Attenuation in Continuously Loaded Coaxial Cables

By GORDON RAISBECK

(Manuscript received May 22, 1957)

The formula for the attenuation of a coaxial line loaded continuously with magnetic materials involves, after a change of variables, only three parameters, even when the effect of dielectric losses is included. If the dimensions of the line have their optimum values, the attenuation is a function of only two parameters. The relation is exhibited both graphically and analytically, in forms which can be applied to practical problems. A simple numerical example is given to illustrate the use of the formula.

INTRODUCTION

The development of ferrite materials having high permeability and low loss at high frequencies has raised again the question of continuous loading of coaxial cables. The question is one of many treated in a recent thorough paper on loading by P. M. Prache.[1,2] He considers, among other things, cables loaded with coaxial cylinders of magnetic material, and attacks the problem of minimizing the attenuation through the regulation of various free parameters.

The present paper is another attempt to find the conditions under which attenuation in a loaded coaxial cable is a minimum.[3] It differs from the previous attempts in several respects. First, a drastic set of changes of variable makes the problem much easier to manage mathematically. Second, the resulting simplification makes it possible to include the effect of dielectric loss in the loading material. Third, a detailed analysis of magnetic losses, such as found in Prache's paper, is ignored and losses are described in terms of Q, the ratio of peak energy stored to energy lost per radian. Special assumptions about the joint restrictions on Q and the permeability (or dielectric constant, as the case may be) are reserved for later steps in the analysis.

This paper is entirely theoretical. The results indicate that substantial reductions in attenuation are possible at frequencies of a few megacycles with magnetic materials having a pQ product (p = relative permeability) of 10^4. Whether this is worth while practically is another matter. For example, ferrites having a pQ product greater than 10^4 and p less than 100 have been made,[4] but these are brittle and rigid. Stringing beads of ferrite or assembling split cylinders about a center wire are quite conceivable, but the mechanical difficulties are enough to dampen the enthusiasm of anyone designing for manufacture. Nevertheless, the analysis in this paper provides a sound basis for comparing the electrical characteristics of various loading methods as they became available, and provides quantitative information about dimensions and electrical properties required if new loading materials are sought.

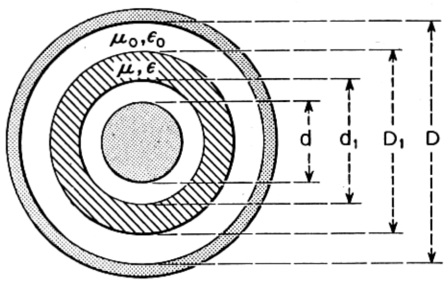

Fig. 1 — Schematic cross section of a coaxial transmission line loaded with magnetic material. The diameters of the inner and outer conductors are d and D, respectively. The inner and outer diameters of the cylinder of loading material are d_1 and D_1, respectively.

CABLE STRUCTURE AND EQUIVALENT CIRCUIT

A cross section of the cable is shown in Fig. 1. The diameter of the inner conductor is d, and the (inside) diameter of the outer conductor is D. The inside and outside diameters of the loading cylinder are d_1 and D_1. The main dielectric has dielectric constant ϵ_0 and permeability μ_0, and the loading material has dielectric constant ϵ in the radial direction and permeability μ in the tangential direction. (Other components of μ and ϵ are not important to the problem; the material may as well be assumed isotropic. However, if the shell has air-gaps, the effective dielectric constant and permeability are not in fact isotropic.) The permeability of the loading material relative to that of the main dielectric is defined as*

$$p = \frac{\mu}{\mu_0},$$

and the reciprocal of the dielectric constant of the loading material rela-

* The notation used in this paper is due mostly to Prache, Reference 2.

tive to that of the main dielectric is defined as

$$q = \frac{\epsilon_0}{\epsilon}.$$

Notice that ϵ_0 and μ_0 need not be the dielectric constant and permeability of free space. It will turn out that the various diameters d, d_1, D_1 and D are not individually important, but their ratios are. Accordingly we adopt the notation

$$N = \frac{D}{d},$$

$$n = \frac{D_1}{d_1}.$$

A convenient equivalent circuit of a unit length of line is represented

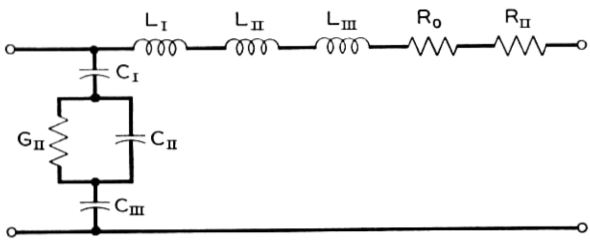

Fig. 2 — Equivalent circuit of a unit length of magnetically loaded line.

in Fig. 2. The series elements are L_{I}, the inductance due to magnetic fields between d and d_1; L_{II}, the inductance due to fields between d_1 and D_1; R_{II}, the loss associated with L_{II}; L_{III}, the inductance due to fields between D_1 and D; and R_0, the series conductor resistance. The shunt elements are C_{I}, the capacitance due to electric fields between d and d_1; C_{II}, the capacitance due to electric fields between d_1 and D_1; G_{II}, the loss conductance associated with C_{II}; and C_{III}, the capacitance due to fields between D_1 and D. The main dielectric is assumed to be free of loss, and the frequency is assumed high enough so that all current flow is at the surface of the conductors. The resulting expressions are:

$$L_{\text{I}} = \frac{\mu_0}{2\pi} \log \frac{d_1}{d},$$

$$L_{\text{II}} = \frac{\mu}{2\pi} \log \frac{D_1}{d_1} = \frac{p\mu_0}{2\pi} \log \frac{D_1}{d_1},$$

$$L_{\text{III}} = \frac{\mu_0}{2\pi} \log \frac{D}{D_1},$$

$$\frac{1}{C_{\text{I}}} = \frac{1}{2\pi\epsilon_0} \log \frac{d_1}{d},$$

$$\frac{1}{C_{\text{II}}} = \frac{1}{2\pi\epsilon} \log \frac{D_1}{d_1} = \frac{q}{2\pi\epsilon_0} \log \frac{D_1}{d_1},$$

and

$$\frac{1}{C_{\text{III}}} = \frac{1}{2\pi\epsilon_0} \log \frac{D}{D_1}.$$

The losses are evaluated through the relations

$$R_{\text{II}} = \frac{2\pi f L_{\text{II}}}{Q'},$$

and

$$G_{\text{II}} = \frac{2\pi f C_{\text{II}}}{Q''}.$$

It is convenient to define the "Q" of the unloaded line as

$$Q = \frac{\mu_0 f \log (D/d)}{R_0}$$

so that

$$R_0 = \frac{2\pi f L_0}{Q}.$$

The inductance L_0 is simply the inductance per unit length which the line would have if it contained no loading material.

The various components of series impedance and shunt admittance are easily added to make an equivalent circuit with one series inductance L, one series resistance R, one shunt capacitance C, and one shunt conductance G, as in Fig. 3. Here

$$\begin{aligned} L &= L_{\text{I}} + L_{\text{II}} + L_{\text{III}}, \\ &= \frac{\mu_0}{2\pi} [\log (N/n) + p \log n], \end{aligned}$$

and

$$\begin{aligned} R &= R_0 + R_{\text{II}}, \\ &= \frac{2\pi f}{Q} \frac{\mu_0}{2\pi} \log N + \frac{2\pi f}{Q'} \frac{p\mu_0}{2\pi} \log n, \\ &= f\mu_0 \left[\frac{1}{Q} \log N + \frac{p}{Q'} \log n \right]. \end{aligned}$$

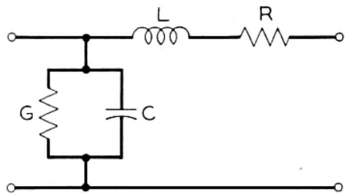

Fig. 3 — Simplified equivalent circuit of a unit length of magnetically loaded line.

If Q'' is assumed to be large compared to unity, then

$$\frac{1}{C} = \frac{1}{C_{\mathrm{I}}} + \frac{1}{C_{\mathrm{II}}} + \frac{1}{C_{\mathrm{III}}},$$

$$= \frac{1}{2\pi\epsilon_0}\left[\log(N/n) + q \log n\right],$$

and

$$G = \frac{2\pi f}{Q''} \frac{\dfrac{1}{C_{\mathrm{II}}}}{\left[\dfrac{1}{C_{\mathrm{I}}} + \dfrac{1}{C_{\mathrm{II}}} + \dfrac{1}{C_{\mathrm{III}}}\right]^2},$$

$$= \frac{2\pi f}{Q''} \frac{2\pi\epsilon_0 q \log n}{[\log (N/n) + q \log n]^2}.$$

TRANSMISSION CHARACTERISTICS

The propagation constant and characteristic impedance of the line are easily described in terms of R, L, C and G. In fact, to a first approximation

$$Z_0 = \text{characteristic impedance}$$

$$= \sqrt{\frac{L}{C}}$$

$$\beta = \text{phase constant}$$

$$= \omega\sqrt{LC}$$

$$\alpha = \text{attenuation per unit length}$$

$$= \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}.$$

It is easy to show that

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{[\log (N/n) + p \log n][\log (N/n) + q \log n]},$$

and

$$\beta = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\frac{\log (N/n) + p \log n}{\log (N/n) + q \log n}}.$$

We shall devote more attention to the attenuation-per-unit-length α. Substituting values for R, L, C and G, one finds

$$\alpha = \frac{f\mu_0}{2}\left[\frac{1}{Q}\log N + \frac{p}{Q'}\log n\right] \frac{2\pi\sqrt{\epsilon_0/\mu_0}}{\sqrt{[\log(N/n) + p \log n][\log(N/n) + q \log n]}} + \frac{2\pi f}{2Q''}\frac{2\pi\epsilon_0\, q \log n}{[\log (N/n) + q \log n]^2}\frac{1}{2\pi}\sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{[\log (N/n) + p \log n][\log (N/n) + q \log n]}.$$

The factor $\pi f\sqrt{\mu_0\epsilon_0}$ is common to both terms. In fact, a little reflection about the unloaded line (set $Q' = Q'' = \infty$, $p = q = 1$) shows that for the unloaded line

$$\alpha_0 = \frac{\pi f\sqrt{\mu_0\epsilon_0}}{Q}.$$

Now make a change of variable

$$\frac{\log N/\log n + q - 1}{p - q} = v.$$

Then

$$\frac{\alpha}{\alpha_0} = kv^{-1/2}(v + 1)^{-1/2} + v^{1/2}(v + 1)^{-1/2} + mv^{-3/2}(v + 1)^{1/2},$$

where

$$k = \frac{1 - q}{p - q} + (Q/Q')\frac{p}{p - q},$$

and

$$m = (Q/Q'')\frac{q}{p - q}.$$

Note that the effect of magnetic loss is included in k, and the effect of dielectric loss in m.

OPTIMUM DIMENSIONS FOR MINIMUM ATTENUATION

In the ordinary case where the conductor resistance R_0 is simply ohmic skin resistance, Q is a well-known function of D and N:

$$Q = \frac{D \log N}{1 + N} \sqrt{\pi \sigma \mu_0 f},$$

where σ is the conductivity of the conducting material. For fixed D, the maximum value of Q, and hence the minimum value of α_0 occurs when

$$1 + N = N \log N,$$

or

$$N = 3.59.$$

If v is kept constant (e.g., by keeping $\log N/\log n$ constant) this also yields the lowest value of α. Hence, unless some mechanical or dimensional restraint limits n, the value $N = 3.59$ is the optimum for attenuation reduction whether loading is used or not.

If we now regard Q, Q', Q'', p, q, d, d_1 and D_1 as independent parameters, we find that all of the new variables are independent of the dimensions save v. Hence it is possible to find the optimum dimensions of the line by differentiating with respect to v, and setting the derivative equal to zero. After solving for v, the result is

$$v = \frac{k + 5m + \sqrt{k^2 - 14mk + m^2 + 12m}}{2(1 - 2k - 2m)}.$$

It is necessary to reject the root with the negative radical, because v is always a positive number. Unfortunately, the expression for α becomes rather complicated if this value is substituted for v. The result is

$$\begin{aligned}\frac{\alpha^2}{\alpha_0{}^2} &= \frac{16}{9}(1 - k)(k - m) + \frac{8}{9}(1 - k)(k - m + \sqrt{\ }) \\ &\qquad + \frac{8(1 - k)(k - m)^2}{9(k - m + \sqrt{\ })}, \\ &= \frac{8(1 - k)(2k - 2m + \sqrt{\ })^2}{9(k - m + \sqrt{\ })}, \\ &= \frac{8}{3}(1 - k)(k - m) - \frac{2(k - m)^3}{27m} + \left[\frac{8}{9}(1 - k) + \frac{2(k - m)^2}{27m}\right]\sqrt{\ },\end{aligned}$$

where

$$\sqrt{\ } = \sqrt{k^2 - 14mk + m^2 + 12m}.$$

Of these, the first seems to be suited to general computation, and the second to slide rule computation. The third form is indeterminate when $m = 0$, which is just the region which it is desirable to investigate. However, if special relations between k and m are assumed, then some easy results are available. For example, suppose dielectric loss can be neglected, i.e., assume $m = 0$. Then

$$\frac{\alpha}{\alpha_0} = kv^{-1/2}(v + 1)^{-1/2} + v^{1/2}(v + 1)^{-1/2}.$$

A family of these curves for various values of k is plotted in Fig. 4. The minimum occurs when

$$v = \frac{k}{1 - 2k}$$

and has the value

$$\frac{\alpha}{\alpha_0} = 2\sqrt{k(1 - k)}.$$

This curve is the lowest curve in Fig. 5.

To include the effect of dielectric loss, one can assume a fixed value of

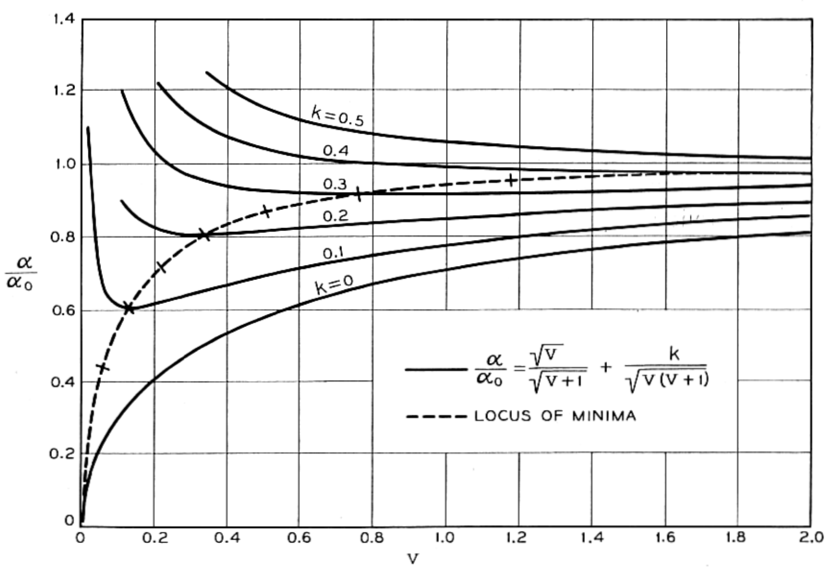

Fig. 4 — Reduction of attenuation in a magnetically loaded line in terms of the normalized dimension parameter v and the normalized magnetic loss parameter k. For each fixed k, the value of v yielding minimum attenuation is shown.

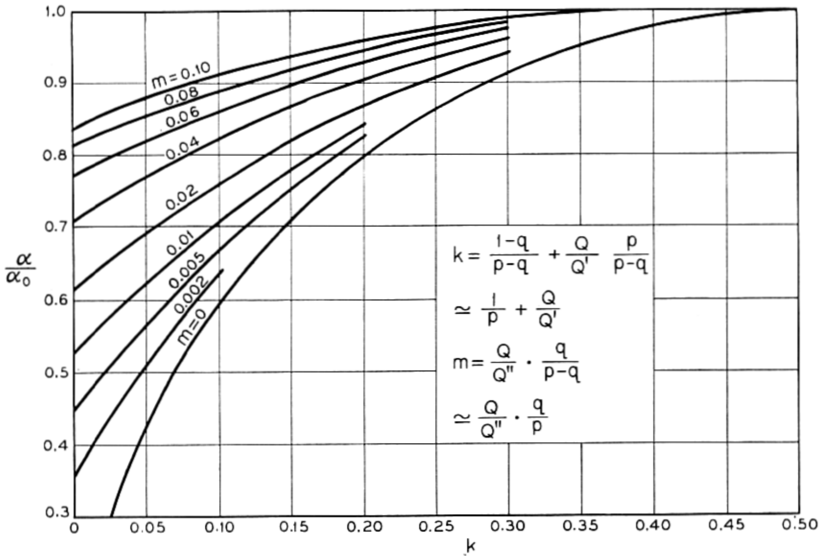

Fig. 5 — Maximum reduction of attenuation in a magnetically loaded line in terms of the magnetic loss parameter k and the dielectric loss parameter m of the magnetic material.

m and proceed as before. The result is a family of curves of α/α_0 versus k for fixed values of m. Such a family is plotted on Fig. 5 for various values of m. These data may be replotted as in Fig. 6 to show m versus k for constant α/α_0. This family of curves is useful to show the extent to which one kind of loss can be traded for another. As α/α_0 decreases, the slope of the curves decreases. This means that the role of m, and hence of dielectric loss, is becoming more and more important compared to that of k, which includes magnetic loss.

REDUCTION IN ATTENUATION WITH OPTIMUM DESIGN

If all dimensions have their optimum values, then the reduction in attenuation can be described in terms of two variables only, k and m. If, furthermore, dielectric loss is known to be negligible, for each value of α there is a unique value of k, and

$$k = \frac{1-q}{p-q} + \frac{Q}{Q'}\frac{p}{p-q} = f\left(\frac{\alpha}{\alpha_0}\right) = \frac{1 - \sqrt{1 - (\alpha/\alpha_0)^2}}{2}.$$

If we assume that p is large compared to q, this reduces to

$$\frac{1}{p} + \frac{Q}{Q'} = \frac{1 - \sqrt{1 - (\alpha/\alpha_0)^2}}{2}.$$

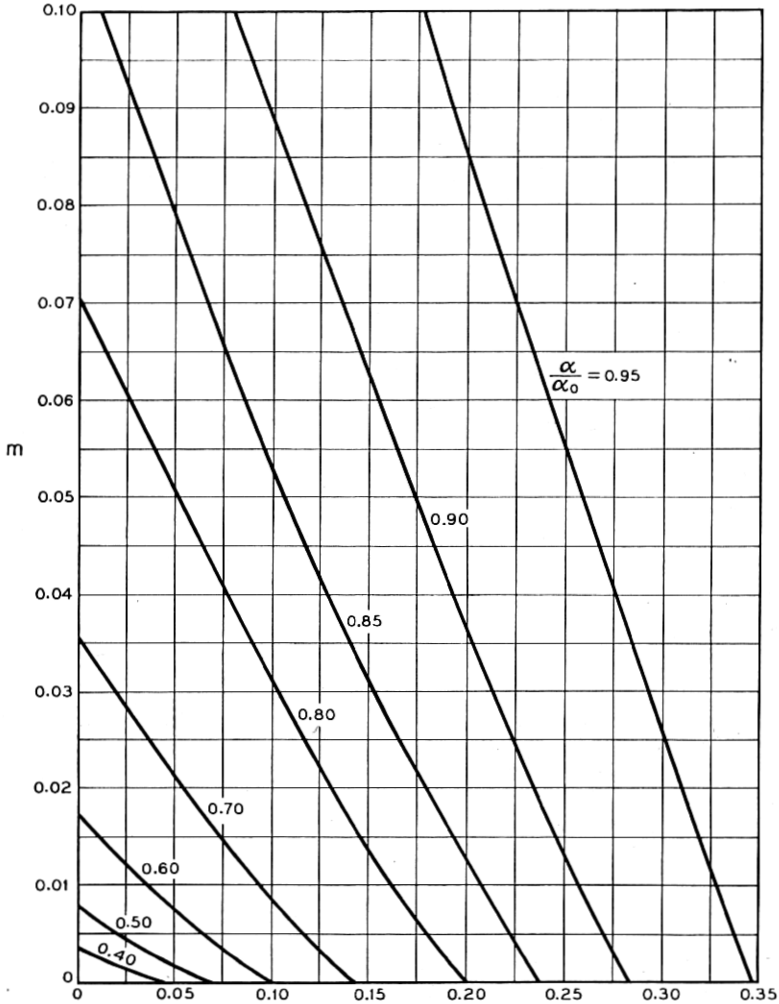

Fig. 6 — The relation between dielectric loss parameter m and magnetic loss parameter k for which the maximum possible reduction of attenuation is the same, plotted for several different values of attenuation reduction.

If we assume further the old rule of thumb that pQ' equals a constant for different types of cores made from the same type of material, then the lowest value of α is found when

$$p = \sqrt{\frac{pQ'}{Q}}.$$

Then

$$k = 2\sqrt{\frac{Q}{pQ'}},$$

and

$$\frac{\log N}{\log n} = 1 + \frac{2}{1 - 2\sqrt{Q/pQ'}} \cong 3$$

if pQ' is large compared to Q. These results, and their counterparts involving q explicitly, were found by Prache.[2] They have been criticized on the ground that pQ' is not a constant for all types of materials. It is clear, however, that the minimum value of α arrived at from the above formula is nearly correct, even if p deviates rather widely from $\sqrt{pQ'/Q}$, whereas the value of $\log N/\log n$ required to realize the minimum attenuation will be altered. In fact, if $p > \sqrt{pQ'/Q}$, $\log N/\log n > 3$, i.e., the shell of loading material is thinner than before. It is unlikely in practical cases that we should want to use $p < \sqrt{pQ'/Q}$, but if we did, a thicker shell of loading material would be required.

The analogous approximation when dielectric loss is not neglected is:

$$k = \frac{1}{p} + \frac{Q}{Q'},$$

$$m = \frac{Q}{Q''}\frac{q}{p}.$$

It is clear from the graphs that if the loading material has dielectric losses, then one should use a thinner shell of material having higher p. The relative variation of p, Q' and Q'', the relative permeability and the magnetic and dielectric "Q" respectively, in one general type of material has not been described in detail, and probably no concise approximation like ($pQ' =$ constant) exists which involves all three. It must be borne in mind furthermore that k and m involve Q and hence depend on the dimensions of the line. For a given line size one could compute k and m for known magnetic materials and deduce from Fig. 5 or Fig. 6 whether it is worth while to load the line with any particular material.

All of the reasoning on previous pages applies, with appropriate modification, to the continuous loading of any transmission link whose cross-section is a conformal transformation of a circular annulus. This broad class includes any uniform transmission line having just two conductors. Some examples are given in the author's patent.[3]

NUMERICAL EXAMPLE

Suppose for the sake of a numerical example that dielectric losses are negligible, that $pQ' = 10^4$ for the loading material to be used, that the conductors are made of copper, and that the dielectric constant of the loading material is high. At frequencies such that the skin depth of currents is small compared to conductor thickness,

$$Q = (D/3.59)\sqrt{\pi\sigma\mu_0 f},$$

where σ is conductivity and all quantities are in MKS units. For copper

$$\sigma = 58.10^6 \text{ mho/meter},$$

and for most dielectrics

$$\mu_0 = 12.57 \times 10^{-7} \text{ henry/meter}.$$

Then

$$Q = 4.23\, D\sqrt{f}$$

when D is in meters and f in cycles per second. If D is measured in inches and f in megacycles per second, then

$$Q = 107 D_{\text{in}}\sqrt{f_{\text{mcps}}}\,.$$

The optimum value of p is

$$p = \frac{10^4}{\sqrt{107 D_{\text{in}} f_{\text{mcps}}^{1/2}}} = 5.44 D_{\text{in}}^{-1/2} f_{\text{mcps}}^{-1/4},$$

and k has the value

$$k = \frac{2}{p} = 0.368 D_{\text{in}}^{1/2} f_{\text{mcps}}^{1/4}.$$

Then

$$\frac{\alpha}{\alpha_0} = 2\sqrt{0.368 D_{\text{in}}^{1/2} f_{\text{mcps}}^{1/4}(1 - 0.368 D_{\text{in}}^{1/2} f_{\text{mcps}}^{1/4})}.$$

The resulting curves are plotted on Fig. 7 for $D = 1''$, $\frac{1}{2}''$, $\frac{1}{4}''$, and $\frac{1}{10}''$, for values of f between 100 kilocycles per second and 10 megacycles per second. It is clear from these curves that the improvement attainable in a one-inch line is modest over this frequency range, but that worthwhile improvements can be attained in a $\frac{1}{10}''$ cable even at many megacycles.

The effect of dielectric loss might be included thus: Take the 0.25″ line at 1 megacycle per second. Imagine that Q'' is constant, say, and that, e.g.

$$Q'' = 200,$$

$$q = 0.10.$$

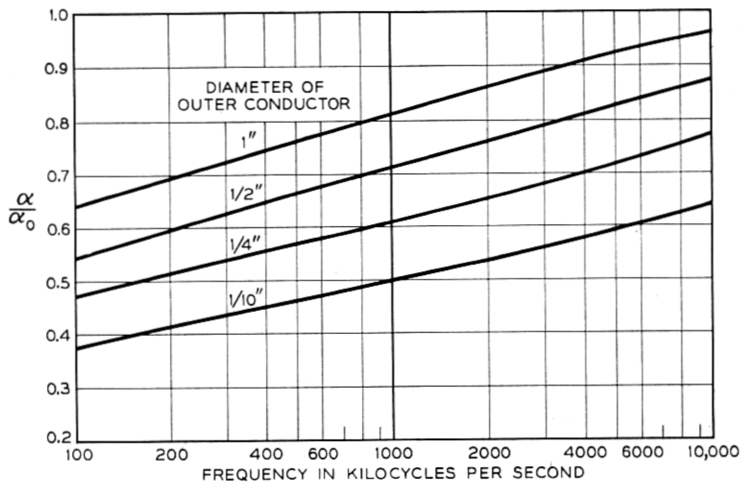

Fig. 7 — Maximum possible reduction in attenuation by magnetic loading with material having a pQ' product of 10^4 for copper coaxial lines of various diameters, plotted as a function of frequency.

Then

$$\frac{Q}{Q''} = \frac{85}{200} = 0.425,$$

$$m = \frac{Q}{Q'}\frac{q}{p}$$

$$= \frac{0.0425}{p}.$$

Neglecting the dielectric loss we find

$$p = 5.44\, D_{\text{in}}^{-1/2} f_{\text{mcps}}^{-1/4} = 10.9.$$

Hence

$$m \leqq 0.0040.$$

From Fig. 5 it is easy to see that α/α_0 is increased from 0.78 to about 0.81 by the dielectric loss. A little experimentation with different values of p shows that this is very nearly the optimum. If, on the other hand, e.g.,

$$Q'' = 20,$$

then

$$m = \frac{0.425}{p},$$

and α/α_0 is increased from 0.78 to 0.90. This can be reduced to 0.88 by choosing $p \cong 20$.

REFERENCES

1. P. M. Prache, Noyaux et Coquilles dans le Domaine des Tele-Communications, Cables et Transmission, **6,** 1 and 2, Jan. and April 1952.
2. U. S. Patent 2,727,945, High Frequency Magnetic Elements and Telecommunication Circuits, issued December 20, 1955 to M. P. Prache.
3. Certain aspects of this analysis are treated slightly differently in U. S. Patent 2,787,656, Magnetically Loaded Conductors, issued April 2, 1957, to the present author.
4. Private communication from F. J. Schnettler. There appears to be no reference to such materials in publications, where emphasis has been placed on high relative permeability p.