Synthesis of Driving-Point Impedances with Active *RC* Networks

By I. W. SANDBERG

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A general method is presented for synthesizing driving-point impedances using RC networks and active elements. The procedure realizes any real rational driving-point function and leads to rather simple structures. Only one active device, a negative-impedance converter, is required. The synthesis of biquadratic impedance functions is considered in detail.

I. INTRODUCTION

It is often desirable to avoid the use of magnetic elements in synthesis procedures, since resistors and capacitors are more nearly ideal elements and are usually cheaper, lighter and smaller. This is especially true in control systems in which, typically, exacting performance is required at very low frequencies. The rapid development of the transistor has provided the network synthesist with an efficient low-cost active element and has stimulated considerable interest in active *RC* network theory during the past decade.[1,2,3,4,5]

The present paper considers the active *RC* synthesis of driving-point impedances. Transfer functions are not treated directly, but are covered at least in principle, since it is always possible, and indeed sometimes convenient, to reduce the synthesis of transfer functions to the synthesis of two-terminal impedances.

It is now well known that any driving-point impedance function expressible as a real rational fraction in the complex frequency variable can by synthesized as an active *RC* network requiring only one ideal active element. Two proofs of this result are already in the literature.[6,7]* The present paper provides a third proof, although its main objective is to present a new and more practical realization network. The synthesis

* Another proposed proof[8] is in fact concerned only with those impedance functions which are positive on some section of the negative-real axis of the complex frequency plane.

technique is similar to that used in the author's first proof,[6] and considerably simpler than those used in the later paper by Kinariwala.[7]

Like the technique previously presented by the author,[6]* the present technique yields the values of the required network elements explicitly and directly, without the solution of simultaneous equations. The general procedure yields networks that contain more capacitors than the absolute minimum required; in return for this, however, it yields structures which, unlike those of Kinariwala, do not need balanced amplifiers or complex resistance networks.†

An alternative procedure applicable to a wide class of biquadratic impedance functions is also described. This procedure leads to structures requiring only two capacitors.

The synthesis techniques presented in this paper are based on networks employing a type of impedance converter that is a generalization of the negative-impedance converter. The converter concept is introduced in a general way in order to properly orient the reader.

II. THE IDEAL IMPEDANCE CONVERTER

Consider a two-port network terminated by an impedance $Z_T(s)$ as shown in Fig. 1. The input impedance at port 1 is

$$Z_1(s) = h_{11} - \frac{h_{12}h_{21}Z_T(s)}{1 + h_{22}Z_T(s)}, \tag{1}$$

where the hybrid parameters are functions of the complex frequency variable defined by

$$\begin{aligned} E_1 &= h_{11}I_1 + h_{12}E_2 , \\ I_2 &= h_{21}I_1 + h_{22}E_2 . \end{aligned} \tag{2}$$

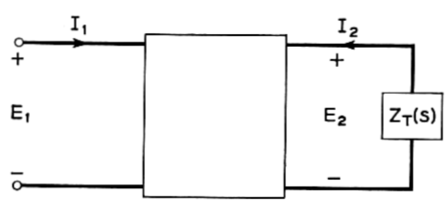

Fig. 1 — Two-port network.

* The particular configuration involved is essentially the one mentioned in Section 3.2 of this paper.

† A more practical procedure in Ref. 7 treats a restricted class of driving-point functions and employs a passive RC two-port network terminated by a negative RC impedance. Other restricted realization techniques are presented in Ref. 8.

For the ideal impedance converter we require that for every $Z_T(s)$

$$Z_1(s) = K(s)Z_T(s), \tag{3}$$

where $K(s)$ is a predetermined fixed function of the complex frequency variable. In order that (1) and (3) be compatible,

$$\begin{aligned} h_{11} &= h_{22} = 0, \\ K(s) &= -h_{12}h_{21} . \end{aligned} \tag{4}$$

An ideal impedance converter, therefore, is a two-port network with a hybrid parameter matrix of the form

$$\begin{bmatrix} 0 & h_{12} \\ h_{21} & 0 \end{bmatrix}. \tag{5}$$

It follows that the impedance at port 2 with the termination $Z_T(s)$ connected to port 1 is

$$Z_2(s) = \frac{Z_T(s)}{K(s)}. \tag{6}$$

A "controlled-source" representation of the unbalanced ideal impedance converter is given in Fig. 2.

The negative-impedance conveter ($h_{12}h_{21} = 1$) has been heavily exploited as a synthesis tool in active RC network theory. The synthesis techniques presented in this paper are based on networks employing a more general type of converter characterized by

$$\begin{bmatrix} 0 & 1 \\ \dfrac{Z_3}{Z_4} & 0 \end{bmatrix}, \tag{7}$$

where Z_3/Z_4 is the ratio of two RC driving-point impedances. This hybrid parameter matrix can be realized with either of the idealized cir-

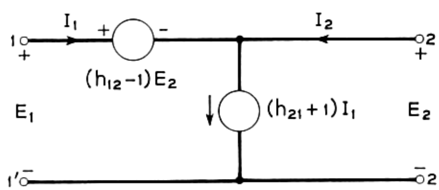

Fig. 2—Representation of the ideal impedance converter.

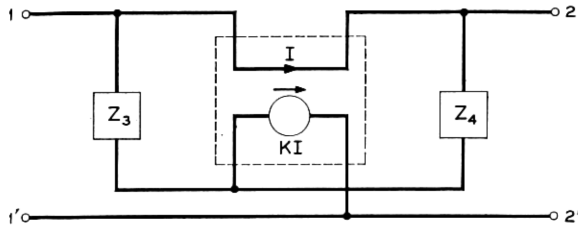

Fig. 3 — Idealized realization of the required converter.

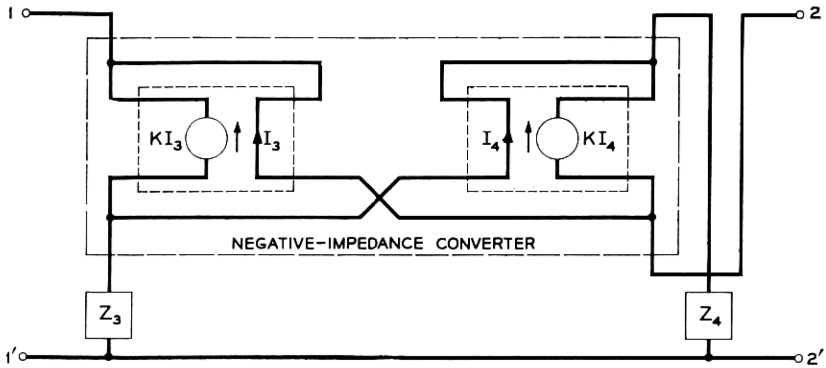

Fig. 4 — Alternate idealized realization of the converter.

cuits shown in Figs. 3 and 4.* The smaller rectangles enclose "infinite gain" current amplifiers. The realization of Fig. 3 is a modification of Larky's idealized current-inversion negative-impedance converter.[9] The larger rectangle in Fig. 4 encloses Linvill's well-known idealized voltage-inversion negative-impedance converter.[10] Hence, (7) can be realized with Linvill's negative-impedance converter and two *RC* driving-point impedances.

A controlled-source representation of (7) is given in Fig. 5. Transistor realizations are discussed in the Appendix.

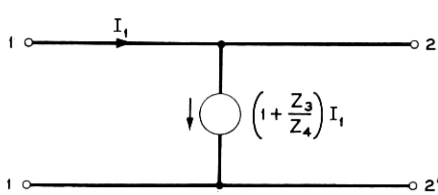

Fig. 5 — Representation of (7).

* Many other realizations are possible.

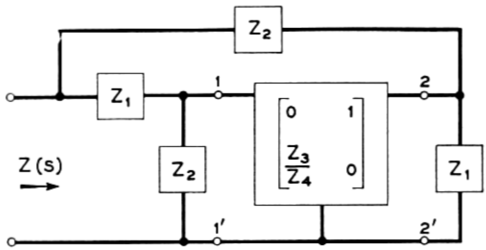

Fig. 6 — Realization one-port.

III. SYNTHESIS OF DRIVING-POINT IMPEDANCES

The driving-point impedance of the one-port network shown in Fig. 6 is*

$$Z(s) = \frac{Z_4 - Z_3}{\dfrac{Z_4}{Z_1} - \dfrac{Z_3}{Z_2}}. \tag{8}$$

The impedance function that is to be synthesized is a real rational fraction in the complex frequency variable

$$Z(s) = \frac{P(s)}{Q(s)}. \tag{9}$$

The synthesis consists of identifying each of the four parameters Z_1, Z_2, Z_3 and Z_4 with a two-terminal RC impedance function. The presence of negative signs in (8) suggests an approach similar to techniques previously proposed for the synthesis of transfer functions.[3,5]

Assume that the prescribed function $Z(s)$ is positive on at least one section of the negative-real axis of the complex frequency plane. Suppose we write

$$Z(s) = \frac{\dfrac{P(s)}{\prod_{i=1}^{N} (s + \sigma_i)}}{\dfrac{Q(s)}{\prod_{i=1}^{N} (s + \sigma_i)}}, \tag{10}$$

where

* This expression can be readily obtained by applying Blackman's equation[11] to the network that results when the converter is replaced by its controlled source representation. The required return differences are computed with respect to the quantity $1 + (Z_3/Z_4)$. A detailed study of Blackman's equation led to the discovery of the realization networks presented in this paper.

$$0 \leqq \sigma_1 < \sigma_2 < \sigma_3 \cdots < \sigma_N ,$$

and the number N is equal to the degree of the polynomial $P(s)$ or of the polynomial $Q(s)$, whichever is greater. The points $-\sigma_i$ are chosen to lie anywhere on the section or sections of the negative-real axis where the function $Z(s)$ is finite and positive.*

We replace both the numerator and denominator of the right-hand side of (10) by their partial fraction expansions and group the resulting terms to obtain

$$Z(s) = \frac{\dfrac{P_1}{Q_1} - \dfrac{P_2}{Q_2}}{\dfrac{P_3}{Q_1} - \dfrac{P_4}{Q_2}} \tag{11}$$

where

$$Q_1 Q_2 = \prod_{i=1}^{N} (s + \sigma_i)$$

and P_1/Q_1 , P_2/Q_2 , P_3/Q_1 , and P_4/Q_2 are each RC driving-point impedances. The function $Z(s)$ is expressible in this form since:

(a) Each of the two partial fraction expansions possesses real residues of like sign at any particular pole.

(b) All RC driving-point impedances are expressible in the form

$$R_\infty + \sum_{k=1}^{M} \frac{a_k}{s + b_k},$$

where R_∞ , a_k and b_k are nonnegative.

Equation (11) is equivalent to

$$Z(s) = \frac{\dfrac{P_1}{Q_1} - \dfrac{P_2}{Q_2}}{\dfrac{P_3 + RQ_1}{Q_1} - \dfrac{P_4 + RQ_2}{Q_2}}. \tag{12}$$

We identify the impedance parameters of (8) as follows:

$$Z_4 = \frac{P_1}{Q_1}, \qquad Z_3 = \frac{P_2}{Q_2},$$
$$Z_1 = \frac{P_1}{P_3 + RQ_1}, \qquad Z_2 = \frac{P_2}{P_4 + RQ_2}. \tag{13}$$

* The choice of the σ_i's influences the spread of element values and the sensitivity of the driving-point impedance to variations in the active and passive elements.

According to the qualifications appended to (11), the functions Z_3 and Z_4 are RC driving-point impedances.

Consider the function Z_1. The degree of the polynomial Q_1 is at least as great as the degree of P_3. Note that, as the parameter R assumes nonnegative real values that increase from zero to infinity, the poles of Z_1 move from the zeros of P_3 to the zeros of Q_1. Recall that the ratio of polynomials P_1/Q_1 is an RC driving-point impedance. Consequently, it is always possible to find a finite value of the parameter R for which the poles and zeros of Z_1 interlace properly.

A similar argument shows that Z_2 also can always be made an RC driving-point impedance. Note that $Z(s)$ need not be a positive-real function.

3.1 *An Example*

Let

$$Z(s) = \frac{s^2 + s + 1}{s^2 + s + 2}. \tag{14}$$

This function is positive on the entire negative-real axis. We choose $\sigma_1 = 1$, and $\sigma_2 = 2$. From (12),

$$Z(s) = \frac{\dfrac{s+2}{s+1} - \dfrac{3}{s+2}}{\dfrac{s(1+R)+3+R}{s+1} - \dfrac{Rs+4+2R}{s+2}}. \tag{15}$$

Employing (13),

$$\begin{aligned} Z_4 &= 1 + \frac{1}{s+1}, & Z_3 &= \frac{3}{s+2}, \\ Z_1 &= \frac{s+2}{s(1+R)+3+R}, & Z_2 &= \frac{3}{Rs+4+2R}. \end{aligned} \tag{16}$$

The set of impedances is realizable for $R \geqq 1$. Choose $R = 1$ so that

$$Z_1 = \frac{1}{2}, \qquad Z_2 = \frac{3}{s+6}. \tag{17}$$

The corresponding network is shown in Fig. 7.

Note that Z_3 and Z_4 can be scaled by the same constant without affecting the impedance $Z(s)$. In a practical design, this degree of freedom would be utilized in order to optimize some figure of merit such as the sensitivity function. The choice of the σ_i's would be similarly influenced.

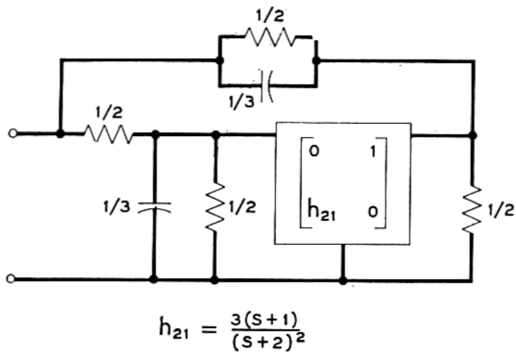

Fig. 7 — Realization of (14).

3.2 *An Alternate Realization*

The class of functions treated in this section can also be synthesized with the network given in Fig. 8. The input impedance of this structure is

$$Z(s) = \frac{Z_8 Z_6 - Z_7 Z_5}{Z_8 - Z_7}. \tag{18}$$

By employing arguments similar to those already discussed, it can be shown that the value of a nonnegative real parameter R can be chosen to ensure that the following set of impedances is realizable:*

$$\begin{aligned} Z_8 &= \frac{P_3 + RQ_1}{Q_1}, \qquad & Z_7 &= \frac{P_4 + RQ_2}{Q_2}, \\ Z_6 &= \frac{P_1}{P_3 + RQ_1}, \qquad & Z_5 &= \frac{P_2}{P_4 + RQ_2}. \end{aligned} \tag{19}$$

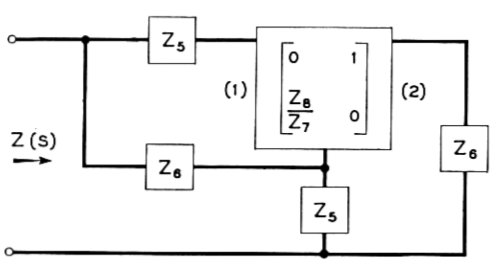

Fig. 8 — Alternate realization one-port.

* A realizable set of impedances can also be obtained in other ways.

This circuit has the disadvantage of requiring a "floating" two-port active network.

3.3 *Restrictions on $Z(s)$*

The methods of synthesis presented in the previous sections are not applicable to functions that are nonpositive on the entire negative-real axis. This is a significant theoretical restriction, since positive-real functions, for example, need not possess the required property. In particular, all reactance functions must be excluded.

The difficulty mentioned above can be circumvented in several ways by modifications of the synthesis technique. Suppose that the prescribed impedance $Z(s)$ is nonpositive on the entire negative-real axis. The function

$$Z'(s) = Z(s) - \frac{a_0}{s + b_0} \qquad a_0 > 0, \quad b_0 \geqq 0 \tag{20}$$

must, however, be positive on one section of the negative-real axis. It can therefore be synthesized by the previously discussed procedure. The impedance $Z(s)$ is obtained by connecting an RC impedance $a_0/(s + b_0)$ in series with the resulting network. An alternative procedure on an admittance basis also applies, the network being modified at the input terminals by the parallel connection of an RC impedance

$$c_0 + \frac{d_0}{s} \qquad d_0 > 0, \quad c_0 \geqq 0.$$

Both methods usually necessitate a larger number of passive components than would be required for the synthesis of $-Z(s)$. For this reason it may be more desirable to employ a negative-impedance converter terminated by $-Z(s)$.

3.4 *Sufficiency of One Active Element or One Negative-Impedance Converter*

Since the realization of the converter requires only one active element or only one Linvill-type negative-impedance converter, the preceding discussion constitutes a proof of

Theorem: Any driving-point impedance function, expressible as a real rational fraction in the complex frequency variable, can be synthesized with a network containing only resistors, capacitors and either a single ideal active element or a single ideal negative-impedance converter.

Note that it is theoretically possible to synthesize impedance functions which approach infinity as any integral power of the frequency variable. Obviously, these functions can be realized by actual networks only over the frequency band where the active two-port is essentially characterized by the controlled-source model of Fig. 5.*

IV. BIQUADRATIC SYNTHESIS

The synthesis of biquadratic impedance functions merits special attention. The network associated with a function of this type, previously considered as an example, requires a total of four capacitors (two in the converter). Two of the capacitors can be eliminated by employing an alternative technique based on the network of Fig. 6 with Z_1 and Z_2 replaced by resistors R_1 and R_2. The structure becomes an impedance converter imbedded in a simple resistance network.

From (8),

$$\frac{Z_3}{Z_4} = \frac{R_2(Z - R_1)}{R_1(Z - R_2)}. \tag{21}$$

The biquadratic function $Z(s)$ is given by

$$Z(s) = \frac{(s - z_1)(s - z_2)}{(s - p_1)(s - p_2)}. \tag{22}$$

It is required that R_1 and R_2 be chosen so that Z_3 and Z_4 are RC impedances. Assume that each of the impedances Z_3 and Z_4 is to be realizable as a resistor in series with a parallel combination of a resistor and capacitor as shown in Fig. 9. This structure is sufficient to realize the most general first-degree RC impedance function. The ratio of these two functions is given by

$$\frac{Z_3}{Z_4} = K\frac{(s + \alpha_1)(s + \alpha_2)}{(s + \alpha_3)(s + \alpha_4)}, \tag{23}$$

where, from (21) and (22),

$$K = \frac{R_2(1 - R_1)}{R_1(1 - R_2)}. \tag{24}$$

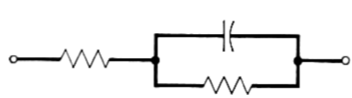

Fig. 9 — Structural form of Z_3 or Z_4.

* It must be remembered that this model will ordinarily be inadequate for stability analyses. For this purpose it must be modified to be valid in the frequency range where the significant active and passive parasitic parameters are influential.

Four possibilities exist for the pole-zero pattern of Z_3/Z_4 as shown in Fig. 10. The zeros in Fig. 10(a) and the poles in Fig. 10(b) may occur with multiplicity two.

Assume tentatively that $Z(s)$ has complex conjugate poles and zeros. The function $Z(\sigma)[-\infty \leqq \sigma \leqq \infty]$, where $s = \sigma + j\omega$, is nonnegative and approaches unity at both extremes of the argument. Since the function is the ratio of two second-degree polynomials in σ, only two points

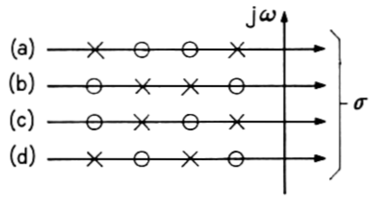

Fig. 10 — Permissible pole-zero patterns for Z_3/Z_4 .

of intersection with a horizontal line are possible. When such an intersection occurs, the intersecting points will be separated by an extremum of the function. Hence, if $Z(s)$ exhibits at least one extremum on the negative-real axis, the parameters R_1 and R_2 can be chosen to provide a pole-zero pattern for Z_3/Z_4 of the type shown in Figs. 10(a) or (b). Since, for both type (a) and type (b), $(1 - R_1)$ and $(1 - R_2)$ have the same sign, the impedances Z_3 and Z_4 would be realizable.

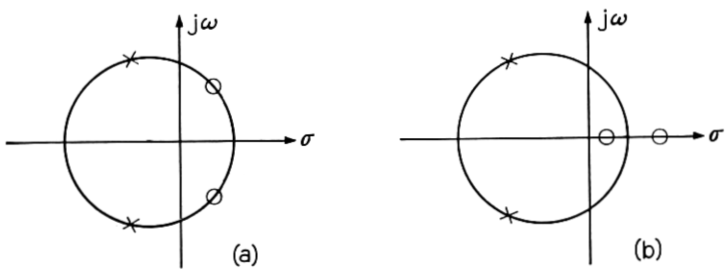

Fig. 11 — Construction of the extremum points of $Z(\sigma)$.

Consider the pole-zero diagram for $Z(s)$. It can easily be shown that the circle passing through the poles and zeros [Fig. 11(a)] will intersect the real axis at the two points where $Z(\sigma)$ has an extremum. Consequently, any open-circuit or short-circuit stable biquadratic impedance function with complex conjugate poles and zeros can be synthesized. Two permissible limiting cases exist when the zeros or poles or both the zeros and poles occur with multiplicity two on the real axis. Another permissible case occurs when the zeros are at infinity and the complex conjugate poles are in the left-half plane.

Suppose that $Z(s)$ has distinct real zeros and complex conjugate poles. It can easily be shown that the circle centered on the real axis at

$$\sigma_0 = \frac{\sigma_p^2 + \omega_p^2 - z_1 z_2}{2\sigma_p - (z_1 + z_2)} \qquad p_{1,2} = \sigma_p \pm j\omega_p \tag{25}$$

and passing through the poles [Fig. 11(b)] will intersect the real axis at the two points where $Z(\sigma)$ has an extremum. The circle must pass between the zeros. It follows from (25) that the center of the circle will lie to the left of the zeros if the poles lie to the left of the point midway between the zeros. This will result in an extremum of $Z(\sigma)$ located to the left of the poles in a region where the function is positive. It follows directly that the synthesis can be accomplished for any biquadratic impedance function with left-half plane complex conjugate poles and real zeros, where the poles are located to the left of the point midway between the zeros.*

The steps in the biquadratic synthesis procedure are: (a) choose R_1 and R_2 so that Z_3 and Z_4 are realizable, and (b) from (21), identify Z_3 and Z_4. The permissible values of R_1 and R_2 can be determined by inspection of $Z(\sigma)$.

It should be noted that this procedure is not limited to the two broad classes of functions considered above. The synthesis can obviously be accomplished if step (a) can be carried out. Hence the applicability of the procedure can be determined by inspection of $Z(\sigma)$.

4.1 *An Example*

Let

$$Z(s) = \frac{s^2 - 3s + 2}{s^2 + s + 1}. \tag{26}$$

It is evident from the graph of $Z(\sigma)$ for this case (Fig. 12) that the choice $R_1 = 4$, $R_2 = 6$ is acceptable. From (21) and (26),

$$\begin{aligned} \frac{Z_3}{Z_4} &= \frac{6}{4}\frac{Z - 4}{Z - 6} = \frac{9}{10}\frac{s^2 + \frac{7}{3}s + \frac{2}{3}}{s^2 + \frac{9}{5}s + \frac{4}{5}} \\ &= \frac{9}{10}\frac{(s + \frac{1}{3})(s + 2)}{(s + \frac{4}{5})(s + 1)}. \end{aligned} \tag{27}$$

Two possibilities exist for the pair of impedances Z_3 and Z_4:

* Biquadratic admittance functions with distinct negative-real poles can be realized with a negative-impedance converter as the difference of two *RC* admittances. For this reason, such functions are not considered in detail here.

$$Z_3 = \frac{9}{10}\frac{(s+2)}{(s+1)}, \qquad Z_4 = \frac{(s+\frac{4}{5})}{(s+\frac{1}{3})}, \tag{28}$$

or

$$Z_3 = \frac{9}{10}\frac{(s+2)}{(s+\frac{4}{5})}, \qquad Z_4 = \frac{(s+1)}{(s+\frac{1}{3})}. \tag{29}$$

4.2 *Synthesis on the Basis of the Network Shown in Fig. 8*

The synthesis technique presented in this section can be extended to apply to the network of Fig. 8 by comparing (8) and (18) and identifying

$$\begin{aligned} Z_7 &= \frac{R_1}{R_2} Z_3, \qquad & Z_8 &= Z_4, \\ Z_6 &= R_1, \qquad & Z_5 &= R_2. \end{aligned} \tag{30}$$

V. CONCLUSION

A general method of synthesizing driving-point impedances has been presented. An impedance converter is required that can be realized by modifying Larky's current-inversion negative-impedance converter. An alternate realization employs Linvill's voltage-inversion negative-impedance converter and two RC impedances. This realization leads to the result that any driving-point impedance function, expressible as a real rational fraction in the complex frequency variable, can be synthesized as a network containing resistors, capacitors and a single nega-

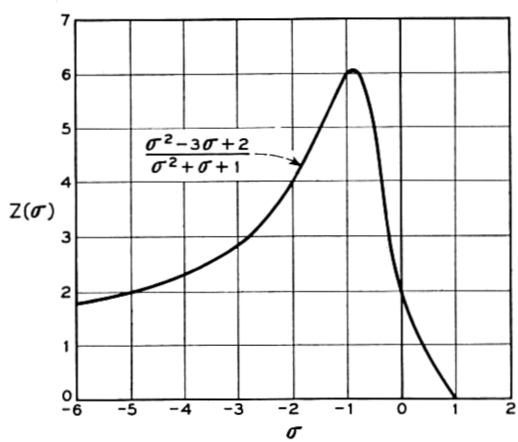

Fig. 12 — Graph of $Z(\sigma)$ for a biquadratic function.

tive-impedance converter. The technique does not require the synthesis of two-port RC or n-port balanced resistor networks, and leads to the direct determination of the required two-terminal RC elements.

The synthesis of biquadratic impedance functions has been given special attention, resulting in structures employing the minimum number of capacitors and a moderate number of resistors. The procedure is applicable to a wide class of functions including, in particular, all open-circuit or short-circuit stable impedances with complex conjugate poles and zeros.

VI. ACKNOWLEDGMENT

The author is grateful to B. McMillan and to P. M. Dollard for their constructive criticism and advice.

APPENDIX

Transistor Circuit Analysis

An approximate model for the transistor in the range where its parameters are essentially independent of frequency is shown in Fig. 13

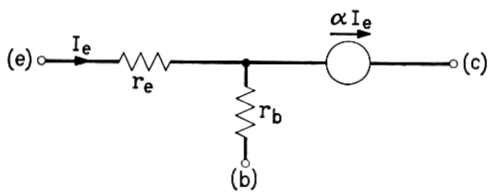

Fig. 13 — Approximate low-frequency transistor model.

The collector resistance is assumed to be infinite, an approximation which is often reasonable—especially for drift transistors. In terms of this model, the hybrid parameters of the impedance converter circuits of Fig. 14 are

Fig. 14(a):

$$
\begin{aligned}
h_{11} &= \frac{r_1(1-\alpha_1)(Z_3+Z_4)}{Z_4(\beta_2\alpha_1+1)+Z_3+(1-\alpha_1)r_1} \simeq 0, \\
h_{12} &= 1, \qquad h_{22} = 0, \\
h_{21} &= \frac{\dfrac{Z_3}{Z_4} - \dfrac{(1-\alpha_1)}{\alpha_1\beta_2}\left(1+\dfrac{r_1+Z_3}{Z_4}\right)}{1+\dfrac{1}{\alpha_1\beta_2}\left[1+\dfrac{Z_3+r_1(1-\alpha_1)}{Z_4}\right]} \simeq \frac{Z_3}{Z_4},
\end{aligned}
\tag{31}
$$

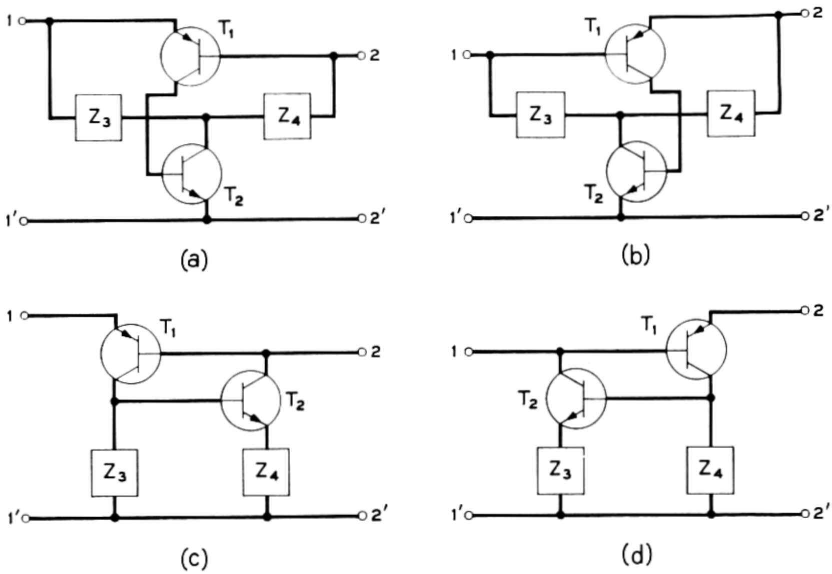

Fig. 14 — Transistor realizations of the converter.

where

$$r = r_b + \frac{r_e}{1 - \alpha} \qquad \beta = \frac{\alpha}{1 - \alpha}.$$

Fig. 14(b):

$$\begin{aligned} h_{11} &= \frac{-r_1(1 - \alpha_1)(Z_3 + Z_4)}{\alpha_1\beta_2 Z_4 - (1 - \alpha_1)(r_1 + Z_3 + Z_4)} \simeq 0, \\ h_{12} &= 1, \qquad h_{22} = 0, \\ h_{21} &= \frac{\dfrac{Z_3}{Z_4} + \dfrac{1}{\alpha_1\beta_2}\left[1 + \dfrac{Z_3 + r_1(1 - \alpha_1)}{Z_4}\right]}{1 - \dfrac{(1 - \alpha_1)}{\alpha_1\beta_2}\left[1 + \dfrac{r_1 + Z_3}{Z_4}\right]} \simeq \frac{Z_3}{Z_4}. \end{aligned} \tag{32}$$

Fig. 14(c):

$$h_{11} = (1 - \alpha_1)r_1 \simeq 0,$$

$$h_{12} = 1, \qquad h_{22} = 0,$$

$$h_{21} = \frac{\alpha_1\alpha_2 \dfrac{Z_3}{Z_4} - (1 - \alpha_1)\left[1 + (1 - \alpha_2)\dfrac{Z_3 + r_2}{Z_4}\right]}{1 + (1 - \alpha_2)\dfrac{Z_3 + r_2}{Z_4}} \simeq \frac{Z_3}{Z_4}. \qquad (33)$$

Fig. 14(d):

$$h_{11} = -(1 - \alpha_1)r_1 \frac{Z_3 + (1 - \alpha_2)(Z_4 + r_2)}{\alpha_1\alpha_2 Z_4 - (1 - \alpha_1)[Z_3 + (1 - \alpha_2)(Z_4 + r_2)]} \simeq 0,$$

$$h_{12} = 1, \qquad h_{22} = 0,$$

$$h_{21} = \frac{\dfrac{Z_3}{Z_4} + (1 - \alpha_2)\left(1 + \dfrac{r_2}{Z_4}\right)}{\alpha_1\alpha_2 - (1 - \alpha_1)\left[\dfrac{Z_3 + (1 - \alpha_2)(r_2 + Z_4)}{Z_4}\right]} \simeq \frac{Z_3}{Z_4}. \qquad (34)$$

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