An Experimental Clogston 2 Transmission Line

By Mrs. R. A. KING

With the aid of two special machines, a 276-foot laminated conductor was assembled by hand around a $\frac{7}{8}$-inch conducting core. The laminations consisted of 100 concentric layers of $\frac{1}{4}$-mil aluminum foil separated by 99 polystyrene cylindrical insulators, each 1.35 mils thick. After fabrication, the cable was shielded by a wrap of 10-mil aluminum foil.

The Clogston 2 dominant mode was propagated. Measurements were made of the mode pattern and the attenuation as a function of frequency up to 25 mc. Cavity-resonator and insertion-loss measurements were in approximate agreement. Measured attenuations exceeded the theoretical values for a uniform Clogston 2 line in the ratio of 2 to 1 for frequencies above 10 mc, but were in agreement with theoretical values for a line showing a systematic lack of uniformity of the laminae. The 276-foot cable was cut into shorter lengths. Measured attenuation-frequency characteristics were proportional to length of cable.

As a by-product, successful methods were evolved both for cutting and joining laminated conductors.

I. INTRODUCTION

Clogston[1] has discovered that it is possible to reduce skin effect losses in transmission lines by properly laminating the conductors and adjusting the velocity of transmission of the waves. During a period in late 1951 and early 1952, a transmission line completely filled with laminated material was constructed and its loss was measured in the frequency range from 1 to 25 mc. This structure is referred to as the Clogston 2 stack to distinguish it from the Clogston 1 described by Black, Mallinckrodt and Morgan.[2] The measured loss was compared with the loss computed by Morgan for a uniform line and was found to match at low frequencies, but to rise to a value of twice the computed loss at 10 mc. The ratio of measured loss to computed loss was nearly 2 to 1 over the

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remainder of the measured frequency range. The explanation of this difference awaited the work of Raisbeck[3] on the effect of lack of uniformity in a laminated line. The measured loss was then compared with the loss computed by Raisbeck and was found to match reasonably well in the frequency range where the computation is expected to be valid.

A description is given of the construction of the line and the technique used for measuring, with some emphasis on the way this line departs from the ideal.

II. CONSTRUCTION OF THE LINE

One of the transmission lines described by Clogston, known as a Clogston 2 cable, is a line in which the space is entirely filled with insulated laminations. For a line of prescribed cross section and top operating frequency, there exists an optimum thickness of insulation which will minimize the loss for a prescribed thickness of metal.[4] It is believed that approximate equality of thickness of metal and insulation will usually be desirable in practice. However, when choosing dimensions for a manually constructed transmission line, practical considerations, based on materials conveniently available, dictated a dielectric layer approximately six times as thick as the conducting layer. Aluminum foil approximately 0.25 mil thick and polystyrene films 0.60 mil thick were chosen. Each dielectric layer was composed of two polystyrene films and a layer of adhesive. The base on which the layers were placed was an aluminum pipe with a $\frac{7}{8}$-inch outside diameter.

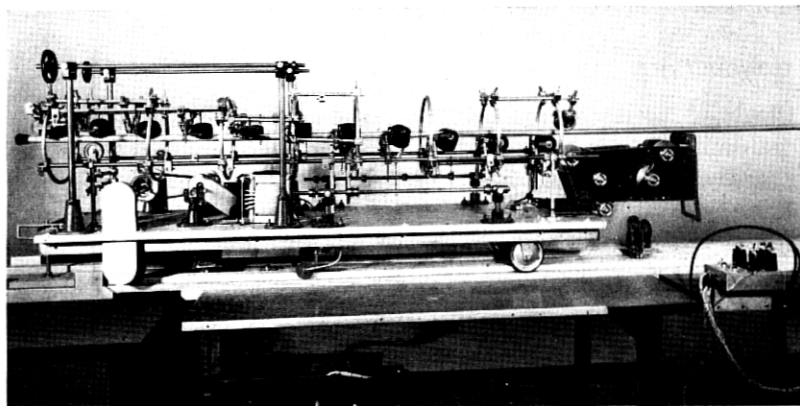The foil laying machine shown in Fig. 1 was equipped with a series
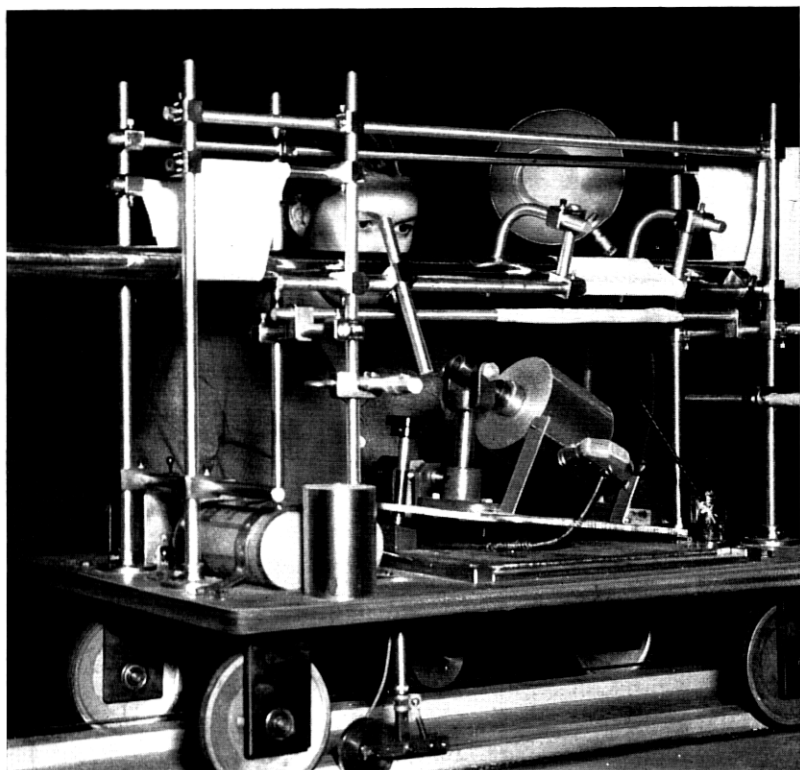

Fig. 1 — Foil laying machine.

Fig. 2 — Insulating layer mechanism.

of metal rollers which reeled off the foil, put a coating of adhesive in a petroleum-ether solution on the inside surface of the foil and then laid the foil longitudinally along the under surface of the pipe, after which a series of rubber rollers progressively formed the foil around the pipe as the machine moved forward. For a short interval before it made initial contact with the pipe, the foil was exposed to a stream of heated air to hasten evaporation of the solvent from the adhesive. In putting the foil layers on longitudinally, the hope was to have a butt joint down the length of the pipe. Actually, this proved difficult and, consequently, the final result had few butt joints and more gaps than laps at the longitudinal seam.

The insulating polystyrene layers were wrapped spirally by a semi-automatic mechanism shown in Fig. 2, which rotated the pipe and synchronously moved a carriage along the deck which supported the pipe.

The feed mechanism for the polystyrene film was mounted on this carriage. To correct for imperfections in match between the rotation of the pipe and the carriage travel, the carriage was equipped with a manually controlled vernier for advancing or retarding its motion. An overlap in the spiral wrap very slightly in excess of 50 per cent was the goal aimed for, which would give an insulating layer of at least two thicknesses of polystyrene film everywhere. This was considered necessary in view of the imperfections in the film. The irregularities in the commercially available material, together with a cut-and-try method of controlling the winding pitch, led to overlaps that exceeded 50 per cent and often increased steadily until the film had to be cut and spliced and a new start made. The same adhesive used on the conducting layers was used for splicing the polystyrene films.

The adhesive coating on the inside surface of the aluminum foil was a 10 per cent solution of Vistac (polyisobutylene) in a petroleum ether solvent. The process of evaporating off the petroleum ether, which is normally rapid, was accelerated by blowing hot air over the coated surface just before it was wrapped and leaving a thin film of Vistac on the foil. This film was estimated to be 0.1 mil thick.

The composition of one full layer is shown in Fig. 3. Its thickness was estimated to be 1.6 mils. While every effort was made to keep the layers as uniform as possible, the foregoing explanation of the methods of applying the layers is sufficient to indicate that the resulting line was far from uniform. It was not known at the time of the experiment what effect nonuniformity would have.

III. TERMINATING PROBLEM

The distribution of currents and voltages in the several conductors has been computed by Morgan[5] for a flat stack one meter wide.* The current density $J_2(y)$ at a distance y from the center of the stack is

$$J_2(y) = \frac{I_0 \pi}{2b} \sin \frac{\pi y}{2b}, \tag{1}$$

where I_0 is the total one-way current flowing in the line and $2b$ is the total thickness of the stack of laminae. The voltage with reference to the inner surface is

$$V(y) = \frac{2b}{\pi} I_0 \sqrt{\bar{\mu}/\bar{\epsilon}}(1 + \sin \pi y/2b). \tag{2}$$

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* The equation for current distribution in a cylindrical medium is (305). In the present paper, the stack is assumed to be flat; i.e., the thickness of the stack is assumed small compared to its radius of curvature. Then the Bessel functions reduce to their asymptotic trigonometric forms.

The total voltage from inner to outer surface is

$$V_0 = \frac{4b}{\pi} I_0 \sqrt{\bar{\mu}/\bar{\epsilon}}. \tag{3}$$

In these formulas $\bar{\mu}$ is the magnetic permeability of the materials in the stack (equal to the permeability of free space) and $\bar{\epsilon}$ is the equivalent dielectric constant of the stack, as described by Morgan, and equals:

$$\bar{\epsilon} = \epsilon_2/(1 - \theta), \tag{4}$$

where ϵ_2 is the dielectric constant of the polystyrene dielectric layers and θ is the proportion of the total stack thickness contributed by the thickness of the aluminum layers.

The current in the 50th foil is zero. Taking foils by pairs from 0 and 100 to 49 and 51, the ratio of potential difference to current was computed and found to be a constant value of 19.4 ohms. The line was assumed to be correctly terminated if 19.4 ohms were connected across each of these pairs. However, convenience dictated a different arrangement and an equivalent termination was worked out with a resistance of appropriate value from each conductor to its neighbors. The computed dc resistance values varied from 0.00475 to 0.00943 ohm. It was overlooked that these resistances apply to a stack one meter wide, and should, in general, be multiplied by the reciprocal of the stack width in meters. Therefore, the resistance values actually used departed from the matching resistances by a factor of approximately 1/11.5.

These resistance values are smaller than any physical resistors ordinarily supplied and some means of fabricating the wanted values was looked for. Tests on aluminum foil of the same thickness as was used for the metal laminations in the cable itself proved that this material, when wrapped over the edge of a strip of polystyrene so as to double back on itself, had the right order of magnitude of resistance. The final result is
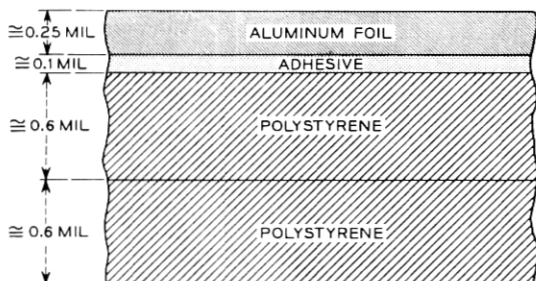

Fig. 3 — One composite layer of foil and insulation.

shown on Fig. 4. A suggestion from D. A. McLean that the points of contact between conducting layers and terminating resistors be arc-welded by discharging a capacitor across them succeeded in reducing the contact resistance to a negligible figure.

At the receiving end, the terminations were left as indicated above and a tab was connected to each junction to permit measurement of the voltages.

At the transmitting end, it was desired to launch a wave as near as possible to the principal mode of transmission. This was accomplished by sending currents through high-impedance padding resistors into each junction on the terminating network. These desired currents vary as the cosine of the distance from the junction to the 50th foil.

IV. LOSS MEASUREMENTS

With 276 feet of cable with a pileup of 100 composite layers having been built and terminated so that the previously determined resistance existed between each layer and its adjacent layers at both ends of the cable, the line was ready for loss measurement.

A signal generator was used to supply all the currents. A transformer with a stepdown ratio of about 3.75 to 1 was used to give a reasonably balanced output voltage. Suitable resistors were determined to connect conducting layers 0 through 49 to one side of the transformer secondary and 51 through 100 to the other side. With the proper distribution established, the ratio of voltage-to-ground to current at each junction was 4.85 ohms when foil 50 was taken as ground. This was one-fourth of the resistance needed between pairs. The termination resistors constituted one half of the conductance and the line the other half.

This value of 4.85 ohms had to be considered in fixing the ratio of the padding resistors. The resistors were chosen as a compromise between
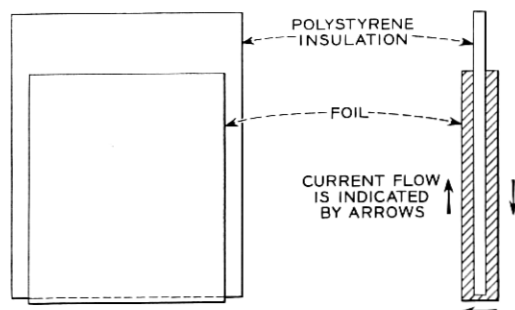

Fig. 4— Termination configuration.

various needs. A high value would have the undesirable result of increasing the dead loss of the pad. On the other hand, a low value would have shunted the termination resistors, which were not adjusted with this in mind and so would have increased the reflection factor. The inductance of the tabs making contact to the conducting layers was not thought to be negligible at high frequencies; the reactance was estimated to be as great as 10 ohms at 20 megacycles. Mutual inductance between tabs was not small either. The use of large resistors would have minimized the effect of inductance in the leads but would have made the capacitance across the resistors more serious.
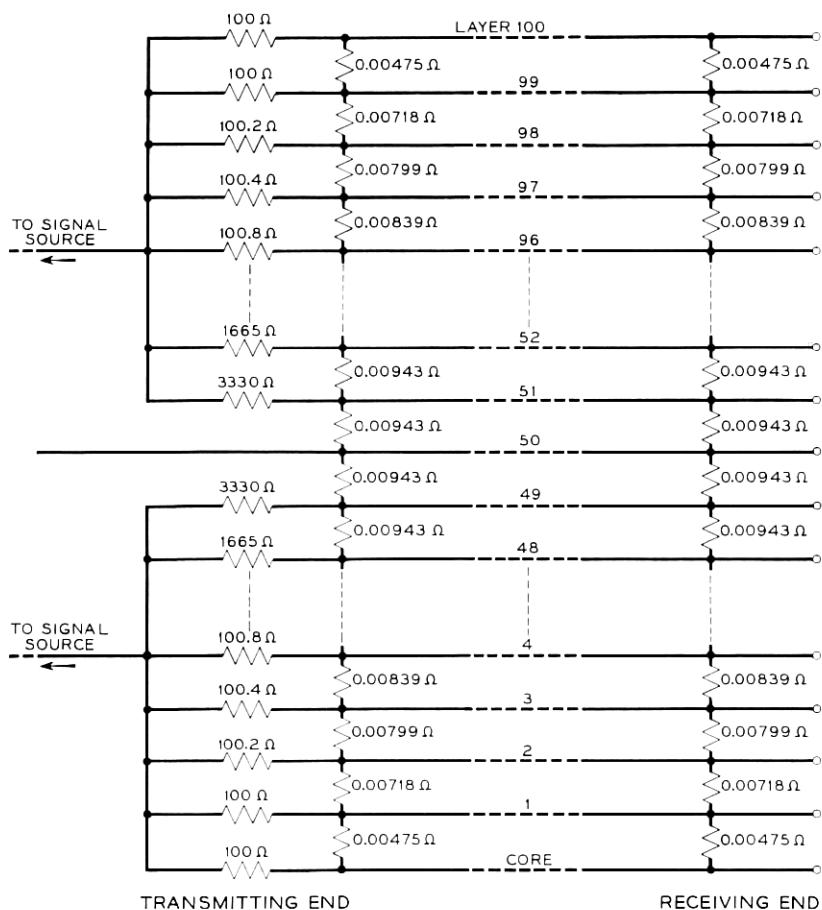

Fig. 5 — Terminating and padding resistor arrangement.

It was decided that 100 ohms would be a good compromise value to use for the resistors to the extreme conductors of the stack. The others were higher. By taking 100 + 4.85 ohms as the total resistance and by applying a cosine variation with distance in to the center of the stack, and then subtracting the constant 4.85 ohms, the values of the padding resistors were obtained. In order to insure that the values were as nearly correct as possible, the molded resistors used were pre-aged by the application of heat and were measured both before and after being connected.

The padding resistors at the sending end introduced a loss of 26.8 db between the transformer secondary and voltage across the transmission line. Fig. 5 shows the arrangement of the terminating and padding resistors at the sending and receiving ends of the line.

The measuring circuit for the terminated line is shown in Fig. 6. A wideband radio receiver feeding a vacuum tube voltmeter was used as a detector. A pad of resistors at the input of the receiver provided a 75-ohm impedance at all frequencies. Transmission loss to any of the
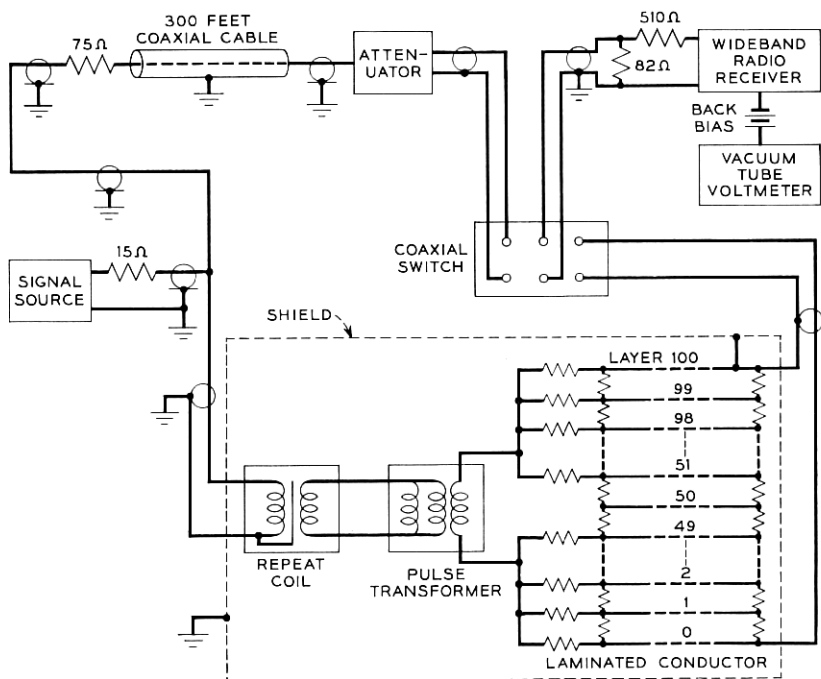

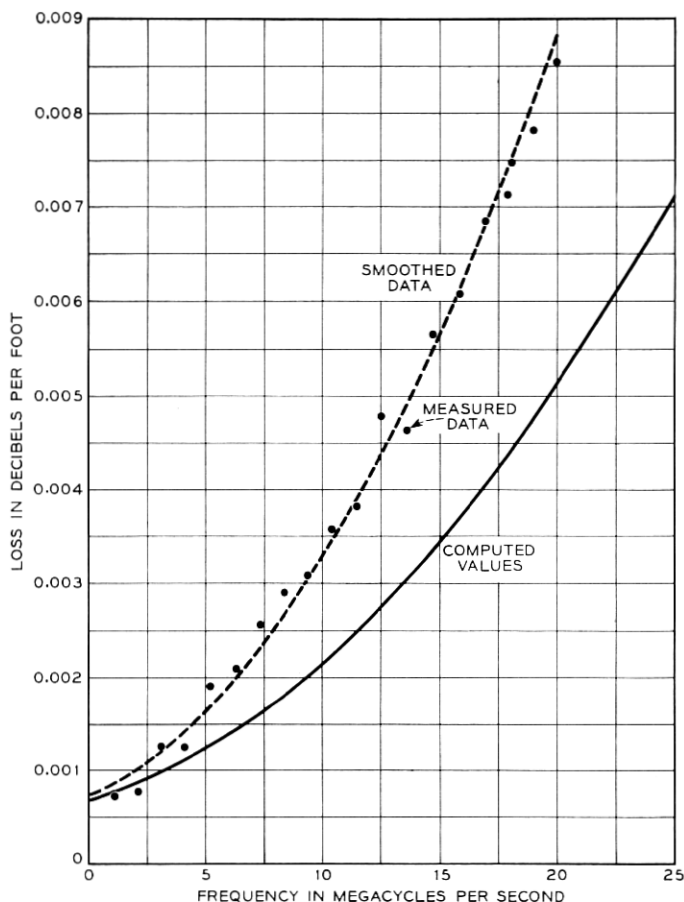Fig. 6 — Measuring circuit for terminated line.

Fig. 7 — Loss measurements on 276-foot terminated laminated transmission line, with line and measuring circuit completely shielded.

available taps could be measured by comparison to the calibrated loss of a 300-foot length of solid-polyethylene dielectric coaxial cable. In order to obtain the net loss of the line, the calibration loss and the 26.8 db loss of the pads back-to-back were subtracted from the measured attenuations.

Before any measurement could be made over the frequency band desired, it was necessary to shield the pipe by adding a longitudinal strip of 10-mil aluminum wrapped as closely as possible. It was also necessary to shield the terminations. Before these shields were applied,

strong fields were set up in the space outside the system and large irregularities in loss occurred at frequencies for which the outside of the pipe was in resonance with free space waves.

The first data taken showed the mismatch introduced by the terminating resistors. A resume of the method used for finding the true loss from the measured values is given in the Appendix. Fig. 7 shows the loss data found for the completely shielded line. A theoretical loss curve found by S. P. Morgan is also shown. At the time when the experiment was conducted, the only computed curve available was that in which irregularities were ignored, and it is seen that the correspondence with experimental values is not satisfactory.

V. RESONANT CAVITY MEASUREMENT

The terminations were cut away and both ends of the pipe were faced off smoothly so that no shorts existed between adjacent layers at the cut ends. A check, using a micromanipulator and a pair of transistor points that made contact to each pair of adjacent layers in turn, revealed a number of shorts, some of which were removed by applying a small voltage across the layers.

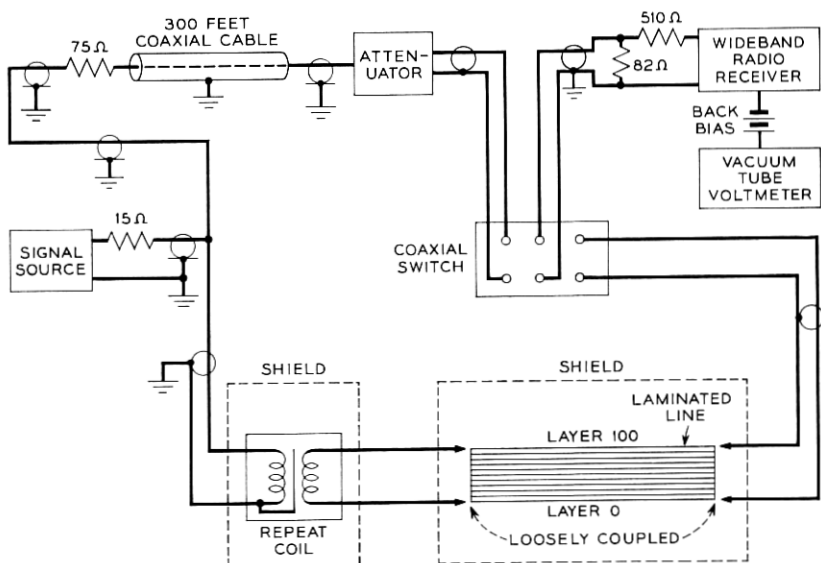The line, now 269 feet in length, was measured as a resonator by

Fig. 8 — Circuit for measuring laminated line as a resonator.

loosely coupling a coaxial lead to each end of the line. The measuring circuit is shown in Fig. 8 and the connector permitting loose coupling is shown in Fig. 9.

Loss measurements were made on a 269-foot length of line with five known shorts, a 252-foot length with three shorts, a 142.5-foot length with three shorts and 106.5- and 103-foot lengths that were short-free. The loss data for the various lengths were consistent enough to lead to the conclusion that the presence of shorts in some of the lengths measured was not affecting the results enough to account for the spread between the measured and theoretical values of loss. In fact, the experiments indicate that, as a practical matter, shorts to this extent could be tolerated.

Fig. 10 shows the loss data for the pieces of cable measured. Fig. 11 shows a curve that is a composite of all the data taken compared with Morgan's values, which were computed assuming a dielectric constant, $e_r = 2.56$ and a loss tangent, $\tan \phi = 0.0015$. The loss curves for an aluminum coaxial cable of the same dimensions with the same dielectric and with an air dielectric are included in Fig. 11.

VI. METHOD OF FINDING LOSS

The measurements of attenuation on the sections of laminated conductor gave a loss vs. frequency curve from which it was necessary to deduce the interaction loss.

The total transmission loss of a network is the sum of four factors:

(a) image transfer constant,
(b) reflection loss at the input,
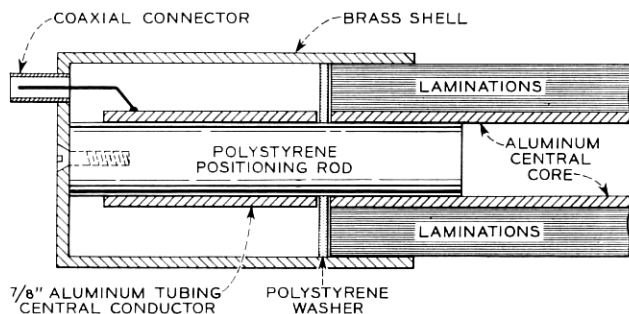(c) reflection loss at the output,
(d) interaction loss or gain.

Fig. 9 — Cross section of the coupler used in measuring laminated line as a resonator.
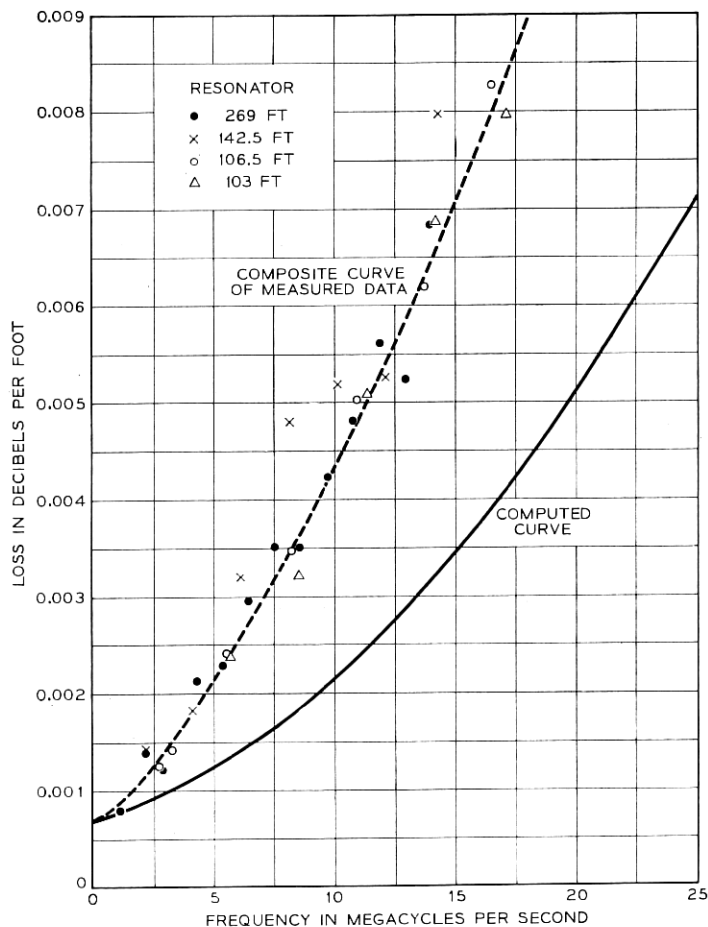
Fig. 10 — Loss measurements on various lengths of laminated transmission line measured as a resonator.

The first three terms will be positive losses but the fourth can be positive or negative. The interaction factor is expressed as

$$\frac{1}{1 - \rho_1\rho_2\epsilon^{-2\theta}} \quad \text{or} \quad \frac{1}{1 - \rho_1\rho_2\epsilon^{-2\alpha}\epsilon^{-2\beta j}}, \tag{5}$$

where ρ_1 and ρ_2 are the reflection coefficients at the sending and receiving ends of the line and $\theta = \alpha + j\beta$ is the image transfer constant. With a high impedance at each end, both reflection coefficients have the same sign and the interaction phase angle is $\beta = \pi f/f_0$, where f_0 is the fre-
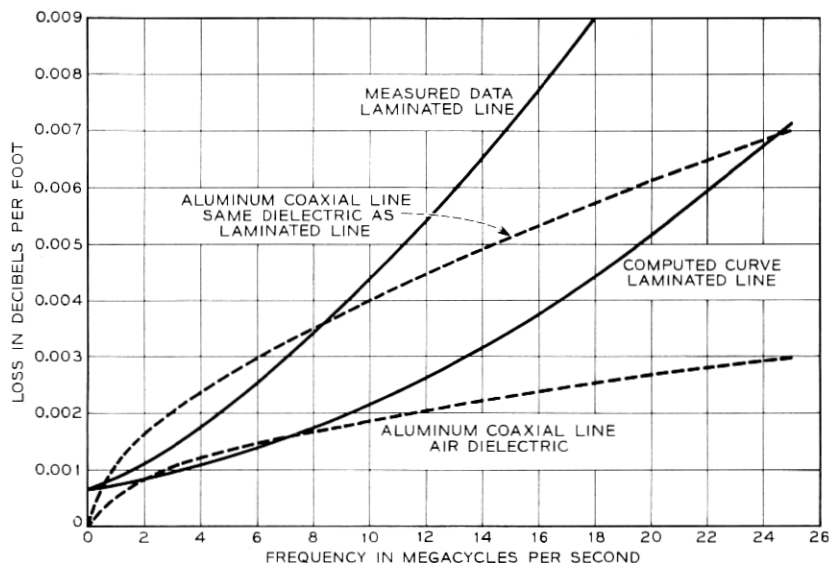
Fig. 11 — Measured and computed losses of a laminated transmission line compared with coaxial lines of the same dimensions.

quency of the first resonance or minimum loss point. When β is an integral multiple of π, $(\beta = n\pi)$, the interaction factor is

$$\frac{1}{1 - \rho_1\rho_2\epsilon^{-2\alpha}},\tag{6}$$

and when $\beta = m\pi/2$ and m is odd, the interaction factor becomes

$$\frac{1}{1 + \rho_1\rho_2\epsilon^{-2\alpha}}.\tag{7}$$

Calculations by J. O. Edson showed that $\rho_1 = \rho_2 = 1$ to an accuracy better than 1 per cent and, for our purposes, can be ignored in the calculation of true loss. Then, α can be computed from the measured attenuation, since

$$\frac{1 + \epsilon^{-2\alpha}}{1 - \epsilon^{-2\alpha}} = K,\tag{8}$$

where $20 \log_{10} K$ is the spread between maximum and minimum loss points in decibels.

In the case under consideration, 2α is not independent of frequency and neither are the reflection losses, but the quantity K may still be ob-
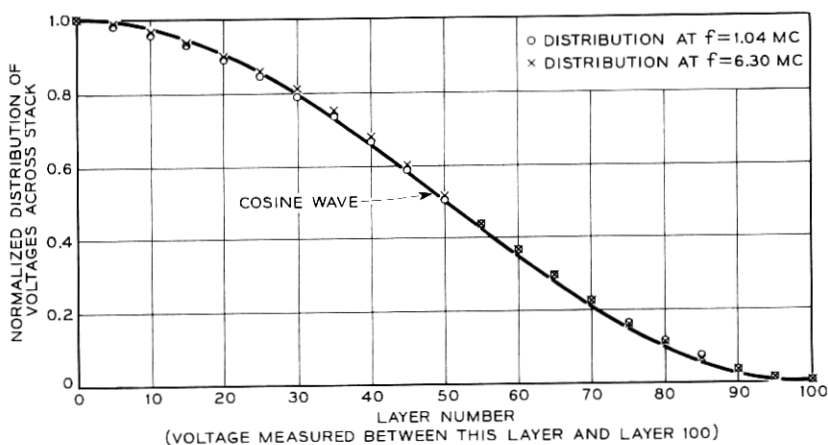
Fig. 12 — Distribution of voltages across 276-foot terminated Clogston 2 line at two minimum-loss points.

tained from a study of the measured loss. The procedure is to draw an envelope of the maximum loss points and an envelope of the minimum loss points and to interpret the spread between the two envelope curves as $20 \log_{10} K$.

VII. MEASUREMENTS ON DISTRIBUTION OF VOLTAGE

Two sets of distribution measurements were made, one on the 276-foot terminated line and one on the 103-foot unterminated length of line. On the 276-foot line, readings were made at two minimum loss points and the normalized distributions were plotted and compared to a cosine wave which is a fair approximation to the expected voltage distribution across the stack. The results are shown in Fig. 12. On a 103-foot length that was short-free, the same kind of measurements were made and compared to a cosine wave. This is shown in Fig. 13. The distributions under the two conditions varied considerably from each other and it was concluded that the terminated line, in spite of the mismatch, forced the principal mode, while the unterminated line distribution shows the presence of other modes. The loss measurements, however, do not show this marked variation although the terminated line loss is somewhat smaller than the loss taken on the line as a resonator.

VIII. VELOCITY OF PROPAGATION

For the different lengths of line measured, the phase velocities were found by using the measured lengths and the observed first resonances

of the standing waves. The velocities were of the order of 0.586 to 0.592 times the speed of light in free space. A check of these figures was made by using the formula

$$v = \frac{1}{\sqrt{\bar{\epsilon}\bar{\mu}}}, \tag{9}$$

where $\bar{\epsilon}$, the effective dielectric constant, is given by

$$\bar{\epsilon} = \epsilon_2 \left(1 + \frac{t_1}{t_2}\right) = \frac{\epsilon_2}{1 - \theta}, \tag{10}$$

where ϵ_2 is the dielectric constant of the insulating material and t_1 and t_2 are the thicknesses of the conducting and insulating layers. This resulted in a velocity equal to 0.574 times the speed of light. The experimental velocity is probably more accurate than the computed velocity, since the frequencies and lengths could be determined somewhat more accurately than the thicknesses of the layers. The agreement is very satisfactory and further supports the view that transmission follows Clogston's prediction.

IX. CAPACITANCE MEASUREMENTS

The loss-frequency curves show that the data on the terminated 276-foot line and the various unterminated lengths show a highly consistent increase in loss over that computed for a uniform line. This disparity between the measured and computed values was noted on the
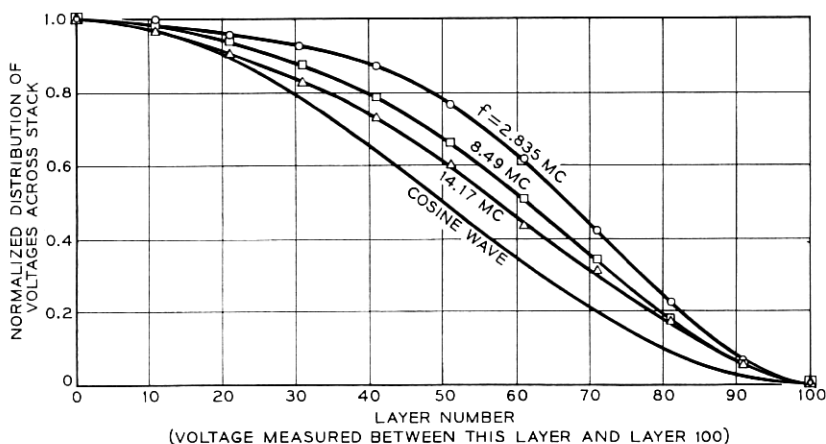

Fig. 13 — Distribution of voltages across unterminated 103-foot Clogston 2 line at three minimum-loss points.

first attenuation-frequency data, and each new set of measurements was made with the hope of finding the reason for and thereby reducing this spread. The increase in loss with frequency remained painfully persistent whether the data were taken on the whole line or on any of the sections into which it was later cut.

In addition to the check on the voltage distributions mentioned above, again using a micromanipulator and a pair of transistor points, the capacitance was measured between each layer and its neighbor, going from the inside to the outside of the stack in order to find a clue to the increase in loss. These capacitance measurements are shown in Fig. 9 of the accompanying paper[3] for the 103- and 142.5-foot lengths of unterminated line. When these capacitances were reduced to a unit area basis, they were found to correlate well with a computed value of 0.000426 microfarads per square inch of area for an assumed thickness of 1.35 mils of dielectric.

It will be noted that the two sets of measurements show similar fluctuations across the stack and at first these fluctuations were felt to be small enough not to be troublesome. However, Morgan,[5] considered the effects of nonuniformity in laminated lines and his work indicates that small systematic variations across the stack can cause substantial variations in transmission properties. Further work by Raisbeck on the analysis of these capacitance fluctuations is reported in the companion paper.[3]

X. SPLICING EXPERIMENTS

Computations by J. G. Kreer and J. O. Edson indicated that a separation of 1 mil between two lengths of Clogston 2 line would give 0.3-db loss for the splice and the loss would increase to about 1.8 db when the separation exceeded the thickness of the stack though the inner pipes and the outer foils of the two sections are connected.

One further experiment was made on the Clogston 2 line. Two lengths of cable, one 142.5 feet long with two known shorts and the other a short-free 103-foot line were faced off as plane as possible, butted together and measured for loss. A second set of measurements was made with a 2-mil spacer at the butt joint.

Interpretation of the data shown in Fig. 14 is made difficult by the fact that reflections occur at the junction as well as at the two ends of the line. The reflections from the individual sections confuse the picture at high frequency. The low-frequency points should be fairly reliable. When the ends were butted together as tightly as possible, the indicated
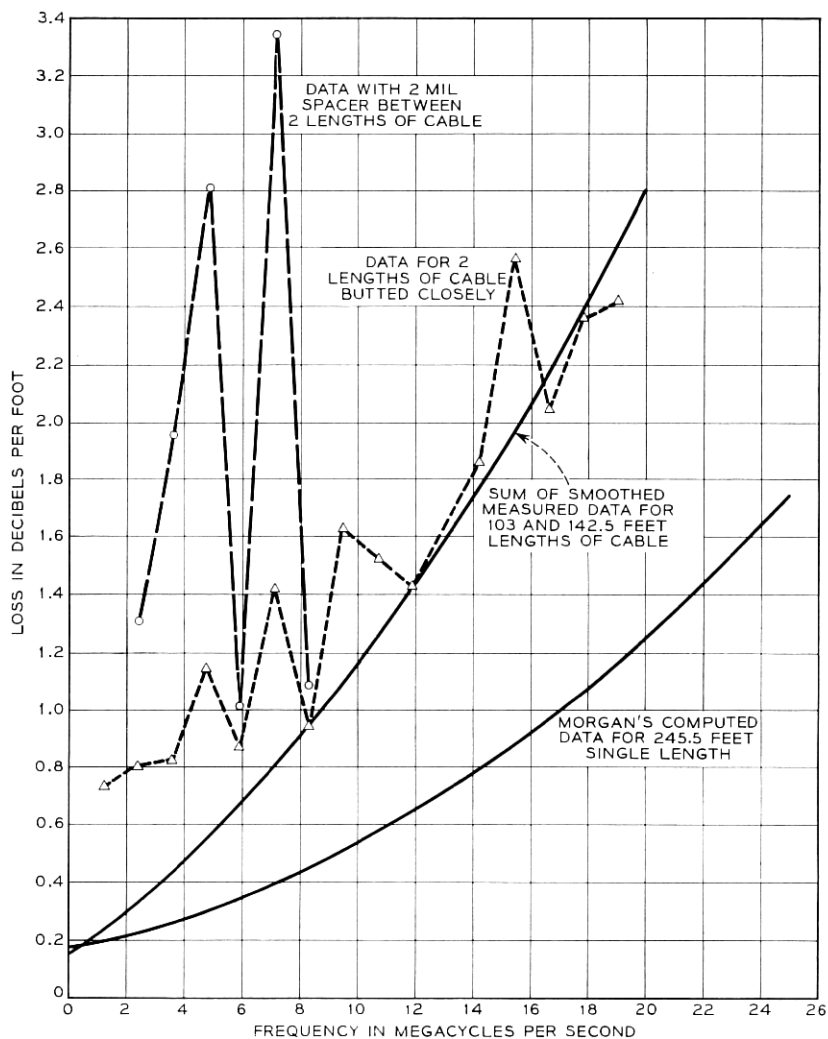
Fig. 14 — Loss measurements on two lengths of laminated conductor spliced together.

loss was about 0.5 db greater than the sum of the losses of the individual sections of line. At higher frequencies, the points are found to be on either side of the sum of the parts. This is not unreasonable, since the loss at the splice is of the nature of a reflection loss rather than a real dissipative transfer loss, and consequently this loss may be reduced by reflections in the individual sections of line. With a 2-mil spacer between

the ends of the two lines, the low-frequency loss of the splice appears to be about 1 db and the variations due to multiple reflections are more violent.

With a machine for cutting the faces plane within closer tolerances than was accomplished here, and with a jig that squeezed the two faces together as closely as possible, this method of splicing Clogston 2 cable appears feasible.

XI. ACKNOWLEDGMENT

The experiment described above was designed by J. O. Edson and J. G. Kreer and was carried out under the supervision of H. S. Black with considerable credit for mechanical ingenuity going to C. R. Meissner. In addition, many other members of Bell Telephone Laboratories contributed to various phases of this project.

APPENDIX

Method Used to Figure Insertion Loss of a Network With a Mismatch

Assume a network of unknown image impedance $Z_0 = k$ ohms and unknown transfer constant $\theta = \alpha + j\beta$ working between a 1-ohm source and a 1-ohm load.

Start by adding $(k - 1)$ ohms in series with the source and call the total applied voltage e_0. Then the open circuit output voltage is $e_0 \epsilon^{-\theta}$ volts and the voltage at 1-ohm load is

$$e_{\text{out}} = \frac{e_0 \cdot 1}{k + 1} \epsilon^{-\theta}. \tag{11}$$

The output resistance can be built up to a match by adding to the 1-ohm load a resistance $(k - 1)$ and a generator

$$e_2 = \frac{k - 1}{k + 1} e_0 \epsilon^{-\theta}. \tag{12}$$

The voltage across the input terminals is then

$$e_1 = \frac{e_0}{2} - \frac{1}{2} e_2 \epsilon^{-\theta} = \frac{e_0}{2} \left(1 - \frac{k - 1}{k + 1} \epsilon^{-2\theta} \right). \tag{13}$$

The voltage across the total source resistance is

$$e_0 - \frac{e_0}{2} + \frac{e_0}{2} \left(\frac{k - 1}{k + 1} \right) \epsilon^{-2\theta}. \tag{14}$$

That across $(k - 1)$ ohms is

$$\frac{k - 1}{k} \frac{e_0}{2} \left(1 + \frac{k - 1}{k + 1} \epsilon^{-2\theta}\right). \tag{15}$$

Associating this part with the excess resistor leaves a net source

$$e_0 \left[1 - \frac{k - 1}{2k} - \frac{(k - 1)^2}{2k(k + 1)} \epsilon^{-2\theta}\right]. \tag{16}$$

The net insertion transmission ratio is one-half the ratio of the net source voltage in series with 1 ohm to the output voltage delivered to 1 ohm. It is

$$\frac{1}{2} \left[1 - \frac{k - 1}{2k} - \frac{(k - 1)^2}{2k(k + 1)} \epsilon^{-2\theta}\right] \div \frac{\epsilon^{-\theta}}{k + 1}. \tag{17}$$

The ratio is

$$\frac{\epsilon^\theta}{2} \left[1 + k - \frac{(k^2 - 1)}{2k} - \frac{(k - 1)^2}{2k} \epsilon^{-2\theta}\right]$$
$$= \epsilon^\theta \left[\frac{1 + k}{2} - \frac{(k^2 - 1)}{4k} - \frac{(k - 1)^2}{4k} \epsilon^{-2\theta}\right]. \tag{18}$$

If $\beta = 0$ in $\theta = \alpha + j\beta$, the ratio is

$$\epsilon^\alpha \left[\frac{1 + k}{2} - \frac{k^2 - 1}{4k} - \frac{(k - 1)^2}{4k} \epsilon^{-2\alpha}\right]$$
$$= \epsilon^\alpha \left[\frac{(k + 1)^2}{4k} - \frac{(k - 1)^2}{4k} \epsilon^{-2\alpha}\right]. \tag{19}$$

If $\beta = \pi/2$, $2\beta = \pi$, which changes sign of the second term in brackets. The insertion loss is minimum for $\beta = 0$. The insertion loss is maximum for $\beta = \pi/2$.

If α is small,

$$\epsilon^{-2\alpha} \cong 1 - 2\alpha,$$

$$\text{minimum loss} \cong \alpha + 20 \log_{10} \left[1 + \frac{(k - 1)^2 2\alpha}{4k}\right], \tag{20}$$

$$\text{maximum loss} \cong \alpha + 20 \log_{10} \left[\frac{k^2 + 1}{2k} - \frac{(k - 1)^2}{4k} 2\alpha\right].$$

The approximations may be made exact if the quantity 2α is interpreted as the difference between $\epsilon^{-2\theta}$ and unity.

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