Pressure at the Interface of Inner Conductor and Dielectric of Armorless Ocean Cable

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Pressure at the interface of the inner conductor and the dielectric of the armorless ocean cable is experimentally determined for plane stress boundary conditions. Polyethylene outer and steel inner concentric cylinders are considered as a mechanical model of the ocean cable. Equations are derived to give the pressure at the interface for plane stress and plane strain conditions. It is found that the theoretical values of the pressure calculated from equations of elasticity are in good agreement with experimental values, even when one of the materials is polyethylene, which has a nonlinear stress-strain relationship in simple tension or compression. From previously measured values of bulk and Young's moduli, it is shown that, for an elastic analysis, Poisson's ratio for polyethylene is very nearly 0.5. An experimental verification of the plane stress solution given in this paper confirms the foregoing value.

I. INTRODUCTION

In the process of designing new armorless coaxial ocean cables it is important to know the pressure at the interface of the inner conductor and the polyethylene dielectric. Determination of pressure at the interface would enable one to study the decay of tension, in a cable-laying machine, from the strength member to the holding surface, and to investigate the pressure effects on the resistance and capacitance of the cable.

A suitable mechanical model of the armorless ocean cable is a polyethylene outer cylinder concentric with a steel inner cylinder. The cable is actually subjected, in three dimensions, to a pressure loading that varies with the depth of the ocean bottom. However, for a section of the cable on a relatively flat ocean bottom one may assume that the cable is subjected to a uniform external pressure.

One may obtain an expression for the pressure at the interface from a two-dimensional model (i.e., for plane stress or plane strain) using the theory of elasticity. However, the result is questionable, since polyethylene has a nonlinear stress-strain relationship in simple tension or compression. Moreover the value of Poisson's ratio for polyethylene, μ_p, ranges from 0.3 to 0.5, and it is not possible to determine μ_p accurately by a simple experiment. The value of the pressure at the interface, as obtained from such an equation, depends considerably upon the particular value of μ_p that is used. Hence, an experimental determination of the pressure at the interface is necessary, to investigate whether an elastic analysis is valid when one of the materials is polyethylene and to establish the value of Poisson's ratio for polyethylene.

The pressure at the interface of the model is determined experimentally for plane stress boundary conditions. Equations are derived, based on the two-dimensional theory of elasticity, to give the pressure at the interface of the model for plane stress and plane strain conditions. It is found that the theoretical values of the pressure calculated from equations of elasticity are in good agreement with experimental values. From previously measured values of bulk and Young's moduli, it is shown that, for an elastic analysis, Poisson's ratio for polyethylene is very nearly 0.5. For this reason and since the area of polyethylene is much greater than that of steel, an extension of the two-dimensional plane strain solution to a three-dimensional case of uniform pressure by the method of superposition would give the pressure at the inner conductor and dielectric interface to be equal, or at most slightly greater, than the external pressure.

II. EXPERIMENTAL MODEL AND PROCEDURE

Initially, the experimental model consisted of a cold-drawn steel tubing 30 inches long, 1.254 inches in outer diameter and 0.083 inch in wall thickness. Two SR-4 strain gages of type A7 were cemented on a 0.010-inch-thick steel ring, which fitted snugly inside the steel tubing shown in Fig. 1. The steel ring containing the gages to which long leader wires were soldered was cemented in the tubing so that the it was at the same distance from either end. The gages were far enough from the ends to eliminate end effects.

The tubing was subjected to an external hydrostatic pressure in increments of 500 psi up to 4500 psi when a maximum strain of approximately 1000 microinches per inch, which is well within the elastic limit for the tubing, was recorded. The pressure was reduced to zero, and upon increasing the pressure again, the same calibration curve was obtained.

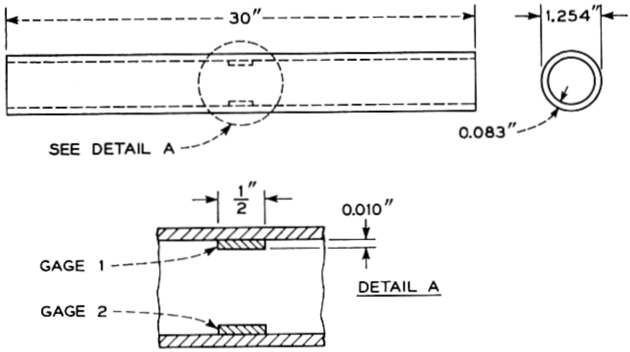

Fig. 1 — Steel tubing, showing location of strain gages.

The experimental arrangement for calibration of gages is shown in Fig. 2.

After calibration of gages, the polyethylene outer cylinder, whose inner diameter was 0.004 inch smaller than the outer diameter of the steel cylinder, was forced onto the steel cylinder in a universal testing machine. The assembled model appeared as in Fig. 3. Before the model was subjected to external hydrostatic pressure, the strain gages were balanced to indicate zero strain by adjusting resistances in a multichannel switching-and-balancing unit. The experimental arrangement for the model is given in Fig. 4. It is not the intent of the present paper to evaluate the pressure produced at the interface by the interference fit. External hydrostatic pressure was applied in increments of 500 psi up to 2500 psi and corresponding strains were observed. The pressure was reduced to zero and the reproducibility of the data was checked.

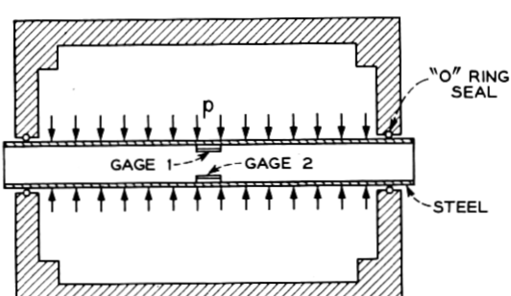

Fig. 2 — Experimental arrangement for calibration.

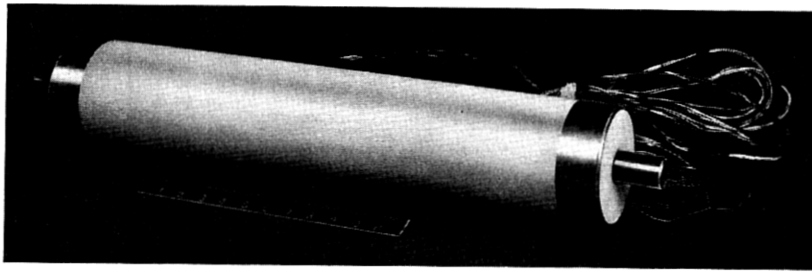

Fig. 3 — Experimental model.

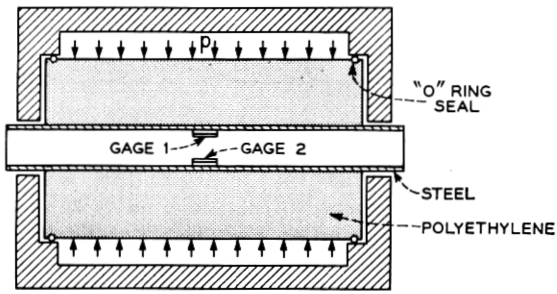

Fig. 4 — Experimental arrangement for the model.

III. EXPERIMENTAL RESULTS

The calibration curves for external hydrostatic pressure versus the strain observed from gages 1 and 2, when the external hydrostatic pressure was applied directly on the pipe, are given in Figs. 5 and 6. Corresponding curves when the external pressure was applied on the model are also plotted in these figures.

The lines pass through the origin because the bridge was initially balanced at zero pressure with the help of blancing resistances in the multichannel switching unit, and in each case the zero reading was checked after the experiment, when the pressure was brought back to zero. The deviation of the experimental points from the straight lines for low values of pressure is probably due to the nonlinearity in the pressure gages, which are designed to be used normally at much higher pressures.

It may be noted that, in the case of Lame's equations for thick-walled cylinders, the sum of the radial and circumferential stresses is a constant at every point in the cross section, so that the deformation of all elements in the direction of the axis of the cylinder is the same and cross sections of the cylinder remain plane after deformation. Hence, in spite of the considerable length of the cylinder, the experimental arrangement gives the verification for plane stress conditions.

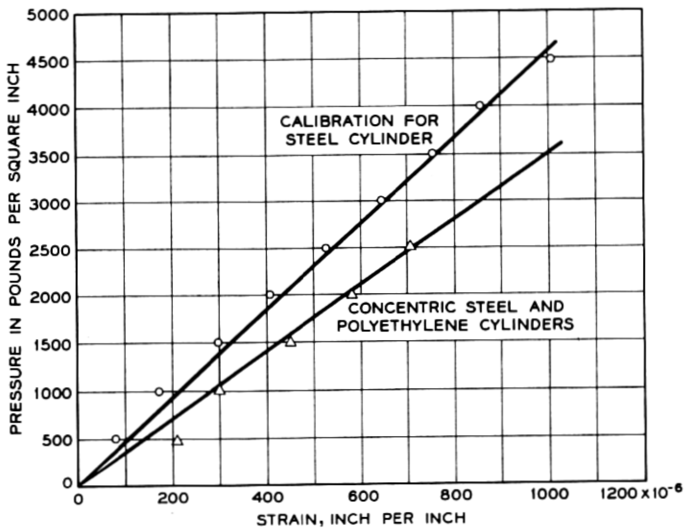

Fig. 5 — Pressure vs. strain from gage 1.

The pressure at the interface is obtained by drawing a vertical line from the observed value of strain to the calibration line for the steel cylinder alone and reading the corresponding pressure. Numerical calculations from Figs. 5 and 6 show that the pressure at the interface, in the case of plane stress is 1.3 times the external pressure.

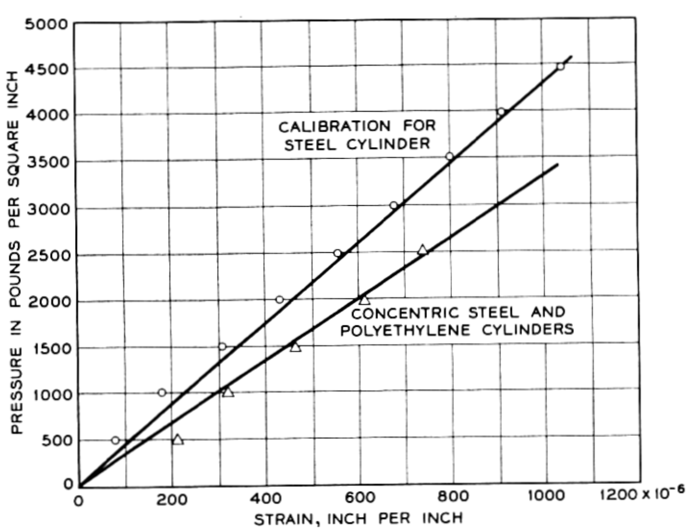

Fig. 6 — Pressure vs. strain from gage 2.

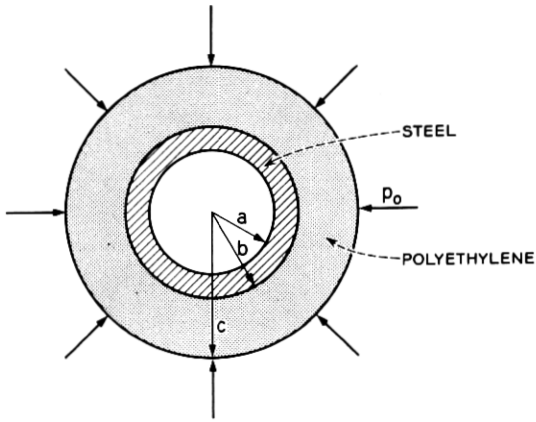

Fig.. 7 — Cross section of the model.

IV. THEORETICAL DERIVATION OF PRESSURE AT THE INTERFACE

The differential equation of equilibrium,[1] in terms of radial displacement, for a thick-walled cylinder subjected to external and intermal pressures is

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = 0. \tag{1}$$

The general solution of the above equation is

$$u = c_1 r + \frac{c_2}{r}. \tag{2}$$

Substituting (2) in the expressions for Hooke's law for plane stress, one obtains for radial stress, σ,

$$\sigma = \frac{E}{1 - \mu^2}\left[c_1(1 + \mu) - c_2\frac{(1 - \mu)}{r^2}\right], \tag{3}$$

where c_1 and c_2 are arbitrary constants and E and μ are Young's modulus and Poisson's ratio respectively. For the model shown in Fig. 7, the following boundary conditions must be satisfied:

$$\sigma_p \big|_{r=b} = \sigma_s \big|_{r=b}, \tag{4}$$

$$\sigma_p \big|_{r=c} = p_0, \tag{5}$$

$$\sigma_s \big|_{r=a} = 0, \tag{6}$$

$$u_p \big|_{r=b} = u_s \big|_{r=b} . \tag{7}$$

In the above equations subscripts p and s stand for polyethylene and steel. Substituting (2) and (3) in (4) through (7), one gets:

$$\frac{E_p}{1 - \mu_p^2}\left[c_1(1 + \mu_p) - c_2\left(\frac{1 - \mu_p}{b^2}\right)\right] = \frac{E_s}{1 - \mu_s^2}\left[c_3(1 + \mu_s) - c_4\left(\frac{1 - \mu_s}{b^2}\right)\right], \tag{8}$$

$$\frac{E_p}{1 - \mu_p^2}\left[c_1(1 + \mu_p) - c_2\,\frac{(1 - \mu_p)}{c_2}\right] = p_0, \tag{9}$$

$$\frac{E_s}{1 - \mu_s^2}\left[c_3(1 + \mu_s) - c_4\,\frac{(1 - \mu_s)}{a^2}\right] = 0, \tag{10}$$

$$c_1 b + \frac{c_2}{b} = c_3 b + \frac{c_4}{b}. \tag{11}$$

The pressure p at the interface is obtained by solving for the constants from the foregoing equations and evaluating

$$\sigma_p \big|_{r=b} = p.$$

Equation (12) gives the pressure at the interface for a plane stress solution.

$$p = p_0\left\{\frac{(b^2 - c^2)\left[\frac{E_s}{E_p}(1 - \mu_p)(b^2 - a^2) - (1 - \mu_s)b^2 - (1 + \mu_s)a^2\right]}{(b^2 - c^2)[(1 - \mu_s)b^2 + (1 + \mu_s)a^2] - \frac{E_s}{E_p}(b^2 - a^2)[(1 - \mu_p)b^2 + (1 + \mu_p)c^2]} + 1\right\}. \tag{12}$$

To obtain the plane strain solution, the constants E and μ in (12) are replaced by E^* and μ^*, with appropriate subscripts given by the following:[2]

$$E^* = \frac{E}{1 - \mu^2}, \tag{13}$$

$$\mu^* = \frac{\mu}{1 - \mu}. \tag{14}$$

V. NUMERICAL EXAMPLE FOR THE MODEL

The values for the various constants are:

$$a = 1.088 \text{ inches},$$

$$b = 1.254 \text{ inches},$$

$$c = 4.621 \text{ inches},$$

$$E_s = 29 \times 10^6 \text{ psi},$$

$$E_p = 19 \times 10^3 \text{ psi},$$

$$\mu_s = 0.3,$$

$$\mu_p = 0.5.$$

Substitution of above values in (12) gives for the plane stress solution

$$p = 1.3(p_0). \tag{15}$$

Replacing E and μ by (13) and (14) and substituting in (12), one obtains for the plane strain solution

$$p = 1.0(p_0). \tag{16}$$

It may be noticed from (12) that the pressure at the interface appears to depend on the values of E_p and μ_p. However, examination reveals that (12) is relatively insensitive to E_p, for the range of known values for low and intermediate density polyethylene. Since it is difficult to determine μ_p, one may use the previously known values of bulk modulus, K_p, for polyethylene and obtain μ_p from

$$K_p = \frac{E_p}{3(1 - 2\mu_p)}. \tag{17}$$

Refs. 3 and 4 give the value of $K_p \approx 0.5 \times 10^6$ psi. Ref. 5 gives for E_p, for low and intermediate density, the range of values from 19×10^3 to 55×10^3 psi. With $K_p = 0.5 \times 10^6$ psi and $E_p = 19 \times 10^3$ or 55×10^3 psi, (17) gives μ_p to be very nearly equal to 0.5. It may be noted that the analysis for concentric cylinders given in this paper agrees with experimental results when μ_p is very nearly equal to 0.5.

VI. CONCLUSION

Experiments indicate that an analysis based on equations of the two-dimensional theory of elasticity for the pressure at the interface of polyethylene outer and steel inner concentric cylinders, under plane stress conditions subjected to a uniform pressure on the curved surface, is valid even for polyethylene, which is a material with nonlinear stress-strain relationship. The agreement between the theoretically calculated and experimentally determined values is very good. It is also shown that the value of Poisson's ratio for low and intermediate density polyethylene, for an elastic analysis, should be very nearly 0.5. The experimental verification of the plane stress solution given in the present paper confirms this value.

An experimental verification of the plane strain solution involves a number of difficulties in satisfying the condition that the axial strain be zero; it has not been attempted in the present paper. However, one may assume that the plane strain solution is also valid, because the plane stress solution obtained by using the same two-dimensional theory of elasticity has been experimentally verified. The plane strain solution gives for the numerical example the result that the pressure at the interface is the same as external pressure.

The cable is actually under a uniform pressure in three dimensions. The two-dimensional plane strain solution may be extended to a three-dimensional case of uniform pressure by a superposition of an axial loading of intensity p_0 and satisfying the appropriate boundary and compatibility conditions. However, since the value of Poisson's ratio for polyethylene is 0.5 and the area of polyethylene is much greater in comparison to the area of the steel, the pressure at the interface of the inner conductor and the polyethylene dielectric, as given by the plane strain solution, would not be much altered. Hence, the pressure at the interface of the inner conductor and the dielectric of the armorless ocean cable is equal to or at most slightly greater than the external pressure.

It may be remarked that a new experimental method employing resistance wire strain gages has been developed to measure interface pressures accurately. The method devised in this paper could be used in other applications to measure interface pressures.

VII. LIST OF SYMBOLS

a = inner radius of steel cylinder (inches).
b = outer radius of steel cylinder (inches).
c = outer radius of polyethylene cylinder (inches).
K_p = bulk modulus for polyethylene (psi).
p = pressure at the interface (psi).
p_0 = external hydrostatic pressure (psi).
u = radial displacement (inches).
r = radial distance (inches).

E = Young's modulus (psi).
E_p = Young's modulus for polyethylene (psi).
E_s = Young's modulus for steel (psi).
μ = Poisson's ratio.
μ_p = Poisson's ratio for polyethylene.
μ_s = Poisson's ratio for steel.
σ = radial stress (psi).
σ_p = radial stress in polyethylene (psi).
σ_s = radial stress in steel (psi).

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