Circular Electric Wave Transmission Through Serpentine Bends

By H. G. UNGER

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An otherwise straight waveguide line with equally spaced discrete supports may deform elastically into a serpentine bend under its own weight. The TE_{01} *wave couples in such bends to the* TM_{11} *and* TE_{1n} *waves. The general solution of coupled lines with varying coupling coefficient is applied to a serpentine bend by an iterative process, and evaluated for the elastic curve resulting from a periodically supported line.* TE_{01}-TM_{11} *coupling causes only a small increase in* TE_{01} *attenuation. Mode conversion to* TE_{1n} *waves can become seriously high at certain critical frequencies when the supporting distance is a multiple of the beat wavelength. In a copper pipe of* $2\frac{3}{8}$ *inch O.D. and 2 inch I.D., the mode conversion to the* TE_{12} *wave at critical frequencies near 5-mm wavelength causes a* TE_{01} *attenuation increase of 90 per cent and a spurious mode level of −7 db. These mode conversion effects can be controlled effectively by inserting mode filters.*

I. INTRODUCTION

In curved sections of round waveguide the TE_{01} - wave couples to the TM_{11} and TE_{1n} waves, and power is converted to these waves when the TE_{01} wave is transmitted through bends. A form of bend which is inherently present even in an otherwise straight and perfect line is the serpentine bend, Fig. 1. Between discrete supports the pipe is deflected by the force of its own weight. The resulting curve is well known from the theory of elasticity. The curvature varies along the axis following essentially a square law. The minimum bending radius occurs at the supports. For the practical example of a copper pipe of $2\frac{3}{8}$-inch O.D., 2.00-inch I.D. and a supporting distance of 15 ft, this minimum bending radius is 992 ft. A uniform bend of this radius would convert most of the power incident in the TE_{01} wave to the TM_{11} wave after a certain length of bend.

Fortunately a serpentine bend with the same order of bending radius does not affect circular electric wave transmission as seriously as does a

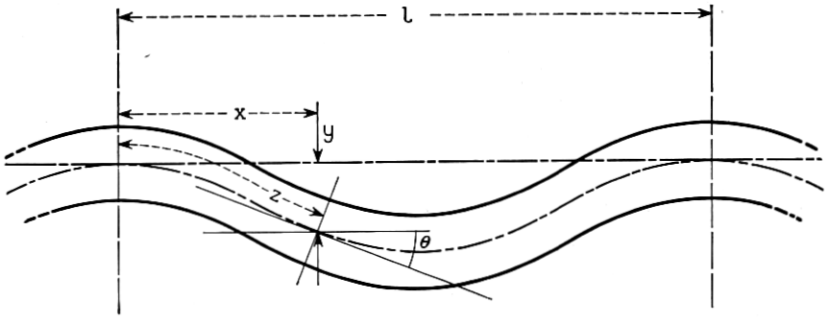

Fig. 1 — Serpentine bends.

uniform bend with the same bending radius. This will be shown in the following analysis.

A treatment of serpentine bends given previously by Albersheim[1] considers only circular and sinusoidal curvatures. Furthermore, it does not show all the effects of serpentine bends that we are interested in. We shall present a more general and complete analysis here. The only restriction we have to make is that the power exchanged in one section of the serpentine bend from the TE_{01} mode to any of the coupled modes or vice versa is small compared to the power in the mode from which it has been abstracted. The curvature may be any function of distance along the serpentine bend.

The general results indicate that normally a serpentine bend causes an additional attenuation to the TE_{01} mode. Part of the TE_{01} power which travels temporarily in one of the coupled modes suffers the higher attenuation of this coupled mode. Formulae for this increase in attenuation constant are obtained for periodically supported guides from the deflection curve given by the theory of elasticity.

The coupling between the TE_{01} and TM_{11} waves causes only a very slight increase in TE_{01} attenuation, since the difference in propagation constant for these two modes is very small. In fact, if there were no difference in propagation constant, as in the round guide of infinite wall conductivity, coupling between TE_{01} and TM_{11} waves in serpentine bends would not affect the TE_{01} transmission at all.

Coupled modes which have a larger difference in phase constant than TM_{11} but still are close to TE_{01} cause a serious increase in TE_{01} attenuation at certain critical frequencies. If a multiple of the beat wavelength between the TE_{01} mode and a particular coupled mode is equal to the supporting distance, power is converted continuously into the coupled mode. For low attenuation in the coupled mode the power transfer may even be essentially complete.

II. SOLUTION OF THE COUPLED LINE EQUATIONS

In a curved round waveguide the TE_{01} mode couples to the TM_{11} mode and the infinite set of TE_{1n} waves. Consequently, the wave propagation is described by an infinite system of simultaneous first order linear differential equations. An adequate procedure is to consider only coupling between TE_{01} and one of the spurious modes at a time. Thus, the infinite system of equations reduces to the well known coupled line equations,[2]

$$\begin{aligned} \frac{dE_1}{dz} + \gamma_1 E_1 - kE_2 &= 0, \\ \frac{dE_2}{dz} + \gamma_2 E_2 - kE_1 &= 0; \end{aligned} \tag{1}$$

in which

$E_{1,2}(z)$ = wave amplitudes in mode 1 (here always TE_{01}) and mode 2 (TM_{11} or one of the TE_{1n}), respectively;

$\gamma_{1,2}$ = propagation constant of modes 1 and 2, respectively, (The small perturbation of γ_1, γ_2 caused by the coupling may be neglected here); and

$k(z) = jc(z)$ = coupling coefficient between modes 1 and 2.

In the curved waveguide the coupling is proportional to the curvature;

$$c = \frac{c_0}{R} = c_0 \frac{d\theta}{dz}, \tag{2}$$

in which R = radius of curvature, and θ = direction of guide axis. (The various coupling coefficients are listed in the appendix.) Without loss of generality we start with $\theta(0) = 0$. We will use the average propagation constant γ,

$$\begin{aligned} \gamma &= \tfrac{1}{2}(\gamma_1 + \gamma_2), \\ \Delta\gamma &= \tfrac{1}{2}(\gamma_1 - \gamma_2). \end{aligned} \tag{3}$$

Several coordinate transformations will change (1) to a form which can be solved approximately. A similar procedure has been used to solve the related problem of the tapered mode coupler.[3] With the first transformation

$$\begin{aligned} E_1(z) &= \tfrac{1}{2}e^{-\gamma z}\,[u_1(z)e^{jc_0\theta} + u_2(z)e^{-jc_0\theta}], \\ E_2(z) &= \tfrac{1}{2}e^{-\gamma z}\,[u_1(z)e^{jc_0\theta} - u_2(z)e^{-jc_0\theta}], \end{aligned} \tag{4}$$

the new coordinates satisfy the equations:

$$\begin{aligned} \frac{du_1}{dz} + \Delta\gamma e^{-j2c_0\theta}\,u_2 &= 0, \\ \frac{du_2}{dz} + \Delta\gamma e^{j2c_0\theta}\,u_1 &= 0. \end{aligned} \tag{5}$$

With the second transformation

$$\begin{aligned} u_1(z) &= \tfrac{1}{2}[v_1(z)\,e^{-\Delta\gamma z} + v_2(z)\,e^{+\Delta\gamma z}], \\ u_2(z) &= \tfrac{1}{2}[v_1(z)\,e^{-\Delta\gamma z} - v_2(z)\,e^{+\Delta\gamma z}], \end{aligned} \tag{6}$$

$v_1(z)$ and $v_2(z)$ satisfy

$$\begin{aligned} \frac{dv_1}{dz} - 2\Delta\gamma\sin^2 c_0\theta v_1 + j\Delta\gamma\sin 2c_0\theta e^{2\Delta\gamma z}v_2 &= 0, \\ \frac{dv_2}{dz} + 2\Delta\gamma\sin^2 c_0\theta v_2 - j\Delta\gamma\sin 2c_0\theta e^{2\Delta\gamma z}v_1 &= 0. \end{aligned} \tag{7}$$

With the third transformation

$$\begin{aligned} v_1(z) &= w_1(z)\exp\left(2\Delta\gamma\int_0^z \sin^2 c_0\theta\,dz'\right), \\ v_2(z) &= w_2(z)\exp\left(-\,2\Delta\gamma\int_0^z \sin^2 c_0\theta\,dz'\right). \end{aligned} \tag{8}$$

We finally get the equations

$$\begin{aligned} \frac{dw_1}{dz} + \xi_1(z)w_2 &= 0, \\ \frac{dw_2}{dz} + \xi_2(z)w_1 &= 0, \end{aligned} \tag{9}$$

in which

$$\xi_{1,2}(z) = \pm j\Delta\gamma \sin 2c_0\theta \exp\left[\pm 2\Delta\gamma\left(z - 2\int_0^z \sin^2 c_0\theta \, dz'\right)\right]. \tag{10}$$

As long as we have the condition $\int_0^z |\xi_{1,2}(z')| \, dz' \ll 1$, approximate solutions of (9) can be written down which proceed essentially in powers of $\xi_{1,2}$ as follows,

$$\begin{aligned} w_1(z) &= w_1(0) - w_2(0)\int_0^z \xi_1(z') \, dz' \\ &\qquad + w_1(0)\int_0^z \xi_1(z')\int_0^{z'} \xi_2(z'') \, dz'' \, dz', \\ w_2(z) &= w_2(0) - w_1(0)\int_0^z \xi_2(z') \, dz' \\ &\qquad + w_2(0)\int_0^z \xi_2(z')\int_0^{z'} \xi_1(z'') \, dz'' \, dz'. \end{aligned} \tag{11}$$

The new coordinates are related to the wave amplitudes by:

$$\begin{aligned} E_1(z) &= \frac{1}{2} e^{-\gamma z}\left\{w_1(z) \exp\left[-\Delta\gamma\left(z - 2\int_0^z \sin^2 c_0\theta \, dz'\right)\right] \cos c_0\theta \right. \\ &\qquad \left. + jw_2(z) \exp\left[\Delta\gamma\left(z - 2\int_0^z \sin^2 c_0\theta \, dz'\right)\right] \sin c_0\theta\right\}, \\ E_2(z) &= \frac{1}{2} e^{-\gamma z}\left\{jw_1(z) \exp\left[-\Delta\gamma\left(z - 2\int_0^z \sin^2 c_0\theta \, dz'\right)\right] \sin c_0\theta \right. \\ &\qquad \left. + w_2(z) \exp\left[\Delta\gamma\left(z - 2\int_0^z \sin^2 c_0\theta \, dz'\right)\right] \cos c_0\theta\right\}. \end{aligned} \tag{12}$$

The solution (12) in combination with (11) is general and may be applied to any form of curvature as long as the converted power remains small compared to the original power in either of the modes.

III. WAVE PROPAGATION IN SERPENTINE BENDS

If we apply (12) to a section of a serpentine bend with the length l we have $\theta(l) = 0$. The output amplitudes are related to the input amplitudes of both modes by a transmission matrix

$$E_i(l) = \| T \| E_i(0). \tag{13}$$

The elements of $\| T \|$ are obtained from (11) and (12):

$$
\begin{aligned}
T_{11} &= \left[1 + \int_0^l \xi_1(z) \int_0^z \xi_2(z')\, dz' dz\right] \\
&\qquad \cdot \exp\left[-\gamma_1 l + 2\Delta\gamma \int_0^l \sin^2 c_0\theta\, dz\right], \\
T_{21} &= -\int_0^l \xi_1(z)\, dz \exp\left[-\gamma_1 l + 2\Delta\gamma \int_0^l \sin^2 c_0\theta\, dz\right], \\
T_{12} &= -\int_0^l \xi_2(z)\, dz \exp\left[-\gamma_2 l - 2\Delta\gamma \int_0^l \sin^2 c_0\theta\, dz\right], \\
T_{22} &= \left[1 + \int_0^l \xi_2(z) \int_0^z \xi_1(z')\, dz'\, dz\right] \\
&\qquad \cdot \exp\left[-\gamma_2 l - 2\Delta\gamma \int_0^l \sin^2 c_0\theta\, dz\right].
\end{aligned} \tag{14}
$$

For a line of iterative serpentine bends we apply the rules of matrix calculus. If $E_{10} = 1$ and $E_{20} = 0$ at the input of the first section, then the output amplitudes of the n^{th} section are:

$$
\begin{aligned}
E_{1n} &= \frac{1}{2}\left[1 + \frac{T_{11} - T_{22}}{\sqrt{(T_{11} - T_{22})^2 + 4T_{12}T_{21}}}\right] e^{-ng_1} \\
&\qquad + \frac{1}{2}\left[1 - \frac{T_{11} - T_{22}}{\sqrt{(T_{11} - T_{22})^2 + 4T_{12}T_{21}}}\right] e^{-ng_2}, \\
E_{2n} &= \frac{T_{12}}{\sqrt{(T_{11} - T_{22})^2 + 4T_{12}T_{21}}} [e^{-ng_1} - e^{-ng_2}],
\end{aligned} \tag{15}
$$

where

$$
e^{-g_{1,2}} = \tfrac{1}{2}[T_{11} + T_{22} \pm \sqrt{(T_{11} - T_{22})^2 + 4T_{12}T_{21}}]. \tag{16}
$$

Two limiting cases for the expressions (15) and (16) are of special interest:

1. $| T_{11} - T_{22} |^2 \gg 4 | T_{12} T_{21} |$

$$
\begin{aligned}
e^{-g_1} &= T_{11} + \frac{T_{12}T_{21}}{T_{11} - T_{22}} \\
E_{1n} &= e^{-ng_1} - \frac{T_{12}T_{21}}{(T_{11} - T_{22})^2} [e^{-ng_1} - e^{-ng_2}] \\
e^{-g_2} &= T_{22} - \frac{T_{12}T_{21}}{T_{11} - T_{22}} \qquad E_{2n} = \frac{T_{12}}{T_{11} - T_{22}} [e^{-ng_1} - e^{-ng_2}]
\end{aligned} \tag{17}
$$

2. $4 | T_{12} T_{21} | \gg | T_{11} - T_{22} |^2$

$$
\begin{aligned}
e^{-g_{1,2}} &= \frac{T_{11} + T_{22}}{2} \pm \sqrt{T_{12}T_{21}} \\
E_{1n} &= \tfrac{1}{2}(e^{-ng_1} + e^{-ng_2}) \\
E_{2n} &= \frac{1}{2} \sqrt{\frac{T_{12}}{T_{21}}} (e^{-ng_1} - e^{-ng_2}).
\end{aligned} \tag{18}
$$

In case 1 the wave amplitude E_{1n} is only affected by a slight change of the propagation constant. The small additional term in the expression for E_{1n} can usually be neglected. The power in the wave E_{2n} is small compared to the E_{1n} power; T_{12} is usually of the same order of magnitude as T_{21}.

In case 2, however, a complete power transfer between E_{1n} and E_{2n} occurs cyclically if the loss is sufficiently low. Consequently, condition (18) has to be carefully avoided. Rather, condition (17) must always be satisfied.

IV. SERPENTINE BENDS FORMED BY ELASTIC CURVES

A section of the waveguide line between two supports deforms like a beam fixed at both ends. Under its own weight, w per unit length, such a beam will bend and form an elastic curve, Fig. 1, whose deflection from a straight line is given by:

$$y = \frac{wl^4}{24EI} \frac{x^2}{l^2} \left(1 - \frac{x}{l}\right)^2, \tag{19}$$

in which E = modulus of elasticity of the beam, and I = moment of inertia of the beam. Since we are concerned with small deflections y only, we have $x = z$ and $\theta = dy/dx$. Hence,

$$\theta = d\left(\frac{z}{l} - 3\frac{z^2}{l^2} + 2\frac{z^3}{l^3}\right), \tag{20}$$

in which $d = wl^3/12EI$. Introducing the elastic curve (20) into the transmission matrix (10) and (14) and performing the integrations with sin $c_0\theta = c_0\theta$ we get for the elements of the transmission matrix:

$$\begin{aligned} T_{11} &= \exp\left[-\gamma_1 l + \Delta\gamma l \frac{c_0^2 d^2}{105}\right]\left\{1 + \frac{c_0^2 d^2}{4\Delta\gamma^6 l^6}\left[9 - 3\Delta\gamma^2 l^2 + \Delta\gamma^4 l^4 \right.\right. \\ &\qquad \left.\left. + \frac{2}{5}\Delta\gamma^5 l^5 - \frac{4}{105}\Delta\gamma^7 l^7 - (3 - 3\Delta\gamma l + \Delta\gamma^2 l^2)^2 e^{2\Delta\gamma l}\right]\right\}, \\ T_{22} &= \exp\left[-\gamma_2 l - \Delta\gamma l \frac{c_0^2 d^2}{105}\right]\left\{1 + \frac{c_0^2 d^2}{4\Delta\gamma^6 l^6}\left[9 - 3\Delta\gamma^2 l^2 + \Delta\gamma^4 l^4 \right.\right. \\ &\qquad \left.\left. - \frac{2}{5}\Delta\gamma^5 l^5 + \frac{4}{105}\Delta\gamma^7 l^7 - (3 + 3\Delta\gamma l + \Delta\gamma^2 l^2)^2 e^{-2\Delta\gamma l}\right]\right\}, \\ T_{12} &= T_{21} = j\frac{c_0 d}{2\Delta\gamma^3 l^3}[(3 - 3\Delta\gamma l + \Delta\gamma^2 l^2)e^{-\gamma_2 l} \\ &\qquad - (3 + 3\Delta\gamma l + \Delta\gamma^2 l^2)e^{-\gamma_1 l}]. \end{aligned} \tag{21}$$

The expressions (21) are hard to evaluate, but for some special cases of interest they can be simplified greatly.

To compute the coupling effects between the TE_{01} wave and the TM_{11} wave we make use of $|\Delta\gamma l| \ll 1$ and get the following approximations:

$$
\begin{aligned}
T_{11} &= \left[1 - \frac{c_0^2 d^2}{315} \Delta\gamma^3 l^3\right] \exp - \left(\gamma_1 - \frac{c_0^2 d^2}{105} \Delta\gamma\right) l, \\
T_{22} &= \left[1 + \frac{c_0^2 d^2}{315} \Delta\gamma^3 l^3\right] \exp - \left(\gamma_2 + \frac{c_0^2 d^2}{105} \Delta\gamma\right) l, \qquad (22) \\
T_{12} &= T_{21} = j \frac{c_0 d}{15} \Delta\gamma^2 l^2 e^{-\gamma l}.
\end{aligned}
$$

In (22) the condition $|T_{11} - T_{22}|^2 \gg 4 | T_{12} T_{21} |$ is satisfied; consequently the wave propagation is described by (17),

$$
\begin{aligned}
E_{1n} &= \exp - \left(\gamma_1 - \frac{c_0^2 d^2}{105} \Delta\gamma\right) nl + \frac{c_0^2 d^2}{450} \Delta\gamma^2 l^2 \sinh \Delta\gamma n l e^{-\gamma n l}, \\
& \qquad (23) \\
E_{2n} &= j \frac{c_0 d}{15} \Delta\gamma l \sinh \Delta\gamma n l e^{-\gamma n l}.
\end{aligned}
$$

In addition to small oscillations, which are negligible, the wave amplitude E_1 suffers an additional attenuation

$$
\Delta\alpha_s = -\frac{c_0^2 d^2}{105} \Delta\alpha. \qquad (24)
$$

Physically this means that to a first approximation there is no net power transfer from E_1 to E_2. The power converted from E_1 to E_2 in one section of the iterative serpentine bends is all reconverted in the same section. But this power, which travels partly in the E_2 wave, suffers the E_2 attenuation and consequently changes the E_1 attenuation.

To evaluate (24) we introduce the coupling coefficient and the difference in attenuation constants between the TE_{01} and TM_{11} wave. Then, the relative increase of TE_{01} attenuation is:

$$
\frac{\Delta\alpha_s}{\alpha_{01}} = 6.39 \times 10^{-3} d^2 \frac{a^2}{\lambda^2} \left(2.69 \frac{a^2}{\lambda^2} - 1\right), \qquad (25)
$$

where α_{01} = attenuation constant of TE_{01},

a = inner radius of pipe,

λ = free-space wavelength.

A numerical example shows that the increase in TE_{01} attenuation caused by coupling to the TM_{11} wave in serpentine bends is small. For a copper pipe with 2.00-inch I.D. and $2\frac{3}{8}$-inch O.D. and a supporting length $l = 15$ ft, we have $d = 1.51 \times 10^{-2}$, and at $\lambda = 5.4$ mm we get $\Delta\alpha_s/\alpha_{01} = 0.19 \times 10^{-2}$.

For coupling between the TE_{01} wave and the waves of the TE_{1n} family the difference in propagation constant is no longer small; the approximation which was valid for the TM_{11} wave can not be made for TE_{1n} waves. Actually, the supporting distance is usually several beat wavelengths. Therefore, no essential simplifications of (21) are possible for the general case of coupling between TE_{01} and TE_{1n}. But closer examination of (21) shows that if

$$2\Delta\beta l = 2m\pi \qquad m = 1, 2, 3, \cdots \tag{26}$$

is satisfied, the difference T_{11}–T_{22} becomes very small. The net power converted to any of the TE_{1n} modes may be small in each section of the iterative serpentine bends, but if (26) is satisfied the contributions from each section add in phase and in a long line with the square of distance more and more power is built up in the particular TE_{1n} wave. Only when the attenuation in this TE_{1n} wave is large enough to damp out the power as fast as it is converted will an undistorted TE_{01} propagation be maintained. This condition for the attenuation constant can be derived from $| T_{11} - T_{22} |^2 \gg 4 | T_{12}T_{21} |$. If $| \Delta\beta | \gg | \Delta\alpha |$ and $| \Delta\alpha l | \gg c_0^2 d^2/10m\pi$ then $T_{11} - T_{22} = -2\Delta\alpha l e^{-\gamma_1 l}$, and since $T_{12}T_{21} = -9\, c_0^2 d^2/m^4\pi^4\, e^{-2\gamma_1 l}$ the condition for undistorted TE_{01} propagation is:

$$| \Delta\alpha l |^2 \gg 9\, \frac{c_0^2 d^2}{m^4\pi^4}. \tag{27}$$

If (27) is satisfied the wave propagation is again described by (17). Neglecting all terms which are small because of (27) the wave amplitudes are:

$$\begin{aligned} E_{1n} &= \left[1 + 9\frac{c_0^2 d^2}{m^4\pi^4}\frac{1}{2\Delta\alpha l}\right]^n e^{-\gamma_1 n l} + 9\frac{c_0^2 d^2}{m^4\pi^4}\frac{1}{4\Delta\alpha^2 l^2}[1 - e^{2\Delta\alpha n l}]e^{-\gamma_1 n l}, \\ E_{2n} &= -j3\,\frac{c_0 d}{m^2\pi^2}\frac{1}{2\Delta\alpha l}\,[1 - e^{2\Delta\alpha n l}]e^{-\gamma_1 n l}. \end{aligned} \tag{28}$$

For not too large values of n, the first term of E_{1n} may be written:

$$\left[1 + 9\frac{c_0^2 d^2}{m^4\pi^4}\frac{1}{2\Delta\alpha l}\right]^n e^{-\gamma_1 n l} = \exp\left[-\left(\gamma_1 - 9\,\frac{c_0^2 d^2}{m^4\pi^4}\frac{1}{2\Delta\alpha l^2}\right) nl\right].$$

The additional attenuation to the E_1 wave as caused by the continuous power abstraction is seen from this expression to be

$$\Delta\alpha_s = -9 \frac{c_0^2 d^2}{m^4\pi^4} \frac{1}{2\Delta\alpha l^2}. \tag{29}$$

The condition (27) may now be written

$$2\Delta\alpha_s \ll |\Delta\alpha|, \tag{30}$$

or, the rate of conversion loss has to be small compared to the difference between E_2 attenuation and E_1 attenuation.

When the wave is travelling through a large number of serpentine bends, power is built up gradually in the E_2 mode to a constant value,

$$\left|\frac{E_2}{E_1}\right| = 3 \frac{c_0 d}{m^2\pi^2} \left|\frac{1}{2\Delta\alpha l}\right|. \tag{31}$$

Both the attenuation increase and the power level in spurious modes can seriously affect the E_1 transmission.

To evaluate (29) and (31) we rewrite them with $m\pi = \Delta\beta l$ and $d = wl^3/12EI$ as follows;

$$\frac{\Delta\alpha_s}{\alpha_{01}} = -\left[\frac{w}{EI}\frac{c_0}{(2\Delta\beta)^2\alpha_{01}}\right]^2 \frac{\alpha_{01}}{2\Delta\alpha}, \tag{32}$$

$$\left|\frac{E_2}{E_1}\right| = \frac{w}{EI}\frac{c_0}{(2\Delta\beta)^2\alpha_{01}}\left|\frac{\alpha_{01}}{2\Delta\alpha}\right|. \tag{33}$$

We note from (32) and (33) that the coupling effects cannot be controlled by changing the supporting distance. Only the number of critical frequencies in a given range decreases with decreasing supporting distance.

The previously cited numerical example of the 2 inch copper pipe yields the following values at the critical frequencies of the two lowest TE_{1n} waves near $\lambda = 5.4$ mm:

$$TE_{11} \quad \frac{\Delta\alpha_s}{\alpha_{01}} = 0.114 \qquad 20 \log\left|\frac{E_2}{E_1}\right| = -23.4\ db,$$

$$TE_{12} \quad \frac{\Delta\alpha_s}{\alpha_{01}} = 0.855 \qquad 20 \log\left|\frac{E_2}{E_1}\right| = -6.85\ db.$$

The mode conversion, especially to the TE_{12} wave, causes a seriously high additional attenuation and spurious mode level.

V. MODE FILTERS IN SERPENTINE BENDS

Periodically spaced supports are a condition for the critical case described by (32) and (33). Accordingly, the coupling effects can be controlled by removing the periodicity of the supports.

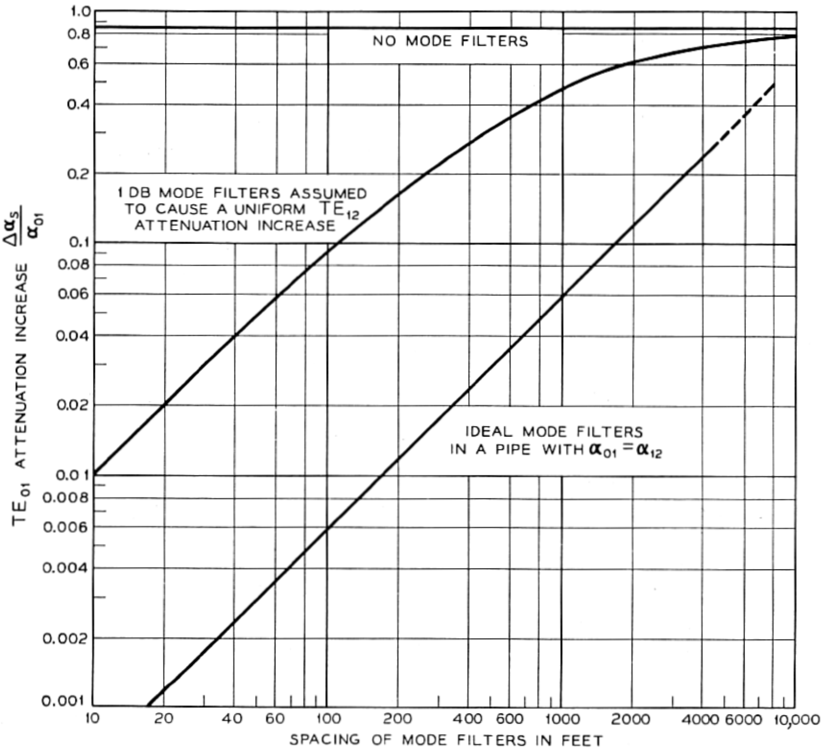

FIG. 2 — TE_{01} attenuation increase of TE_{01}–TE_{12} coupling in serpentine bends with mode filters; 2-inch I.D. $2\frac{3}{8}$-inch O.D., $\lambda = 5.4$ mm. Serpentine bends are caused by equally spaced supports and the elasticity of the copper pipe. The supporting distance is a multiple of the beat-wavelength between TE_{01} and TE_{12} .

An alternative to control the coupling effects is insertion of mode filters into the line. Mode filters which pass the TE_{on} waves without loss but attenuate all other modes have been developed in various forms. To estimate the amount of mode conversion control that can be achieved by mode filters, we make two different assumptions. Only the critical case of the supporting distance equal to a multiple of the beat wavelength is considered here.

A. The mode filters are ideal; they present infinite attenuation to all unwanted modes. At the input of a section between two mode filters we have a TE_{01} wave only and at the output the spurious mode level has risen to

$$\left|\frac{E_2}{E_1}\right| = \frac{w}{EI}\,\frac{c_0}{(2\Delta\beta)^2}\,L. \tag{34}$$

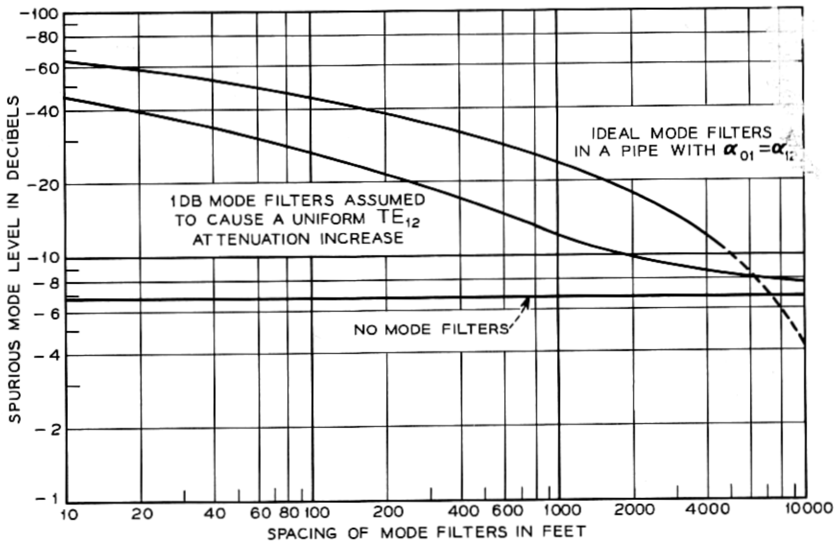

Fig. 3 — Spurious mode level of TE_{01}–TE_{12} coupling in serpentine bends with mode filters; 2-inch I.D. $2\frac{3}{8}$-inch O.D., $\lambda = 5.4$ mm. Serpentine bends are caused by equally spaced supports and the elasticity of the copper pipe. The supporting distance is a multiple of the beat-wavelength between TE_{01} and TE_{12} .

The loss to the TE_{01} wave caused by the mode conversion is equivalent to an increase in TE_{01} attenuation

$$\Delta\alpha_s = \frac{1}{2}\left[\frac{w}{EI}\frac{c_0}{(2\Delta\beta)^2}\right]^2 L. \tag{35}$$

L is the spacing between two successive mode-filters. In (34) and (35) the attenuation constants of E_1 and E_2 are assumed to be equal. Furthermore (34) and (35) hold only as long as the E_2 power level is small compared to the E_1 power level.

Ideal mode filters is a rather optimistic assumption. Practical mode filters present only a limited attenuation to the unwanted modes. Therefore a more realistic procedure is to represent the effect of the mode filters by a uniform increase of unwanted mode attenuation.

B. The mode filters with a loss A are considered to cause a uniform increase in E_2 attenuation $\Delta\alpha = A/L$. Equations (32) and (33) modified to include this attenuation increase are:

$$\frac{\Delta\alpha_s}{\alpha_{01}} = \left[\frac{w}{EI}\frac{c_0}{(2\Delta\beta)^2\alpha_{01}}\right]^2 \frac{\alpha_{01}}{\mid 2\Delta\alpha \mid + \dfrac{A}{L}}, \tag{36}$$

$$\frac{E_2}{E_1} = \frac{w}{EI} \frac{c_0}{(2\Delta\beta)^2 \alpha_{01}} \frac{\alpha_{01}}{| 2\Delta\alpha | + \frac{A}{L}} . \quad (37)$$

.n evaluation of the TE_{01}-TE_{12} coupling in the previously described 2 inch copper pipe for critical frequencies near $\lambda = 5.4$ mm is shown in Figs. 2 and 3. The mode filter loss of 1 db can be achieved for the TE_{12} wave in an 18-inch long helix waveguide. A 100-foot spacing of the mode filters reduces the increase of TE_{01} attenuation to 9 per cent and the spurious mode level to -26 db.

APPENDIX

The coupling coefficient for the wave coupling between the TE_{01} wave and the TE_{11}, TE_{12}, TE_{13} waves as well as the TM_{11} wave have been calculated by S. P. Morgan.[4] If $c = c_o/R$, the factor c_o is for the various waves

$$TM_{11} \; c_0 = 0.18454\beta a,$$

$$TE_{11} \; c_0 = \frac{0.09319(\beta a)^2 - 0.84204}{\sqrt{\beta_{01} a \beta_{11} a}} + 0.09319 \sqrt{\beta_{01} a \beta_{11} a},$$

$$TE_{12} \; c_0 = \frac{0.15575(\beta a)^2 - 3.35688}{\sqrt{\beta_{01} a \beta_{12} a}} + 0.15575 \sqrt{\beta_{01} a \beta_{12} a},$$

$$TE_{13} \; c_0 = \frac{0.01376(\beta a)^2 - 0.60216}{\sqrt{\beta_{01} a \beta_{13} a}} + 0.01376 \sqrt{\beta_{01} a \beta_{13} a},$$

where a = radius of waveguide,

β = free-space phase constant,

β_{nm} = phase constant of TE_{nm} wave.

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