A New Measuring Set for Message Circuit Noise

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A new message circuit noise measuring set is described. The instrument is an easily portable, visual-indicating and direct-reading device with which rapid and accurate noise measurements may be made. The set is fully transistorized and battery powered, small in size and light in weight. It can be used for bridging, terminating and noise-to-ground measurements with a variety of plug-in weighting networks, one of which is the new C-message weighting designed for the 500-type telephone set. The sensitivity of the set is such that it can detect −90 dbm. This value is chosen as the new reference for measurement and is defined as "zero dbrn."

I. INTRODUCTION

Various forms of electrical disturbances in a telephone circuit result in acoustic noise at the output of the receiver of the telephone instrument. As noise, capable of detection by the human ear, it may be merely slightly annoying or it may be obtrusive enough to seriously impair the reception of a telephone message. If good service to the telephone user is to be maintained, it is evident that the plant forces must be provided with means for measuring electrical noise on a scale which may be correlated with transmission degradation.

An ideal instrument for this purpose would measure and combine, to a high degree of precision, those physical attributes of a noise voltage that contribute to the over-all subjective effect on the listener at the circuit terminal. Such a device would be complex indeed—so complex, in fact, that it would not be practical from an operational standpoint. However, it has been found feasible to provide a noise meter, based upon only the most important qualities of the human hearing mechanism, which has proved most useful to the plant maintenance forces. For the past 20 years the 2B Noise Measuring Set has served this purpose.

In recent years it became evident that there was a need for an improved set that was simpler to use and more nearly met the needs of the present plant, and by utilizing today's advanced technology, was lighter, more portable and lower in cost. The 2B is essentially a modified version of its predecessor, the 2A Set. Each included features to permit volume and sound level measurements, which today are found in other standard instruments and are thus not needed in a noise measuring device. Furthermore, the modifications necessary to create the 2B Set from the earlier 2A added greatly to its complexity and required increased knowledge on the part of the user in order for him to obtain meaningful data. Lastly, the 2B Set is not sufficiently sensitive for many of today's uses: it does not provide adequately for the measurement of noise with the present 500-type telephone set; it is too heavy to be readily portable; and some of its components are now obsolete.

Accordingly, work was recently undertaken to develop a new noise measuring set to replace the 2B. This set, the 3A, is now in production by the Western Electric Company, and many units are already in use by the operating telephone companies. This paper will review the fundamental objectives in message circuit noise measurement, show the connection between the objectives and the requirements, and present some of the more important engineering aspects of the measuring instrument.

II. OBJECTIVES IN MESSAGE CIRCUIT NOISE MEASUREMENT*

The objective in message circuit noise measurement is to quantitatively characterize the effects of noise on the listener such that two noises that are judged to be equally interfering are assigned the same numerical magnitude. The realization of this objective in a measuring instrument imposes two types of requirements:

(a) those that imitate the important qualities of the hearing mechanism (subjective), and

(b) those that make the set compatible with the telephone system environment for the purpose of measurement (objective).

The subjective requirements must account for the manner in which noise interferes during the presence and absence of speech. This necessitates a characterization of the relative interfering effect of single-frequency noise components in the telephone channel, and an approximation of the manner in which the ear adds these components as a function of frequency and time to indicate the total effect.

Since noise effects are complicated, depending on both the character

* For a more detailed account of this topic the reader is referred to the companion paper.[1]

and type of noise, it is necessary to compromise these requirements. For example, the total interfering effect differs not only as a function of absolute noise level, but also as a function of the immediate environment (such as the room noise in the location of the telephone) and as a function of the telephone conversation level itself. Thus, it is the practice to characterize the effects of noise "on the average," and specify the requirements for the set accordingly. The characterization of relative interfering effect specifies the frequency response; the particular (average) frequency response is achieved by a weighting network. Knowledge of the pertinent integrating characteristics of the hearing mechansim determines the type of detector and the dynamic response of the indicating meter. Hence, a noise measurement made with such a device does not reflect the degree by which a given amount of noise interferes with speech transmission. Rather, the actual magnitudes are numerical entities normalized with respect to nominal conditions of noise interference.

This is not a serious drawback. After a noise set is developed and physically realized, one need only relate the measured noise magnitude to a subjective scale indicative of the degree of noise interference. The subjective tests needed for this purpose differ from the type that are used to determine relative interfering effect in that they establish the severity of measured noise level relative to ease of communication.

The objective requirements for the noise set are dictated principally by noise measuring needs. In particular, these requirements serve to transform the subjective reactions of the user (evaluated at the output of the system) to particular points in a circuit where it is most appropriate to make noise measurements. To illustrate, suppose we are in the process of deriving a weighting characteristic, that is, a subjective evaluation of the relative interfering effect of single-frequency tones as heard in a telephone. Such a test may prescribe that the observer adjust the level of each tone until it has the same interfering effect as a standard reference tone. For the measurement of noise at the line terminals of the station set, it is convenient to express the level of each of the tones in terms of relative noise voltage developed across a nominal resistance that is substituted for the telephone instrument. The relation of this voltage to frequency is the inverse of the characteristic of the desired weighting network, referred to the point of measurement (in this case, the line terminals).

The functions to be performed by a noise measuring set may be represented schematically as shown in Fig. 1. The first block consists of various inputs. As discussed above, these are chosen in conjunction with particular weighting characteristics so that the combination (input and

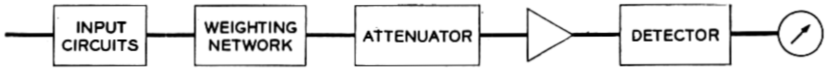

Fig. 1 — Block schematic of electronic noise measuring set.

weighting network) refer the subjective effect of noise to a convenient and appropriate point in the circuit for measurement. An attenuator and an amplifier are followed by a suitable detector, whose transfer characteristics are chosen so that the output is approximately proportional to the way the ear integrates single frequency noise components. Finally, the output is indicated on a suitable decibel meter with response characteristics similar to the ear.

III. REQUIREMENTS FOR THE 3A NOISE MEASURING SET

3.1 *General*

In accord with the fundamental objective of message circuit noise measurement, a noise measuring set must give approximately the same reading on noises that are judged to be equally interfering. In addition to this need, there are requirements that it be easy and understandable to operate.

Since the 2B Noise Measuring Set gave satisfactory answers under the conditions for which it was designed, the requirements for the 3A Set followed those of the 2B Set in principle. In addition, some of the operational features of the 2B were carried over to the 3A as well. In setting down the broad requirements, it was suggested that the new set be an easily portable, visual-indicating and direct-reading device with which rapid and sufficiently accurate noise measurements might be made.

To conform with this principle and yet adhere to the highlighting features of the 2B Set, it was decided that the new set should basically be capable of weighted and unweighted noise-metallic (bridging and terminating) and noise-to-ground measurements. In comparison to the 2B, this meant deleting volume, sound level and noise-receiver measurements. Moreover, it meant that a new weighting characteristic would have to be provided for noise measurements relative to the 500-type station set, and also that the root-sum-square law of addition (rms detection) for weighted single frequency components used in the 2B be re-examined. In addition, it was felt that the new set should be built around a flat 30-to-15,000-cycle amplifier with a variety of interchangeable plug-in weighting networks. Lastly, it was decided that the metering circuit should permit the standard 200-millisecond noise measurement and also a long-time (averaged) indication for rapidly fluctuating noise—both referred to a 1000-cycle noise reference of −90 dbm at the input.

With these considerations in mind, it was felt the new set would be sufficiently versatile to meet most of the present and anticipated future voice-frequency noise measuring needs.

3.2 *Subjective Requirements*

As discussed above, three important qualities of the hearing mechanism must be simulated in a noise measuring device: the way the ear weights single frequency noise components from the standpoint of interfering effect; the way the ear combines these frequencies to indicate the total effect; and the way the ear responds to sounds as a function of time. The first and second of these three phenomena are evaluated for average noise conditions.

Since the relative importance of single frequencies is dependent on the telephone instrument, a new weighting characteristic was derived using 500-type sets. Two tests were conducted, one in the absence and one in the presence of speech (the latter being taken at the average received level). In the absence of speech a group of observers was asked to adjust the loudness of 14 different frequencies between 180 and 3500 cycles until the sound of each was judged to be equal in annoyance to a −59 dbm 1000-cycle reference tone. Then, in the presence of speech, the same group of observers was asked to adjust the level of each of these frequencies so that the resulting impairment tended to be equivalent to the effect produced by the 1000-cycle reference tone. Throughout these experiments the room noise level was kept at 50 db referred to 0.0002 microbar.

In both cases the various single frequency levels were measured into a 900-ohm resistance, which was substituted for the 500 set. The results of each of these tests were averaged at each frequency, combined and smoothed as shown in Fig. 2.

Another test was then performed to determine the way the ear adds a larger number of single frequency noise components, weighted in accord with this particular characteristic. For earlier weighting, root-sum-square, or power addition (rms detection), was found to be a good approximation—thus the square-law detector in the 2B Set. In the present case, this form of summation was also found to be appropriate. The tests were conducted as follows:

Different pairs of bands of thermal noise were presented to various observers, who were then asked to change the level of one until it was

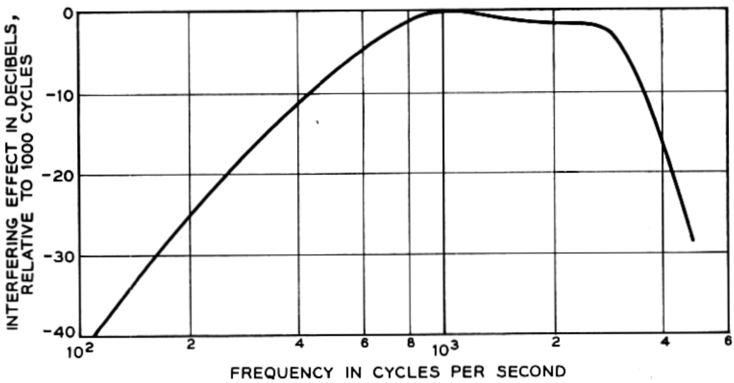

Fig. 2 — Response vs. frequency of new C-message weighting characteristic.

equal in interference to the other. These same bands were also compared objectively by computing the difference between their weighted (with the new characteristic) spectra on a power ($10^{db/10}$), voltage ($10^{db/20}$) and high-level loudness ($10^{db/40}$) basis. The best agreements between subjective and objective difference were those based on power addition. As a consequence, both power addition and the weighting shape of Fig. 2 were specified for the new set. Moreover, the same weighting was proposed for noise measurements on toll trunks in conjunction with a 600-ohm input impedance. In this respect, it was argued that, for the given weighting, the effect of distortion of the subscriber's loop would not be significant for noise originating in the toll portion of the circuit.

The third quality of hearing to be specified is the time constant of the meter circuit. Since the 200-millisecond integration time is well established,[2] this value was specified for the new set without review.

3.3 *Objective Requirements*

In general, the objective requirements for the new set were dictated by past experience with the 2B. Most important of these are (a) size and weight, (b) sensitivity and internal noise, (c) noise reference level, (d) amplifier gain and stability, (e) means of internal calibration, (f) other standard weightings, (g) inputs, (h) meter scale and attenuator steps and (i) output arrangements. For purposes of completeness, we will now discuss the particular choice of requirements for each of these features:

3.3.1 *Size and Weight*

An important consideration in the measurement of noise is the noise survey. This requires that the noise set be carried by hand for considerable distances. Measurements are made not only in telephone offices but also at cable terminals and along rights-of-way. Therefore it is most important that the noise set be small in size and light in weight. This requirement was realized in the new set by the use of miniature components and transistors. Its weight was kept to 15 pounds, its size with cover is $8 \times 11 \times 7$ inches.

3.3.2 *Sensitivity and Internal Noise*

Experience has indicated that the 2B Noise Measuring Set is not sufficiently sensitive for noise measurement on many of today's low-noise circuits. In order to assure that the new set would not have this difficulty, it was decided that the sensitivity (unweighted) be such that it could detect -90 dbm (24.5 microvolts across 600 ohms) at a maximum temperature of 50°C. In order that the new set might detect -90 dbm with some degree of meaning, the set noise (internal) is about 20 db below this figure (or -110 dbm).

3.3.3 *Noise Reference Level*

The original reference level for noise measurement with electronic measuring sets was 10^{-12} watts of 1000-cycle power at the input (-90 dbm). This reference was pertinent to noise measurements with 144-type weighting. The unit was called "dbrn" (decibels above reference noise, weighted either with 144-line or 144-receiver weighting).

With the advent of the 302-type telephone set, the 144 weightings were no longer applicable. New weightings were, therefore, derived: the F1A-line weighting for noise measurement on toll circuits, the HA1-receiver weighting for the measurement of noise across the receiver of the 302-type set. Both, however, admitted a substantially wider band of frequencies. The difference in shape was such that with the 10^{-12}-watt noise reference, the newer weightings would assign 5 db more magnitude to 3 kc flat thermal noise than the 144 weightings.

To have comparable magnitudes (for approximately equal interfering effect), a new reference was proposed for the newer weightings. This reference was a 5 db upward adjustment from 10^{-12} to 3.16×10^{-12} watts. The new reference was designated dba (dbrn adjusted).

Since the C-message weighting admits a wider equivalent spectrum than does the F1A line weighting, a similar approach would require a still higher (less sensitive) equivalent reference. This was considered for the new set, but it was deemed unwise. Using a higher equivalent reference with a more sensitive device would result in "negative" noise magnitudes on low noise circuits. To circumvent this undesirable outcome, it was decided to revert to the lower 10^{-12}-watt 1000-cycle reference for the new 3A Set. While this reference is numerically equal to the original one at 1000 cycles, and while the scale in the new set is designated dbrn, the two dbrn units are not equivalent when viewed in terms of weighted noise measurement. It is important, therefore, to designate all C-message weighted measurements with the new set as "dbrn C-message".

3.3.4 *Amplifier Gain and Stability*

For the given sensitivity it was estimated that the net voltage gain of the amplifier should be approximately 90 db. This was to assure adequate output signals for metering, monitoring and recording purposes on any measurable input signal. At full scale deflection, the open-circuit ac output is approximately 1.2 volts rms.

While the amplifier in the 2B Noise Measuring Set has fairly good long-time stability, it is usually necessary to recalibrate about every hour. In the new set, increased amplifier stability is realized by improved circuit techniques. It is not necessary to calibrate the instrument more than twice each day.

3.3.5 *Means of Internal Calibration*

The new 3A (as does the 2B Set) provides for internal calibration at 1000 cps. The reason for this choice of frequency is twofold: (a) the insertion loss for all frequency weightings is the same at 1000 cps, and (b) this is the frequency of the standard milliwatt reference generator, found in most offices, which provides the means for primary calibration of the set.

3.3.6 *Weighting Characteristics*

In addition to the basic C-message weighting, it is desirable to make various other weighted noise measurements. For flexibility, these are packaged to be used on a "plug-in" basis. Past experience has indicated the need for 3-kc flat, 15-kc flat and 5-kc weighted program noise measurements. These should provide for all message circuit noise measuring needs.

The nominal frequency response of each of these weightings is given in Figs. 2, 3 and 4.

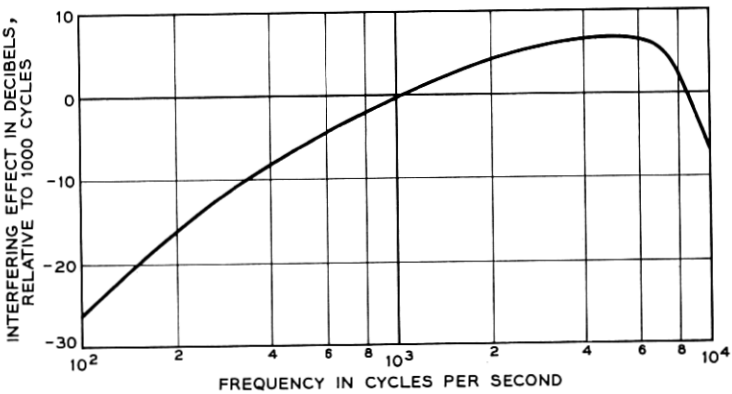

Fig. 3 — Response vs. frequency of standard 5-kc program circuit weighting.

3.3.7 *Inputs to the New Set*

Circuit arrangements for the inputs to the set are similar to many found in the 2B. The inputs are listed in Table I, in conjunction with pertinent weightings (and their purpose).

The longitudinal input circuit balance for each of the metallic input arrangements is more than 85 db at 60 cycles and more than 55 db at 15 kc.

Each input is obtained by connecting the circuit under test through a

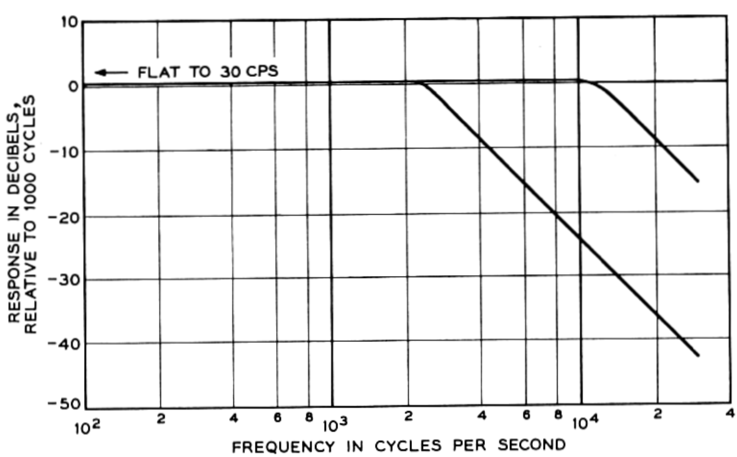

Fig. 4—Response vs. frequency of 3-kc flat and 15-kc flat weighting characteristics.

TABLE I

Input	Input Impedance, ohms	Purpose
Noise-metallic (N_m)		
Bridging	Nominal 10,000	Weighted or unweighted noise-metallic measurements on message or program circuits
600	Nominal 600	
900	Nominal 900	
Noise-to-ground (N_g)	Nominal 100,000	Weighted or unweighted noise-to-ground measurements on open wire or cable facilities

standard telephone jack or by connection to a pair of binding posts, selecting the input by a function switch and inserting the desired weighting network.

3.3.8 *Meter Scale and Attenuator Steps*

From an operational standpoint the meter scale-attenuator feature in the new set is like that in the 2B. The major difference is that the meter scale is calibrated over a zero-to-12 db range and the attenuator in 5 db steps from 0 to 85 db. The 12-db range of the meter resulted from calibrating as much of the usable range of the meter as practicable. This arrangement allows noise measurements from 0 to 97 dbrn. As mentioned earlier, no corrections are needed for any one of the various possible measurements.

3.3.9 *Output Arrangements*

As we have seen above, the measurement of noise has qualitative aspects. It is therefore desirable to be able to listen to the noise being measured. For this purpose, an ac output and a headphone are provided with the set.

It is also convenient in noise survey work to make a continuous record of the noise levels at a point in the telephone system. Therefore a dc output is also provided on the new set, which is capable of driving a variety of recorders.

IV. CONSIDERATIONS OF TRANSISTORS

To make the 3A Noise Measuring Set a truly portable device, the use of transistors as the active elements was investigated as a first consideration. The minimum power to be measured made the transistor noise figure of primary concern. It is rather elementary to find the maximum permissible noise figure for the set—and consequently for the initial stage of amplification—since we know the three necessary numbers: the minimum power to be measured (10^{-12} watts), the maximum bandwidth to be used (15 kc), and the maximum operating temperature (50°C).

Thermal power is equal to kTB, where k is Boltzman's constant, T is absolute temperature and B is bandwidth. Numerically, this is $(1.38 \times 10^{-23})(323)(15{,}000)$, which is approximately 6.7×10^{-17} watts or -131.7 dbm. If the measuring set is to measure -90 dbm, then the internal noise should be 20 db below this number (even the 20-db difference will slightly affect the accuracy of the set). This results in a maximum noise figure of $(-90 - 20) + 131.7 = 21.7$ db. This is certainly not an unrealistic noise figure requirement for the noise measuring set, but it is, nevertheless, low enough that it could not be neglected in the design.

Requirements other than noise figure also influence the consideration of transistors as the active elements in the set. Such things as bandwidth (15 kc), voltage gain (about 90 db), and the required temperature and time stability have been obtained earlier by many people using present-day feedback techniques. The only remaining quantity in the list of requirements is the proposed output voltage and overload margin. The requirements indicate that one volt (rms) open circuit at the output is desired for a full-scale meter indication. They also indicate that the set should not overload for signals 20 db higher than full scale. This means, then, that the output voltage should be as high as 10 volts (about 14 volts peak) without distortion. These numbers, in themselves, are by no means impossible, but it was found during the development of the set that they necessitated a very large battery. Since this would defeat the purpose of using transistors (portability), the overload requirements were eased to 15 db.

V. THE DEVELOPMENT OF THE 3A

The general noise measuring device consists of six black boxes: input circuit, attenuator, amplifier, frequency weighting, detector and meter. The development problem might be defined as designing these black boxes and physically ordering them in the measuring set. The input circuit, by its nature, is first, and the detector and meter are last. This leaves the attenuator, the amplifier and the frequency weighting yet to be placed.

We have seen that the amplifier must have a relatively low noise figure (about 21 db maximum). A low noise figure may be achieved by satisfying certain bias conditions on the first stage of amplification.

Montgomery[3] has shown that low base-to-collector voltage and low emitter current lead to a low noise figure for an alloy transistor. (Since this work was done, lower noise transistors have become available, which would have rendered this problem less severe.) These low-bias conditions impose the necessity of good dc stability. Also required of the amplifier is a high input impedance, since the new set is to be basically a bridging instrument. These factors, and the realization that the design of feedback amplifiers with more than three stages becomes somewhat burdensome, dictated the use of two amplifiers.

The first of these was built with the noise figure as the primary objective. In doing this, and striving for sufficient feedback for the necessary stability, the voltage gain was set at about 25 db. This, with a 9-db loss allotted to the frequency-weighting networks, sets the voltage gain of the second amplifier at 74 db (giving the required 90 db over-all voltage gain). The two amplifiers were built, with an eye for economy, using standard techniques to obtain the required bandwidth, stability and gain characteristics.

Because of the bias conditions on the first stage of amplification it follows that the input signal should be kept quite small in order to avoid nonlinear operation. This consideration places the attenuator between the input circuits and the first amplifier

It is not advisable to put the weighting network ahead of the first amplifier, because its loss will degrade the noise performance of the set. Furthermore, the input of the first amplifier provides a better termination for the attenuator than would the network. Finally, terminating the weighting networks in the two amplifiers eases the network design problem. The final layout, then, becomes as shown in Fig. 5.

The input circuit has associated with it a switch, controlled from the front panel, known as the function switch. The first position is OFF. In this position the battery is disconnected and the meter is shorted for protection in handling. The next two positions provide battery and calibration checks. They have been positioned here to encourage the user to make these checks when turning the set on. The remaining posi-

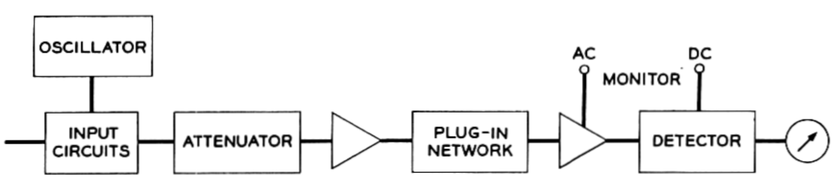

Fig. 5 — Block schematic of 3A Noise Measuring Set.

tions select one of four input conditions: N_g (noise-to-ground measurements), BRDG (bridging input impedance 10,000 ohms), 600 and 900 (terminating impedance of 600 ohms or 900 ohms).

Another switch on the set, called the DBRN switch, controls the loss in the attenuator. The attenuator is constructed of simple pi and tee networks of resistors, which are combined by the switch to give loss from 0 to 85 db in 5-db steps. Again with an eye on economy, the attenuator was built using only 14 resistors and a three-deck switch for its construction. The 5-db increment was determined by the largest practical logarithmic scale on the meter. Since this is about 10 db, the 5-db attenuator increments are necessary for convenient meter readings.

In order to give assurance that the meter reading is correct, an internal calibration source is provided. This source is a 1000-cycle transistorized Hartley-type oscillator using the common collector configuration. A breakdown diode is used to maintain the dc base-to-collector voltage at a nearly constant value. The level of oscillation is determined by this voltage, and is consequently quite stable.

The networks, which were designed to be used on a plug-in basis with the set, were developed using standard network synthesis techniques. The nominal frequency characteristics of these networks are pictured in Figs. 2, 3 and 4. These networks, of course, make the difference between the 3A Set being a voltmeter and its being a noise measuring set. As such, they form one of the two basic elements of the set, the other being the detector.

As discussed earlier, the proper summation of the spectral components of the noise can best be approximated by root-sum-square addition. A detector responding to the root-mean-square value of the complex waveform resulting from these components will accomplish this. The other restriction calls for a 99 per cent response to a noise burst in 200 milliseconds. Thermocouples and thermistors, which have rms response, are too slow for this application. The desired response was therefore approximated by a rectifier circuit. This type of detector was used in the 2B Set, but was achieved through the use of selected diodes. Analysis showed that it was possible to design a rectifier of satisfactory characteristics. This is covered in the Appendix.

The complete set is shown in Fig. 6. Its performance meets all requirements for a modern noise measuring set.

VI. CONCLUDING REMARKS

While the instrument described here performs the job of noise measurement, there are other features of importance in the design of a noise

Fig. 6 — The 3A Noise Measuring Set.

set for practical application in the telephone industry. Three of these features are cost, operational simplicity and versatility. Of these, cost was possibly the most important element influencing the development of the set. But certain things cannot be overlooked because of their initial "cost." One of these things is operational simplicity. This is achieved through at least two considerations. One is keeping the number of controls down and their arrangement logical. The other is the arrangement of the operating steps in a logical order. Perhaps both of these can best be demonstrated by describing how a measurement is made.

After the operator connects the circuit to be tested to the input of the noise set, he selects the proper weighting network and plugs it into the set. He then turns the function switch through the BAT and CAL positions (assuring the set is in proper working order) to one of the four input impedances. Finally, he turns the attenuator switch (called the DBRN switch) until he can observe a meter indication. The noise reading is then the sum of the DBRN switch indication and the meter indication. A typical reading might be "23 dbrn—C-message", where 23 dbrn is the reading and C-message is the frequency weighting that was used.

TABLE II

Type of Noise	2B, dba	3A, dbrn	Difference, db
Power harmonics, 180, 300 and 540 cycles	20	25	5
Modulation product noise	20	27	7
Panel office switching noise	20	26	6
3-kc thermal noise, power hum and switching noise	20	27	7

Note, however, that the 3A Noise Measuring Set will read higher (using C-message weighting) than the 2B Noise Measuring Set (using F1A-weighting), as shown in Table II. Although the numerical difference depends on the character of the noise, it is found in general that the reading is about 6 db higher on the 3A. This relationship should be remembered during the transition period while the 3A is superseding the 2B.

The final item of interest in the design of the set is versatility, which is exemplified by the frequency-weighting networks. For instance, if a new frequency weighting should become necessary due to some change in human attitudes, the character of the interference, or the noise sensitivity of the telephone plant, a new network could be designed and built into a package that would fit into the present set. Another aspect of versatility is reflected in the provision of ac and dc outputs, which may be used to drive a variety of other instruments.

VII. ACKNOWLEDGMENTS

The authors wish to thank D. W. McLellan for performing the measurements relating the performance of the 3A to the 2B. We also wish to acknowledge the help given by A. J. Aikens and T. C. Anderson in the preparation of this paper. The networks used in the 3A Set were synthesized by H. Simon and H. M. Thomson.

APPENDIX

Suppose that $f(t)$ is a voltage applied to the input of the circuit of Fig. 7(a). If the RC product is much larger than the period of the lowest spectral component of $f(t)$, then the circuit will "peak-detect" the input voltage. For this case $\hat{E}$ will be the dc voltage corresponding to the peak value of $|f(t)|$.

If the same signal is applied to the input of Fig. 7(b), and if $\bar{E}$ is the dc voltage across R, then $\bar{E}$ corresponds to the average of $|f(t)|$.This might be called "average" detection.

Since the rms value of a positive function lies between the average and peak value, it is instructive to investigate the action of a detector which gives a dc voltage corresponding to something between average and peak. It might be termed "quasi-rms". Such a detector might be built by properly combining Figs. 7(a) and 7(b). The result is given in Fig. 7(c).

In Fig. 7(c), let $\bar{E}_1$ and $\bar{E}_2$ be the dc components of the voltages across R_1 and R_2, respectively. If the R_2C product is long (as above), then the voltage across R_2 is practically constant and equal to $\bar{E}_2$.

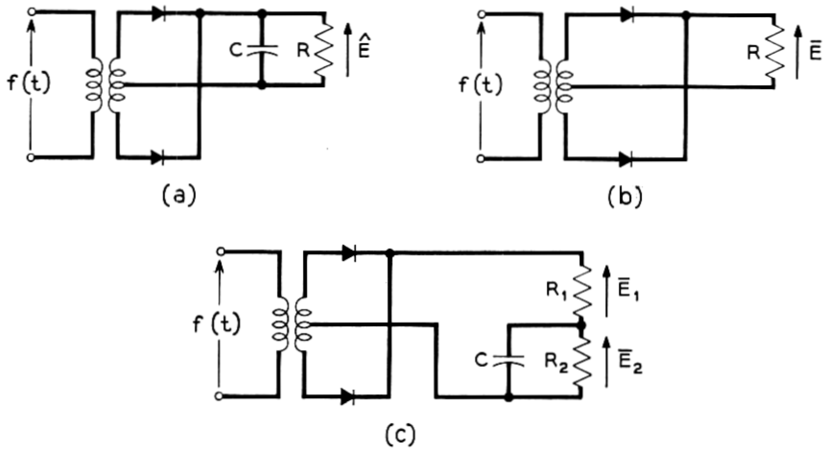

Fig. 7 — (a) Peak detection; (b) average detection; (c) "quasi-rms" detection.

Then, when $| f(t) |$ is less than $\bar{E}_2$, the diodes are back-biased and $\bar{E}_1$ is zero. The waveform across R_1 and R_2 is as pictured in Fig. 8(a) (ideally). Since $\bar{E}_2$ is the voltage across R_2, then $\bar{E}_1$ is the average of the unshaded portion of Fig. 8(a), or

$$\bar{E}_1 = \frac{1}{T} \int_{t_1}^{t_2} [| f(t) | - \bar{E}_2] \, dt, \tag{1}$$

where $f(t_1) = f(t_2) = \bar{E}_2$. Assume, for simplicity,

$$f(t) \geqq \bar{E}_2 \qquad \text{when } t_1 \leqq t \leqq t_2,$$

$$f(t) < E_2 \qquad \text{otherwise.}$$

This is not a necessary assumption, since the integral in (1) could be expressed so that the assumption is true.

Let $\bar{E}$ be the detected voltage; i.e,

$$\bar{E} = \bar{E}_1 + \bar{E}_2 .$$

Then, from (1),

$$\bar{E} = \bar{E}_2 + \frac{1}{T} \int_{t_1}^{t_2} [| f(t) | - \bar{E}_2] \, dt. \tag{2}$$

Now, let E_0 be the true rms value of $f(t)$ and let $KE_0 = \bar{E}$. Then K is the factor by which $\bar{E}$, the detected voltage, differs from E_0, the true rms voltage. From (2):

$$KE_0 = \bar{E}_2 + \frac{1}{T} \int_{t_1}^{t_2} [| f(t) | - \bar{E}_2] \, dt. \tag{3}$$

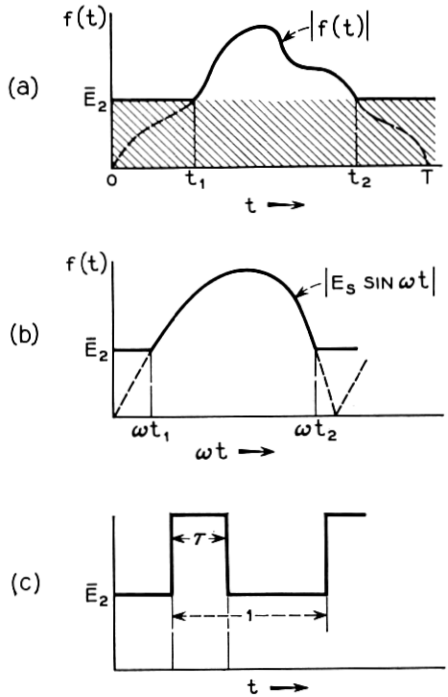

Fig. 8 — (a) Voltage across R_1 and R_2 with $f(t)$ into the detector; (b) sine wave into the detector; (c) rectangular wave into the detector.

If we let $E_0 = 1$ for all $f(t)$'s, then for a given $f(t)$ we have K in terms of $\bar{E}_2$. There is another relation between K and $\bar{E}_2$, which may be found by observing from Fig. 7(c) (and $E_0 = 1$) that

$$\frac{\bar{E}_2}{\bar{E}_1 + \bar{E}_2} = \frac{\bar{E}_2}{K} = \frac{R_2}{R_2 + R_1}. \tag{4}$$

The problem is now to find $R_2/(R_1 + R_2)$ such that $K = 1$ for a given $f(t)$. Possibly the simplest procedure is to proceed graphically by first finding K in terms of $\bar{E}_2$ from (3) and assigning values to $\bar{E}_2$ to determine K. Then, from (4), $R_2/(R_1 + R_2)$ may be found. Finally, plot K versus $R_2/(R_1 + R_2)$. Several examples follow:

Sine Wave

In Fig. 8(b), the amplitude of the sine wave, E_s, is given as $\sqrt{2}$ since we want $E_0 = 1$. From the figure, it is clear that

$$\omega t_1 = \arcsin \frac{\bar{E}_2}{\sqrt{2}}$$

and

$$\omega t_2 = \pi - \arcsin \frac{\bar{E}_2}{\sqrt{2}}.$$

Then, from (3), using the symmetry of the function about $\pi/2$, we have

$$K = \bar{E}_2 + \frac{2}{\pi} \int_{\arcsin(\bar{E}_2/\sqrt{2})}^{\pi/2} [\sin \omega t - \bar{E}_2] \, d(\omega t). \tag{5}$$

The evaluation of (5) gives:

$$K = \frac{2}{\pi} \left[\sqrt{2 - \bar{E}_2^2} + \bar{E}_2 \arcsin \frac{\bar{E}_2}{\sqrt{2}} \right]. \tag{6}$$

By simply assigning values to $\bar{E}_2$ and using (4), the curve labeled "sine wave" on Fig. 9 is obtained. Note that $R_2/(R_1 + R_2) = 0$ gives $R_2 = 0$. This reduces Fig. 7(c) to Fig. 7(b); then the detection is "average." For $R_2(R_1 + R_2) = 1$, $R_1 = 0$ giving "peak" detection.

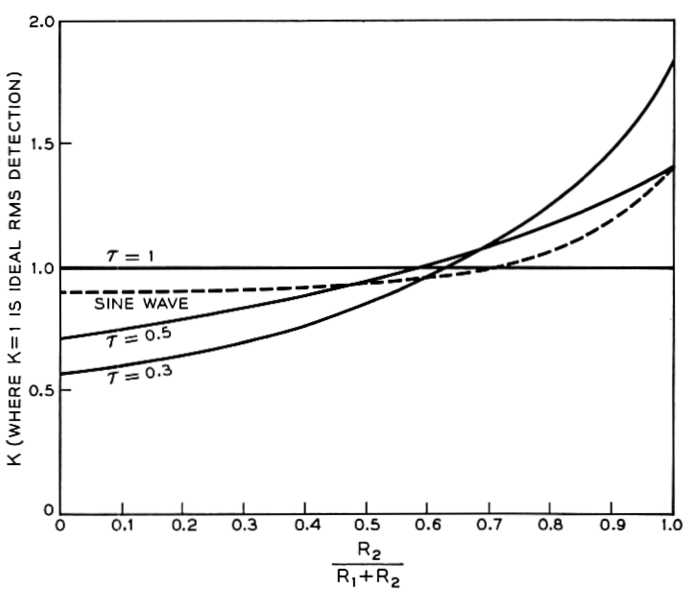

Fig. 9 — Response of the detector to a sine wave and rectangular waves of duty cycle τ.

Rectangular Wave

The rms value of the waveform given in Fig. 8(c) is given by $E_{max}\sqrt{\tau}$. Since we want $E_0 = 1$, then we choose $E_{max} = 1/\sqrt{\tau}$. The expression for K easily follows from (3):

$$K = \sqrt{\tau} + E_2(1 - \tau)$$

Using the procedure described for the sine wave, the curves labeled "τ" are given on Fig. 9 for several values of τ.

Gaussian Noise

For gaussian noise, $f(t)$ in (3) holds no meaning. Remember, however, that the integral term of (3) represents the average value, or dc component, of the voltage across R_1 in Fig. 7(c). The average value of a doubly rectified long-term sample of gaussian noise is:

$$E_{ave} = \int_0^\infty \frac{2e}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{e^2}{2\sigma^2}\right) de, \tag{7}$$

where

$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{e^2}{2\sigma^2}\right)$$

is the probability density function for the voltage e. The factor of 2 is present, since double rectification simply "folds" the negative half of the density function across the mean value of e (which is zero). The rms value of such a signal is σ. Since we want E_0 (the rms value) $= 1$, we set $\sigma = 1$. The average value of the voltage across R_1 in Fig. 7(c) can be expressed in a fashion similar to (7). In this case, however, $\bar{E}_2$ is subtracted from the value of the instantaneous noise voltage, e. Then (7) becomes

$$\bar{E}_1 = \int_{\bar{E}_2}^\infty (e - \bar{E}_2) \frac{2}{\sqrt{2\pi}} \exp\left(-\frac{e^2}{2}\right) de, \tag{8}$$

where the lower limit is $\bar{E}_2$, since the voltage across R_1 cannot be negative.

From (3) we have:

$$K = \bar{E}_2 + \int_{\bar{E}_2}^\infty (e - \bar{E}_2) \sqrt{\frac{2}{\pi}} \exp\left(-\frac{e^2}{2}\right) de. \tag{9}$$

Evaluation of (9) gives:

$$K = \sqrt{\frac{2}{\pi}}\left[\exp\left(\frac{\bar{E}_2^{\,2}}{2}\right) + \bar{E}_2 \int_0^{\bar{E}_2} \exp\left(-\frac{e^2}{2}\right) de\right].$$

As in the two previous examples, values were assumed for $\bar{E}_2$, and K was determined. Then, from (4), $R_2/(R_1 + R_2)$ was found using $\bar{E}_2$ and K. The results for gaussian noise are given in Fig. 10.

Conclusion

Similar curves may be plotted for a wide variety of functions using the techniques given in the examples. Fig. 11 is an illustration of the curves given for many examples including those of this Appendix. However, since the detector will be calibrated with a sine wave, the curves of Fig. 9 are referred to a sine wave. The figure represents, then, the error in the detection of the rms value of a set of functions when the detector is calibrated to read correctly the rms value of a sine wave. The error is given in decibels.

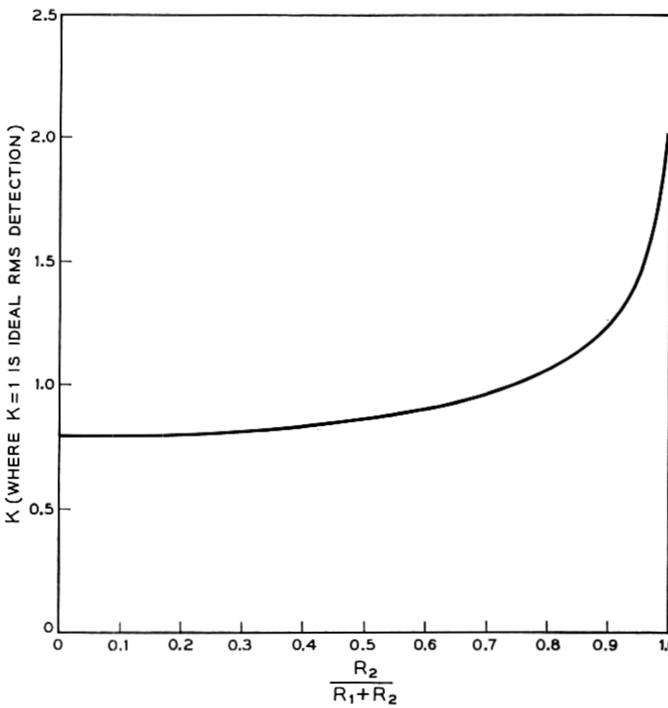

Fig. 10 — Response of the detector to gaussian-distributed noise.

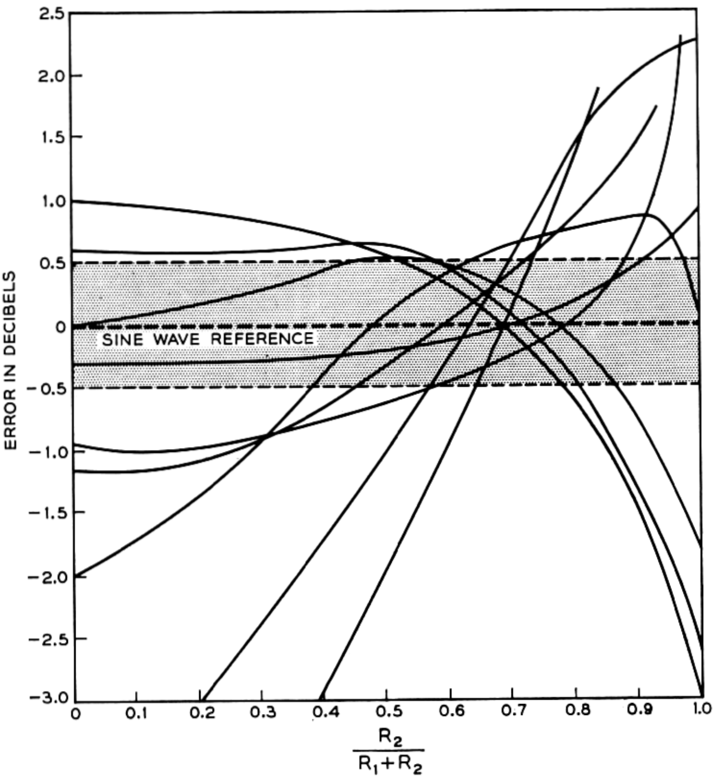

Fig. 11 — "Quasi-rms" detector error for several functions.

With only a few exceptions, the curves of Fig. 11 have a striking similarity in that they all fall within 0.5 db of "true rms" for $R_2/(R_1 + R_2)$ equal to about 0.6. Thus, if the resistors of Fig. 7(c) are properly chosen it is theoretically possible to detect the rms value of many different waveforms with an error as small as 0.5 db.

Although the assumptions of a perfect transformer, perfect diodes and $\bar{E}_2$ to be a constant may seem to be restrictive in the analysis, experimental results verified the calculations.

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