System Design of the Flying Spot Store

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The flying spot store is a versatile and efficient semipermanent information storage system developed for use in the electronic telephone switching system. Because of the form of the store, many variables enter into system design. In this paper the factors which control speed, capacity, number of channels, physical size and probability of error in readout are discussed. It is shown that these factors are related to the intended application for the store, to the form of the store and to device parameters. A number of scaling operations which involve exchanges between system parameters such as speed and capacity or between system and device parameters are given.

I. INTRODUCTION

1.1 *Purpose and Organization of the Paper*

This paper discusses physical factors which control the speed, capacity, number of channels, physical size and probability of error in readout in the system design of the flying spot store. These factors are related to the form of the store, to the intended application and to device characteristics.

All of the factors which enter into system design interact strongly. For this reason, a number of scaling relationships which involve exchanges between system variables such as speed and capacity, or between system variables and device parameters, are given. A graphical representation for the flying spot store is developed that demonstrates the relationships between the factors entering into physical system design. This representation is used in discussing the range of realizable designs.

In system design, devices and operating conditions are chosen to meet the requirements of speed, capacity, etc., set by the intended application. Electronic circuit design then proceeds and, if necessary, the parameters chosen are modified on the basis of circuit requirements. In all the development to date, very little modification of the physical system parameters has been necessary in order to permit electronic circuit realization.

1.2 *Form of the Flying Spot Store*

The form of the flying spot store is shown in Fig. 1. Information is stored in the film plane as patterns of transparent and opaque spots on developed photographic emulsion. Access for writing and for reading is gained by means of a number of light beams generated by a single cathode ray tube (CRT) and an array of objective lenses operated in parallel, each lens imaging the working area of the CRT screen on a separate photographic storage area in the film plane. When the image of the cathodoluminescent spot on the phosphor screen of the CRT falls on a transparent spot on the developed emulsion, light passes through the film plane to the photomultiplier tube (PMT) detector. When the image falls on an opaque area, no light reaches the PMT. Light signals are converted into current signals at the photocathode, and these current

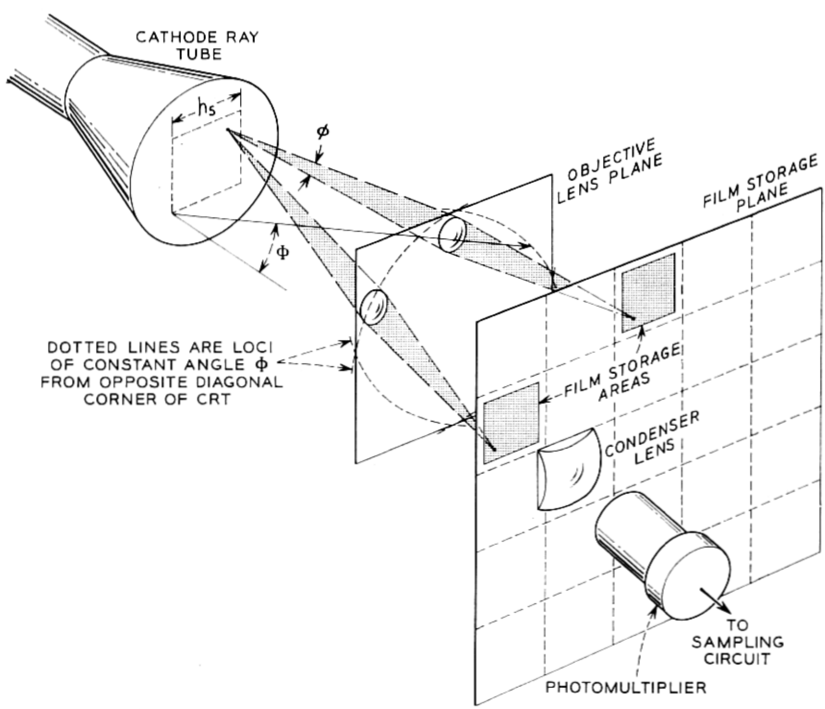

Fig. 1 — Form of the flying spot store.

signals are then amplified in the secondary-emission multiplier stages (dynodes) of the PMT. Thus, the combination of the CRT, optical system and PMT is used to provide an electrical output which indicates whether each spot is transparent or opaque. Transparent spots are used to represent binary "1's," opaque spots to represent binary "0's". The associated objective lens, photographic storage area, condenser lens and PMT are physically located within the solid angle that is subtended at the center of the CRT by the photographic storage area. These, together with the information sampling circuit, constitute an information channel of the system. Fig. 1 shows the system half-field angle Φ, the channel field angle φ, and the array size h_s.

In this paper it is assumed that the electron beam of the CRT is under the control of a feedback beam-positioning system and that a number of channels of the system are used in the optical beam encoder element of this feedback system. The information stored in these channels (called servo channels) is arranged to give a continuous indication of beam position. The mode and speed of operation of the feedback beam-positioning system have been described in an earlier article[1] and additional considerations are given elsewhere in this issue.[2]

1.3 *Speed of Operation*

Cycle time is the time required to complete a readout operation from the store. It is the sum of the beam-positioning time and the sampling time. Beam-positioning time is made as short as possible, consistent with system requirements. The minimum beam-positioning time which has been obtained to date is of the order of 2 microseconds. Information channel sampling has been done, and is practical at 0.3 microsecond. Thus, beam-positioning time is the major part of cycle time. In the design of the servo channels the relative aperture of the lens and the channel magnification are chosen, along with other system parameters, to obtain minimum beam-positioning time and required beam-positioning accuracy. On the other hand, there are a number of very fruitful exchanges which can be made between sampling time and other channel parameters in the design of the information storage channels. These exchanges, which affect sampling time but have a relatively small effect on total cycle time, are discussed in this paper.

1.4 *Capacity and Word Length*

The capacity of the system is given by the product of the number of information channels and the number of spots in the array at the CRT.

It is desirable to provide the total capacity in a number of channels equal to the word length. However, the total capacity required or other system requirements may dictate the use of a larger or smaller number of channels.

The number of bits per channel is the square of the number of spots in one row of the array at the CRT. In almost all cases, this number is set at much less than the total capacity required in the store. This limitation on the number of bits per channel will be shown to be due in part to energy conditions which must be satisfied, in part to device limitations and in part to system requirements. The array size will almost always be less than 1000 × 1000 spots; it now appears practical to design for 256 × 256 spots.

Because of the relatively small capacity per channel and the fact that 16 or more servo channels are used in stores of this type (to define 256 rows and 256 columns), systems with one or few information storage channels are of little interest. Likewise, systems with a number of channels less than the word length require a register to assemble the complete word. Two or more cycle times are required per word readout and, unless the two or more bits of each word in each channel are located remotely from each other, the very desirable property of independence of the bits of the stored word is lost. For these reasons, principal consideration is given in this paper to systems with a total number of channels of 40 or more.

II. SYSTEM DESIGN

2.1 *System Design for the Efficient Use of the Flux from the CRT*

A number of factors combine to set the maximum flux density (radiant power density) in the cathodoluminescent spot and the maximum number of spots which can be resolved on the CRT screen. The maximum flux density, together with the spot size, fixes the maximum flux per spot. The spot size and the maximum usable area of the CRT screen together determine the number of bits which can be stored in each channel. Thus, it is desirable to use as large a CRT screen area as possible. In all tubes we have tested to date it has been possible to obtain sufficient flux per spot in the smallest spot which can be resolved (approximately 0.015 inch). Thus, this organization has been used, since it results in maximum capacity per channel and minimum equipment for given total capacity.

Each channel of the system must be designed to provide a minimum flux level at the photocathode, which is determined by the sampling time (inversely proportional to channel bandwidth) and the signal-to-noise requirement of the intended application.

Since there are limits on the maximum flux per spot, the maximum number of spots in the array at the CRT and the minimum flux level at the photocathode in each channel, the first consideration in system design is to arrange the optics so that the available flux is used efficiently. The most efficient arrangement is one in which the greatest fraction of the flux from each spot is intercepted by the apertures of the objective lens array. The field angle for which the lenses are designed limits the area in the lens plane, and the ratio of lens aperture to lens mount diameter limits the percentage of the light falling in the plane which is actually used. Any reduction in the flux gathered into the channels of the system results in loss in maximum attainable system speed or capacity.

2.2 *Power and Signal-to-Noise Requirements*

2.2.1 *Noise Sources*

As readout proceeds in any one channel of a flying spot store, distributions in amplitude are found in the flux signals, incident on the photocathode of the PMT that represent "1's" and "0's." These signal distributions are due largely to spot-to-spot variations and imperfections in the system. Examples are variations in the flux from the CRT screen as a function of beam position, variations with field angle of illumination within the channel and film blemishes. These distributions are very nearly independent of absolute flux level. In the photocathode, the flux signals are converted into current signals, and fluctuations in time may be added to the signal arising from reading of any spot on the film. These fluctuations may be due to shot noise, secondary emission noise or regeneration noise. The most important of these will be shown to be shot-noise-in-signal, which depends on the magnitude of the signal current from the photocathode and, hence, on the flux level at the photocathode, photocathode efficiency and channel bandwidth. Both types of fluctuation — spatial and temporal — as well as the performance of the information sampler circuit, must be taken into account in the channel design. In a design for a high-capacity system with a requirement that the probability of error be very small, it may be necessary to set the flux level so high that shot noise fluctuations introduced in the detector are negligible. In this case, the probability of error depends only on the sampler performance and on the amplitude distributions determined by transmission factors and physical imperfections. In another case, where the distribution in amplitudes of the incident flux was smaller or where a higher probability of error would be acceptable, some of the total error might be allotted to shot noise fluctuations introduced in the PMT and, in this case, a lower flux level than that of the first example would suffice at the same sampling time. Thus, it is clear that the flux level at the photocathode must be set so that the required electrical signal-to-noise ratio fixed by the system application is obtained in the readout of the most poorly illuminated storage location.

2.2.2 *Electrical Signal-to-Noise Ratio*

The signal-to-noise ratio in the currents leaving the photocathode is given by:

$$\frac{\bar{I}_{pc}}{I_{n,pc}} = \left[\left(\frac{\bar{I}_{pc}}{2e\Delta f}\right)\left(\frac{1}{1 + \bar{I}_d/\bar{I}_{pc}}\right)\right]^{1/2}, \tag{1}$$

where $\bar{I}_{pc}$ and $\bar{I}_d$ are the mean signal and dark emission currents respectively, $I_{n,pc}$ is the total rms shot noise in the photocathode current, e is the electronic charge, and Δf is the bandwidth in which the observation is made. $\bar{I}_{pc}$ is directly proportional to the incident flux and photocathode sensitivity. $\bar{I}_d$ depends on the area, type and temperature of the photocathode. For a 2-inch diameter, type S-11 (Cs-O-Sb) photocathode of the type commonly used in the flying spot store, $\bar{I}_d = 1.7 \times 10^{-15}$ amperes at 25°C. The first factor on the right-hand side of (1), $(\bar{I}_{pc}/2e\Delta f)^{1/2}$, is the signal-to-shot-noise-in-signal ratio, while the second accounts for the reduction in total signal-to-noise ratio due to shot noise in the dark emission current $\bar{I}_d$. As $\bar{I}_{pc}$ varies through the range $\infty \geqq \bar{I}_{pc} \geqq \bar{I}_d$, the second factor in (1) goes from 0.707 to 1.

At all bandwidths of interest for flying spot store information channels, $\bar{I}_{pc}$ must be made large compared to $\bar{I}_d$ to obtain a satisfactory over-all signal-to-noise ratio. The maximum sampling time of interest is of the order of 1 millisecond. The channel bandwidth required in this case is 350 cps. To obtain a signal-to-shot-noise-in-signal ratio of 10 at this bandwidth, the flux level at the photocathode must be set to produce $\bar{I}_{pc} = 1.1 \times 10^{-14}$ amperes. Thus, at this limit $\bar{I}_{pc} = 6.5\, \bar{I}_d$ and it is seen that the shot-noise-in-signal is the limiting noise. Other factors limit scaling the system in capacity by increasing sampling time before the flux level that corresponds to a sampling time of 1 millisecond at a signal-to-noise ratio of 10:1 is reached. Thus, it is clear that shot-noise-in-signal dominates shot noise in dark current in flying spot store channels.

Equation (1) gives the signal-to-noise ratio at the photocathode. The signal-to-noise ratio in the currents at the anode is found to be slightly less than that at the photocathode due to fluctuations in secondary emission ratio in the first few dynode multiplier stages.[3] Under the operating conditions used in the flying spot store, the maximum reduction in signal-to-noise ratio due to this additional noise source is of the order of one-third. It has been found that regeneration contributes little additional noise under the PMT operating conditions which are used in the flying spot store.

The nearly noise-free current amplification obtained in the dynodes makes it possible to amplify the signal and noise currents originating at the photocathode to a sufficiently high level that the signal-to-noise ratio is not further reduced by Johnson noise in the anode circuit resistance. The condition that the rms shot noise voltage in the output shall be at least r times the rms Johnson noise voltage leads to the following requirement on the minimum value of anode circuit resistance, R_a :

$$R_a \geqq \frac{2kT}{e} \frac{r^2}{G^2} \frac{1}{\bar{I}_{pc}} = \frac{1}{20} \frac{r^2}{G^2} \frac{1}{\bar{I}_{pc}}, \tag{2}$$

in ohms, where G is the total current amplification of all of the dynodes, k is Boltzmann's constant and T is the absolute temperature of the anode circuit resistor. Note that, in the case of a phototube ($G = 1$), excessively high anode circuit resistance would be needed and that this large resistance, together with tube and wiring capacitance, would result in severe limitation of bandwidth. In photomultiplier tubes, current amplification of 10^6 can be obtained. This makes it possible to choose a value of anode circuit resistance compatible with the bandwidth needed in the channel without degrading the signal-to-noise ratio.

It is for this reason that PMT's must be used in the flying spot store. In a typical case, where a channel bandwidth of 1 mc and a signal-to-noise ratio of 10 are required by the application, an anode circuit resistance of 10,000 ohms is chosen in order to provide this bandwidth. The solution of (2) reveals that a minimum current amplification of 820 must be provided in the PMT if the electrical signal-to-shot-noise ratio is not to be reduced by more than 10 per cent by Johnson noise in the anode resistor. In practice, current amplification has been used that is greater than that specified by (2) for the size of the anode circuit resistor R_a chosen. This increases the output current level and decreases the gain required in the amplifier stages following the PMT.

From the foregoing, it is seen that the limiting noise of a fundamental nature is the shot-noise-in-signal and that it is possible to obtain an over-all channel signal-to-noise ratio only a little less than this. Channel electrical signal-to-noise ratio is given by:

$$\frac{I_a}{I_{n,a}} = \frac{1}{m}\left(\frac{\bar{I}_{pc}}{2e\Delta f_c}\right)^{1/2}, \tag{3}$$

where the factor $1/m$ accounts for the maximum reduction due to secondary emission fluctuation noise and Δf_c is the channel bandwidth. Since the bandwidth of the dynode stages is of the order of several hundred megacycles, channel bandwidth is almost always determined by the anode circuit or, as will be shown later, by the frequency response characteristic of the phosphor.

2.2.3 *Minimum Photocathode Current*

We now find the minimum flux at the photocathode required to produce a specified signal-to-noise ratio in channel bandwidth, Δf_c. The total flux incident on the photocathode is given by:

$$F_{pc} = P_{\max} \int_{\lambda_1}^{\lambda_2} \left(\frac{P}{P_{\max}}\right)_\lambda d\lambda, \text{ watts.} \tag{4}$$

The photocathode current is given by:

$$\bar{I}_{pc} = \int_{\lambda_1}^{\lambda_2} P_\lambda S_\lambda d\lambda = P_{\max} S_{\max} \int_{\lambda_1}^{\lambda_2} \left(\frac{P}{P_{\max}}\right)_\lambda \left(\frac{S}{S_{\max}}\right)_\lambda d\lambda, \text{ amperes,} \tag{5}$$

where $(P/P_{\max})_\lambda$ is the relative spectral emission characteristic of the phosphor, $P_{\max}$ is the maximum flux per unit wavelength interval (watts/Å) at the photocathode, $(S/S_{\max})_\lambda$ and $S_{\max}$ are the relative and maximum photocathode sensitivities respectively, and λ_1 and λ_2 are chosen to include the entire wavelength region in which the spectral emission and photocathode sensitivity are both nonzero.

The minimum flux which will produce an over-all channel signal-to-noise ratio of $(\bar{I}_a/I_{n,a})$ in channel bandwidth Δf_c is found by combining (3), (4) and (5). It is given by:

$$F_{pc,\min} = (m)^2 \frac{2e\Delta f_c}{S_{\max} k_m} \left(\frac{\bar{I}_a}{I_{n,a}}\right)^2, \text{ watts,} \tag{6}$$

where k_m is a normalized factor which depends only on the spectral match between the emission characteristic of the phosphor and the sensitivity characteristic of the photocathode, and is given by:

$$k_m = \frac{\int_{\lambda_1}^{\lambda_2} \left(\frac{P}{P_{\max}}\right)_\lambda \left(\frac{S}{S_{\max}}\right)_\lambda d\lambda}{\int_{\lambda_1}^{\lambda_2} \left(\frac{P}{P_{\max}}\right)_\lambda d\lambda}. \tag{7}$$

2.2.4 *Required Transmission per Channel*

A number of factors, singly or in combination, set the maximum flux density in the cathodoluminescent spot at the CRT. Examples are the thermal velocity limitation on the maximum current density in the electron beam, accelerating voltage, phosphor type and efficiency and the maximum phosphor loading which can be used without burn. Likewise, system requirements on the number of bits per channel or device limitations may set the maximum number of spots in the array at the CRT. Once the maximum flux density and spot size have been set, the maximum flux per spot at the CRT screen, $F_{s,\max}$ is fixed. The minimum flux level at the photocathode which provides the required signal-to-noise ratio in the channel is given by (6). Clearly, therefore, each channel of the system must be designed so that a fraction, $k_g \geqq F_{pc,\min}/F_{s,\max}$ of the light emitted from the cathodoluminescent spot is transmitted to the photocathode. The minimum value of k_g has been arrived at through consideration of system requirements (sampling time, signal-to-noise, number of bits per channel, etc.) and CRT, phosphor and PMT characteristics. In the further consideration of device characteristics in Section III it will be seen that this is a reasonable approach, since there are a limited number of fast-decay cathodoluminescent phosphors, all of which are quite well matched to the relative spectral sensitivity characteristic of the most efficient photocathode available. Thus, once system speed, capacity, signal-to-noise and number of channels are set, we are led rather quickly to a choice of CRT size, phosphor and photocathode type.

Physically, the value of k_g is determined by the relative aperture of the channel objective lens and the channel magnification. Thus, we turn next in this paper, as is done in design, to consideration of the range of channel optical designs which will provide a specified value of k_g. It will be seen that the great flexibility of flying spot store systems derives from the fact that very many different optical systems can be chosen which will provide a specified value of k_g, set on the basis of system speed, capacity and performance. This allows wide latitude in number of channels, capacity and choice of lens design.

2.3 *Geometrical Factors in System Design*

2.3.1 *Transmission and Loss Factors*

The fraction of the light emitted from the cathodoluminescent spot which is transmitted to the photocathode in each channel, k_g, may be written as the product of two factors: k_{g1}, which represents reflection and absorption losses in the optics, and k_{g2}, which is the fraction of the light from the cathodoluminescent spot that is intercepted by the effective aperture of the objective lens of the channel. We define k_{g2} as the optical acquisition factor.

In general, the range of variation of k_{g1} is small and in no case is it a controlling factor in design. On the other hand, k_{g2} varies through an enormous range and is the principal variable of optical system design.

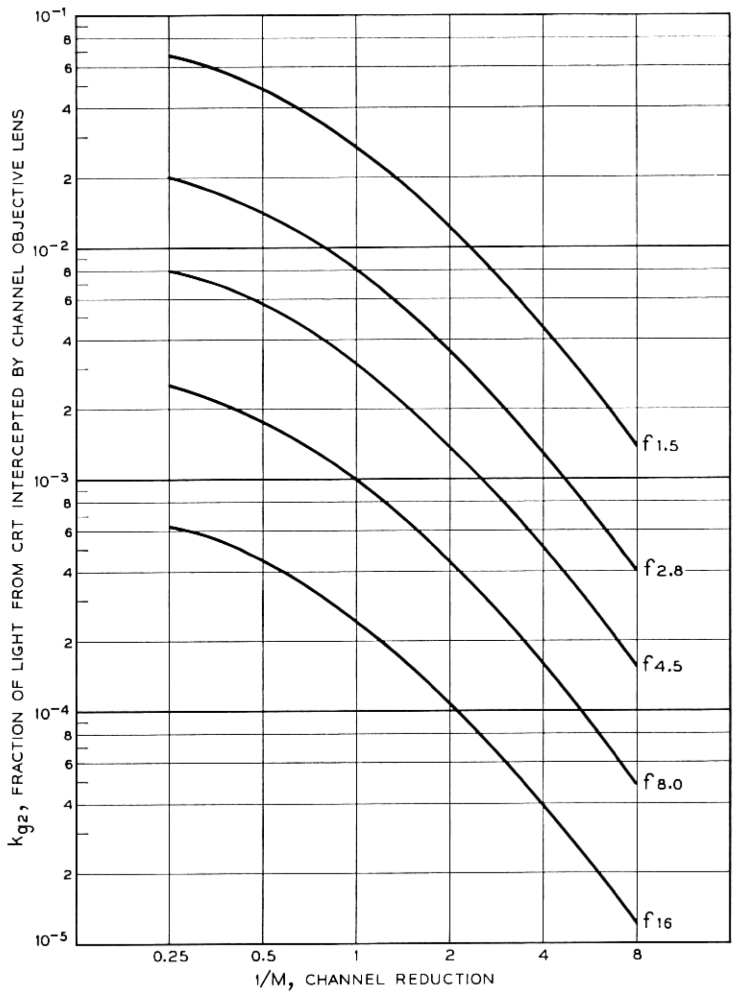

Fig. 2 — The system plane — individual channel designs are represented by points at the channel reduction, $1/M$, and channel objective lens f-number.

At the CRT screen the distribution in intensity has been found to be given approximately by $I = I_0 \cos \theta$, $\theta \leqq 40°$, where θ is the angle between the viewing direction and the normal to the screen. Including the effect of this distribution the optical acquisition factor k_{g2} is given by:

$$k_{g2} = \frac{1}{1 + 4f^2(M + 1)^2/M^2}, \tag{8}$$

where f is f-number of the lens used in the channel (f = focal length/effective lens aperture) and M is the channel magnification.

2.3.2 *Graphical System Design*

A plot of the optical acquisition factor, k_{g2}, as a function of $1/M$, the reduction between CRT screen and photographic storage area, is given in Fig. 2 for lenses of various relative aperture. From this plot it is seen at once that the fraction of the flux intercepted in each channel increases with channel magnification and lens relative aperture. Each point of Fig. 2 represents a channel design. Channel designs which satisfy signal-to-noise requirements are those which lie above the values of k_{g2} given by

$$k_{g2} \geqq \frac{k_g}{k_{g1}} = \frac{1}{k_{g1}} \frac{F_{pc,\min}}{F_{s,\max}}. \tag{9}$$

In order to establish the range of required values of k_{g2} we now consider the following example taken from the case of a practical system. System parameters are as follows: Cathode ray tube — accelerating voltage, 10 kv; beam current, 10 microamperes; spot size, 0.015 inch (90 per cent of current within 0.015 × 0.015 inch area); phosphor, P-16; efficiency, 1.5 per cent. Photomultiplier tube — S-11 photocathode; $S_{\max}$, 0.050 ampere/watt; spectral match factor k_m, 0.88; secondary emission fluctuation noise factor m, 1.5. Optics — loss factor k_{g1}, 0.25. The minimum values of k_{g2} required for sampling in 0.1, 0.3 and 1 microsecond at electrical signal-to-noise ratios of 10:1 and 20:1 are as follows:

Sampling Time, in microseconds	Signal-to-Noise Ratio	Required Value of k_{g2}
0.1	10	1.5×10^{-5}
0.1	20	6.0×10^{-5}
0.3	10	5.0×10^{-6}
0.3	20	2.0×10^{-5}
1.0	10	1.5×10^{-6}
1.0	20	6.0×10^{-6}

Flying spot store designs fall within the range $4 \geqq M \geqq \frac{1}{8}$. The reasons for this are easily seen. At $M > 4$ the very great size of each individual storage area (area = 16 times the area of spot array at the CRT screen) limits systems of reasonable dimensions to few channels and results in inefficient use of the photographic storage medium. At $M \leqq \frac{1}{8}$ three factors enter. First, for even the largest CRT spot sizes contemplated, the small spot size in the film plane makes the system increasingly susceptible to errors due to film blemishes. Second, the problem in repositioning the storage plate in the store after development becomes more difficult in inverse proportion to the size of the bit storage cell. Finally, before this level is reached it is possible to obtain a large enough number of channels for any application now foreseen.

2.3.3 *Nonoverlapping Channels*

In general, a solid angle slightly greater than that required to prevent overlap of images must be alloted to each channel in order to allow mounting of the film storage area, condenser lens and PMT. This requirement leads to a limit on the minimum value of channel magnification which can be used with any particular channel design. This limit is expressed by:

$$\frac{1}{M} \geqq \frac{h_s}{h_{lc}} - 1 = \frac{f}{c}\frac{h_s}{\text{f.l.}} - 1, \tag{10}$$

where h_s is the CRT array size, f is the f number and f.l. is the focal length of the channel lens. h_{lc} is the distance between lens centers, and c is the ratio of the distance between lens centers to lens aperture.

Fig. 3 shows the minimum value of magnification which can be used for values of the ratio of CRT array size to lens focal length, h_s/f.l., from 0.25 to 2.0, and for lens-spacing parameter c, values of 1.5 and 2.5, along with the data plotted in Fig. 2. Channel designs which satisfy the nonoverlap requirement are those to the right of the curve for the values of h_s/f.l. and c chosen for the system.

2.3.4 *Optical System Length*

System length is directly proportional to focal length of the objective lens. It is given by:

$$L_s = \left(M + 2 + \frac{1}{M}\right)\text{f.l.} \tag{11}$$

Minimum optical system length is found for the case where $M = 1$,

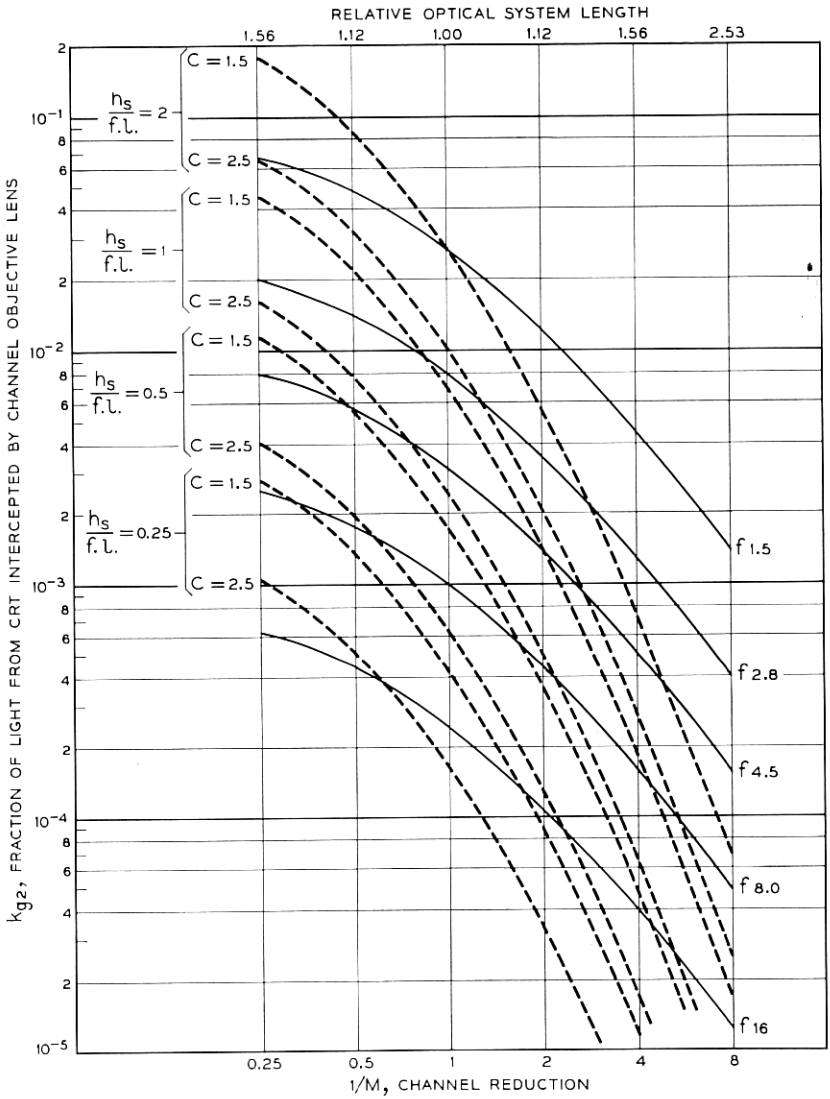

Fig. 3 — Minimum value of magnification which can be used for values of ratio of CRT array size to lens focal length.

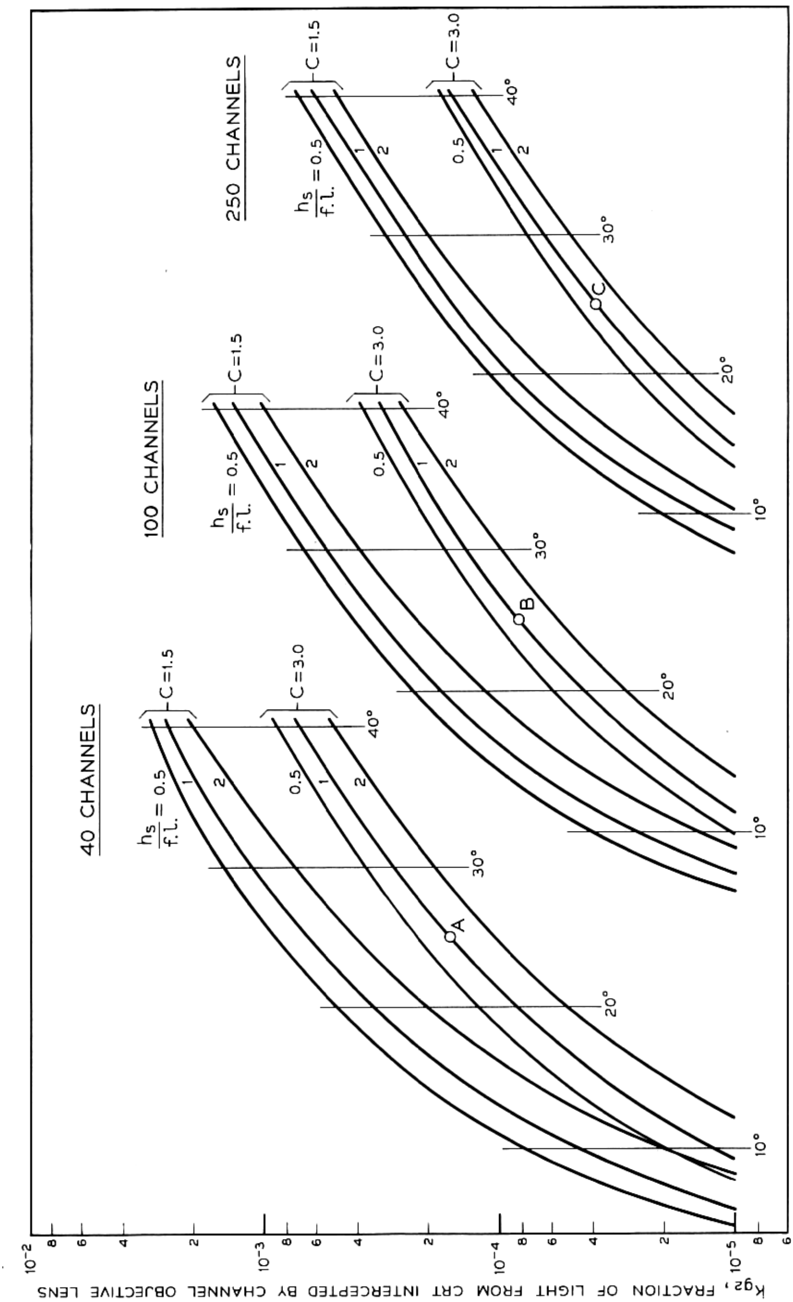

Fig. 4 — Field angle needed to get a specified optical acquisition factor in a specified number of channels.

but system length does not increase strongly with variation in magnification around unity. Thus, at $M = \frac{1}{4}$ and $M = 4$ the optical system length is 1.56 times the minimum length. As noted previously, h_s, the size of the array at the CRT, is usually made as large as possible, since system capacity and the number of bits per channel both increase with this dimension. When h_s has been fixed by the choice of a CRT the curves for h_s/f.l. in Fig. 3 become curves of constant focal length and define the minimum focal length which can be used for specified magnification ratio.

2.3.5 *Number of Channels*

The number of channels which can be combined into a system which provides a specified value of optical acquisition factor, k_{g2}, depends on the field angle of good correction of the lens, the ratio of CRT array size to focal length, h_s/f.l., and the ratio of lens mount diameter to operating aperture, c. Fig. 4 shows the lens half-field angle which must be obtained in order to get 40, 100 and 250 nonoverlapping channels. Each curve is drawn for specified values of h_s/f.l. and c. In finding these curves, we have first found the area in the lens plane which can be used. This is the area defined by the intersection with the lens plane of cones of constant half-angle Φ, erected from the corners of the CRT array as shown in Fig. 1. This area is a function of the ratio h_s/f.l. as well as of the lens focal length and channel magnification. The calculation assumes that this area is approximately filled by a rectangular array of objective lenses, spaced on centers, by c times the working aperture of the lens. The conditions listed thus far do not define a curve, but rather define a set of curves, since a given value of k_{g2} can be obtained with a large variety of different values of channel magnification and lens f-number, as shown in Fig. 2. The nonoverlap condition has therefore been used to find the minimum value of $1/M$ and lens f-number which can be used for the specified values of h_s/f.l. and c in each case. The set of curves shown in Fig. 4 has been obtained using both this condition and those previously stated.

Thus, each curve gives the value of k_{g2} which can be obtained in the stated number of channels as a function of lens half-field angle Φ, with the following additional conditions: (a) values of the ratios h_s/f.l. and c are fixed, and (b) the value of k_{g2} is obtained with the minimum values of channel parameters, $1/M$ and lens f-number which can be used without violating the nonoverlap condition.

The curves of Fig. 4 illustrate several important features of flying spot store systems. First, it is important to note that values of $k_{g2} \geqq$

10^{-5} can be obtained in as many as 250 nonoverlapping channels, even at extreme values of the lens spacing parameter c and the ratio h_s/f.l., with lens half-field angles $\leqq 30°$. As shown in an earlier example, the flux level obtainable at the photocathode in a practical system with $k_{g2} = 10^{-5}$ is such that sampling can be done in 0.1 microsecond with a signal-to-rms-noise ratio of 10. Most designs operate at a value of k_{g2} greater than 10^{-5}. Furthermore, sampling time may be increased by a factor of 10 or more without seriously affecting cycle time, since beam positioning requires a minimum of 2 microseconds. This increase in sampling time can be used where necessary to compensate for reduced flux level at the photocathode or to increase the signal-to-noise ratio.

A second fact shown in Fig. 4 is that k_{g2} does not vary rapidly with number of channels, if Φ, h_s/f.l. and c remain constant. Thus, k_{g2} typically decreases by a factor somewhat less than the factor by which the number of channels is increased. The curves also show that the number of channels which can be obtained with a fixed value of k_{g2} and h_s/f.l. increases quite rapidly with the half-field angle of the channel objective lens. In considering the effect of increasing field angle on the number of channels obtainable at constant k_{g2} (moving horizontally to the right across the curves of Fig. 4), it is noted that the value of c increases, since lens designs providing larger field angle usually have a greater value of barrel-to-aperture ratio than do those operating at narrow field angles. Thus, the value of the lens spacing parameter, c, might well be 1.5 for a simple triplet design operating at a field angle of 15° and 3.0 for a wide-angle design which could be used up to a half-field angle of 30°. The curves of Fig. 4 show that a 40-channel system with $k_{g2} = 10^{-4}$ can be constructed using a maximum half-field angle of 12°, with lenses spaced by 1.5 times the operating aperture ($c = 1.5$), and h_s/f.l. $= 0.5$. The use of a wide-angle design with maximum half-field angle of 24.5° and lenses spaced by three times the operating aperture allows us to construct a 100-channel system providing the same value of k_{g2}, with h_s/f.l. $= 0.5$.

2.3.6 *Scaling the System in Number of Channels*

One particularly useful method of scaling a flying spot store system to obtain a greater number of channels, starting with an existing design, assumes that the lens design, CRT design and CRT operating conditions remain constant. Thus, system field angle Φ, lens focal length and the ratios h_s/f.l. and c remain constant. In this scaling operation we proceed along a line of constant half-field angle in Fig. 4, between curves for different numbers of channels with the common values of h_s/f.l. and *c*. In order to provide a larger number of channels than in the starting design, within the same half-field angle and with fixed lens focal length, the channel magnification and/or the lens aperture must be reduced. Both result in decreased values of k_{g2}.

As an example, consider scaling the system which has the following fixed parameters: $\Phi = 25°$; h_s/f.l. $= 1$; $c = 3$. The points labeled A, B and C in Fig. 4 correspond respectively to 40-, 100- and 250-channel systems having these parameters. The values of f and $1/M$ which yield given values of k_{g2} can be found from Fig. 3. The attainable values of k_{g2}, lens f-number and channel magnification for these three solutions are given by this table:

Number of Channels	f-Number of Channel Objective Lens	Channel Magnification	Attainable Value of k_{g2}
40	11	0.44	2.2×10^{-4}
100	12	0.30	1.1×10^{-4}
250	13	0.25	5.0×10^{-5}

Thus, it is seen that increasing the number of channels and the capacity by a factor of 6.25 results in a decrease in flux level at the photocathode by a factor of 4.4. This reduction could easily be compensated for by increasing the sampling time by a factor of 4.4. In most cases, this increase would have negligible effect on cycle time.

This method of scaling is particularly useful, since it requires a minimum number of device changes: CRT design and operating conditions remain fixed, the lens design and focal length also remain fixed and no difficulty should be experienced in modifying the given lens design for the smaller aperture and slightly reduced magnification ratio which result from scaling to a system with more channels.

III. DEVICE CHARACTERISTICS

3.1 *Phosphor*

3.1.1 *Frequency Response and Efficiency*

Cathodoluminescent phosphors differ widely in efficiency and decay characteristics. For the flying spot store, efficient materials in which the luminesence centers have short mean lifetimes to emission are used. Mean lifetime to emission is an important parameter, since the time for the flux from a given area to decay when the beam is moved to a new location limits the speed of position sensing and of readout. The type of decay characteristic is also important. A number of materials which have been studied for use in the flying spot store show an exponential decay following the cessation of excitation. When such a phosphor is used, its effect on the over-all frequency response characteristic of the channel is precisely the same as that of a circuit which contributes a 6 db/octave cutoff above a corner frequency given by $f = 1/(2\pi\tau)$, where τ is the phosphor decay time constant. By the use of an equalizing amplifier, this characteristic can be exactly equalized to provide a flat frequency response to a frequency higher than f. The amount of equalization which can be obtained depends on the gain-bandwidth product of the active element used in the equalizer.

A number of other phosphors of possible interest show hyperbolic or bimolecular decay characteristics. These are harder to equalize. The required gain-frequency response in the equalizing amplifier can only be approximated, and, unlike materials which show exponential decay, the constants of the decay characteristic are functions of beam current density and screen temperature.

Efficiency and mean lifetime are not independent variables, since efficiency can be exchanged for frequency equalization.

Thus, in comparing phosphors and in the design of circuits in which the phosphor is one element in the transmission path, it is useful to have a phosphor "frequency response" curve. Such frequency response curves plot the amplitude of the ac component of the flux as a function of frequency, at constant average beam power and beam power density, and with 100 per cent beam current modulation. Thus the ordinate at low frequency is proportional to the dc efficiency of the phosphor. The ac component of flux drops off above a cutoff frequency which is inversely proportional to the mean lifetime of the excited states in the phosphor. This is illustrated by the frequency response curves shown in Fig. 5 for two phosphors with different decay time constants (both are assumed to decay exponentially) and efficiencies. At frequencies up to f_2, electrical equalization can be employed at the output of the PMT to bring the frequency response curve for the system using phosphor A into coincidence with that for the system using phosphor B. If this were done, the signal level and signal-to-noise ratios would be identical in the two systems. The chief disadvantage, in practice, in the use of phosphor A would be the higher PMT output current that would result, since PMT life decreases with increasing anode current. However, if frequency cutoffs of f_2 and f_1 were sufficiently high for the servo and information channels respectively, a real advantage would accrue to the system through the use of phosphor type A, since the total available flux in the information storage

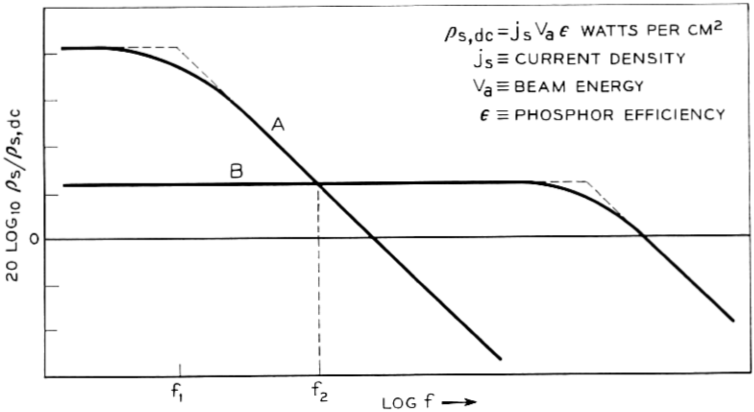

Fig. 5 — Frequency response curves for two phosphors with different decay time constants and efficiencies.

channels would be greater than that obtainable with phosphor type B in the ratio of their dc efficiencies.

Viewed on the basis of frequency response, efficiency and type of decay, P-16 phosphor has a distinct advantage over the slower but more efficient P-5 and P-11 phosphors, for systems requiring very fast beam-positioning times. Its decay time constant is not a function of current density and it does not saturate at any beam current that can be used without damaging the screen. The ultraviolet component of the emission from type P-15 phosphor has an extremely short decay time constant ($\tau \doteq 0.01$ microsecond) but it is much less efficient than the slower P-16 material. Thus, even in a channel requiring uniform frequency response up to 10 mc, P-16 is superior, since its higher efficiency results in a higher output after equalization.

The efficiencies and decay time constants at 10 kv of three commonly used phosphors are given in Table I. The efficiencies are for aluminized phosphor screens, not for the materials themselves. These are for specific

TABLE I

Type	Material	Screen Efficiency	Decay Time Constant	Spectral Match to S-11 photo-cathode Factor, k_m
P-11	ZnS:Ag	6%	20–40 μsec	0.92
P-5	$CaWO_3$	2%	7 μsec	0.89
P-16	$2(CaO)(MgO)2(SiO_2):Ce^{+3}$	1.5%	0.05 μsec	0.88

tubes, and it should be borne in mind that the screen efficiency obtained depends strongly on the method of phosphor and screen preparation.

3.1.2 *Spectral Emission Characteristic*

The constant k_m, defined by (7), is a measure of the quality of the match between phosphor spectral emission characteristic and photocathode relative spectral sensitivity characteristic. Thus, if all of the emission of the phosphor occurs at the wavelength of maximum spectral sensitivity of the photocathode, $k_m = 1$. This factor has been evaluated for the actual spectral responses of the three phosphors listed in Table I, and the type S-11 photocathode, and is also tabulated in Table I.

3.1.3 *Phosphor Loading*

Phosphor loading is defined as the power density in the incident electron beam. In quite reasonable electron optical densigns, power density in the electron beam at the CRT screen can reach one kilowatt/cm^2 or more. The phosphor loading which can be employed without damage to the screen depends on the phosphor type, screen thickness and method of preparation. Phosphor loading rather than electron optical limitations usually sets the maximum current density in the electron beam and, as a result, the maximum flux density in the spot. Very much higher current densities could be used if the exposure of each spot were limited so that temperature rise would not be enough to damage the screen, and if it were possible to insure that the time between exposures of any single spot of the screen would be comparable to or greater than the thermal time constant of the screen. These conditions cannot be met in a random-access memory without slowing the system. However, phosphor loading could be increased in serial-access systems. A maximum phosphor loading of 200 watts/cm^2 can be used with a P-16 screen, provided that no spot is continuously exposed to the beam for more than 1000 microseconds, and that the average loading at any spot is less than 5 watts/cm^2.

A special "time out" circuit has been incorporated in the store which prevents screen exposure of greater than 1000 microseconds at any one point, by deflecting the beam off the working area whenever no new address is received within this period of time.

3.2 *Cathode Ray Tube*

3.2.1 *System Considerations*

System considerations strongly influence the choice of CRT parameters. The CRT screen is made large, since total available flux and number of spots both increase with screen area. Fast-feedback beam-positioning systems introduce the requirement for electrostatic deflection systems. Accelerating voltage, and therefore phosphor efficiency and flux density, is limited to set a bound on deflection amplifier power (this power is very significant — in the wideband deflection amplifiers used in maximum-speed feedback beam-positioning systems it is a large fraction of the total power consumption). Finally, phosphor loading, rather than the thermal velocity or space charge limitations, sets the maximum current density in the cathode ray tube beam.

3.2.2 *Spot Size Spacing and Uniformity*

Spot size may be set to produce the required flux per spot, given a maximum flux density, or the flux level attainable in the minimum size spot that can be resolved may be accepted and the rest of the system adjusted to work at that level. In both cases, spot spacing and uniformity must be set to provide satisfactory discrimination between stored "1's" and "0's" in readout.

The ability of the system to distinguish between stored "1's" and "0's" depends on the fraction of the total flux in the reading beam that is intercepted by the individual square bit storage cell in the photographic storage area. Flux falling outside the cell decreases the amplitude of the signal representing a "1" and increases the amplitude of a signal representing a "0". The size of the bit storage cell is chosen so that satisfactory discrimination can be obtained between the lowest-amplitude "1" and the maximum-amplitude "0". Many factors contribute to the spatial distribution of flux in the reading beam. Among these are the distribution in intensity in the cathodoluminescent spot, optical aberrations, variations in illumination and local imperfections. Since the distribution in intensity in the cathodoluminescent spot is only one of the factors contributing to the spread in the reading image, spot spacing on the CRT screen must be set at a greater fraction of spot diameter than would be the case if this were the only distribution that had to be considered.

In the flying spot store application, the relevant measure of CRT spot size is the size of the centrally located square within the cathodoluminescent spot, with sides parallel to the sides of the bit storage cell, which includes within it a specified fraction of the total flux from the spot. All measurements of CRT spot size have been done with equipment which allows us to find the fraction of flux as a function of the length of side of the central square. Spot spacing has been set so that a minimum of 90 per cent of the total flux falls within the square area at the CRT screen, corresponding to a bit storage cell. Since the spot spacing at the CRT is set to meet this criterion in the case of the worst spot on the screen, it can be seen that efficient use of the CRT screen demands good uniformity of spot size over the surface.

The principal factors that reduce spot uniformity are defocusing due to the cylinder lens effect of the deflection plates and deflection aberrations. In order to minimize both types of distortion, deflection angles must be kept small.

Spot uniformity is also an important variable in the exposure process. Variations in the image area due to spot defocusing result in variations in density and size in the developed image and also reduce discrimination ratio in readout.

3.2.3 *Tube Length and Magnification Ratio*

The distance between the deflection plates and the screen in the CRT is determined by the array size and the maximum deflection angle. Since array size tends to be large and deflection angles small, CRT's for flying spot stores tend to be long. In order to get the long throw between deflection plates and screen without excessive total length electrostatically deflected CRT's usually image the first crossover at a magnification of 3 to 5.

3.2.4 *Beam Current and Cathode Loading*

In the usual electrostatically deflected CRT, operated at an accelerating voltage of 10 kv, the maximum current density which can be obtained at the screen is set by thermal velocity limitations. The current density which can be obtained at this limit, with conservative cathode loading that gives good CRT life, is considerably in excess of the current density which produces the maximum allowable phosphor loading. Likewise, the space-charge limitation on the maximum current density in the beam is not approached. We have tested tubes which provide maximum current densities in the focused spot of 20 to 100 ma/cm^2 and beam currents in the range from 5 to 30 microamperes.

These considerations might be interpreted to mean that the optimum CRT design would be one in which the accelerating voltage had been reduced to the point where electron optical and phosphor limitations are met at the same point. This is not the case, since uniformity of flux output and efficiency of the phosphor fall off rapidly with decreasing voltage.

3.2.5 *Cathode Ray Tube Performance*

The performance of a cathode ray tube depends strongly on the fabrication techniques used in its assembly as well as its design. Therefore a "benchmark" is given at this point rather than a theoretical prediction of spot size. We have tested CRT's at an accelerating voltage of 10 kv and a beam current to the screen of 5 microamperes, which produce an undeflected spot size of 0.015 inch (90 per cent of the light within a 0.015-inch square). Spot size in these tubes varies by at most 1.5 to 1 in a 256 $\times$ 256 array.

3.3 *Optics*

3.3.1 *Requirement for Correction over a Wide Field Angle*

The basic optical system arrangement discussed here, in which the lenses are mounted in a plane with their axes parallel to the normal to the cathode ray tube screen, results in considerable simplification in the mounting of components and allows for more than one photographic storage area on each photographic plate. This arrangement requires that the lenses be corrected over the field angle between the ray from one corner of the CRT array to the lens farthest away from that corner in the diagonal direction. Thus, the lens must be well corrected over a half-field angle, which typically is 25° instead of the 10° or less subtended in the individual channel.

3.3.2 *Requirement on Uniformity of Illumination*

It is necessary that the illumination in the poorest channel of the system meet minimum requirements imposed by sampling considerations. A more critical requirement is that of uniformity of illumination within each channel, since variations here cause signal amplitude to vary from point to point and contribute to the probability of error in readout.

The average illumination in a channel varies with the field angle of the channel to the center of the CRT. Within the channel, illumination varies with spot position on the CRT screen. The variation in any one channel is greater than the cube of the cosine of the channel field angle and may amount to 10 per cent in an on-axis channel and to 20 per cent or more in a channel at the edge of the field. An improvement in this per-channel variation has been obtained by placing a stop in front of the lens to vignette the high illumination side of the field.

3.3.3 *Requirement on Condenser Lens*

Photocathodes show variations in sensitivity from point to point over their surface. Likewise, electron collection efficiency varies with position over the photocathode. Thus if the reading beam passing through the film were allowed to fall directly onto the photocathode, large fluctuations in signal amplitude as a function of beam position would result. The condenser lens is introduced to prevent this. It images the aperture of the lens onto the photocathode, and all flux passing through the photographic storage area in the channel falls within this area. Ideally, this image is stationary and independent of the position of the reading beam. Condenser aberrations tend to cause the image on the multiplier to move with variations in position of the reading beam. This motion must be small compared to the image size on the photocathode.

3.3.4 *Optical Requirements of the Optical Beam Encoder*

In many cases, it is desirable to use lenses in the servo channels that are faster than those in the information storage channels, since the feedback beam-positioning system operates at wide electrical bandwidths (e.g. 10 mc in maximum speed feedback beam-positioning systems) and because the beam is half covered by the servo edge at the readout position. Uniformity of illumination is more critical here, since it affects the spacing of the spots in the array at the CRT through the action of the feedback system.

3.3.5 *Lens Designs*

Almost all flying spot store designs result in the choice of lenses which are not diffraction-limited. This is a result of the fairly wide spacing of stored information used to minimize the effect of blemishes on the developed photographic storage plate. Thus, illumination and aberration performance are the critical lens characteristics.

The number of channels that may be used is limited by the field angle of good correction and good illumination that may be obtained with a given design. It also depends on the outside diameter of the lens mount. A lens designed to cover a relatively narrow half-field angle such as 15° will be much smaller in mount diameter than one which will cover 30° without vignetting. Thus, it may be desirable in some cases to use different lens designs and channel spacings in different areas of the lens array.

Two design modes are possible. A relatively narrow field design, such as a triplet, may be used over its limited field of view with efficient channel packing, due to the small barrel-to-aperture ratio in this design. At the other extreme, a very wide angle lens may be used over a wide field with poorer channel packing efficiency.

A moderate field angle lens of the six-element double-Gauss type has been used in experimental work with excellent results. A 25° half field may be obtained at $f/8$ relative aperture and channel magnification in the range from 4:1 to 6:1. The diameter and length of this design are relatively large, and illumination has been found to fall off at a rate greater than the cube of the cosine of the field angle. Tessar types have also been used which show only slightly poorer performance at these field angles.

The image characteristic important in lenses for a flying spot store is measured in terms of contrast at the spacing of the stored information in a flat focal plane. The lens must be designed to concentrate the maximum flux into the individual bit storage cell rather than to concentrate it into a sharp core surrounded by flare. The flare light in such a lens may not deteriorate photographic performance, due to its low density, but it does reduce the difference between the minimum "1" and the maximum "0", since it may represent a sizable fraction of the total light in the reading beam. All measurements of lens performance have been made in terms of the sine wave response or by methods which allow us to determine the fraction of the light falling within a specified area. The method of measurement is described in another paper in this issue.[4]

Distortion is not critical, because the stored information is written through the lens that it is later read through.

3.4 *Photomultipliers*

3.4.1 *Photocathode Sensitivity*

The type S-11 photocathode has a wide region of high sensitivity and is well matched to most of the phosphors suitable for use in flying spot stores. This photocathode is the most efficient of the types commercially available. Photocathode sensitivities of 0.040 microamperes/microwatt, averaged over the photocathode, can be obtained at 3900 Å. This corresponds to a quantum efficiency of 12.7 per cent.

3.4.2 *Noise and Current Amplification*

Noise in the photomultiplier detector plays an extremely important role. Provided it is no greater than shot noise, a very wide range of

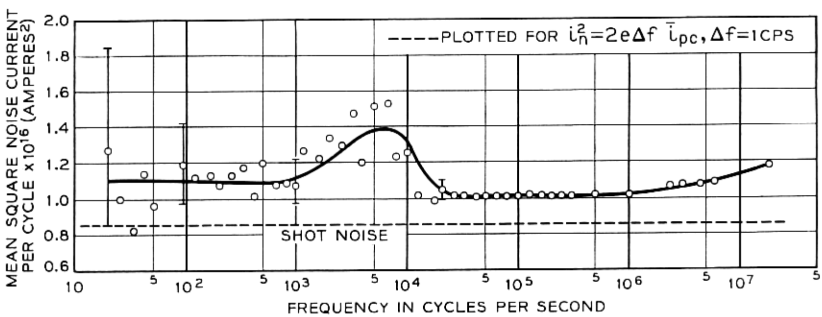

Fig. 6 — Noise power spectrum for photomultiplier.

systems can be built. Early studies on photomultipliers revealed noise in the anode current which was in excess of the expected shot noise by 20 db or more, plus sudden changes of total amplification by a factor of two or more. These anomalies caused great concern, since they would have seriously limited both the range of stores which could be built and the system reliability. It was found that the anomalous noise was associated with the piling up of "after-pulses." In a new design, "after-pulsing" and the sudden amplification changes in time have been largely eliminated. Tubes of this design show noise which is very little in excess of shot noise under the operating conditions commonly encountered in the flying spot store. Current amplification up to 10^6 at anode current levels of 350 microamperes can be obtained in this design without increase in noise power at the output due to regeneration. A measured noise power spectrum typical of those obtained for tubes of this type is given in Fig. 6.

IV. SAMPLING CONSIDERATIONS

4.1 *Sources of Error*

Fluctuations in signal amplitude which affect probability of error in readout from an information channel include spot size and intensity variations at the CRT, phosphor blemishes, noise and gain variations in the PMT, blemishes on the developed photographic plates, CRT beam-positioning error, optical error and information sampler error. This section discusses means which have been used to minimize the probability of error due to these factors. Approximate criteria for allowable error probabilities are given, and the way in which early sampler designs were arrived at is described.

4.1.1 *Variation in CRT Spot Size and Shape*

Local variations in spot size as well as variations which are a function of deflection angle are found in electrostatically deflected CRT's. In general, the distribution of intensity in the spot is not cylindrically symmetric. The straightforward way used to control error due to this variation is to set spot spacing on the basis of the poorest spot in the array.

4.1.2 *Intensity Variations at the CRT Screen*

It has been found that the flux level varies from spot to spot on the CRT screen. When one set of spots is used more frequently than the others in a flying spot store the situation is further aggravated, since the phosphor fatigues more rapidly at these spots. In the case of a P-16 screen, the initial fatigue rate is rapid, but the rate decreases with time and a quasi-equilibrium state is reached after a few weeks. Thus, preaging of the tubes helps reduce signal fluctuations due to this cause. Even so, after a few weeks of continuous operation spot-to-spot flux variations of as much as ± 15 per cent have been observed.

In the flying spot store, a narrowband feedback control system which acts on beam current is used to hold average intensity constant to within ± 1 per cent. However, this circuit does not compensate for spot-to-spot variations in intensity. Such variations affect probability of error in two ways. The feedback beam-positioning system locks the image of the beam to a mechanical reference edge located in one of the servo channels.[1,2] An electrical "half-light" reference determines the final beam position. If a constant electrical signal were used as reference, beam position with respect to the storage location would vary with spot intensity. Beam-positioning error is especially serious, since it results in highly dependent bit-error probabilities. To reduce this serious source of error, one channel of the system is used to sense the flux level at each spot. Its electrical output is used to provide a dynamic half-light reference for the feedback beam-positioning system, which is nearly correct for all spots within the range from $+20$ per cent to -50 per cent of average intensity.

Even when the beam is correctly positioned, spot-to-spot variations in intensity contribute to signal amplitude fluctuations in the information storage channels. The sampler circuit has been designed to operate properly with these distributions in signal amplitude.

4.1.3 *Phosphor Blemishes*

The best screens made by commercial setting techniques have been found to have a few phosphor blemishes of a maximum diameter of 0.004

inch per tube. Phosphor screen blemishes give rise to dependent bit-error probabilities in a word, which cannot be reduced by means of redundancy in the stored word. The use of spot spacings larger than the spot diameter is a straightforward way of minimizing phosphor blemish effects, but this is very costly in terms of capacity.

Phosphor blemish specifications are arrived at by considering their effect on readout. Clearly, no CRT screen with a phosphor blemish exceeding one spot diameter in extent may be used, since a blemish of this size would displace the writing beam enough to superpose some of the stored information. Some smaller blemishes can be tolerated, but the probability of occurrence of two such blemishes separated by one spot diameter or less must be negligible.

4.1.4 *PMT Current Amplification Variations*

Variations in current amplification in the PMT can easily introduce error by causing some of the signals representing "1's" and "0's" to fall outside the corresponding discrimination levels in the sampling circuit. PMT's generally show an initial increase in current amplification followed by a slow, steady decrease over a long time, which is due to decrease in the secondary emission ratio of the high-current dynodes. A stabilizer circuit has been built which holds the over-all PMT current amplification constant to within ±10 per cent without adjustment for relatively long periods of time.

4.1.5 *PMT Noise*

As has been shown in Section 2.2.2, the electrical signal-to-noise ratio is determined very largely by the shot-noise-in-signal. The standard deviation of the electrical fluctuation noise in the channel is given by $\sigma = m(2e\Delta f_c \bar{I}_a)^{1/2}$. In a typical application, $\Delta f_c = 1$ mc, $\bar{I}_a = 350$ microamperes and $m = 1.5$. The calculated value of σ in this case is 5.8 microamperes. A $\pm 3\sigma$ variation then corresponds to about ±7.5 per cent.

4.1.6 *Flux Variations Within and Between Channels*

Variations in flux level within and between channels lower both the optical discrimination ratio and window, and contribute to the probability of error. By the use of vignetting blades and varying stop sizes, flux variations within and between channels are limited to ±10 per cent. This allows the use of a constant exposure time for all spots in platemaking, and makes it possible to use the same sampler circuit with both on-axis and off-axis channels.

4.1.7 *Storage Plate Blemishes*

A few blemishes are found on the photographic storage plates after development. These may be due to defects in the material or may originate during handling or processing.

One can arrive at a storage plate blemish[4,5] criterion from system operation requirements. In this discussion, a parallel channel arrangement is considered in which one bit of each word is placed in each storage area of the film plane. A storage plate blemish can then affect at most one bit of the stored word. Use of a single-error-correcting, double-error-detecting Hamming code makes it possible to detect single or double errors and correct single errors in each word.[6] System requirements may impose the condition that all words must be correct or correctable. Thus, when the photographic storage plates are tested after preparation, any plate in which one or more words contain a double error is rejected. The practical problem is, therefore, that the probability of making a satisfactory set of plates in a small number of trials be high.

As an example, we consider the case of a system in which one block of stored information consists of 30,000 19-bit words. Table II gives a summary of an arrangement which might be used.

The efficiency is the amount of correct information obtainable from a set of plates over the total information on the plates (redundant bits plus stored bits). In order to employ the Hamming code, six additional channels are added and the complete word, including the redundant bits, is obtained in one readout. We now calculate the probability of the occurrence of a noncorrectable word. Assuming that the probability of an individual bit being in error, p, is small ($p^2 \ll p$), the probability P of two errors occurring within a 25-bit word is given by

$$P = \binom{25}{2} p^2.$$

If this expression is multiplied by the total number of words, w, the

TABLE II — SINGLE-ERROR-CORRECTING, DOUBLE-ERROR-DETECTING HAMMING ARRANGEMENT — SYSTEM PARAMETERS

CRT Array Size	173 × 173
Number of Channels	25
Word Length	19 bits
Redundant Bits	6 bits
Single-Error Correction	5 bits
Double-Error Detection	1 bit
Readouts per Word	1
Efficiency	0.760

TABLE III — PROBABILITY OF MAKING AN ACCEPTABLE SET OF PLATES

Probability of a noncorrectable word, $P = 300\ p^2$

Average Number of Plates, U	Individual Bit Probabilities, $p \times 10^{-4}$
2	2.8
4	3.9
6	4.5

average number of noncorrectable words, a, is obtained. That is,

$$a = \binom{25}{2} p^2 w,$$

which for this example becomes $9 \times 10^6 p^2$. A reasonable and convenient assumption — that the distribution of double errors on the plates is Poisson — is made. It follows from this assumption that the probability of having no double errors on the plate is $a^0 e^{-a}/0! = e^{-a}$ and that the average number of plates, U, which must be made to obtain an acceptable plate, is just e^a.

Table III gives the value of p corresponding to $U = 2, 4$ and 6. Thus, from simple considerations of system requirements, a reasonable criterion for maximum individual bit-error probabilities due to storage plate blemishes can be found. If we wish to obtain a satisfactory set of plates in two or less trials on the average, p must be $\leqq 2.8 \times 10^{-4}$ under the conditions of our example. Recent laboratory studies of the storage plate blemish problem indicate that this criterion will be easily met and in fact exceeded in practical systems.[4]

Plate acceptance can also be based on a count of the single errors on a plate, without regard to the location of these errors, the acceptance count being such as to virtually guarantee that no double errors exist in a single word. Plate verification by this means is also faster.

4.2 *Estimate of Nondependent Error Rate*

An estimate of allowable bit-error rates from all nondependent error sources is now made. Since two plate errors do not occur in the same word in an acceptable plate, the most probable uncorrected errors result from the combination of a storage plate error and a single-bit error due to one or more nondependent error sources, or of two such nondependent single-bit errors occurring within the same word. Thus, let p_1 be the probability of a bit being in error from all sources, other than storage plate blemish, which give rise to sensibly independent bit-error probabilities in a word. Assume that one uncorrected error every four hours in the store readout is the maximum tolerable. For the 25-bit word of our previous example, the probability of a noncorrectable error is given by:

$$\binom{25}{2} \times (2pp_1 + p_1^2) = 600pp_1 + 300p_1^2.$$

In four hours, the number of incorrect readings is given by the product of the number of words read out and the probability that a word has a noncorrectable error. In a typical system, 2×10^5 words might be read every second. Therefore, in our example, $1 > (600\ pp_1 + 300\ p_1^2) \times 2 \times 10^5 \times 14{,}400$. If p is 10^{-4}, $p_1 < 5.8 \times 10^{-9}$; if p is 10^{-5}, $p_1 < 5.8 \times 10^{-8}$.

As a "benchmark", it can be noted that, for Gaussian noise and ideal conditions, the calculated values of p_1 correspond to a minimum signal to rms noise power ratio of 21 db.[7]

One method of fast and relatively uncomplicated system operation involves the abandonment of the double-error detection feature in reading out the information. The additional redundant channel is still used for parity check. Should the parity check fail, single-error correction is initiated. Nonfailure of the parity check is taken to mean that the word is correct.

4.3 *Beam-Positioning Error*

Beam-positioning error can give rise to highly dependent bit-error probabilities. In the preceding discussion, this error was assumed to be zero. The feedback beam-positioning systems which have been built generally position the beam to within $\pm\frac{1}{10}$ spot diameter of the center of the bit storage cell. It has been found that this degree of mispositioning causes little degradation in over-all system signal-to-noise ratio.[4] However, when an occasional larger misposition occurs due to noise in the servo system, many bits in a word can be affected. Dependent errors of this type cannot be completely overcome by the means that have just been described. If the sampling circuit responds more slowly than the positioning circuit it is possible to hide rapid short duration mispositions from the information channels. If it can be determined that a misposition error has occurred during sampling, the system can be arranged to repeat the sample. The error rate in an experimental system has been reduced using these techniques.

4.4 *Optical Discrimination Ratio and Optical Discrimination Window*

The information sampler circuit must be designed to work with the flux distributions representing "1's" and "0's" at the photocathode.

These distributions are a function of spot size; intensity variations and the distribution in intensity in the flux from the cathodoluminescent spot at the CRT screen; spot spacing in the CRT array; phosphor blemishes; halo and reflections in the CRT faceplate; optical aberrations; exposure time; film density and contrast and many others. At the time design was started these distributions were not known. However, experimental data were available on the minimum amplitude signal for a "1" and the maximum amplitude signal for a "0" when the "worst" spots were read by a spot of average flux level at the CRT. Two parameters chosen to be used with these data are optical discrimination ratio, R_{optical}, and optical discrimination window, W_{optical}. Optical discrimination ratio is given by $R_{\text{optical}} = S/N$, where S is the photocathode current corresponding to the lowest-amplitude "1" read by a spot of average flux level at the CRT screen, and N is similarly defined for the highest-amplitude "0". Optical discrimination window is given by $W_{\text{optical}} = S - N$, measured in terms of photocathode current.

4.5 *Worst-Circuit Approach to Combination of Errors*

Since the amplitude distributions are not precisely known and since limits are known in most cases, the "worst-circuit" approach has been used in the design of the sampler circuit.

Let the maximum variations of each of k individual noise sources be $\pm\alpha_i$. Then the minimum discrimination ratio that is presented to the sampler is:

$$\frac{S(1-\alpha_1)(1-\alpha_2)\cdots}{N(1+\alpha_1)(1+\alpha_2)\cdots} = \frac{S}{N}\prod_{i=1}^{k}\frac{(1-\alpha_i)}{(1+\alpha_i)}. \tag{12}$$

Likewise, it is seen that the discrimination window at the sampler circuit is reduced from $A(S - N)$ amperes to:

$$A\left[S\prod_{i=1}^{k}(1-\alpha_i) - N\prod_{i=1}^{k}(1+\alpha_i)\right] \tag{13}$$

amperes, where A is the total amplification (in PMT and sampler circuit) up to the point where the sampler actually discriminates between a "1" and a "0". It is clear that the sampler discrimination levels for a "1" and "0" must be set within this window. These considerations are illustrated in Fig. 7.

Economic samplers having the bandwidths required for use in flying spot stores (~1 mc) do not have zero window width and, in addition, constitute a small but nonnegligible error source, due to component

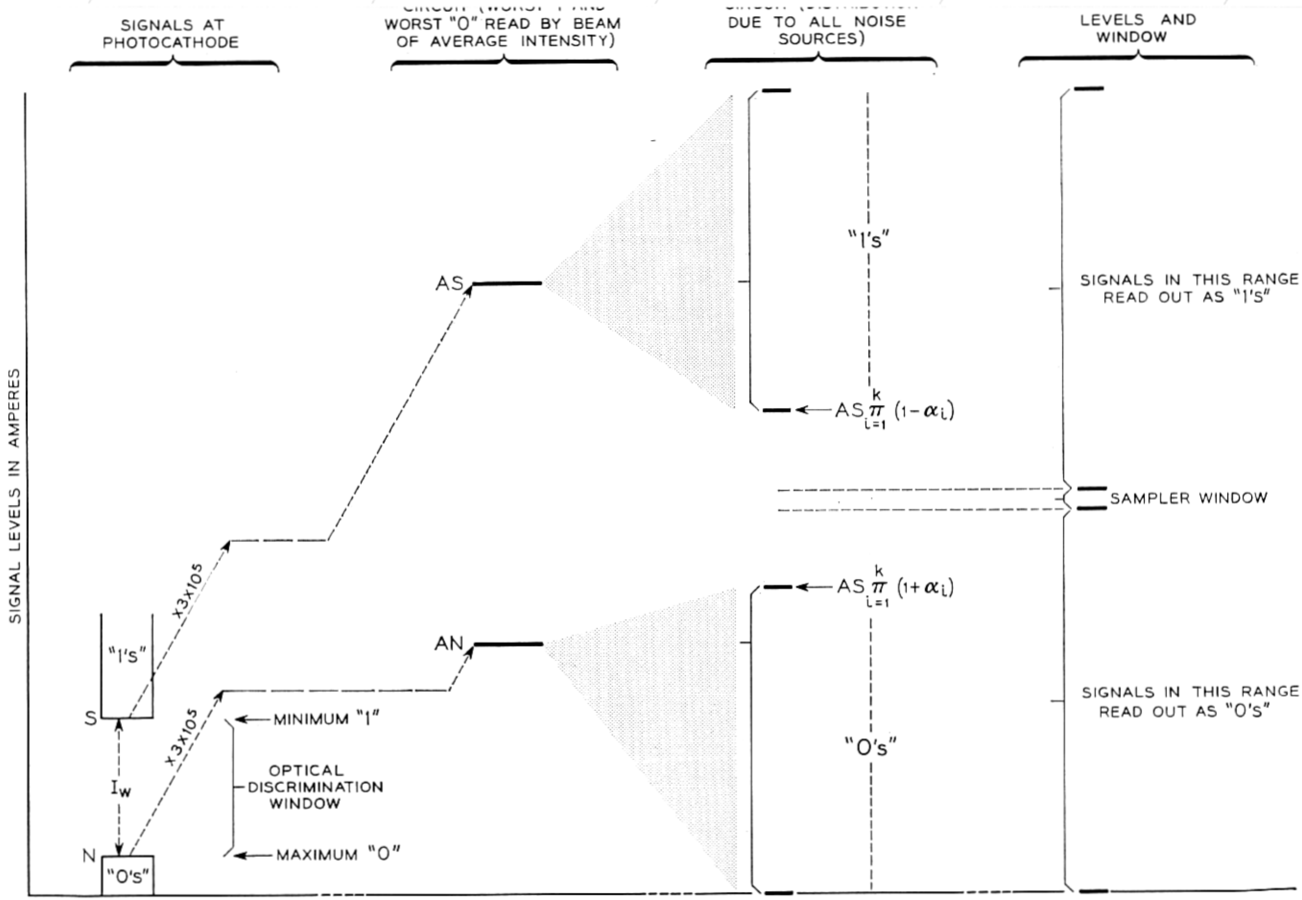

Fig. 7 — Signals representing "1's" and "0's" throughout the system, and optical discrimination window.

aging, power supply drift and residual error due to past history. An example follows:

Estimated Error Source Variations

Photomultiplier Noise	±7.5%
Photomultiplier Gain Variation	±10%
Light Variation Due to Optics (Within and Between Channels)	±10%
Light Variation Due to Phosphor (With Intensity Servo)	±5.0%
Positioning and Sampler Error	±10%
Minimum Optical Discrimination Ratio, R_{optical}	23:1
Minimum Optical Discrimination Window, W_{optical}	1.5×10^{-9} amperes

Other Design Information

Minimum Window at Photomultiplier output, I_w	3.0×10^{-4} amperes

The discrimination levels for "1" and "0" in the information sampler circuit are set by the circuit design. Sufficient amplification must be provided so that the signal distributions found at the photocathode fall outside of the sampler discrimination levels as shown in Fig. 7. The total PMT and circuit gain required is given by

$$A = \frac{I_w(R - 1)}{W\left[R \prod_{i=1}^{k} (1 - \alpha_i) - \prod_{i=1}^{k} (1 + \alpha_i)\right]}. \tag{14}$$

For the example above $A = 3.32 \times 10^5$.

An approach somewhat better than worst circuit can be taken by assuming normality of the distributions when physical reasoning tells us that the true distributions cannot be far from normal. However, the complete problem involves not only the probability that a signal shall exceed certain bounds for a certain percentage of the time but also the indicator, or sampler response. Thus, the requirements on the sampling circuit response prior to and during the sample period must have satisfied a certain criterion. In a simple instance, the criterion might be that the integral of the signal pulse noise in the period of sampling remain within a certain bound.

4.6 *Information Sampler Design*

An early electron tube sampler design is shown in Fig. 8. The first stage supplies gain and frequency equalization for the PMT anode circuit cutoff, and sets the bias voltage level at the suppressor grid of the following stage. The second stage employs a vacuum tube with two control grids. Coincident gating is accomplished in this stage by applying

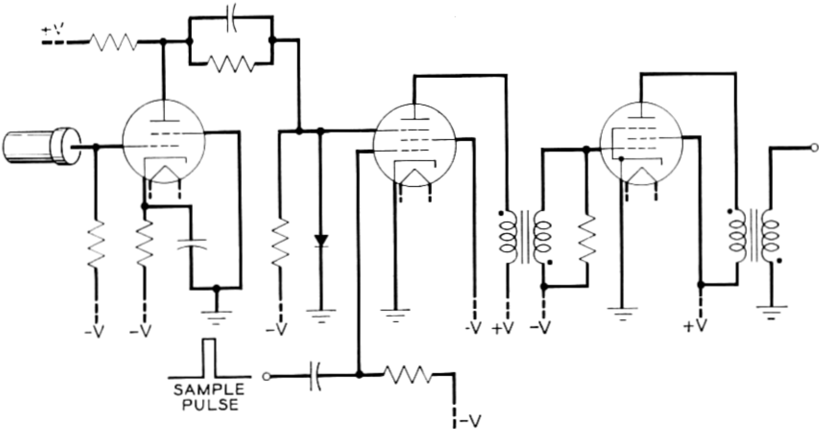

Fig. 8 — Information sampler circuit.

the signals representing "1's" and "0's" to the suppressor grid and a sampling pulse to the control grid. The bias set at the suppressor grid by the first stage is such that the maximum amplitude "0" does not take the tube above cutoff. Likewise, the minimum amplitude "1" carries the tube into the region of operation where output signal is only a function of the size of the sample pulse. The output stage is a pulse amplifier arranged to drive a low-impedance cable.

V. DISCUSSION

5.1 *Introduction of System Requirements*

Maximum store cycle time is set by the intended application, and the phosphor type is then chosen on the basis of its speed of response and efficiency, because of the strong effect which these variables have on beam-positioning time. The phosphor type sets the maximum screen loading which can be used. In high-speed systems, the type S-11 photocathode is usually chosen because of its high efficiency and good match to the emission spectrum of the fast phosphors. The minimum CRT-accelerating voltage, consistent with the requirements on the number of spots to be resolved, spot size uniformity and flux uniformity, is then selected, since this minimizes deflection amplifier power and since, in any case, the additional beam current and power density available at higher accelerating voltages could not be used due to the phosphor loading limit.

Since the number of spots which can be resolved and the total flux both increase with the size of the CRT screen, this dimension is usually made as large as space or the existing technology will allow and, in order to obtain maximum capacity with minimum equipment, the number of spots on the screen is often set at the maximum number which can be resolved. The degree of resolution required is set by sampling considerations. Channel capacity is set at the maximum number of bits which can be resolved, to provide the maximum number of stored words of a specified length. These factors set spot size and, together with phosphor efficiency and screen loading, they determine the maximum flux per spot.

From the foregoing, it is seen that, once store cycle time and channel capacity have been fixed by the intended application, the only independent variable left in the determination of the optical acquisition factor, k_{g2}, is the sampling time, since signal-to-noise ratio in sampling is also set by system requirements. Fortunately, fairly wide latitude is possible in the choice of k_{g2} in the case where sampling time is a small part of total cycle time.

5.2 *Determination of Channel Parameters*

The usable area in the lens plane approaches the area intercepted by a cone of half angle equal to the maximum field angle of the lens for low values of the ratio of screen height to focal length, where the variation in spot position in the array accounts for little of the total field angle. Screen height, h_s, is made large to maximize flux and channel capacity. If the ratio h_s/f.l. is made small, long optical systems result (assuming $1/M > 1$). Thus, we see that efficient production of flux demands the use of a large CRT and that efficient use of this flux precludes the use of very short optical systems. Therefore, increasing capacity and speed almost invariably result in increasing system dimensions.

Having fixed the value of h_s/f.l. and c, the minimum value of $1/M$ (and therefore the minimum system length) and the maximum f-number lens which can be used in the channel while meeting the requirement of nonoverlapping film storage areas are found by proceeding along the ordinate in Fig. 3 at the required value of k_{g2} to the intersection with the curve drawn for the given value of h_s/f.l. and c, the lens spacing parameter. All points to the right require a lens with larger aperture, and hence smaller f-number, and result in a longer system.

The curves of Fig. 4 show the half-field angle which must be obtained to set a specified value of k_{g2} in 40, 100 and 250 nonoverlapping channels, and demonstrate the relationships between h_s/f.l., c and number of channels. They also demonstrate that a wide range of systems in terms of capacity and number channels can be realized, all providing a value of k_{g2} sufficiently high to permit sampling in fractional microsecond times at high signal-to-noise ratios.

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