow space-charge wave, the power flow is negative, while for the circuit wave and the fast space-charge wave the power flow is positive. Thus, for coupling between the slow

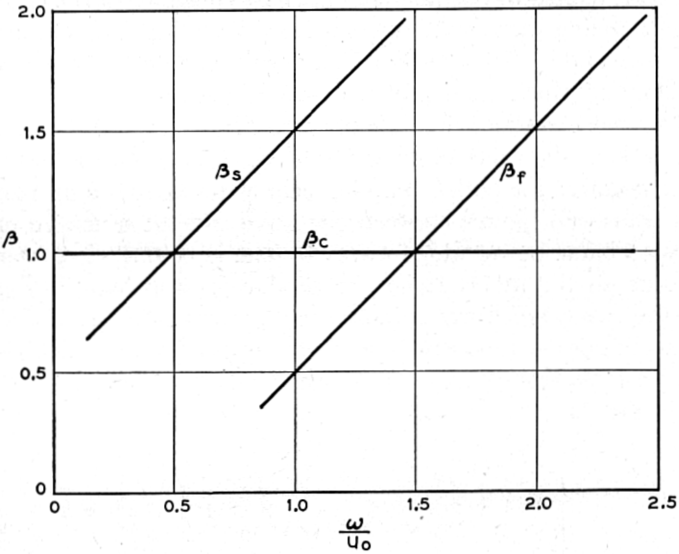

Fig. 10 — Suppose that at a constant radian frequency ω we change the electron velocity u_0 in a traveling-wave tube. If the waves of the electron stream were not coupled to the waves of the helix, the phase constants, β_c of the forward circuit wave, β_s of the slow wave, and β_f of the fast wave, would vary approximately as shown.

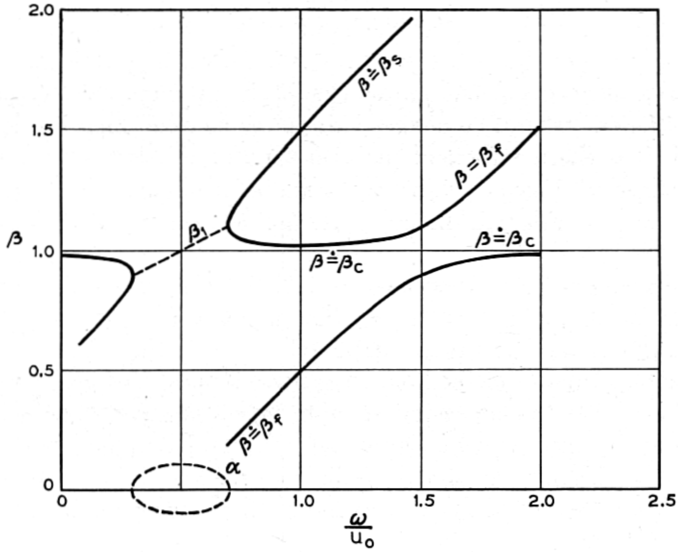

Fig. 11 — Because of coupling of the space-charge waves to the forward circuit wave, gain is produced near $\beta_c = \beta_s$, while the curves sheer off from one another near $\beta_c = \beta_f$.

space-charge wave and the circuit wave we can have a stop band, while for coupling between the circuit wave and the fast space-charge wave we cannot.

The consequences of the couplings between the circuit wave and the space-charge waves near the intersections of β_c with β_s and β_f are illustrated in Fig. 11.

We see from Fig. 11 that near synchronism between the circuit wave and the fast space-charge wave ($\beta_c = \beta_f$ for no coupling) these waves combine so that for any given value of ω/u_0 there are always two distinct real values of β. This is typical for coupling between modes with power flows of the same sign. At $\beta_c = \beta_f$ each of the two mixed waves has equal energies in the circuit and in the electron stream.

Near synchronism between the circuit wave and the slow space-charge wave ($\beta_c = \beta_s$ in absence of coupling) these two waves combine so that over a range of ω/u_0 near $\beta_c = \beta_s$, β has two complex values with the same real part and with equal and opposite imaginary parts. We can write this as

$$\beta = \beta_1 \pm j\alpha; \qquad -j\beta = -j\beta_1 \mp \alpha$$

This corresponds to an attenuated and a growing wave with the same phase velocity. In Fig. 11, β_1 and α are plotted as dashed lines.

Over the range of ω/u_0 for which the waves are attenuated ($\alpha \neq 0$) the net power flow in each of the modes is zero. The power flow in the electron stream is equal and opposite to that in the circuit. Such behavior is characteristic when two modes with power flows of opposite signs are coupled. It is characteristic of the stop band of an electric wave filter.

The curves of Fig. 11 exhibit the same behavior that has been found by other means, although similar curves are sometimes plotted somewhat differently.

When an input signal is applied to the helix of a traveling-wave tube, all three forward waves are set up. The increasing wave grows until it predominates, and it forms the amplified output of the tube.

The total ac power of the increasing wave is zero. How can we obtain power from it? In the increasing wave we have a positive electromagnetic power flow in the circuit and an equal negative power flow in the electron stream. If we terminate the helix we can draw off the electromagnetic power; the electron stream is left with less power than it had on entering the helix.

DOUBLE-STREAM AMPLIFIERS

A double-stream amplifier makes use of two streams of electrons which have different velocities, as shown in Fig. 12. The behavior of a double-stream amplifier is very similar to that of a traveling-wave tube. In such a device each electron stream supports a slow, negative-energy wave and a fast, positive-energy wave. At a constant frequency ω let the velocity u_1 of one stream be kept constant and let the velocity u_2 of the other stream be varied. The behavior of the phase constants β of the waves is shown qualitatively in Fig. 13. β_{s1} and β_{f1} are the phase constants of the slow and fast waves of the constant-velocity stream, and β_{s2} and β_{f2} are the phase constants of the slow and fast waves of the stream whose velocity is changed. There are two ranges of velocity u_2 for which gain is obtained; for u_2 a little larger or a little smaller than u_1.

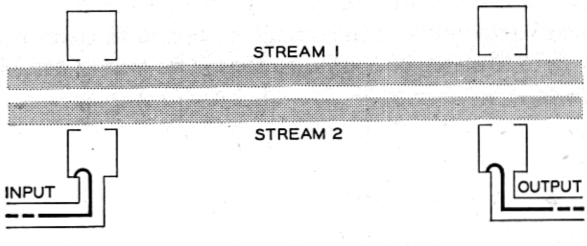

Fig. 12 — Two nearby electron streams of different velocities u_1 and u_2 constitute a double-stream amplifier.

NOISE WAVES ON ELECTRON STREAMS

Consider the electrons of the beam as they leave the cathode. If the velocity distribution is Maxwellian, and if the electrons leave independently, there will be a mean-square fluctuation in convection current, i^2, given by

$$i^2 = 2eI_0B$$

and an uncorrelated mean square fluctuation in ac velocity, v^2, given by

$$v^2 = (4 - \pi)\left(\frac{e}{I_0}\right)\left(\frac{kT_c}{m}\right)B$$

Here I_0 is beam current, e and m are electron charge and electron mass' k is Boltzmann's constant, T_c is cathode temperature and B is bandwidth·

Usually, space-charge-limited flow is used. In this case the beam current is only a part of the emitted current; the rest is turned back at the potential minimum. In this case we may use the above relations, counting I_0 as the beam current, as some sort of approximation for the current passing the potential minimum.

The wave picture we have been discussing may be seriously inaccurate near the cathode where the relative spread in electron velocities is large. Suppose that we hope for the best and apply it. We find that in the most general case our electron stream will have on it a noise standing-wave pattern. If $i_{\min}$ and $i_{\max}$ are the minimum and the maximum noise currents,

$$\frac{|i_{\min}||i_{\max}|}{2eI_0B} = \frac{1}{2}\alpha\left(\frac{\omega}{\omega_q}\right)\left(\frac{kT_c}{eV_0}\right)$$

Here α is a constant near to unity.

The noise pattern is made up of two uncorrelated noise standing-wave patterns, one from i at the cathode and the other from v at the cathode; these patterns have amplitudes i_1 and i_2 at their maxima; the minima are of course zero. We have

$$|i_{\min}||i_{\max}| = |i_1||i_2|\sin\Psi$$

Here Ψ is the relative phase angle of the standing-wave patterns associated with i_1 and i_2. That is, if the maximum of the i_2 pattern is at that of i_1, $\Psi = 0$, while if the maximum of the i_2 pattern is midway between maxima of i_1, then $\Psi = \pi/2$.

The first of these theorems says something about the noise current at the maximum and that at the minimum, but it does not directly say how

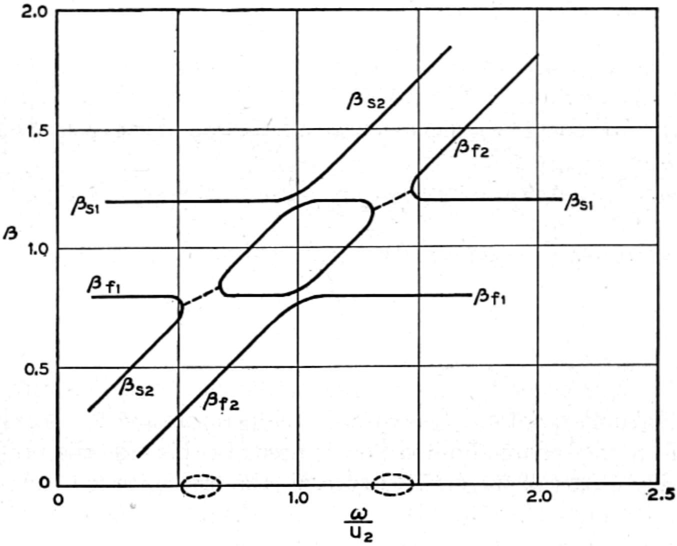

Fig. 13 — In a double-stream amplifier, gain is obtained when the phase constants of the slow wave of the faster stream and the fast wave of the slower stream are nearly equal.

large the maximum is. For an ordinary two-potential electron gun, i_{max} is very large compared with i_{min} .

NOISE DEAMPLIFICATION

Early traveling-wave tubes made use of a two-potential electron gun spaced a critical distance from the circuit, as shown in Fig. 14. More recently it has been found possible to reduce the noise figure considerably by the use of space-charge-wave amplification, as discussed in the section on "The Space-Charge-Wave Amplifier." The structure used is indicated in Fig. 15. The gun has a low-potential anode followed by a

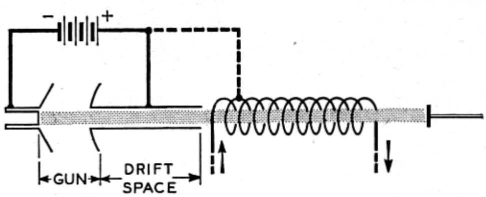

Fig. 14 — When a simple, two-potential electron gun is used, the noise figure of a traveling-wave tube can be optimized by adjusting the drift-space between the gun anode and the helix.

drift tube. At the point where the noise current is a minimum the voltage is "jumped" to the helix voltage. A second drift tube follows, so that there is a critical distance between the jump and the helix.

The effect of this "voltage jump" gun is to deamplify the component of the space-charge waves which is associated with the noise current at the current maximum. In space-charge-wave amplifier terms, this component sets up the decreasing wave only. Thus, in the second drift tube the ratio $| i_{max}/i_{min} |$ is smaller than in the first.

By using a single velocity jump, traveling-wave tubes with noise figures around 8 db have been made.

The use of more velocity jumps has been proposed. It can be shown, however, that as i_{max} is deamplified, i_{min} must be amplified. This sets a theoretical limit of around 6 db to the noise figure attainable by means of space-charge-wave deamplification alone.

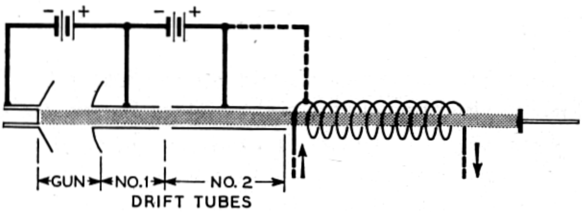

Fig. 15 — When a two-potential or "velocity jump" gun is used, the noise figure can be reduced by space-charge-wave deamplification of the noise on the electron stream.

NOISE CANCELLATION

It would be highly desirable to build a traveling-wave tube such that the electromagnetic input would excite an increasing wave, but the noise in the electron stream would excite only some combination of the decreasing and the unattenuated waves. If we succeeded in this, the noise introduced by the tube could be made as small relative to the signal as desired, merely by making the tube long enough. Can we accomplish this by means of some special structure near the input end of the tube?

We can represent the noise on the electron stream at some reference point by means of a velocity fluctuation v and a current fluctuation i; we have seen that neither can be zero. Because the system is linear, superposition applies, and the amplitudes of the growing, attenuated and unattenuated waves which are excited are the sums of the amplitudes excited by i and v independently.

Suppose that $v = 0$. Then the beam carries no power. Thus, i cannot excite the unattenuated wave, for that wave carries power. Let us assume that it excites the decreasing wave alone, which, when present alone, carries no power. So far there is no contradiction, and we can believe that it is possible to arrange matters so that the current alone does not excite the increasing wave.

Suppose we have so arranged matters that i does excite the decreasing wave only. Consider what happens when $i = 0$. Can v alone excite the decreasing wave only if i alone excites the decreasing wave only? If it can, then v and i together must excite the decreasing wave only. But suppose v and i are of the same frequency and in phase. Then the beam carries power. But, the decreasing wave alone cannot carry power, and hence what we have assumed is impossible. If the i excites the decreasing wave only, then v must excite at least a component of the growing wave. Hence, we cannot cancel out the noise from the beam completely.

FINAL COMMENTS

We have seen that the properties of space-charge waves and the behavior which must follow when space-charge waves are coupled to other space-charge waves or to circuit waves can be used to explain the operation of seemingly diverse types of microwave tubes. The wave picture gives a clear and quantitative picture of energy relations and power flow. It enables us to understand simply the effect of thermal velocities on the operation of tubes through their effect on the phase constants of the space-charge waves. It is useful in detailed considerations of noise, and in one case it has enabled us to draw a general conclusion without resorting to formal mathematical manipulation. It may well be that the wave picture can be of further use both in calculating detailed behavior of tubes and in understanding their general properties.

Appendix A

SPACE-CHARGE WAVES

Consider a narrow electron stream in which we may assume that electron velocity and charge density do not vary across the stream, and in which the electrons are free to move in the z-direction only. An axially symmetrical electron focusing system immersed, cathode and all, in a strong magnetic field approximates this.

Let all ac quantities contain the factor

$$e^{-j\beta z}e^{j\omega t}$$

and let the total charge density, current and electron velocity be made up of dc and ac parts as follows:*

charge density: $-\rho_0 + \rho$

convection current density: $-I_0 + i$

velocity: $u_0 + v$

Here ρ_0, I_0 and u_0 are positive dc quantities. The quantities on the right are the ac components.

We have from the definition of convection current

$$(-I_0 + i) = (-\rho_0 + \rho)(u_0 + v) \tag{A1}$$

In the case of very low level operation, we neglect products of ac quantities in comparison with products of ac and dc quantities. Doing this, we obtain from (A1) the dc and ac convection currents

$$I_0 = \rho_0 u_0 \tag{A2}$$

or

$$i = -\rho_0 v + u_0 \rho \tag{A3}$$

$$\rho = \frac{i + \rho_0 v}{u_0} \tag{A4}$$

We can apply the continuity equation, or, the equation of conservation of charge, to the ac convection current

$$\begin{aligned} \frac{\partial i}{\partial z} &= -\frac{\partial \rho}{\partial t} \\ \beta i &= \omega \rho \end{aligned} \tag{A5}$$

$$\begin{aligned} \beta i &= \left(\frac{1}{u_0}\right)(j\omega i - \rho_0 j\omega v) \\ i &= \frac{-\omega \rho_0 v}{\omega - \beta u_0} \end{aligned} \tag{A6}$$

Thus, if we have a wave with a given phase constant β, and if we know ρ_0 and ω, (A6) gives the convection current in terms of ac electron velocity. How can we find what β will be? To find this we must consider the effect of the electric field on the electrons. Consider an ac electric field E_z in the z direction, which also varies with time and distance as

* It will be convenient elsewhere to use $-I_0$ and i as currents rather than current densities and $-\rho_0$ and ρ as charge per unit length.

do the other ac quantities. We can write

$$\frac{dv}{dt} = -\frac{e}{m} E_z \tag{A7}$$

Here e/m, the charge-to-mass ratio of the electron, is taken as a positive quantity.

In (A7), dv/dt is the rate of change of v with respect to t for a single electron; that is dv/dt observed riding along with the electron. If we ride along with the electron for a time dt we move along distance dz

$$dz = (u_0 + v)\, dt$$

For small signals we neglect v in this expression and write

$$dz = u_0\, dt$$

Hence, the total change dv in the velocity of the electron in the time dt is

$$dv = \frac{\partial v}{\partial t}\, dt + \frac{\partial v}{\partial z}\, u_0\, dt$$

Hence, we find that

$$\frac{dv}{dt} = j(\omega - \beta u_0)v \tag{A8}$$

Using (A7) and (A8), we see that

$$v = \frac{j \dfrac{e}{m} E_z}{(\omega - \beta u_0)} \tag{A9}$$

We can combine (A9) with (A6) and write for the convection current density

$$i = \frac{-j \dfrac{e}{m} \omega \rho_0 E_z}{(\omega - \beta u_0)^2} \tag{A10}$$

Let us now consider a special, hypothetical case in which the electric field is in the z direction only, so that there are no transverse electric fields and no transverse displacement current. Then the total ac current density i_t is the sum of the convection current density and the displacement current density, or,

$$i_t = i + j\omega\epsilon E_z$$

$$i_t = \left(\frac{-\frac{e}{m}\rho_0}{\epsilon(\omega - \beta u_0)^2} + 1\right) j\omega\epsilon E_z \tag{A11}$$

Let us use a quantity ω_p, which was long ago named the plasma frequency (radian frequency)

$$\omega_p^2 = \frac{\frac{e}{m}\rho_0}{\epsilon} \tag{A12}$$

Using ω_p, (A10) can be written as

$$i_t = \left(-\frac{\omega_p^2}{(\omega - \beta u_0)^2} + 1\right) j\omega\epsilon E_z \tag{A13}$$

According to Maxwell's equations the divergence of the total current is zero. Both components of i_t vary with z. If, as we have assumed, there is no current normal to the z direction, then i_t must be zero. If this is to be so, we must have

$$(\omega - \beta u_0)^2 = \omega_p^2$$

$$\beta = \frac{\omega}{u_0} \pm \frac{\omega_p}{u_0} \tag{A14}$$

In actual electron beams there is transverse electric field away from the beam, and hence i_t is not zero. It is found, however, that when ω_p is small compared with ω, we can write quite accurately

$$\beta = \frac{\omega}{u_0} \pm \frac{\omega_q}{u_0} \tag{A15}$$

Here ω_q, which is known as the *effective plasma frequency*, is smaller than ω_p. As ω is raised, so that the wavelength of the space-charge waves becomes smaller compared with the diameter of the electron beam, the electric field tends to become largely longitudinal and ω_q approaches ω_p.

The upper sign in (A15) gives the phase constant of the slow wave, a wave with a phase velocity less than that of the electrons. The lower sign gives that of the fast wave, a wave with a phase velocity faster than that of the electrons.

From (A15) and (A6) we note that

$$i = \pm \frac{\omega}{\omega_q} \rho_0 v \tag{A16}$$

The upper sign holds for the slow wave; the lower sign for the fast wave.

It has been convenient to use $-I_0$ and i as current densities and $-\rho_0$ and ρ as charge densities. In subsequent work and in the text, $-I_0$ and i will be used as beam current and $-\rho_0$ and ρ as charge per unit length. All the relations of this appendix except (A11)–(A13) will hold if the quantities are so interpreted.

Appendix B

POWER FLOW IN SPACE CHARGE WAVES

The purpose of this appendix is to justify the expression for power flow in the beam.

Consider that the electron beam is acted on over a short distance by an ac voltage. Imagine, for instance, that the beam passes through two very closely spaced grids which form a part of a resonator, and that a voltage ΔV appears between the grids. What does the voltage do to the beam?

The voltage ΔV changes the velocity of the electrons but it does not change the convection current. To find out how much the velocity is changed we need only consider the case in which the beam has no ac velocity on reaching the grids, since in a linear system the change will be the same in all other cases. The total velocity $u_0 + v$ is given in terms of the total accelerating voltage $V + \Delta V$ by

$$u_0 + v = \sqrt{2 \frac{e}{m} (V_0 + \Delta V)} \tag{B1}$$

We assume ΔV to be small, so that

$$v = \frac{\Delta V \frac{e}{m}}{\sqrt{2 \frac{e}{m} V_0}} = \frac{\Delta V \frac{e}{m}}{u_0} \tag{B2}$$

The change ΔU in the "voltage" U is

$$\Delta U = -\frac{u_0 v}{\frac{e}{m}} = -\Delta V \tag{B3}$$

The convection current i flows against the voltage ΔV, so that a power ΔP is transferred from the beam to the resonator which is attached to the grids.

$$\Delta P = -Re\Delta Vi^* \tag{B4}$$

Thus, the change in the power in the beam in passing through the grids must be $-\Delta P$

$$-\Delta P = Re\,(-\,\Delta Vi^*) = Re\Delta Ui^* \tag{B5}$$

$$\Delta P = -Re\Delta Ui^*$$

According to the expression we have used in calculating beam power, if the "voltage" of the beam on reaching the grids is U, and the convection current is i, then the beam power P_1 on reaching the grids is

$$P_1 = ReUi^* \tag{B6}$$

After passing through the grids, U is increased by an amount ΔU while the current is unchanged, so that the power P_2 of the beam leaving the grids is

$$P_2 = Re(U + \Delta U)i^* \tag{B7}$$

The loss of power in the beam, ΔP, is

$$\Delta P = P_1 - P_2 = -Re\Delta Ui^* \tag{B8}$$

This agrees with (B5), in which ΔP was calculated as the power lost from the beam to the resonator.

Appendix C

THE EFFECT OF THE VELOCITY DISTRIBUTION IN THE ELECTRON BEAM ON THE EFFECTIVE PLASMA FREQUENCY

Consider an electron beam in which electron motion is confined to the z-direction, and in which the electrons have a velocity spread with a mean square deviation $\langle u^2 \rangle$ about the mean value u_0. If ω_{q0} is the effective plasma frequency for $\langle u^2 \rangle = 0$, then taking the velocity spread into account, the effective plasma frequency ω_q is given approximately by

$$\omega_q^2 = \omega_{q0}^2 \left(1 + 3 \frac{\langle u^2 \rangle}{u_0^2} \left(\frac{\omega}{\omega_{q0}}\right)^2\right)$$

If the velocity distribution is the same as for electrons accelerated individually by a voltage V_0, that is, if electron interactions do not affect the velocity distribution appreciably (as they probably do not)

$$\frac{\langle u^2 \rangle}{u_0^2} = \frac{1}{4}\left(\frac{kT_c}{eV_0}\right)^2 = \frac{1}{4}\left(\frac{T_c}{11{,}600\ V_0}\right)^2$$

Here k is Boltzman's constant and T_c is cathode temperature. Thus, from this assumption

$$\omega_q^2 = \omega_{q0}^2 \left(1 + \frac{3}{4}\left(\frac{kT_c}{eV_0}\right)^2 \left(\frac{\omega}{\omega_{q0}}\right)^2\right)$$

Following our wave picture, we can take into account the thermal velocity spread by using this corrected value for the effective plasma frequency in all our formulae. For all practical purposes, the change in effective plasma frequency due to thermal velocities is negligible.

In a paper which will appear in the Journal of Applied Physics, D. A. Watkins has used a somewhat different approach in treating the effect of thermal velocities on the operation of traveling-wave tubes.

Appendix D

PHASE AND ATTENUATION CURVES FOR COUPLED MODES

When two unattenuated modes of propagation are coupled together periodically in a lossless manner, they combine to form two new modes. For each of these new modes the amplitude is changed in one period of the coupling structure by a factor

$$Me^{-j(\theta_q - \theta_p)/2} e^{j(\theta_3 + \theta_1)/2} \tag{D1}$$

where M is a root of

$$M^2 - 2\sqrt{1 \mp k^2} \cos \frac{(\theta_q - \theta_p + \theta_1 - \theta_3)}{2} + 1 = 0 \tag{D2}$$

Here k is a coupling coefficient which is zero for zero coupling. The upper sign applies if the power flow in the two modes have the same signs while the lower sign applies if the power flows have opposite signs. θ_q and θ_p are phase lags per coupling period associated with the two original modes and θ_1 and θ_2 are phase angles associated with the coupling device.

We can treat the case of continuous coupling by letting the period of coupling L be very short, the angles θ_q, θ_p, θ_1, θ_3 be very small, and the coupling per period, k, be very small. In this case the cosine can be represented by the first terms of a power series and we find that the phase constants β of the modes are given by

$$\beta = \frac{\beta_a + \beta_b}{2} \pm \left(\frac{\beta_a - \beta_b}{2}\right)\sqrt{1 \pm \left(\frac{2K}{\beta_a - \beta_b}\right)^2} \qquad \text{(D3)}$$

Here β_a and β_b are the phase constants for $K = 0$ (zero coupling)

$$\beta_a = \frac{\theta_p - \theta_1}{L} \qquad \text{(D4)}$$

$$\beta_b = \frac{\theta_q - \theta_3}{L} \qquad \text{(D5)}$$

and K is the coupling per unit length

$$K = \frac{k}{L} \qquad \text{(D6)}$$

As before, the upper sign in the radical applies when the power flows have the same signs and the lower sign when the power flows have opposite signs.

In applying (D3) to the case of traveling-wave tubes and backward-wave oscillators, the effect of all but two modes was of course neglected when the two phase constants would have had nearly the same value in the absence of coupling; the curves for such regions were then joined smoothly to give the overall plots of Figs. 11 and 13.

In Fig. 11 the parameters chosen arbitrarily were:

$$\beta_c = 1$$

$$\beta_s = \omega/u_0 + \tfrac{1}{2}$$

$$\beta_f = \omega/u_0 - \tfrac{1}{2}$$

$$K = 0.1$$

The complex portion of the phase constant, or, the real portion of the propagation constant, in a stop band caused by the coupling of two modes with power flows of opposite signs is designated by α and is plotted as the dashed ellipses about the horizontal axis.

Appendix E

This appendix comments briefly on various sections and cites references, which are listed at the end of the appendix. The list of references is not exhaustive, but it should enable the interested reader to follow work back to its source.

1. Space-Charge Waves

Space-charge waves of the general sort considered are related to the plasma oscillations of Tonks and Langmuir.[1] Waves in long beams were first discussed by Hahn[2] and Ramo.[3] The effects of a velocity distribution are discussed by Pierce[4] and by Bohm and Gross.[5] The negative energy of the slow space-charge wave has been reported by Chu[6] and by Walker.[7] Chu gave the effective "voltage" U and the characteristic impedance K for the waves.

2. The Klystron

Beck[8] gives an adequate description of and references to klystrons.

3. The Resistive Wall Amplifier

The effect has been discussed by Pierce,[9] and a tube using it has been described by Birdsall, Brewer and Haeff.[10]

4. The Easitron; Increasing Wave in a Lossless System

The original easitron was a tube built by L. R. Walker at Bell Telephone Laboratories; it was a 3-cm tube using half-wave wires as resonant elements. It has not been described in the literature. Pierce has discussed the operation of this sort of multi-resonator klystron on page 195 of *Traveling Wave Tubes*[11] and elsewhere.[9]

5. Coupling of Modes of Propagation

The operation of traveling-wave tubes was first explained in terms of coupling between an electromagnetic wave and a space-charge wave by C. C. Cutler in unpublished work. Mathews has made an analysis in these terms.[13] Such coupling has been considered in general terms by Pierce.[12]

6. The Space-Charge-Wave Amplifier

This tube was invented by Tien, Field and Watkins[14] and is described in more detail by Tien and Field.[15]

7. The Traveling Wave Tube

Adequate descriptions and references are available in work by Kompfner,[16] Pierce,[11] and Beck.[8]

8. Double-Stream Amplifiers

Descriptions and references are given by Pierce[11] and by Beck.[8]

9. Noise Waves in Electron Streams

Cutler and Quate have published experimental results.[17] The theorems quoted are given by Pierce.[18]

10. Noise Deamplification

This was suggested by Tien, Field and Watkins[14] and is described in detail by Watkins[19] and Peter.[20]

11. Noise Cancellation

Noise cancellation was first proposed by C. F. Quate.[21]

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