Contribution of Statistics to the Development Program of a Transformer for the L3 Carrier System

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Statistical methods played a significant part in the development program of the L3 system. Experiments were designed to assist in improving the manufacture of the input and output transformers of the amplifiers. Detailed analysis of a few of these experiments is presented.

I. INTRODUCTION

In previous issues of The Bell System Technical Journal the problems in design, development and manufacture that were encountered in building the L3 coaxial carrier system are described. This system provides 1,860 one-way telephone channels or 600 one-way telephone channels plus one TV channel over each coaxial tube. The L3 system is capable of transmitting a television signal over a distance of approximately 1,000 miles and telephone signals, approximately 4,000 miles.

From the start of the development program, statistical methods have played a significant part. Special acceptance procedures have been set up to assure that the shipped product would meet certain distribution requirements.[1] Control chart techniques were generously applied both in the manufacture of component parts and for subassemblies.[2] This paper gives in part a case history of one of the difficult components. The viewpoint is that of the experiments designed to overcome difficulties in the initiation of the manufacturing process and to explore possibilities of improvement of the component.

A detailed discussion of the present manufacturing techniques of this component, the input and output transformer of the amplifier, has already been presented by Earle.[3] That paper will be used freely to provide the technical details and pictures necessary for an understanding of the experiments. No basically new statistical designs were employed in this development. The main interest lies in the fact that these experiments together with the engineering design and the manufacturing operations, including the appropriate process controls and inspection techniques, were integrated in the development program.

An endeavor is made in this paper to point out the logical link between the statistical analysis and the engineering consequences. Advantages of the use of statistical methods in experimental work are as follows:

1. In designing an experiment (the adjective "statistical" will be implied from now on), the type of analysis to be performed on the data is a major consideration from the start. In some experiments one might wish to determine one or several of a larger number of factors which have an important effect. In this case the analysis should yield a statement about the significance of the effects of the operating factors, with a predetermined small risk of being wrong. In other cases one looks for quantitative measures of one or more properties and then the statistician will estimate intervals within which, on the basis of the experimental results, one can expect with a high probability, the true (unknown) value of these measures to lie.

2. Under the limits set by the requirements in the preceding paragraph the design will be such that the experimental effort is minimized.

3. The design will take into account the adverse effects on the precision of the experiment caused by known ambient conditions which are not completely under control of the experimenter.

4. In so far as possible, safeguards against effects from unknown factors will be incorporated in the designs.

The preceding points require that quantitative notions be introduced as much as possible, not only for the things measured but also for the operating factors and disturbances. The experimenter and the statistician try to agree on a statistical model, describing the expected behavior of the physical items in the experiment. Given the model, the statistician can suggest experimental arrangements, in an efficient way with respect to the experimental effort, which should yield reliable information about the problem at hand.

In many cases it turns out, when the observations become available, that the model has to be modified or that the experiment has not been performed according to the design. This usually increases the burden on the analysis. It happens occasionally that the data do not show definite results, and further experimentation is needed. In that case the careful statistical analysis might yield clues in what direction to proceed as well as useful quantitative information about disturbing factors, experimental errors, etc.

It has been pointed out that a difference between agricultural and industrial experiments lies in the time factor involved.[4] Extension or repetition of agricultural experiments is in most cases only possible at yearly intervals. In industry the time schedule is much less restricted. Therefore it pays to use involved designs in agriculture even at the cost of complex analyses. Where it is comparatively easy to start a new or partly new experiment, complexity may be too high a price to pay. Moreover when experimentation goes on parallel to a production process, speed in obtaining the results of an experiment is of prime importance. Simplicity of design is also valuable when the underlying model is not yet well understood, as in the early stages of exploratory development.

In the early stages of the manufacture of a complex component, the actual specification has to be written on the basis of the results on a comparatively small number of samples. It can hardly be expected that these samples are fully representative of the production items which will be manufactured. Nevertheless the design engineer will have to determine workable limits to give the manufacturer the opportunity to get his production rolling without producing too many items not acceptable for use. In the L3 system, studies of the over-all requirements of the system had indicated in which way they had to be broken down into the requirements for the components and subassemblies in order to assure satisfactory operation. In the case of the transformer under discussion the electrical transmission requirements were more or less fixed. It was the task of the design engineer to translate these requirements into mechanical tolerances which could be controlled during manufacture. On the basis of the equivalent diagram (Fig. 1) for the transformer, extensive calculations had been made to determine the relation between the variations of the electrical parameters and the over-all transmission response.[5, 6, 10] Each of the electrical parameters as shown in Fig. 1, a simplified picture of the equivalent diagram, does not necessarily correspond to a discrete part of the physical transformer, but the diagram can be considered to represent a model, which lends itself to mathematical treatment. Mathematical considerations, statistical or otherwise, on the basis of the model, help to establish the mechanical requirements for the manufacture, as will be shown later.

A few of the experiments performed to quantify the underlying relationships will be presented in a logical order. Although, through the pressure of circumstances, the actual experiments did not proceed in a strictly orderly fashion, the general line of experimentation was that described in this article. Production was progressing in parallel with this experimental program and, as described elsewhere,[2] control charts showed several assignable causes of variation in the parameters, which were removed by improvements in manufacturing techniques.

The experiments selected to illustrate the development program will be discussed in some detail. In terms of their most important results these experiments can be described as follows:

1. Pinpointing the input and output network (Fig. 2) as the major source of variation. The transformer (Fig. 3) is the main component in

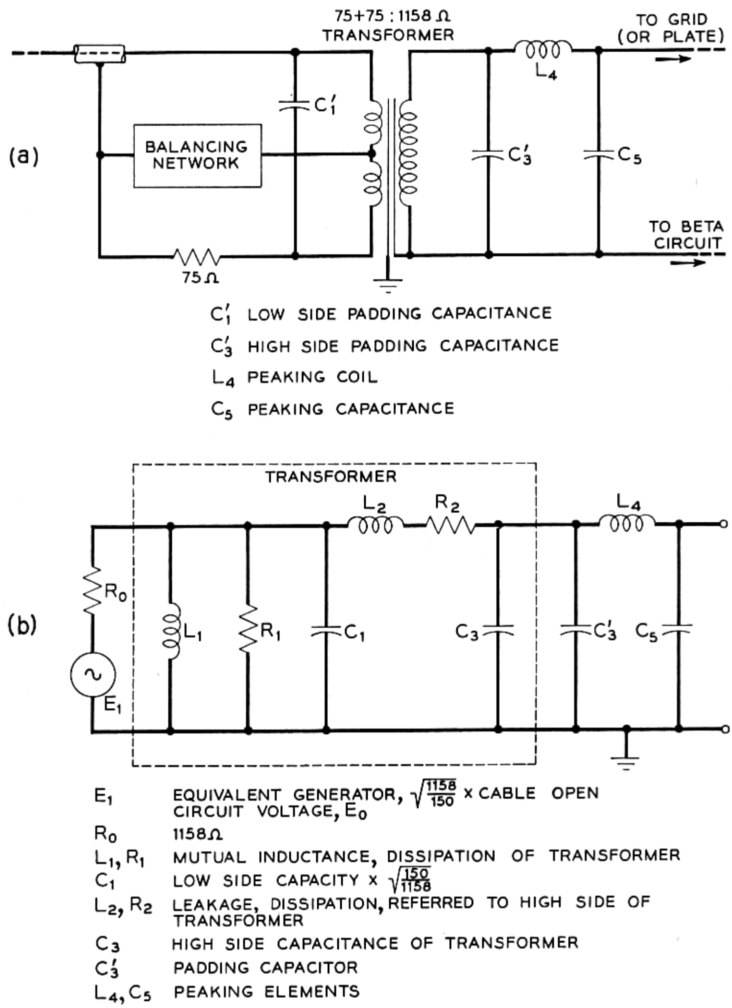

Fig. 1 — Coupling networks circuits. (a) Physical elements. (b) On ground equivalent circuit, adequate for gain and feedback computations in an amplifier configuration employing ground coupling networks.

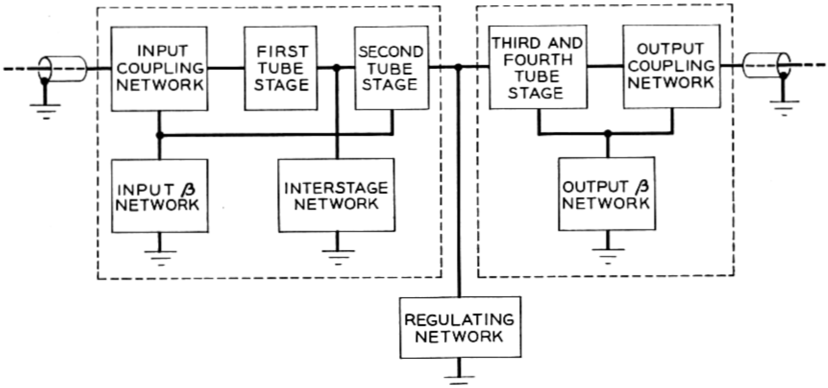

Fig. 2 — Block diagram of amplifier.

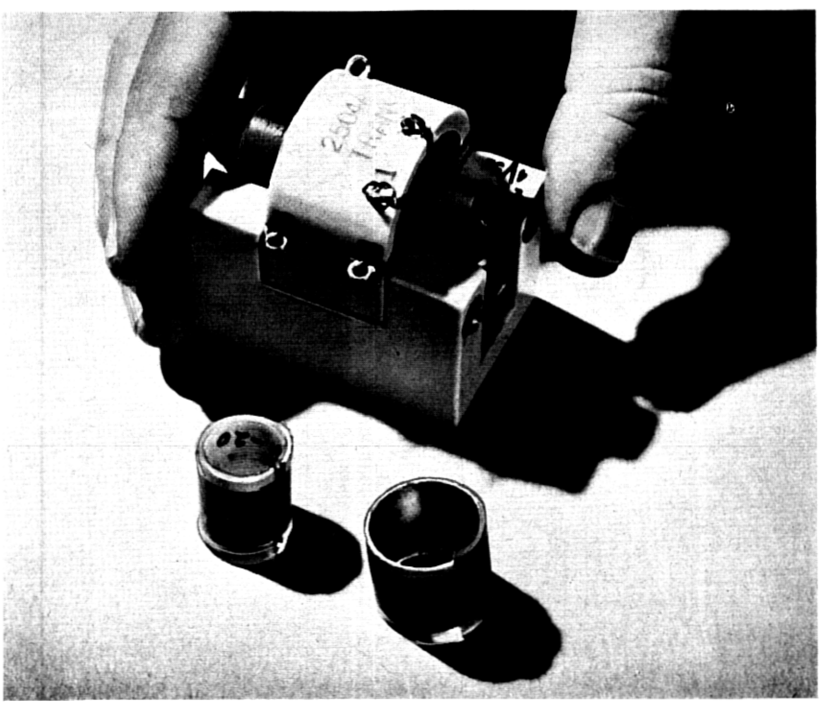

Fig. 3 — Transformer and separate inner and outer winding forms with windings.

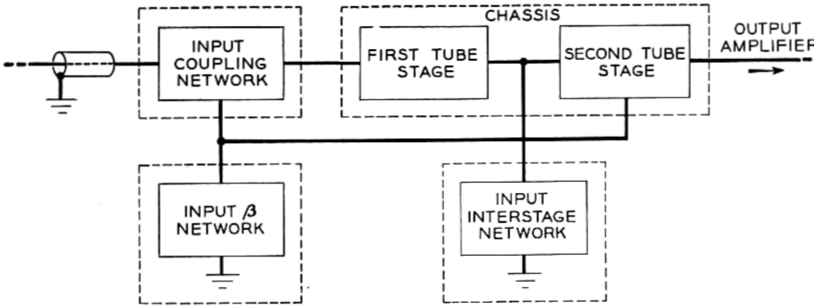

Fig. 4 — Input (sub) amplifier block diagram subdivision for hyper-graeco-latin square experiment.

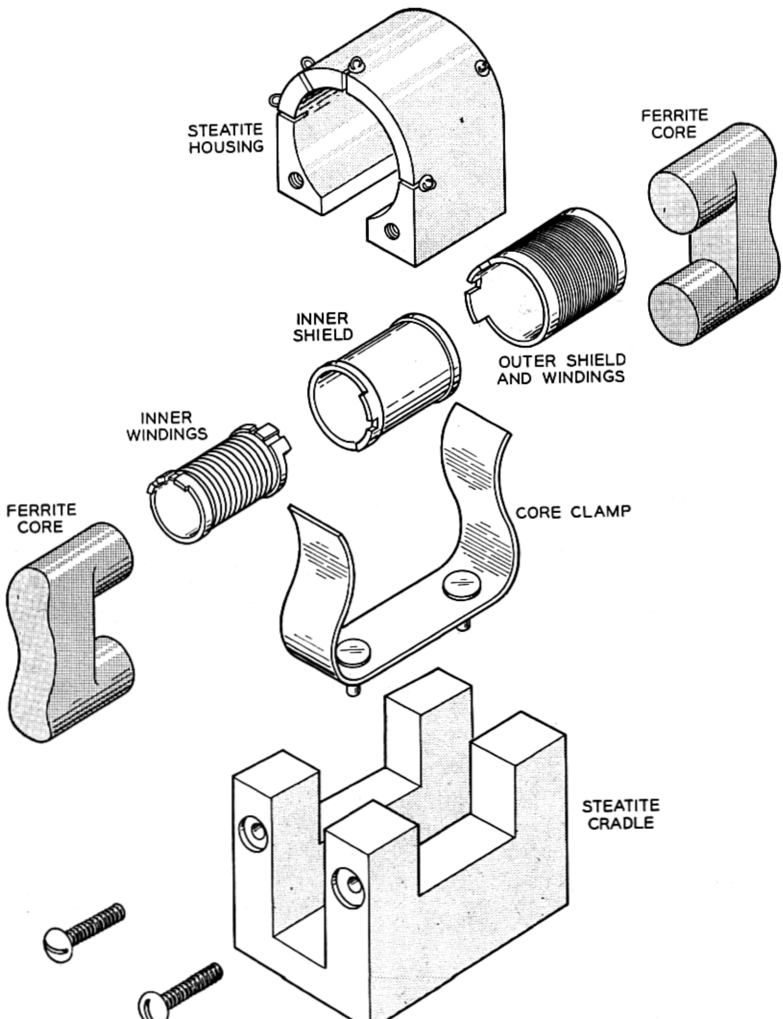

Fig. 5 — Exploded schematic of the 2504A transformer.

these networks so that subsequent experimentation was concentrated on the transformer.

2. Determining the required manufacturing limits for the wall thickness of the outer winding form of the transformer, (Fig. 5, 6).

3. Determining the required manufacturing limits for the "cutback" of the shield under the outer winding of the transformer. (The term "cutback" will be explained later.)

4. Comparing the over-all measured response of the complete amplifier with its predicted performance as based on a detailed knowledge of the components obtained from the designed experiments.

II. FINDING THE NETWORK CAUSING MOST OF THE UNWANTED VARIATIONS

From the first series of amplifiers manufactured, it appeared that the differences between the measured transmission gain curves for the various amplifiers were larger than could be tolerated.

For this discussion it is sufficient to represent the amplifier as in Fig. 2. The blocks represent subassemblies which are mechanically designed so that a high degree of reproducibility in the location of the components and the connected wiring is achieved. It is therefore feasible to inquire if one or two of the subassemblies are responsible for the bulk of the variability in measured gain. It is worth noting that the "large" variations are not large when compared to the capabilities of the measuring equipment. The over-all admissible amplifier gain variations are in the order of 0.2 to 0.3 db corresponding to voltage variations of less than 3 per cent.

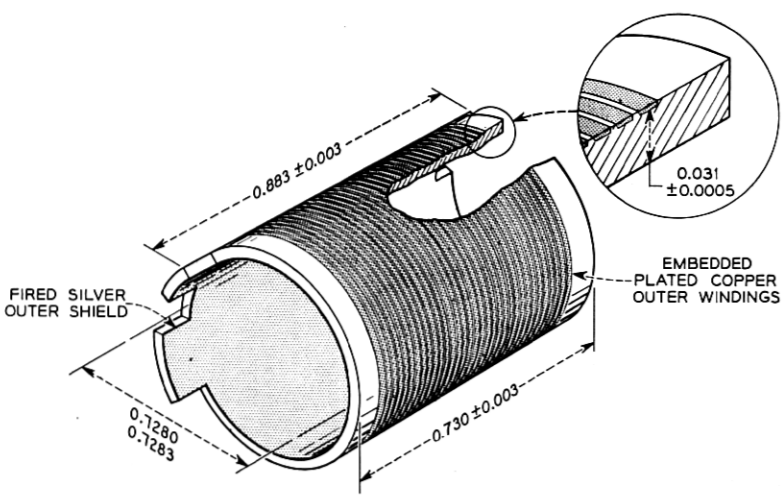

Fig. 6 — Outer winding form and detail to show "wall thickness."

Consequently, to be able to discriminate between the contributions of the individual components one must be able to measure reliably to as close as, say, 0.01 db, i.e., to detect voltage variations in the order of 0.1 per cent. This approaches the presently attainable precision of these types of measurements. Finally, these subassemblies are fairly expensive and were not in plentiful supply at the time these experiments had to be run.

Practically, it was reasonable to treat the input and output amplifiers, as indicated in Fig. 2, as separate entities. Each of these two subamplifiers can be measured accurately for its transmission gain in the same way as can be done with the completed amplifier. In this fashion a direct relationship exists between the results of sub- and complete amplifiers. This favorable condition does not exist with respect to the relationship between sub-amplifiers and its subassemblies which are also indicated in Fig. 2. To determine if the subassemblies meet the over-all requirements, it is necessary to combine them into sub-amplifiers and measure those.

Input and output amplifiers consist basically of the same subassemblies. The type of designed experiment used for both sub-amplifiers was identical so that a detailed example for the input-amplifier tells the main story. It was felt from engineering considerations that interactions between the various subassemblies in an input or output amplifier would be of a considerably smaller magnitude than the variations of interest and therefore could be neglected.

Four types of subassemblies make up a sub-amplifier, so these four should enter as factors in our experiment. As was pointed out above, a set of subassemblies has to be assembled into an amplifier to make transmission measurements possible. To evaluate this procedure, every time the set of available subassemblies was combined into sub-amplifiers it was considered a run. This gives the following factors to be used in the experiment:

Runs
Coupling Networks
Interstage Networks
Beta Networks
Chassis

The number of levels for each of the factors is determined below.

The experimental design should incorporate five factors and minimize the number of required subassembly units; however it does not have to measure interactions. An experimental design that lends itself to this type of situation is a hyper graeco-latin square.[7]

Assigning, as is shown in Table I, the rows to the different runs and

TABLE I. — HYPER GRAECO-LATIN SQUARE LAYOUT

Run No.	Chassis						
	1	2	3	4	5	6	7
1	A1α	B2β	C3γ	D4ϑ	E5ϵ	F6ζ	G7η
2	D3β	E4γ	F5ϑ	G6ϵ	A7ζ	B1η	C2α
3	G5γ	A6ϑ	B7ϵ	C1ζ	D2η	E3α	F4β
4	C7ϑ	D1ϵ	E2ζ	F3η	G4α	A5β	B6γ
5	F2ϵ	G3ζ	A4η	B5α	C6β	D7γ	E1ϑ
6	B4ζ	C5η	D6α	E7β	F1γ	G2ϑ	A3ϵ
7	E6η	F7α	G1β	A2γ	B3ϑ	C4ϵ	D5ζ

Latin letters—Coupling Networks
Greek letters—Beta Networks
Numerals—Interstage Networks

the columns to the different chassis, we can allocate the coupling networks, identified by latin letters, so that each occurs exactly once in each column and row. This results in a latin square. If we add to this structure two more arrays, one composed of greek letters, identifying the beta networks and one composed of numbers identifying the interstage networks, such that each letter or number occurs only once with each other symbol we have an (incomplete) system of "orthogonal squares". Data from such a pattern will allow us to obtain unbiased estimates of the main effects of the five factors incorporated, in the absence of interactions. Moreover, the estimates for one factor will be statistically uncorrelated with those for other factors.

The square in Table I is of size 7 × 7. This is the smallest practical size that could be applied. For 5 factors a square of size 5 × 5 could in theory be used as four different orthogonal squares of this size exist,[8] but we would have only four degrees of freedom to estimate our error.

No orthogonal squares of size 6 × 6 exist. In a 7 × 7 we have 49 observations and 18 degrees of freedom for error. For this experiment 7 units of each type had to be assembled 7 times into a set of 7 amplifiers each. The first set of 7 amplifiers was numbered 1 to 7 in random order, thus at the same time identifying the subassemblies. The complete layout of the experiment is given in Table I.

Measurements on the completed input amplifiers were made at the highest frequency of interest in the transmission band, 8.3 mc, and are listed in Table II. The analysis of variance computed in the usual manner from these data is presented in Table III. Apparent measurement standard deviation $\hat{\sigma} = \sqrt{0.000254} = 0.016$ db.

It is evident from the sums of squares column in the latter table that the coupling networks contribute a very sizeable part of the total variation. The experimental error as estimated from the residual mean

TABLE II. — TRANSMISSION MEASUREMENTS AT 8.3 MC IN DB

Run No.	Chassis No.						
	1	2	3	4	5	6	7
1	4.739	4.799	4.935	4.713	4.824	4.998	4.870
2	4.759	4.841	5.044	4.820	4.870	4.852	4.896
3	4.819	4.749	4.878	4.933	4.719	4.873	4.986
4	5.003	4.749	4.866	5.001	4.797	4.761	4.836
5	4.978	4.824	4.722	4.820	4.945	4.797	4.898
6	4.804	4.910	4.774	4.916	5.013	4.819	4.714
7	4.897	5.056	4.861	4.701	4.827	4.913	4.743

TABLE III. — ANALYSIS OF VARIANCE

Source	D/F	Sums of Squares	Mean Square	Significance Level
Coupling Networks	6	0.376359	0.062726	$\leqq 1\%$
Interstage Networks	6	0.037422	0.006237	$\leqq 1\%$
Beta Networks	6	0.003410	0.000568	not significant at 5% level
Chassis	6	0.003075	0.000512	not significant at 5% level
Runs	6	0.003381	0.000564	not significant at 5% level
Residual	18	0.004634	0.000254	
Total	48	0.428281		

squares amounts to 0.016 db. This disregards the effect of reassembling, as indicated by runs, which, however, is not significant at the 5 per cent level. It would be possible to pool the run, sum of squares, with that for error as estimated from the residual mean square to get more degrees of freedom for error but no new insight would be gained by this procedure. In the type of investigations described a level of significance of 5 per cent or smaller is generally applied. This implies that the chances are 5 per cent or less that, on the basis of the analysis, effects would be singled out for further engineering consideration when actually these effects are nonexistent.

To further illustrate the engineering implications, the results of Table III can be written in terms of the projected model for this experiment. It was assumed that the effects of the members of each of the subassemblies on the amplifier gain were normally distributed. The average value of the amplifier gain can be interpreted as the performance of an amplifier consisting of subassemblies of exact nominal values. The interesting part, however, is the gain variation from amplifier to amplifier, caused by the deviations from nominal of the subassemblies. These deviations

TABLE IV. — STANDARD DEVIATION ESTIMATES FOR THE VARIATIONS DUE TO THE DIFFERENT NETWORKS

Coupling Networks	0.094 db
Interstage Networks	0.029 db
Beta Networks	0.007 db
Chassis	0.006 db
Runs	0.007 db

TABLE V. — APPROXIMATE 90 PER CENT CONFIDENCE LIMITS FOR THE VARIATIONS DUE TO THE DIFFERENT NETWORKS

	Lower Limit (db)	Upper Limit (db)
Coupling Networks	0.065	0.181
Interstage Networks	0.019	0.056
Beta Networks	0.0	0.016
Chassis	0.0	0.015
Runs	0.0	0.016

can be measured by the standard deviation of their respective distributions. These standard deviations as derived from Table III are listed in Table IV and their approximate 90 per cent confidence limits in Table V.[9]

It appears again that the coupling networks contribute most to the variations in the transmission of the subamplifier. The interstage networks are of secondary importance, whereas the other three factors can be neglected. A similar picture emerged from the companion experiments on the output amplifier. It was therefore logical to concentrate first on trying to decrease the variability of the coupling network of which the transformer was the main part.

III. WALL-THICKNESS STUDIES ON THE OUTER COIL FORM OF THE TRANSFORMER

The transformer, even in its simplified form as in the equivalent circuit of Fig. 1, involves many parameters. By numerical evaluation the changes in transmission gain due to specified changes in these parameters were calculated on the basis of this circuit.[5, 6] As has already been pointed out, not all of the parameters in the equivalent diagram are directly represented in the physical transformer; therefore a relationship between the parameters and physical dimensions is not easy to establish.

From evaluation of the electrical circuit it was felt that the capacitance at the high inductance side of the transformer, C_3 in Fig. 1, would be a major contributor to the gain variation. Direct correlation between the behavior of this capacitance and various mechanical properties on the basis of control charts did not yield sufficiently strong clues, partly due to the fact that the measurement accuracy in the production process was marginal in view of the small variations concerned. On the basis of engineering experience one of the strongly suspected mechanical variables was the wall thickness of the outer coil form of the transformer. The exploded views in Fig. 5 and Fig. 6 show that the outer form carries the winding with the highest number of turns. These turns are ground into the vycor glass body and they are subsequently copper plated. A silver shield is sprayed on the inside of the vycor glass form and fired subsequently. The "thickness" of the wall as measured between the bottom of the groove and the inner face is about 0.031″ and the geometry of the situation leads us to expect a strong dependence of the high side capacity on the wall thickness. (Fig. 6.)

The experiment to estimate the quantitative influence of wall thickness variations on electrical properties was set up as follows:

Two batches of 9 transformers each were produced in accordance with current production specifications except that batch "A" contained outer coil forms with "thick" walls and batch "B" with "thin" walls. On a nominal thickness of about 0.031″ batch A was on the average about 6 ten thousandths thicker than batch B. Due to the difficult grinding process it was impossible to make all coil forms of the same batch exactly alike to the limit of measurement, i.e., to within half a ten thousandth. The resulting variation in this thickness within a batch is indicated by the standard deviation of 1.5×10^{-4}.

All these transformers were measured in the same standard amplifier and the gain was observed at a number of frequencies. In addition, various short-circuit and open-circuit impedances were determined on the isolated transformers. Since these impedances bear a direct relation to the magnitude of the parameters in the equivalent diagram, one obtains information about the variations in the parameter values from the observed variations in the impedances. Allowing for these variations in predicting the performance of the circuit on the basis of the equivalent diagram, it is possible to compare the observed gain with that predicted. An example of such a comparison will be discussed later.

After a complete first run of measurements had been made on the transformers as manufactured, a second run was performed after the thick walled and thin walled coil forms had been interchanged between the transformers of batch "A" and "B".

Identifying the transformers without a coil form by capital letters and the forms by lower case ones in accordance with the batch to which they originally belonged, the actual set-up is given in Table VI. This table

TABLE VI. — BASIC DESIGN FOR WALL THICKNESS DEPENDENCY DETERMINATION

Coil Form	Transformer Batch	
	A	B
a	Run 1	Run 2
b	Run 2	Run 1

represents the experiment only "batchwise". It is important to note with respect to the model given below, that the interchange of one pair of coil forms (one thick and one thin) did not in general take place within one pair of transformers (one from batch A and one from batch B). If this had been done, a different analysis could have been performed on the same amount of data.

The mathematical model underlying this design takes into account the following effects:

μ = average level

β_j = batch $\quad j = 1, 2.$

$\varphi_{i,j}$ = transformer i in batch $j \quad i = 1, 2, \cdots, 9$; $j = 1, 2.$

ω_k = wall thickness $\quad k = 1, 2.$

ρ_l = runs $\quad l = 1, 2.$

$\epsilon_{i,j,k,l}$ = residual, being the difference between the measurements of the i^{th} transformer in the j^{th} batch and its prediction from wall thickness, batch and run effect.

With these definitions the observations $y_{i,j,k,l}$ can be expressed as follows:

$$y_{i,j,k,l} = \mu + \beta_j + \varphi_{i,j} + \omega_k + \rho_l + \epsilon_{i,j,k,l}\,.$$

From Table VI it is apparent that the wall thickness is measured by the row differences, the batch effect by the column differences and the run effect by the diagonal differences. The latter is indistinguishable from the row by column interaction, but there were reasons to believe that the interactions were of a smaller order of magnitude than the run effect.

The results of the gain measurements at one of the frequencies employed, 8.3 mc, are presented in Table VII, which gives only the fractional db, expressed in thousandths of db. A constant whole number of db is omitted throughout. This incorporates the fixed gains and attenuations of the measuring set up.

TABLE VII. — GAIN MEASUREMENTS AT 8.3 MC. EFFECT OF DIFFERENT WALL THICKNESS OF OUTER FORM

	Batch A			Batch B	
	Transformer	× 0.001 db		Transformer	× 0.001 db
Run 1 "Thick" Wall	1	744	"Thin" Wall	1	531
	2	778		2	510
	3	723		3	437
	4	698		4	487
	5	738		5	447
	6	644		6	608
	7	711		7	562
	8	670		8	476
	9	604		9	470
Run 2 "Thin" Wall	1	645	"Thick" Wall	1	674
	2	582		2	700
	3	556		3	634
	4	577		4	711
	5	582		5	512
	6	524		6	725
	7	550		7	658
	8	483		8	680
	9	547		9	676

TABLE VIII. — ANALYSIS OF VARIANCE OF WALL THICKNESS EXPERIMENT

Source	Sum of Squares	Degrees of Freedom	Mean Square	Significance Level
Between batches	20 449	1	20 449	5%+
Between transformers, within batches	75 126	16	4 695	1%
Between runs	880	1	880	not significant at 5% level
Between wall thickness	203 401	1	203 401	<1%
Within transformers corrected for runs and wall thickness (error)	20 888	16	1 305	
Total	320 744	35		

The analysis of variance of these data is presented in Table VIII.

It is readily seen from Table VIII that the wall thickness accounts for most of the variations, and that the effect of runs is indistinguishable from the error. It is possible just as was done in Table V to calculate the variance components for these effects and its limits. Both however are only based on one degree of freedom which makes this procedure hardly profitable. The batch effect is tested against the "transformer within batches" variation, and the level of significance is a little over 5 per cent. This indicates that there was a systematic difference between the two batches.

An estimate of residual variation can be obtained from the two observations on the same transformer corrected for the estimated differences due to wall thickness and run effects. The standard deviation for error is $\hat{\sigma} = \sqrt{1305} = 36$ or 0.036 db in actual units. This can be compared to the stated goal of 0.01 db and the result of the preceding experiment 0.016 db. The two averages computed for the different wall-thickness groups, $y_{\cdot\cdot k}$, provides us with an estimate of the effect of the average change in wall thickness on the gain:

For the "thick" wall the estimated gain is 0.682 db.

For the "thin" wall the estimated gain is 0.532 db.

Average increase of 0.006″ in wall thickness results in an increase of 0.150 db at 8.3 mc. In order to find out if the experiment was sensitive enough to find the dependence on wall thickness of the transmission measurements of the individual transformers, the residuals, as calculated from the equation on page 35, are plotted against the measured wall thickness, Fig. 7. The measurements of the wall thickness could be read to the nearest 0.00005″, but as seen in Fig. 7, the variations are too great to show any significant correlation with the fine structure of the wall thickness.

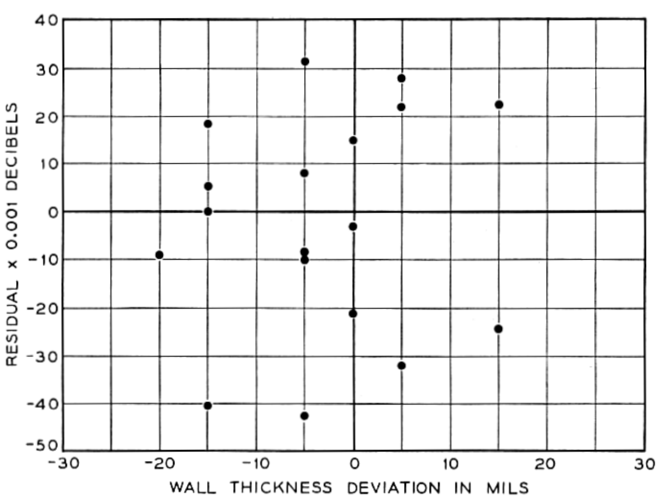

Fig. 7 — Residual variations, after the systematic effects have been removed, as a function of the wall thickness variation.

This experiment showed that it was necessary to control the wall thickness as closely as would be economical. The practical limit was known and the resulting transmission variations as estimated from the findings in this experiment, would be satisfactory from the over-all systems point of view.

IV. STUDY OF SHIELDING AND WINDING TERMINATION

Another mechanical variable to be considered is related to the termination of the winding on the outer form. One side of the winding (terminal No. 4) is connected to the shield that covers the inside of the coil form (Fig. 8). The other end has to be connected to one of the terminals (No. 5) on the body of the transformer. Electrically this latter point is sensitive and should be shielded as much as possible. On the other hand, in order to be able to connect the terminal lead to the winding a tab is inserted on the form. The shield must be cut back sufficiently to avoid short circuiting the winding via the tab. Originally a 0.150″ cutback was employed. Mechanical limitations make variations around the nominal cutback value unavoidable. The following experiment was set up to find out which nominal cutback value would result in the smallest variations in the transmission gain of the transformer.

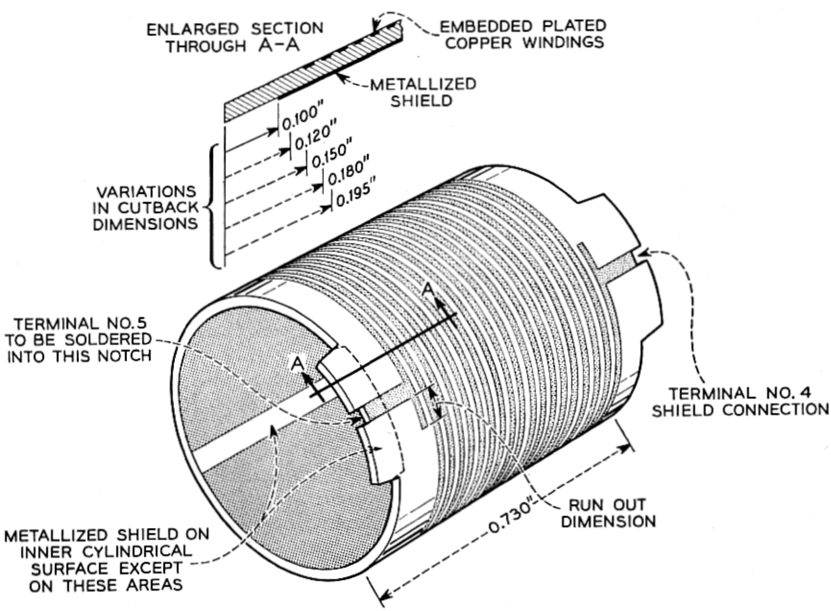

Fig. 8 — Side view of outer cylindrical spool, as per Fig. 6.

Another variable had been introduced into the problem inadvertently in the manufacturing process. This variable was related to the same sensitive point of the winding, and consisted of the amount of run-out or extra winding cut by the grinder beyond the point where the terminal tab No. 5 was connected to the winding. The run-out is measured in degrees of arc. Originally the run-out was kept close to 28°. After some manufacturing changes required for other reasons, the run-out variations became much larger. It was thought important to examine cutback and run-out at the same time to find any interaction effects if present.

An experiment to determine effects of cutback and run-out faces a difficulty similar to the previous one. The only hope to detect these effects is to try out the same transformer with different cutback and run-out values. This implies disassembling and re-assembling the transformers as many times as changes in the variables are made. In addition the change in variables can only go in one direction: the cutback can be increased by taking away a little bit of the shield and the run-out can be decreased by removing part of the run-out winding.

In accordance with these conditions an experiment was designed as indicated in the flow chart of Fig. 9, covering the possible combinations of applied changes in cutback and run-out in a systematic manner.

The cutback value of 0.150″ and the 28° run-out were the standard values in the manufacture at the time of the experiment. The stages of

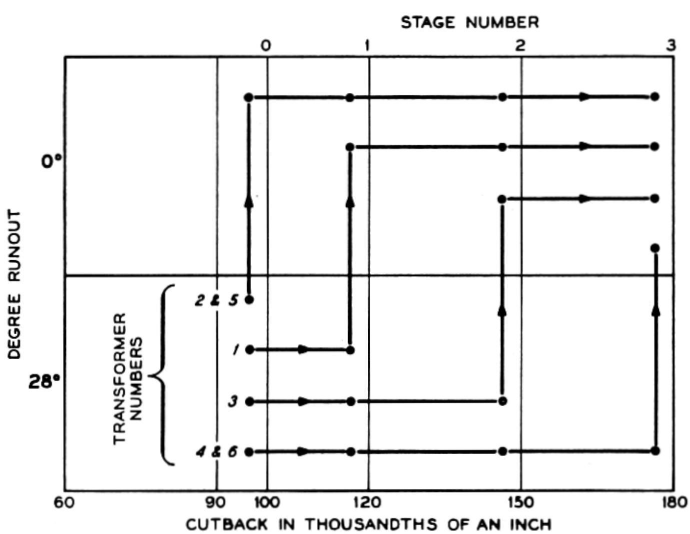

Fig. 9 — Flow chart of applied changes.

reassembly are indicated in order. The starting point for each transformer was 0.100″ cutback and 28° run-out.

This is an example of an experiment where several mishaps distorted the original design — a not unusual occurrence. Due to the time and costs involved the experiment was not repeated but a special effort was made to recover the information sought.

As in the previous experiment the transformers were measured in an amplifier to determine the gain characteristic as a function of frequency. In addition a few characteristic parameters were measured on the transformer itself.

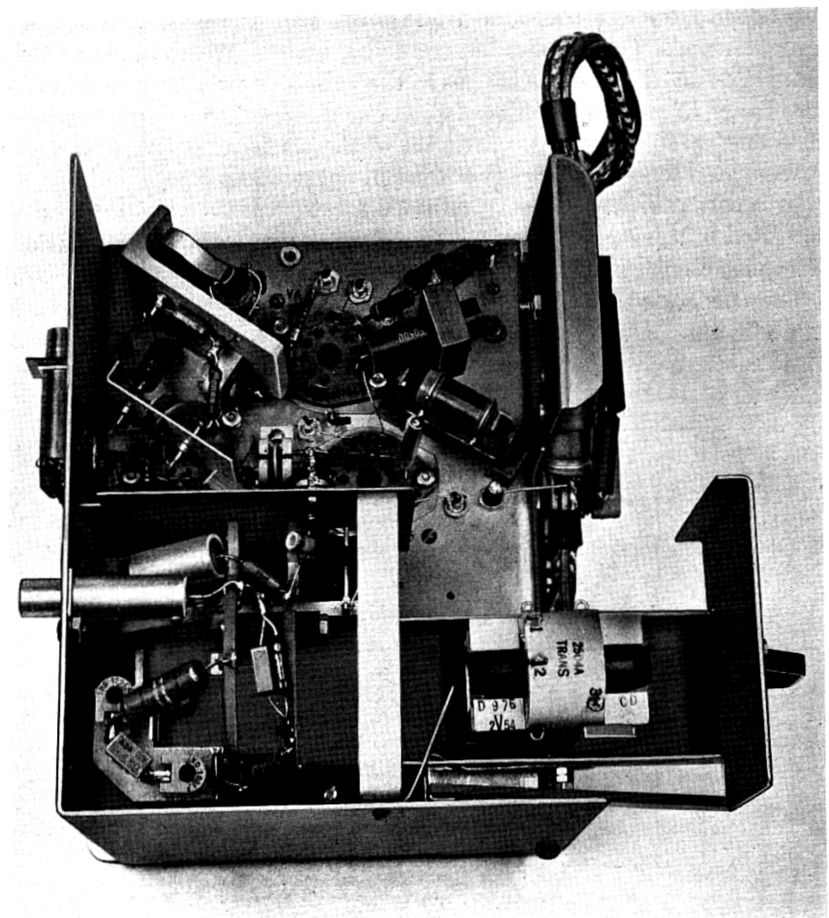

Fig. 10 — Jig for transformer measurement.

TABLE IX. — GAIN MEASUREMENTS AT 7.3 MC IN THOUSANDTHS OF DB

Transformer Number	Run-Out	Cut-Back × 0.001″				
		100	120	150	180	195
1	0	—	203	226	—	377
	28	—	—	—	—	—
2	0	154	166	216	—	360
	28	—	—	—	—	—
3	0	—	—	242	—	344
	28	—	216	240	—	—
4	0	—	—	—	—	351
	28	—	193	264	340	—
5	0	243	184	227	—	333
	28	—	—	—	—	—
6	0	—	—	—	—	377
	28	—	184	242	324	—

At the second stage of the experiment, Fig. 9, it appeared that the precision of measurement was rather poor due to the differences occurring when the transformer was disconnected from the amplifier and after the change in cutback and/or run-out reconnected by means of soldering. It was therefore decided to construct a contact fixture allowing the transformer to be plugged in and out of the amplifier.

For the first time after the fixture shown in Fig. 10 became available the transformers were measured twice — once soldered into the amplifier and once plugged in. This was done after the second reassembly and the previous measurements were adjusted to the fixture readings on the basis of this comparison. Almost all of the initial measurements (State 0) had to be discarded.

An additional deviation from the design occurred in the final stage when some of the transformers were cut back too far, to 0.195″ instead of 0.180″.

As an example the gain measurements at 7.3 mc are listed in Table IX.

When considering results such as in Table IX for further analysis the question arises what type of model should be fitted to the data. It goes without saying that apart from fitting the data the choice of the model must primarily make sense from an engineering standpoint. For designs like the hyper graeco-latin square of Section II and balanced designs in general the computational part of the analysis is small, measured in man-hours on a desk calculator. Changing the model in those designs by incorporating more factors or discarding alleged superfluous ones is simple, as the estimates of the effects of these factors in balanced situations are independent of the others.

In a case like in Table IX where no reasonable balance is left but where the operating factors (cutback, etc.) are measurable or quasi measurable, regression models are indicated. The computational effort on a desk calculator to estimate the parameters in the regression model is considerable for three operating factors, as in our case. To explore a sufficient set of modifications of a model for four or more factors is only practical if an automatic computer is available.

As a first step in the analysis a linear multiple regression equation on three variables was calculated, the independent variables being:

x_1 : number of resolderings

x_2 : run-out

x_3 : cut-back.

The model fitted was:

$$Y - \bar{y} = \beta_1(x_1 - \bar{x}_1) + \beta_2(x_2 - \bar{x}_2) + \beta_3(x_3 - \bar{x}_3).$$

Estimates b of the β's resulted in

$$b_1 = -0.023 \text{ db/step}$$

$$b_2 = -0.0028 \text{ db/degree}$$

$$b_3 = 0.0052 \text{ db/mil.}$$

The corresponding analysis of variance table is Table X. Having a set of numbers it is always possible to go through the calculations and obtain estimates of the β's. The important part, however, is to determine how well the model fits. Looking at the analysis of variance Table X it appears in this case that a substantial part of the total observed variation as measured by the total sum of squares is explained by the model. The variations taken care of by the model are accounted for by the sum of squares for regression. The remainder measures our error. The estimated $\hat{\sigma}$ from the residual is $\sqrt{0.000630} = 0.025$ db.

Table X. — Analysis of Variance for Linear Regression on 3 Variables

Source	*SS*	*D/F*	*MS*
Regression	0.102845	3	0.034282
Residual	0.012596	20	0.000630
Total	0.115441	23	

TABLE XI. — ANALYSIS OF VARIANCE FOR LINEAR REGRESSION ON X_1 AND X_3

Source	*SS*	*D/F*	*MS*
Regression x_3 alone	0.096059	1	0.096059
Improvement due to x_1 added	0.002085	1	0.002085
	0.098144	2	
Residual	0.017297	21	0.000824
	0.115441		

Estimated $\hat{\sigma} = 0.029$.

It is of importance to find out the magnitude of the contribution by the individual independent variables x_i to our model. The general way of doing this is to drop one or more of the independent variables, recompute the estimates for the regression coefficients for the remaining variables and study the result in a new analysis of variance table.

As an example consider the simplified model

$$Y - \bar{y} = \beta_3(x_3 - \bar{x}_3)$$

and ask for the importance of incorporating the reassembly variate x_1 into this model. We can list the results as in Table XI. The improvement due to the addition of x_1 is not significant at the 5 per cent level.

Fig. 11 illustrates this procedure for a number of possibilities. Whatever model for fitting is chosen the total sum of squares is the same. The horizontal line at the top of the picture corresponds to this value of 0.115441 $(\text{db})^2$ (Table X). The length of the bars shows the part that is explained by incorporating in the model the variables listed at the bottom of each bar.

The run-out x_2 by itself does not appear to contribute anything appreciable, although in combination with resoldering x_1 it shows up a little. Cutback x_3 alone accounts for the bulk of the variation. Resoldering x_1 also shows up alone, but once x_3 is incorporated, addition of x_1 is not too important. This behaviour corresponds to the very strong correlation (correlation coefficient = 0.93) between the independent variates x_1 and x_3. This correlation stems from the fact that an increase in cutback necessarily corresponds to a later resoldering.

Engineering considerations suggested that the amount of non-linearity due to the cutback variable x_3 should also be examined. Cutbacks smaller than about 0.150″ do not reach under the first turn of the winding (Fig. 8) so they do not influence the shielding operation as strongly as when the cutback exceeds 0.150″.

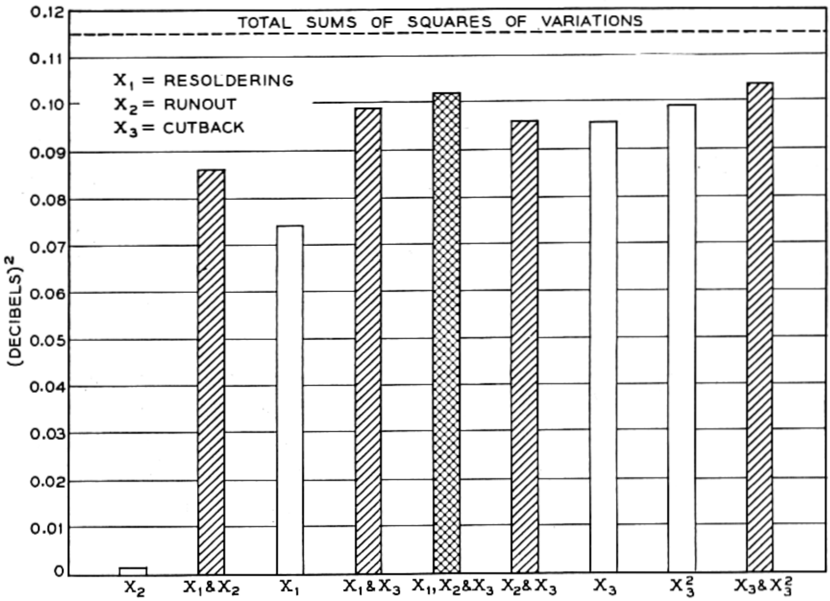

Fig. 11 — Contributions of various factors to the sums of squares of regression.

Introducing a quadratic term in the model

$$(Y - \bar{y}) = \beta_1(x_3 - \bar{x}_3) + \beta_{11}(x_3^2 - \overline{x_3^2})$$

gives the best fit to date as shown in Fig. 11. Run-out and resoldering are now left out, the former making no significant contribution and the latter being sufficiently taken care of by its correlation with the cutback. After all the resoldering was only of interest in the experimental situation, and did not occur in actual production.

Estimating the parameters yields

$$Y - 0.470 = 0.005\ x_3 + 0.000023\ x_3^2 \text{ db}$$

when x_3 is the cutback in 0.001″. The residual error standard deviation $\hat{\sigma} = 0.023$ db. Predicting some values

Cutback	*Gain*
0.120″	0.201 db
0.150″	0.238 db
0.180″	0.315 db

shows that 0.030″ less cutback with respect to 0.150″ makes a difference of about 0.04 db, whereas 0.030″ increase changes the gain by almost

0.08 db. Since gain should be insensitive to the variations in cutback which occur in manufacture, it was decided to keep the nominal cutback value at 0.120″.

In the analysis of each of the above experiments only one set of measurement results has been discussed. With the particular type of measuring set used, the gain of the amplifier is obtained as a continuous curve over the whole frequency range of interest. At about ten different frequencies ranging from 0.3 to 8.5 mc the results have been analyzed in the way described. In addition several discrete impedances in the transformer closely related to the elements in the equivalent diagram, Fig. 1, were measured directly.

In such a situation a very important check can be made about the assumptions underlying the experimentation and the analytical approach. On the one hand, we have the measurements of the performance of the transformer in the circuit and the measurements of various impedances connected with leakage, stray capacitances, etc. of the transformer. On the other hand, we have the analytical study of the model in the form of the equivalent diagram, Fig. 1, which provides us with a prediction of the over-all performance from the values of these impedances. If this prediction is sufficiently close to the measured over-all performance we can use control of the impedances to control the performance. In addition we can use the model for studying the consequences of contemplated major changes in the design.

From the point of view of guaranteeing reliability of complex systems it seems to be essential that a model as close to reality as possible be employed for prediction.

Comparisons between prediction from the equivalent diagram, Fig. 1, and measured curves have been made for the different experiments in the development program. Fig. 12 presents such a comparison for the previously described "wall-thickness" experiment. The changes in impedances observed corresponding to a change in wall thickness of 0.0006″ were fed into the formulas derived[5, 6] for the equivalent diagram. The resulting predicted gain values, together with the measured gain values, are plotted as a function of frequency in Fig. 12. Remembering the order of magnitude of the estimates for the error standard deviation, a few hundredths of a db in this type of transmission measurements, the agreement is satisfactory.

V. FINAL EVALUATION OF THE TRANSFORMER

The results of experiments like the ones described contributed to the tying down of specifications and controls in the manufacture of the

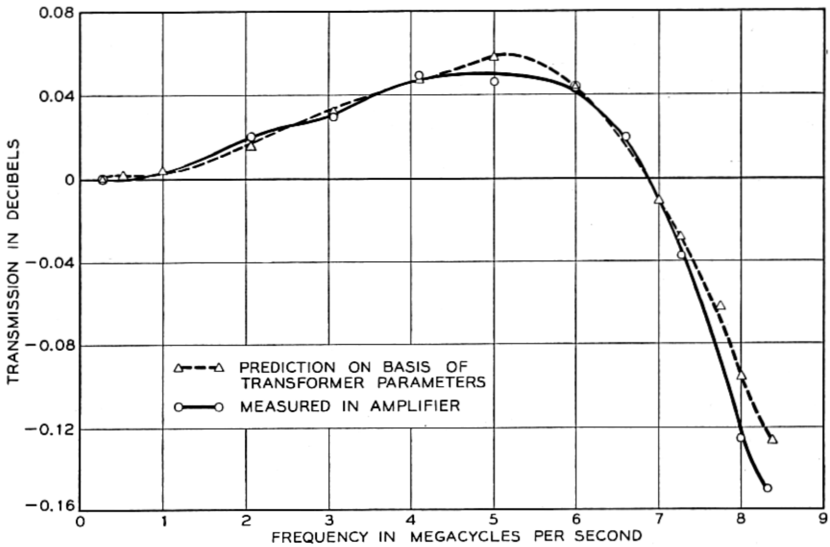

Fig. 12 — Comparison between measured and predicted transmission for a 0.6 mil increase in wall thickness.

transformer. As the measures derived from each of the experiments related only to a detail of the transformer, it was considered necessary to set up an experiment incorporating the results of the various tests, in order to examine the over-all performance of the transformer, in a complete amplifier.

In other words, it would be useful to confirm that the gain variations in the amplifier dependent on the (uncontrollable statistical) variations in the electrical parameters of the transformer are small enough to satisfy the systems designer. The experimental scheme adopted for this purpose called for a fair sized number of transformers basically belonging to two groups:

a. One group of transformers conforming to the current specifications and of recent manufacture at the time of this experiment.

b. One group of transformers consisting of recent rejects and all other old transformers that could be found, all having one or more parameters outside the specifications.

These transformers would be very carefully measured in the Laboratories, taking special care and using the best measuring equipment available. (The previous experiments described in this paper had been conducted in Western Electric factories.)

From the measured values of the parameters such as leakage inductance, stray capacitances, etc., the predicted gain would be computed, again using the formulas derived on the basis of the equivalent diagram. The computed gains would finally be compared to the measured ones. It was hoped that this experiment would show two things:

1. That recently produced transformers which showed satisfactory parameter-measurement results would yield good amplifiers.

2. That the parameters chosen for control measurements in the transformer manufacturing process were adequate to reasonably predict the over-all transmission performance in the amplifier.

The experiment was preceded by a pilot experiment to test the gain-measuring equipment. In both steps of experimentation two jigs for gain measurements were to be used, consisting of almost identical sub-amplifiers, and measurements at 15 frequencies between 0.3 and 8.5 mc were to be made. The pilot experiment was designed such that an estimate of the jig differences and of the influence of time could be made. In addition the magnitude of residual error could be determined.

Eight transformers were measured twice in each of the two jigs in the following sequence. (Table XII.)

As an example let us again choose the results at a high frequency, as the sensitivity of the transformer and amplifiers for small deviations from the ideal increases with frequency.

The time effect will be judged by the difference between the first and second half of the experiments, called H_1 and H_2 respectively.

Disregarding the time sequence in each half, which can always be recovered if so desired by examining the residuals, the results coded as before in thousandths of db are given in Table XIII. The analysis of variance is given in Table XIV. Using the three-way interaction as a measure of residual variation Table XIV shows that the transformer by time and the jig by time interactions are unimportant. The transformer by jig interaction although not significant at the 5 per cent level is disturbing in an experiment of this kind. This might indicate that contact trouble exists between the transformer and the jig. The transformers were not soldered in the jigs but contact was made by means of springs.

TABLE XII. — TRANSFORMER NUMBERS IN TIME SEQUENCE OF MEASUREMENT FROM LEFT TO RIGHT

	H_1				H_2			
Jig 1	1, 2, 3, 4		6, 5, 8, 7		7, 8, 5, 6		4, 3, 2, 1	
Jig 2		5, 6, 7, 8		2, 1, 3, 4		3, 4, 1, 2		8, 7, 6, 5

TABLE XIII. — PILOT EXPERIMENTS 8.3-MC GAIN MEASUREMENTS ON "MICROBEL" TEST SET. UNITS 0.001 DB

	Jig 1		Jig 2	
	H_1	H_2	H_1	H_2
Tr. 1	765	777	890	888
2	652	672	797	777
3	812	814	920	910
4	760	747	915	927
5	832	840	961	938
6	775	743	909	887
7	756	757	889	878
8	698	705	832	820
Average for Jig 1	756		Average for Jig 2	884

TABLE XIV. — ANALYSIS OF VARIANCE OF PILOT EXPERIMENT

Source	SS	D/F	MS	Significance Level
Between Jigs	129159	1	129159	$\ll 1\%$
Between Transformers	79930	7	11419	$\ll 1\%$
Between Time	256	1	256	$>10\%$
Transf × Jigs	2684	7	383	$\approx 7\%$
Jig × Time	229	1	229	$>10\%$
Transf × Time	602	7	84	$>25\%$
Transf × Jigs × Time	803	7	115	
Total	213663	31		

In the main experiment following this pilot one, contact trouble arose again. Moving up in the table the time effect appears negligible. The significant differences between transformers do not have to be considered as this reflects only the differences in their nominal gain, but the jig effect is highly significant even with respect to the transformer by jig interaction.

It would have been unrealistic to expect the jigs to be equal because of their complexity. What was hoped was that the difference between the two would be substantially constant. From the averages listed in Table XIII, we estimate the difference between Jig 1 and Jig 2 as 0.128 db, with 90 per cent confidence limits of 0.114 to 0.142 db based on standard deviation for the average difference of 0.008 db with 15 degrees of freedom. For this latter estimate the jig interactions were pooled with the "error" variance.

If the variations between jigs would remain within the above limits in the main experiment yet to be made, this would be reasonable. However, the jig by transformer interaction tells us to be on guard.

The main experimental design following this pilot study is presented in Table XV. The intent was to obtain units with as wide a spread of properties as possible. Then, as explained in the beginning of this section, we could see if the formulas which predict the over-all gain from the detailed impedances of the transformer would hold over a wide enough range. In each period all the transformers listed were measured in one jig and then in the other. The jig sequence was varied from period to period. Transformers meeting specifications and rejects were collectively randomized over serial numbers. Therefore 50 good transformers of recent production were combined with 33 rejected ones. The latter were rejected for a variety of reasons and over a considerable period of time. In principle, no special design is necessary to obtain observations for comparing detailed measurements of a transformer to the

TABLE XV. — MEASURING SCHEDULE FOR TRANSFORMERS IN TERMS OF THEIR SERIAL NUMBERS

Runs = Days	1		2		3		4	
	Jigs							
	1	2	1	2	1	2	1	2
Morning	1	10	22	25*	43	50	64	72
	2	3	23	26	44	46	65*	73
	3	2	24*	27	45	44	66*	71
	4	1	25*	28	46	45	67	64
	5	6	25	22	47	49*	68	66*
	6	5*	27	29	48*	47	69	67
	7	4	28	30	49*	48*	70	65*
	8	8	29	31	50	51	71	69
	9*	7	30	23	51	52	72	70
	10	9	31	24*	52	43	73	68
	24*	40*	49*	57*	65*	80*	9*	20*
	35*	25*	59*	48*	77*	66*	14*	5*
Afternoon	11	20*	32	35*	53	59*	74	79
	12	13	33	36	54	57*	75	80*
	13	11	34	34	55	58	76	78
	14*	12	35*	38	56	62	77*	82
	15	16	36	32	57*	60	78	83
	16	14	37	39	58	61	79	74
	17	15	38	37	59*	63	80*	81
	18	19	39	42	60	53	81	76
	19	21	40*	33	61	56	82	77*
	20	17	41	40*	62	54	83	84
	21	18	42	41	63	55	84	75
	40*	24*	57*	49*	80*	65*	20	9
	25*	35*	48*	59*	66*	77*	5	14

performance of an amplifier containing the same transformer. But the time involved in measuring more than 80 transformers in each of two jigs is several days, so the possibility of time effects had to be watched. First, the numbers in the design were assigned at random to the pool of good and rejected transformers. Second, to keep a running check on the precision of the measurements a number of observations were repeated on different days (runs). In each pair of adjacent runs, and in the last and the first, a set of four transformers was replicated both in Jig 1 and Jig 2. From Table XV it can be seen that these linking sets are the following:

Run I and II	Transformers	24, 25, 35, 40
Run II and III	Transformers	48, 49, 57, 59
Run III and IV	Transformers	65, 66, 77, 80
Run IV and I	Transformers	5, 9, 14, 20

As a further precaution, which it was found not necessary to use in the analysis, half of the transformers in the sets above were replicated in the same period of the day, the other half in different periods. For Runs I and II we find from Table XV, in Jig 1, transformers 24 and 40 in the same periods, transformers 25 and 35 in different periods, in Jig 2, transformers 25 and 35 in the same periods, transformers 24 and 40 in different periods. A typical analysis for one linking set disregarding the period allocation, is shown in Table XVII for the observations taken at 8.3 mc and listed in Table XVI.

Both the interactions of jigs and runs and jigs and transformers are significant at the 5 per cent level. The run main effects mean square is not significant but the interactions with the jigs are disturbing. These interactions showed up to a greater or lesser extent in all the comparisons, both in those similar to this one and in the pilot experiment. The importance of the jig by run interaction can be illustrated if we list the

TABLE XVI. — TYPICAL SET OF LINKING MEASUREMENTS INCLUDED IN MAIN EXPERIMENT. UNITS IN 0.001 DB

Transformer	Run III		Run IV	
	Jig 1	Jig 2	Jig 1	Jig 2
65	4	230	15	195
66	−4	191	10	195
77	−65	92	−47	75
80	−45	152	−18	148

TABLE XVII. — ANALYSIS OF VARIANCE. TYPICAL LINKING SET IN MAIN EXPERIMENT

Source	*SS*	*D/F*	*MS*	Significance Level
Between Jigs	127449	1	127449	$\ll 1\%$
Between Runs	20	1	20	$>25\%$
Between Transformers	22975	3	7658	$\ll 1\%$
Jigs × Runs	931	1	931	$<5\%$
Jigs × Transformers	2262	3	754	$<5\%$
Transformers × Runs	337	3	112	20%
Jigs × Runs × Transf.	170	3	57	

TABLE XVIII. — JIG COMPARISON

	Jig 2 − Jig 1 (in db)	90% Confidence Limits in db
Pilot	0.128	0.114 to 0.142
Run I & II	0.114	0.054 to 0.174
Run II & III	0.149	0.037 to 0.201
Run III & IV	0.170	0.120 to 0.220
Run IV & I	0.121	0.040 to 0.201

average differences between the jigs as observed in the various pairs of runs and in the pilot experiment. In Table XVIII are also calculated 90 per cent confidence limits for the jig difference based on a variance estimate incorporating the variances for the jig interactions. It was originally hoped to use an estimate of difference between the jigs to eliminate the jig effect from all the individual observation. The wide confidence limits of the jig difference estimates compared to the 0.01 db order of magnitude we are interested in, do not allow us to do this. Therefore the subsequent analysis was made separately for both jigs.

In addition to the gain measurements the following impedances were observed on all transformers: Resistive and Reactive component of leakage (R_R and R_L); Capacitance over the high winding (C_H); Stray Capacitances (C_{S_1} and C_{S_2}). These impedance results introduced in the formulas for the equivalent diagram of the amplifier yield a predicted gain, which should represent, if everything is all right, the measured gain values.

Using the coefficients m_i, $i = 1, 2, \cdots 5$, as computed from the equivalent diagram, we predict the transmission gain to be:

$$Y = m_0 + m_1 R_R + m_2 R_L + m_3 C_H + m_4 C_{S_1} + m_5 C_{S_2}.$$

Here, m_0 is an arbitrary constant, not important in these considerations, as in measuring amplifiers of this type, frequency-independent loss networks are often introduced, which add an additional constant in m_0.

Calling the measured transmission gain y, we will try to fit the model

$$y = \alpha + \beta Y.$$

The Y is taken as the independent variable as the transformer parameter measurements are more precise than the transmission measurements. In general for this type of regression line fitting the independent variable should be known without error.

If the equivalent diagram is adequate β should be equal to 1; our estimates b of β therefore should not differ significantly from that value. Table XIX lists for 8.3 mc the estimates of the slopes, their standard deviations, and the estimated standard deviations of the residual variations not accounted for by the regression. The intercept α like the parameter m_0 in the prediction equation, is of no interest as explained above.

It is seen that the agreement of the slopes with the theoretical value 1.00 is reasonably good, especially for Jig 2.

The rejects selected for this experiment fall into two classes, those in one set of recent manufacture not meeting the manufacturing specifications, but not too far removed from them, and the others left-over from the development program. Even for such groups with wide variations in their parameters not meeting the end requirements the agreement between prediction and measurement is reasonable. The Jig 1 results gen-

Table XIX. — Comparison Between the Regression Parameters Estimated from the Measurements in Both Jigs. Frequency 8.3 Mc

	Slope b db/db		Standard error of slope s_b db/db		Standard error residual (db)	
	Jig 1	Jig 2	Jig 1	Jig 2	Jig 1	Jig 2
Standard production 50 units	1.38	0.97	0.18	0.11	0.04	0.02
Rejects from production 18 units	0.90	1.11	0.11	0.07	0.18	0.08
Rejects from development 15 units	0.78	0.82	0.18	0.08	0.07	0.03
All 83 units pooled	0.84	1.05	0.06	0.04	0.05	0.02

erally show a bigger deviation from the ideal value of 1 for the slope and, also, larger residual variations as indicated by the estimates of the variance. It will be remembered that from the pilot experiment and the "built-in" control in the main experiment it appeared that the difference between Jig 1 and Jig 2 was not constant. Subsequently a poor contact in Jig 1 was identified. However, the general result of the experiment was satisfactory, in that the feasibility of maintaining the overall performance of the amplifier within the required limits by controlling the parameters of the transformer was demonstrated.

VI. CONCLUSION

The foregoing describes some highlights in the statistical aspects of the development program of one of the critical components in the L3 system. It will be clear that statistics can be a very powerful help, when integrated in the engineering efforts.

VII. ACKNOWLEDGEMENTS

Many individuals throughout the Bell Laboratories and the Western Electric departments concerned contributed to the success of the L3 system. Most intimately concerned with the development of the transformer were C. W. Thulin and W. L. Brune. The systems aspects, both in development and measurements, were the responsibility of B. J. Kinsburg and G. R. Leopold. J. H. Bash performed most of the measurements discussed. F. E. Stehlik and S. A. Levin performed the equivalent-network studies. N. E. Earle and his co-workers represent the Western Electric effort, and J. W. Tukey and M. E. Terry of the Mathematical Research Department contributed substantially to the design and analysis of the experiments.

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