Noncylindrical Helix Waveguide

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Small uniform deformations of the cross section of helix waveguide perturb the circular electric waves slightly. From these perturbations the added circular electric wave loss is found in a uniformly deformed helix waveguide. For a nonuniformly deformed helix waveguide Maxwell's equations are converted into generalized telegraphist's equations. By an approximate solution for small deformations, mode conversion and circular electric wave loss are found.

Random imperfections with small correlation distance cause an average circular electric wave loss that is nearly independent of the wall impedance which the helix jacket presents to the waveguide interior. It is therefore nearly the same as in metallic waveguide. Near 50 kmc, the rms value of elliptical diameter differences should not be more than 0.0015 inch in order that on the average not more than 10 per cent of TE_{01} loss in a perfect 2-inch inside diameter copper pipe is added to the TE_{01} loss in a helix waveguide of the same inside diameter.

I. INTRODUCTION

Helix waveguide composed of closely wound insulated copper wire covered with a jacket of dielectric material and surrounded by a coaxial metallic shield is a good transmission medium for circular electric waves.[1] In long distance communication with these waves helix waveguide is useful as a mode filter, for negotiating bends and particularly as a transmission line proper. The different applications of helix waveguide require different properties of jacket and shield. Corresponding design rules have been worked out.[2]

The loss of circular electric waves in a metallic waveguide decreases steadily with increasing frequency only if the guide is perfectly round. The same is true for the helix waveguide. To maintain the low-loss properties of the circular electric wave, the helix waveguide must be manufactured to a high degree of roundness and uniformity.

As long as the guide is cylindrical, i.e., any deviation from roundness is independent of distance along the guide, increased circular electric wave loss is the only effect of such deviation from roundness. But if at the same time this deviation changes with length, the transmission characteristics of the guide will be further degraded by mode conversion-reconversion effects. At any change of cross-sectional shape of the guide, power of the circular electric wave will be scattered into unwanted modes, and vice versa. The amount of power scattered depends not only on the magnitude of change but also on the rate of change with length of these deviations from roundness.

Two cases, that of the uniform noncircular helix waveguide and that of the nonuniform helix waveguide, will be analyzed separately. In the first case, a perturbation of the normal modes of the round waveguide will give a simple answer. In the second case, however, Maxwell's equations will be converted into generalized telegraphist's equations,[3] and the results appear to be much more involved.

This paper partly represents an extension of an analysis of noncylindrical metallic waveguide[4] to helix waveguide, and partly uses the results of a mode-conversion analysis which was made more recently.[5,6]

II. THE UNIFORM NONCIRCULAR HELIX WAVEGUIDE

The mathematical model with which helix and surrounding jacket structure is represented in this analysis is an anisotropically conducting sheath. The sheath conducts perfectly in circumferential direction and has a surface impedance Z in longitudinal direction. A cylindrical coordinate system (r,φ,z) will be used, in which $r = 0$ coincides with the axis of the guide. At present the inner radius of the guide is a function of φ only:

$$a = a_0[1 + \delta(\varphi)]. \tag{1}$$

The anisotropic sheath imposes the following boundary conditions at $r = a$:

$$E_\varphi + E_r \frac{d\delta}{d\varphi} = 0, \tag{2}$$

$$E_z = \frac{-Z}{\sqrt{1 + \left(\frac{d\delta}{d\varphi}\right)^2}} \left(H_\varphi + H_r \frac{d\delta}{d\varphi}\right). \tag{3}$$

The deviation from the nominal radius a_0 is assumed to be small and smooth:

$$\delta \ll 1 \qquad \text{and} \qquad \frac{d\delta}{d\varphi} \ll 1. \tag{4}$$

Then the electromagnetic field can conveniently be represented as a perturbation of the field in the round guide of radius a_0:

$$\begin{aligned} E &= E_0 + e, \\ H &= H_0 + h. \end{aligned} \tag{5}$$

Furthermore the fields at $r = a$ can be written in terms of the fields at $r = a_0$:

$$E_0(a,\varphi) = E_0(a_0, \varphi) + a_0\delta(\varphi) \frac{\partial E_0(a_0, \varphi)}{\partial r}. \tag{6}$$

If the unperturbed field is of circular electric form with $E_{0z} = E_{0r} = H_{0\varphi} = 0$, then, upon substituting from (5) into the boundary condition (2), the Taylor series (6) can be used. The perturbation field can then be written in terms of the unperturbed field of the circular electric wave:

$$e_\varphi = -a_0\delta(\varphi) \frac{\partial E_{0\varphi}(a_0)}{\partial r}. \tag{7}$$

The boundary condition (3) imposes an additional requirement on the perturbation field

$$e_z = -Zh_\varphi(a_0). \tag{8}$$

Conditions (7) and (8) suffice to calculate the complete perturbation field.

A circular electric wave that carries unit power in positive z direction has an electric field:

$$E_{0\varphi} = -\sqrt{\frac{2\omega\mu}{\pi\beta_0}} \frac{J_1(\chi_0 r)}{aJ_0(k_0)} e^{-j\beta_0 z}, \tag{9}$$

where

$$k_0 = \chi_0 a_0, \qquad J_1(k_0) = 0$$

and

$$\beta_0^2 = \omega^2\mu\epsilon - \chi_0^2.$$

Here, μ and ϵ are permeability and permittivity of the waveguide interior; ω is the angular frequency.

The perturbation δ of the nominal radius is a periodic function of φ. A Fourier expansion is therefore in order:

$$\delta(\varphi) = \sum_p \delta_p \cos p\varphi. \tag{10}$$

Terms with sin $p\varphi$ have been omitted from (10). They would only add identical perturbations with different polarization. Substituting from (9) and (10) into (7):

$$e_\varphi(a_0) = \sqrt{\frac{2\omega\mu}{\pi\beta_0}}\, \chi_0 e^{-j\beta_0 z} \sum_p \delta_p \cos p\varphi. \tag{11}$$

The expression suggests an expansion of the perturbation fields into terms which individually satisfy Maxwell's equations and have the φ and z dependence of the terms in (11). Such a field is obtained from wave functions

$$\begin{aligned} T_{(p)} &= \sum_p a_{(p)} J_p(\chi_0 r) \sin p\varphi e^{-j\beta_0 z}, \\ T_{[p]} &= \sum_p a_{[p]} J_p(\chi_0 r) \cos p\varphi e^{-j\beta_0 z} \end{aligned} \tag{12}$$

and the following formulae:

$$\begin{aligned} e_r &= -\frac{\beta_0}{\omega\epsilon}\frac{\partial T_{(p)}}{\partial r} - \frac{\partial T_{[p]}}{r\partial\varphi}, \\ e_\varphi &= -\frac{\beta_0}{\omega\epsilon}\frac{\partial T_{(p)}}{r\partial\varphi} + \frac{\partial T_{[p]}}{\partial r}, \\ e_z &= \frac{\chi_0^2}{j\omega\epsilon} T_{(p)}, \\ h_r &= \frac{\partial T_{(p)}}{r\partial\varphi} - \frac{\beta_0}{\omega\mu}\frac{\partial T_{[p]}}{\partial r}, \\ h_\varphi &= -\frac{\partial T_{(p)}}{\partial r} - \frac{\beta_0}{\omega\mu}\frac{\partial T_{[p]}}{r\partial\varphi}, \\ h_z &= \frac{\chi_0^2}{j\omega\mu} T_{[p]}. \end{aligned} \tag{13}$$

Equating $e_\varphi(a_0)$ from (13) with $e_\varphi(a_0)$ from (11) and comparing in this equation the coefficients of cos $p\varphi$, a relation between $a_{(p)}$, $a_{[p]}$ and δ_p is obtained:

$$-\frac{\beta_0}{\omega\epsilon}\frac{p}{k_0} J_p(k_0) a_{(p)} + J_p'(k_0) a_{[p]} = \sqrt{\frac{2}{\pi}\frac{\omega\mu}{\beta_0}}\, \delta_p. \tag{14}$$

Another relation between $a_{(p)}$ and $a_{[p]}$ is obtained by substituting for e_z and h_φ from (13) into (8):

$$\frac{\chi_0^2}{j\omega\epsilon} J_p(k_0)a_{(p)} = Z\left[\chi_0 J'_p(k_0)a_{(p)} - \frac{\beta_0}{\omega\mu}\frac{p}{a_0} J_p(k_0)a_{[p]}\right]. \qquad (15)$$

Equations (14) and (15) can be solved for $a_{(p)}$ and $a_{[p]}$. For example:

$$a_{(p)} =$$

$$-\sqrt{\frac{2\beta_0}{\pi\omega\mu}}\frac{Z\delta_p}{J_p(k_0)} \frac{p}{\dfrac{k_0^2}{j\omega\epsilon a_0}\dfrac{J_p(k_0)}{J_p(k_0)} - k_0 Z\left[\dfrac{J'^2_p(k_0)}{J_p^2(k_0)} - \dfrac{p^2}{k_0^2} - \dfrac{\beta_0^2}{\omega^2\mu\epsilon}\right]}. \qquad (16)$$

With $a_{(p)}$ and $a_{[p]}$ the perturbation fields of circular electric waves are known as functions of the δ_p's. Thus the quasi-circular electric waves in any slightly deformed round waveguide can be written in terms of the normal wave and perturbation fields.

The propagation constant remains unchanged and equal to $j\beta_0$ in this first-order approximation. Now it is just the effect of a deformation on the propagation constant and especially on its real part, the attenuation constant, which is most important. Ordinarily a higher order of approximation would be necessary to determine this attenuation. But here, as in all electromagnetic problems where the dissipated energy is small compared to the stored or propagated energy, the losses may be calculated from a lower order of approximation.[7] The attenuation constant is the ratio of power P_d dissipated per unit length to the power carried by the wave:

$$\alpha = \frac{P_d}{2P}.$$

Power is dissipated by the perturbation field through the anisotropic shield into the wall impedance Z:

$$P_d = \tfrac{1}{2}\,\mathrm{Re}\left(\frac{1}{Z}\right)\int_s e_z e_z^*\, ds\,|_{r=a}.$$

This integral along the actual inner radius of the guide is to first order equal to the integral along the nominal radius a_0:

$$P_d = \frac{\mathrm{Re}\,(Z)}{2\,|\,Z\,|^2}\int_0^{2\pi} e_z e_z^* a_0\, d\varphi. \qquad (17)$$

In (9) the power flow of the circular electric wave was assumed to be unity. Substituting for e_z from (13) into (17) and using (16), it is found that each Fourier component of the mechanical deformation contributes α_p to the total loss α:

$$\alpha = \sum_p \alpha_p ,$$

where

$$\alpha_p = \tfrac{1}{2} \operatorname{Re}(Z) \frac{\beta_0}{\omega\mu} \frac{\left[p\delta_p \dfrac{J_p(k_0)}{J_p'(k_0)} \right]^2}{\left| 1 + jZ \dfrac{\omega\epsilon a_0}{k_0} \left[\dfrac{\beta_0^2}{\omega^2\mu\epsilon} \dfrac{p^2}{k_0^2} \dfrac{J_p(k_0)}{J_p'(k_0)} - \dfrac{J_p'(k_0)}{J_p(k_0)} \right] \right|^2} . \tag{18}$$

This expression for the added circular electric wave attenuation in a deformed helix waveguide agrees with some obvious facts: Any deformation of a purely reactive wall does not cause any circular electric wave attenuation. δ_0 and δ_1 represent changes in diameter and transverse displacement, respectively, of an otherwise round guide. The circular electric wave configuration is not changed by them. Consequently $\alpha_0 = \alpha_1 = 0$.

Equation (18) is valid for but one special case. The absolute value in the denominator is zero whenever the characteristic equation (61) (of Appendix A) for helix waveguide modes of pth azimuthal order is satisfied by k_0. Whenever a mode of pth azimuthal order has the same propagation constant as the circular electric wave, δ_p, however small it may be, causes a substantial change of the normal circular electric mode that can no longer be described by the perturbation expression of (18).

The propagation constant of any of the asymmetric modes, to be equal to $j\beta_0$, requires a purely reactive wall impedance. Because of finite loss, practical wall impedance values will always be at least slightly resistive; (18) will therefore be valid for all practical cases.

For some typical cross-sectional deviations, (18) can be simplified:

δ_2 represents an elliptical deformation:

$$\alpha_2 a_0 = \tfrac{1}{2} \operatorname{Re}(Z) \frac{\beta_0}{\omega\mu} \frac{k_0^2 \delta_2^2}{\left| 1 + j \dfrac{2Z}{\omega\mu a_0} \right|^2} ; \tag{19}$$

δ_3 represents a trifoil deformation:

$$\alpha_3 a_0 = \tfrac{1}{2} \operatorname{Re}(Z) \frac{\beta_0}{\omega\mu} \frac{k_0^2 \delta_3^2}{\left| \dfrac{k_0^2}{12} - 1 + j \tfrac{1}{2} Z \omega\epsilon a_0 \left(1 - \dfrac{k_0^2}{24} - \dfrac{6}{\omega^2 \sqrt{\mu\epsilon}\, a_0^2} \right) \right|^2} ; \tag{20}$$

δ_4 represents a quadrufoil deformation:

$$\alpha_4 a_0 = \frac{1}{2}\,\mathrm{Re}\,(Z)\,\frac{\beta_0}{\omega\mu}\,\frac{k_0^{\,2}\delta_4^{\,2}}{4\left|\dfrac{k_0^{\,2}-12}{24-k_0^{\,2}} + j8Z\,\dfrac{\omega\epsilon a_0}{k_0^{\,2}}\left[\dfrac{\beta_0^{\,2}}{4\omega^2\mu\epsilon} - \left(\dfrac{k_0^{\,2}-12}{24-k_0^{\,2}}\right)\right]\right|^2}; \tag{21}$$

etc., for any multifoil deformation.

III. NONUNIFORM HELIX WAVEGUIDE

Here the relative deformation δ of the guide radius will not only be a function of φ but it will also change with z. In Appendix A Maxwell's equations are converted into generalized telegraphist's for this structure.

The deformation δ is first assumed to be independent of z. The fields in the deformed but cylindrical waveguide are represented in terms of normal modes of the perfectly round helix waveguide. This series representation for the field components is then substituted into Maxwell's equations. With the boundary conditions (2) and (3) and an orthogonality relation between normal modes of the helix waveguide, a set of simultaneous first-order differential equations is obtained, which determines the z-dependence of the coefficients of this series expansion. If the coefficients are chosen so that they represent amplitudes A and B of forward and backward traveling waves of the round guide modes, then the system of equations for the A's and B's can be written as

$$\begin{aligned} \frac{dA_m}{dz} + jh_m A_m &= -j \sum_n c_{nm}(A_n + B_n), \\ \frac{dB_m}{dz} - jh_m B_m &= +j \sum_n c_{nm}(A_n + B_n). \end{aligned} \tag{22}$$

If the perturbation δ of the nominal radius is expanded into a Fourier series (10), then the coupling coefficients are determined by the coefficients of this Fourier expansion:

$$\begin{aligned} p \neq 0: \qquad c_{[0m][pn]} &= \frac{\sqrt{\pi}}{2} N_n \sqrt{\frac{h_{pn}}{h_{0m}}}\,\frac{k_{0m}k_{pn}}{ka_0^2}\, p\, \frac{J_p^{\,2}(k_{pn})}{J_p'(k_{pn})}\,\delta_p\,, \\ p = 0: \qquad c_{[0m][0n]} &= \frac{k_{0m}k_{0n}}{a_0^{\,2}\sqrt{h_{0m}h_{0n}}}\,\delta_0\,. \end{aligned} \tag{23}$$

The metallic waveguide is the limiting case of the helix waveguide with zero wall impedance. The normal modes of the helix waveguide degenerate into TE_{pn} and TM_{pn}. The separation constant: $k_{pn} = \chi_{pn} a_0$ is the root of $J_p'(k_{pn}) = 0$ for TE_{pn} modes and the root of $J_p(k_{pn}) = 0$ for TM_{pn} modes. The coupling coefficients (23) reduce to $c = 0$ for interaction between TE_{0m} and TM_{pn} modes. For interaction between TE_{0m} and TE_{pn} modes the coupling coefficients are:

$$c_{[0m][pn]} = \frac{k_{0m}k_{pn}}{a_0^2\sqrt{2h_{0m}h_{pn}}} \frac{k_{pn}}{\sqrt{k_{pn}^2 - p^2}} \delta_p . \tag{24}$$

In a nonuniform helix waveguide the coupling coefficients c in (22) are functions of z. Then (22) is a system of first-order linear differential equations with varying coefficients. For small deformations and consequently small coupling coefficients, solutions of (22) can be found by successive approximations. To simplify the representation, the B's of (22) are included in the A's. There are then always pairs of A's associated with propagation constants jh_m and $-jh_m$ and coupling coefficients jc_{nm} and $-jc_{nm}$. Thus the two equations of (22) can be replaced by the first alone. The transformation

$$A_m = e^{-jh_m z} E_m \tag{25}$$

eliminates a common propagation factor:

$$\frac{dE_m}{dz} = -j \sum_n c_{nm} e^{-j(h_n - h_m)z} E_n . \tag{26}$$

The only initial conditions of practical interest are

$$E_1(0) = 1,$$
$$E_n(0) = 0 \qquad \text{for } n \neq 1.$$

A TE_{01} wave of unit amplitude is launched into a nonuniformly deformed helix waveguide. A first-order solution of (26) under these initial conditions is:

$$\begin{aligned} E_1(z) &= 1, \\ E_n(z) &= -j \int_0^z c_{1n} e^{-j(h_1 - h_n)s} \, ds. \end{aligned} \tag{27}$$

The first-order solution is substituted into (26) for a second-order solution:

$$E_1(z) = 1 - \sum_n \int_0^z c_{n1} e^{-j(h_n - h_1)s} \int_0^s c_{1n} e^{-j(h_1 - h_n)t} \, dt \, ds, \tag{28}$$

and so on.

As a typical example, a TE_{01} wave will be launched into a waveguide that has a constant deformation δ between $z = 0 \cdots l$ and is round everywhere else. The waveguide is thus uniform except for two discontinuities at $z = 0$ and $z = l$. The wave amplitudes at any point $z > l$ are, from (27) and (28),

$$E_1(z) = 1 - j \sum_n \frac{c_{1n}^2}{(h_1 - h_n)^2} [(h_1 - h_n)l + j(e^{j(h_1-h_n)l} - 1)], \quad (29)$$

$$E_n(z) = \frac{c_{1n}}{h_1 - h_n} (e^{-j(h_1-h_n)l} - 1). \quad (30)$$

The converted wave amplitudes E_n may be regarded as being generated from the TE_{01} wave at the two discontinuities $z = 0$ and $z = l$. Then the conversion at one such discontinuity is:

$$\left| \frac{E_n}{E_1} \right| = \left| \frac{c_{1n}}{h_1 - h_n} \right|. \quad (31)$$

From (23) and (31), with $\delta = \delta_0$, a formula for mode conversion between circular electric waves at diameter changes is obtained:

$$\frac{E_{0n}}{E_{0m}} = \frac{k_{0m}k_{0n}}{a_0^2 \sqrt{h_{0m}h_{0n}} (h_{0m} - h_{0n})} \delta_0 . \quad (32)$$

Likewise, a formula for mode conversion in offsets of helix waveguide with $\delta = \delta_1 \cos \varphi$ can be written down. In the case of $Z = 0$, the formula describes mode conversion at offsets of a metallic guide:

$$\frac{E_{1n}}{E_{0m}} = \frac{k_{0m}k_{1n}}{a_0^2 \sqrt{2h_{0m}h_{1n}} (h_{0m} - h_{1n})} \frac{k_{1n}}{\sqrt{k_{1n}^2 - 1}} \delta_1 . \quad (33)$$

Thus, from (31), mode conversion at an arbitrary discontinuity in helix waveguide can be calculated.

Mode conversion at an arbitrary nonuniform deformation of the helix waveguide, however, is found from (27).

IV. TOLERANCES OF HELIX WAVEGUIDE FOR CIRCULAR ELECTRIC WAVE TRANSMISSION

The all-important question may be asked now: What deformations can be tolerated in a helix waveguide without any excessive degradation of the TE_{01} transmission characteristics? There are two factors which degrade the TE_{01} transmission: (a) Additional normal mode loss in a deformed helix waveguide, as calculated in Section II and described by (18), increases the overall TE_{01} transmission loss. (b) Mode conversion and reconversion in nonuniform sections of helix waveguide, as calculated in Section III and described by (27) and (28), cause mode conversion loss and reconversion distortion of the TE_{01} characteristic.

4.1 *Normal Mode Loss*

The normal mode loss of a uniformly deformed waveguide will be considered first. Helix waveguide in current experimental use at the Bell Telephone Laboratories has a nominal inner radius of $a_0 = 1$ inch. A median frequency of the planned operating range is 55.5 kmc. To optimize various transmission characteristics, the surrounding jacket has been made to present a real wall impedance to the interior that is half of free space impedance $Z = \frac{1}{2}\sqrt{\mu/\epsilon}$. For these values, expressions (19), (20), (21) for the added circular electric wave loss have been evaluated:

$$\begin{aligned} \alpha_2 a_0 &= 3.64\ \delta_2^2, \\ \alpha_3 a_0 &= 0.458\ \delta_3^2, \\ \alpha_4 a_0 &= 0.516\ \delta_4^2. \end{aligned} \tag{34}$$

By far the largest losses are caused by an elliptical deformation. The theoretical loss of TE_{01} in a perfect copper waveguide of 2-inch inside diameter at 55.5 kmc is

$$\alpha_0 a_0 = 2.77 \times 10^{-6}.$$

In order that the increase of attenuation be not more than 10 per cent of this theoretical loss, the elliptical deformation should be

$$\delta_2 < 0.276 \times 10^{-3}.$$

The elliptical diameter differences in a 2-inch helix waveguide should not exceed 1 mil. This is quite a strict requirement.

It is interesting to compare these figures with losses in a deformed metallic waveguide:

$$\begin{aligned} \frac{\alpha_2}{\alpha_0} &= \frac{\beta_0^2 a_0^2}{2}\,\delta_2^2, \\ \frac{\alpha_3}{\alpha_0} &= 10\beta_0^2 a_0^2 \delta_3^2, \\ \frac{\alpha_4}{\alpha_0} &= 1.5\beta_0^2 a_0^2 \delta_4^2. \end{aligned} \tag{35}$$

In a metallic waveguide it is the trifoil deformation which causes most loss. In order that such a trifoil deformation not cause more than 10 per cent of the theoretical TE_{01} loss in a 2-inch metallic waveguide at 55.5 kmc, this deformation should be:

$$\delta_3 < 3.42 \times 10^{-3}.$$

4.2 *Mode Conversion Loss*

Equations (34) and (35) describe the added TE_{01} loss correctly only in a waveguide with uniform, z-independent deformation δ. When the deformation is a function of z, as is the case in an imperfect waveguide, the general expression (28) describes the transmission. Changing the order of integration in (28), a more suitable form is obtained:

$$E_1(z) = 1 - \sum_n \int_0^z e^{j(h_1-h_n)u}\, du \int_0^{z-u} c_{1n}(s)c_{1n}(s+u)\, ds. \quad (36)$$

The loss can be expressed in terms of the geometrical imperfections δ with $c_{1n} = C_n\delta$. For sufficiently small δ,

$$|E_1| = 1 - \Lambda,$$

with the loss

$$\Lambda = \sum_n \int_0^z e^{\Delta\alpha_n u}(P_n \cos \Delta\beta_n u - Q_n \sin \Delta\beta_n u)\, du \cdot \int_0^{z-u} \delta(s)\delta(s+u)\, ds, \quad (37)$$

where

$$C_n^2 = P_n + jQ_n$$

and

$$j(h_1 - h_n) = \Delta\alpha_n + j\Delta\beta_n .$$

In general, the geometric imperfections will not be known, only their statistical properties. Rowe and Warters[5] have determined with a relation like (36) the statistics of the loss in terms of the statistics of the guide imperfections. Use of their analysis is made here.

The deformation is assumed to be a stationary random process with covariance $R(u)$ and spectral distribution $S(\zeta)$

$$R(u) = \langle\delta(z)\delta(z+u)\rangle, \quad (38)$$

$$S(\zeta) = \int_{-\infty}^{+\infty} R(u)e^{-j2\pi\zeta u}\, du. \quad (39)$$

In (38), $\langle x \rangle$ is the expected value of x.

Taking the expected value on both sides of (37), the average added loss is obtained in terms of the covariance $R(u)$ is

$$\langle\Lambda\rangle = \sum_n \int_0^z e^{\Delta\alpha u} R(u)(z - u)(P_n \cos \Delta\beta_n u - Q_n \sin \Delta\beta_n u)\, du. \quad (40)$$

For the following analysis, a special form for the covariance must be assumed. Since existing experimental information is rather vague, Rowe[6] assumes $R(u)$ to be exponential as reasonable physically and to simplify the calculation

$$R(u) = \frac{\pi S_0}{L_0} e^{-2\pi|u|/L_0}. \quad (41)$$

Then the spectral distribution of δ becomes

$$S(\zeta) = \frac{S_0}{1 + (L_0\zeta)^2}, \quad (42)$$

where $S(\zeta)$ is nearly flat with spectral density S_0 for mechanical frequencies in distance smaller than

$$\zeta_0 = \frac{1}{L_0}. \quad (43)$$

At ζ_0 the spectral distribution is down 3 db and falls very rapidly above ζ_0; L_0 may be regarded as the cutoff mechanical wavelength.

Substituting (41) for the covariance in (40) and performing the integration over a length $z \gg L_0$, the average added loss is:

$$\langle\Lambda\rangle = \pi S_0 z \sum_n \frac{P_n(2\pi - \Delta\alpha_n L_0) - Q_n \Delta\beta_n L_0}{\Delta\beta_n^2 L_0^2 + (2\pi - \Delta\alpha_n L_0)^2}. \quad (44)$$

For $\Delta\alpha_n L_0 \gg 2\pi$, (44) reduces to

$$\langle\Lambda\rangle = \frac{\pi S_0 z}{L_0} \sum_n \frac{-P_n \Delta\alpha_n - Q_n \Delta\beta_n}{\Delta\alpha_n^2 + \Delta\beta_n^2},$$

or with

$$\frac{\pi S_0}{L} = R(0) = \langle\delta^2(z)\rangle$$

the added average loss is for this special case:

$$\frac{\langle\Lambda\rangle}{z} = \langle\delta^2\rangle \sum_n \operatorname{Re}\left[\frac{-C_n^2}{j(h_1 - h_n)}\right]. \quad (45)$$

As seen from (29), a long waveguide with a uniform deformation $\delta =$

$\sqrt{\langle\delta^2\rangle}$ would have the same added loss. Equation (45) then is the added normal mode loss. It is also much simpler than that described by (18). But only when the differential loss $\Delta\alpha$ of every single coupled mode is very large in the cutoff mechanical wavelength L_0 will the added loss be described by (18) with $\delta = \sqrt{\langle\delta^2\rangle}$.

The L_0 for waveguide deformation is probably small, certainly not much larger than 1 foot. Certain coupled modes might have a very high differential loss per foot, but then there would always be coupled modes with low differential loss.

Consequently, the condition leading to (45) is not satisfied for cross-sectional deformation in helix waveguide. Expressions (34) cannot be used to determine cross-sectional tolerances. As shown by Rowe,[6] this conclusion is true for a wide class of covariance functions.

The correct expression for mode conversion in helix waveguide is (44). Written as added loss per wavelength, it reads:

$$\frac{\langle\Lambda\rangle}{z} = \langle\delta^2\rangle L_0 \sum_n \frac{P_n(2\pi - \Delta\alpha_n L_0) - Q_n \Delta\beta_n L_0}{\Delta\beta_n^{\,2} L_0^{\,2} + (2\pi - \Delta\alpha_n L_0)^2}. \tag{46}$$

For real coupling coefficients in a lossless structure, (46) reduces to

$$\frac{\langle\Lambda\rangle}{z} = \langle\delta^2\rangle L_0 \sum_n \frac{2\pi C_n^{\,2}}{4\pi^2 + \Delta\beta_n^{\,2} L_0^{\,2}}, \tag{47}$$

and for very small L_0 from (46)

$$\frac{\langle\Lambda\rangle}{z} = \langle\delta^2\rangle \frac{L_0}{2\pi} \sum_n P_n. \tag{48}$$

For a very short correlation distance, however, a more general expression than (48) for the average added loss can be found. In this case $R(u)$, whatever function it may be, has substantial values only in the immediate vicinity of $u = 0$. Then, instead of (40),

$$\langle\Lambda\rangle = z \int_0^z R(u)\, du \sum_n P_n$$

and, with (39),

$$\frac{\langle\Lambda\rangle}{z} = \tfrac{1}{2} S(0) \sum_n P_n \tag{49}$$

for any spectral distribution $S(\zeta)$ of geometric imperfections with small correlation distance.

Equation (47) has been evaluated in Appendix B for cross-sectional deformations in a helix waveguide with an infinitely high wall impedance.

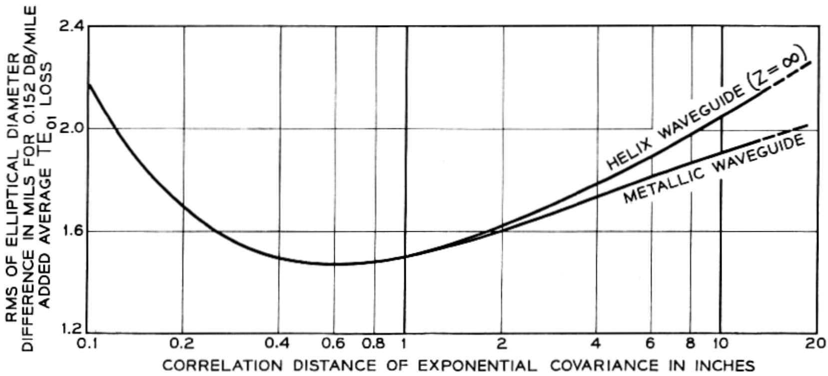

Fig. 1 — TE_{01} loss in round waveguide with random ellipticity; 2 inch inside diameter, at 55.5 kmc.

This particular helix waveguide design minimizes circular electric wave loss and mode conversion in bends.[2] In Fig. 1 is plotted the average ellipticity $\sqrt{\langle\delta_2^2\rangle}$ as a function of the correlation distance L_0 for an additional average loss equal to 10 per cent of the TE_{01} loss in a perfect copper pipe. For comparison, the same curve is plotted for $Z = 0$ representing metallic waveguide.

Both curves coincide for small values of the correlation distance and differ only slighly over the practical range of L_0. Though Fig. 1 is only drawn for a particular helix waveguide and a particular set of covariance functions, it is fairly safe to generalize: Random ellipticity of the cross section causes nearly as much average circular electric wave loss in helix waveguide as it does in metallic waveguide.

A more exact statement has been made for the case of vanishing correlation.[8] When L_0 is small enough for (48) to be valid, the average added TE_{01} loss is independent of the wall impedance and the same as in metallic waveguide.

Manufacturing imperfections usually have a small correlation distance. Therefore helix waveguide has to be manufactured to as close cross-sectional tolerances as metallic waveguide for circular electric wave transmission.

V. CONCLUSIONS

Cross-sectional deformations of the helix waveguide perturb circular electric wave propagation. In a slightly but uniformly deformed helix waveguide circular electric waves propagate with slightly changed field pattern. Power is dissipated into the helix jacket. Consequently, the added circular electric wave loss in a uniformly deformed helix waveguide is considerably larger than it is in a copper waveguide of the same uniform deformation.

Nonuniform deformations cause mode conversion and added TE_{01} loss. Manufacturing imperfections are expected to be random deformations with small correlation distance. Such imperfections increase the average circular electric wave loss nearly independently of the wall impedance which the helix jacket presents to the waveguide interior. The average added loss is therefore nearly the same as it is in metallic waveguide with the same imperfections. For example, ellipticity was assumed to be a stationary random process along the guide with exponential covariance. Then, even at a correlation distance of 1 foot, the added average TE_{01} loss at 55.5 kmc in a 2-inch inside diameter helix waveguide of infinite wall impedance is only 16 per cent smaller than it is in metallic waveguide.

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APPENDIX A

Generalized Telegraphist's Equations for Deformed Helix-Waveguide

Maxwell's equations in cylindrical coordinates (r,φ,z) are:

$$\frac{1}{r}\frac{\partial E_z}{\partial \varphi} - \frac{\partial E_\varphi}{\partial z} = -j\omega\mu H_r\,, \tag{50}$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu H_\varphi\,, \tag{51}$$

$$\frac{1}{r}\frac{\partial (rE_\varphi)}{\partial r} - \frac{1}{r}\frac{\partial E_r}{\partial \varphi} = -j\omega\mu H_z\,, \tag{52}$$

$$\frac{1}{r}\frac{\partial H_z}{\partial z} - \frac{\partial H_\varphi}{\partial z} = j\omega\epsilon E_r\,, \tag{53}$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = j\omega\epsilon E_\varphi\,, \tag{54}$$

$$\frac{1}{r}\frac{\partial (rH_\varphi)}{\partial r} - \frac{1}{r}\frac{\partial H_r}{\partial \varphi} = j\omega\epsilon E_z\,. \tag{55}$$

The electromagnetic field in the helix waveguide can be derived from two sets of wave functions T_n and T'_n given by

$$\begin{aligned} T_n &= N_n J_p(\chi_n r) \sin p\varphi, \\ T'_n &= N_n J_p(\chi_n r) \cos p\varphi. \end{aligned} \tag{56}$$

The T_n and T'_n satisfy the wave equation

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial \varphi}\left(\frac{1}{r}\frac{\partial T}{\partial \varphi}\right)\right] = -\chi^2 T, \tag{57}$$

where χ is a separation constant which takes on discrete values for the various normal modes. The transverse field components are written in terms of these functions:

$$\begin{aligned} E_r &= \sum_n V_n \left(\frac{\partial T_n}{\partial r} + d_n \frac{\partial T'_n}{r\partial \varphi}\right), \\ E_\varphi &= \sum_n V_n \left(\frac{\partial T_n}{r\partial \varphi} - d_n \frac{\partial T'_n}{r\partial \varphi}\right), \\ H_r &= \sum_n - I_n \left(\frac{\partial T_n}{r\partial \varphi} - d_n \frac{h_n^2}{k^2}\frac{\partial T'_n}{\partial r}\right), \\ H_\varphi &= \sum_n I_n \left(\frac{\partial T_n}{\partial r} + d_n \frac{h_n^2}{k^2}\frac{\partial T'_n}{r\partial \varphi}\right). \end{aligned} \tag{58}$$

Substituting from (58) into (55) and taking advantage of (57), an expression for the longitudinal electric field is obtained:

$$E_z = j\omega\mu \sum_n I_n \frac{\chi_n^2}{k^2} T_n , \tag{59}$$

where k is the intrinsic propagation constant of the waveguide interior; d_n and the propagation constant h_n are chosen so that the boundary conditions of the round helix waveguide

$$\begin{aligned} E_\varphi(a_0) &= 0, \\ E_z(a_0) &= -ZH_\varphi(a_0) \end{aligned}$$

are satisfied by the individual terms of (58). Only then do the individual terms of (58) represent normal modes of the helix waveguide.

From $E_\varphi(a_0) = 0$:

$$d_n = \left.\frac{\dfrac{\partial T_n}{r\partial \varphi}}{\dfrac{\partial T'_n}{\partial r}}\right|_{a_0} = \frac{pJ_p(k_n)}{k_n J'_p(k_n)}, \tag{60}$$

where $k_n = \chi_n a_0$. The prime at the Bessel function denotes differentiation with respect to the argument. The remaining boundary condition between E_z and H_φ leads to the following (characteristic) equation:

$$\frac{1}{k_n}\frac{J_p'(k_n)}{J_p(k_n)} - \frac{p^2 h_n^2}{k_n^3 k^2}\frac{J_p(k_n)}{J_p'(k_n)} = \frac{-j}{\omega\epsilon a_0 Z}. \tag{61}$$

The characteristic equation, together with

$$k_n^2 = (k^2 - h_n^2)a_0^2,$$

determines the separation constant k_n. The transverse field components of any two different modes are orthogonal to each other in that:

$$\begin{aligned}\frac{1}{V_n I_m}\int_S (E_{tn} \times H_{tm})\, dS = \\ \int_S \frac{\epsilon}{\epsilon_0}\Bigg[\left(\frac{\partial T_n'}{\partial r} + d_n \frac{\partial T_n'}{r\partial\varphi}\right)\left(\frac{\partial T_m}{\partial r} + d_m \frac{h_m^2}{k^2}\frac{\partial T_m'}{r\partial\varphi}\right) \\ + \left(\frac{\partial T_n}{r\partial\varphi} - d_n \frac{\partial T_n'}{\partial r}\right)\left(\frac{\partial T_m}{r\partial\varphi} - d_m\frac{h_m^2}{k^2}\frac{\partial T_m'}{\partial r}\right)\Bigg] dS = \delta_{nm},\end{aligned} \tag{62}$$

where δ_{nm} is the Kronecker symbol. The integration is to be extended over the cross section of the waveguide. For $n = m$ equation (62) determines the normalization factor:

$$\begin{aligned}N_n = \frac{\sqrt{2}}{\sqrt{\pi}\, J_p(k_n)}\Bigg[\frac{h_n^2}{k^2} p^2(k_n^2 - p^2)Y_n^2 + \frac{1}{Y_n^2} \\ + k_n^2\left(1 - \frac{p^2}{k^2 a_0^2}\right) + 2\left(\frac{1}{Y_n} - p^2 Y_n\right)\Bigg]^{-\frac{1}{2}},\end{aligned} \tag{63}$$

with

$$Y_n = \frac{J_p(k_n)}{k_n J_p'(k_n)}.$$

All quantities in (56) and (58) have now been determined except the current and voltage coefficients. To find relations for them the field components from (58) are substituted into Maxwell's equations and these then are converted to generalized telegraphist's equations.

Add

$$-\left(\frac{\partial T_m}{r\partial\varphi} - d_m \frac{h_m^2}{k^2}\frac{\partial T_m'}{\partial r}\right)$$

times (50) and

$$\frac{\partial T_m}{\partial r} + d_m \frac{h_m^2}{k^2}\frac{\partial T_m'}{r\partial\varphi}$$

times (51) and integrate over the cross section. The result is:

$$
\begin{aligned}
\frac{dV_m}{dz} + j\,\frac{h_m^{\,2}}{\omega\epsilon}\, I_m = {} & \\
& \int_S (\text{grad } E_z)(\text{grad } T_m)\, dS + d_m \frac{h_m^{\,2}}{k^2} \int_S (\text{grad } E_z)(\text{flux } T_m')\, dS \\
& - j\omega\mu \sum_n I_n \frac{\chi_n^{\,2}}{k^2} \Bigg[\int_S (\text{grad } T_n)(\text{grad } T_m)\, dS \\
& \qquad + d_m \frac{h_m^{\,2}}{k^2} \int_S (\text{grad } T_n)(\text{flux } T_m')\, dS \Bigg],
\end{aligned} \tag{64}
$$

where the gradient and flux of a scalar are defined by:

$$
\begin{aligned}
\text{grad}_r\, T &= \frac{\partial T}{\partial r}, \qquad & \text{grad}_\varphi\, T &= \frac{1}{r}\frac{\partial T}{\partial \varphi}, \\
\text{flux}_r\, T &= \frac{1}{r}\frac{\partial T}{\partial \varphi}, \qquad & \text{flux}_\varphi\, T &= -\frac{\partial T}{\partial r}.
\end{aligned} \tag{65}
$$

After partial integration on the right-hand side of (64),

$$
\begin{aligned}
\frac{dV_m}{dz} + j\,\frac{h_m^2}{\omega\epsilon}\, I_m = {} & \\
& \int_0^{2\pi} E_z \left(\frac{\partial T_m}{\partial r} + \frac{d_m}{a_0}\frac{h_m^{\,2}}{k^2}\frac{\partial T_m'}{\partial \varphi} \right) a_0\, d\varphi + \chi_m^{\,2} \int_S E_z T_m\, dS \\
& - j\omega\mu \sum_n I_n \frac{\chi_n^{\,2}}{k^2} \Bigg[\int_0^{2\pi} T_n \left(\frac{\partial T_m}{\partial r} + \frac{d_m}{a_0}\frac{h_m^{\,2}}{k^2}\frac{\partial T_m'}{\partial \varphi} \right) a_0\, d\varphi \\
& \qquad + \chi_m^{\,2} \int_S T_n T_m\, dS \Bigg].
\end{aligned} \tag{66}
$$

In special cases when the helix waveguide degenerates into a perfectly conducting metallic waveguide, the individual terms for E_z in (59) are zero for $r = a_0$, while E_z itself, because of the boundary condition (3), is different from zero. Then (59) is a nonuniformly convergent series, which describes E_z only in the open interval $0 \leqq r < a_0$. Term-by-term differentiation will make the series diverge. Therefore the series had not been substituted for E_z in (64). In (66), (59) may now be substituted in the integral over the cross section. In the line integral, E_z from the boundary condition (3) may be substituted. The fields at $r = a$ can by a Taylor series be written in terms of fields at $r = a_0$. Neglecting higher-order terms:

$$E_z(a_0) = -Z\left[H_\varphi(a_0) + \frac{\partial H_\varphi(a_0)}{\partial r} a_0\delta + H_r(a_0)\frac{d\delta}{d\varphi}\right] - \frac{\partial E_z(a_0)}{\partial r} a_0\delta. \tag{67}$$

Thus, instead of (66):

$$\frac{dV_m}{dz} + j\frac{h_m^2}{\omega\epsilon} I_m = -a_0\int_0^{2\pi}\left[a_0\delta\frac{\partial}{\partial r}(E_z + ZH_\varphi) + ZH_r\frac{d\delta}{d\varphi}\right]\left[\frac{\partial T_m}{\partial r} + \frac{d_m}{a_0}\frac{h_m^2}{k^2}\frac{\partial T_m'}{\partial\varphi}\right]d\varphi. \tag{68}$$

For the other of the two sets of generalized telegraphist's equations, add

$$-\left(\frac{\partial T_m}{\partial r} + \frac{d_m}{r}\frac{\partial T_m'}{\partial\varphi}\right)$$

times (53) and

$$-\left(\frac{1}{r}\frac{\partial T_m}{\partial\varphi} - d_m\frac{\partial T_m'}{\partial r}\right)$$

times (54) and integrate over the cross section. The result is:

$$\begin{aligned}\frac{dI_m}{dz} + j\omega\epsilon V_m = {} & -\int_S (\text{grad } H_z)(\text{flux } T_m)\, dS + d_m\int_S (\text{grad } H_z)(\text{grad } T_m')\, dS \\ & + j\omega\epsilon\sum_n V_n d_n\frac{\chi_n^2}{k^2} \\ & \cdot\int_S [(\text{grad } T_n')(\text{flux } T_m) - d_m(\text{grad } T_n')(\text{grad } T_m')]\, dS.\end{aligned} \tag{69}$$

After partial integration on the right-hand side of (69),

$$\frac{dI_m}{dz} + j\omega\epsilon V_m = d_m\chi_m^2\int_S H_z T_m'\, dS - j\omega\epsilon\sum_n V_n d_n d_m\frac{\chi_n^2\chi_m^2}{k^2}\int_S T_n' T_m'\, dS. \tag{70}$$

To replace H_z, substitute E_r from (58), in (52), multiply (52) by T_m' and integrate over the cross section. The series (58) for E_φ is nonuniformly convergent and cannot be used in (52). After partial integration,

$$-j\omega\mu \int_S H_z T_m' \, dS = \int_0^{2\pi} E_\varphi T_m' a_0 \partial\varphi + \sum_n V_n d_n \chi_n^2 \int_S T_n' T_m' \, dS. \quad (71)$$

With the boundary condition (2) as Taylor series at $r = a_0$:

$$E_\varphi(a_0) = -E_r(a_0) \frac{d\delta}{d\varphi} - \frac{\partial E_\varphi(a_0)}{\partial r} a_0 \delta.$$

Equation (70) can be written as:

$$\frac{dI_m}{dz} + j\omega\epsilon V_m = -j \frac{d_m \chi_m^2}{\omega\mu} a_0 \int_0^{2\pi} \left(E_r \frac{d\delta}{d\varphi} + \frac{\partial E_\varphi}{\partial r} a_0 \delta \right) T_m' \, d\varphi. \quad (72)$$

Partial integration on the right-hand side,

$$\int_0^{2\pi} E_r \frac{d\delta}{d\varphi} T_m' \, d\varphi = -\int_0^{2\pi} \delta \left(\frac{\partial E_r}{\partial \varphi} T_m' + E_r \frac{\partial T_m'}{\partial \varphi} \right) d\varphi,$$

and substitution of the series expressions (58),

$$-\frac{\partial E_r}{\partial \varphi} + a_0 \frac{\partial E_\varphi}{\partial r} = \sum_n V_n d_n \chi_n^2 a_0 T_n',$$

reduces (72) to

$$\frac{dI_m}{dz} + j\omega\epsilon V_m = -j \frac{a_0^2}{\omega\mu} \sum_n V_n d_n d_m \chi_n^2 \chi_m^2 \int_0^{2\pi} T_n' T_m' \delta \, d\varphi$$
$$+ j \frac{a_0}{\omega\mu} d_m \chi_m^2 \int_0^{2\pi} E_r \frac{\partial T_m'}{\partial \varphi} \delta \, d\varphi. \quad (73)$$

The interest is limited here to the propagation characteristics of circular electric waves. Therefore, only terms that describe direct interaction between circular electric and other waves need to be retained in (68) and (73). When V_m and I_m are voltage and current amplitudes of circular electric waves, then T_m and $\partial T_m'/\partial\varphi$, and consequently the right-hand side of (68) and the last term on the right-hand side of (73), are zero. When V_m and I_m are amplitudes of other modes, then the same terms in (68) and (73) are zero, since E_z, H_φ, E_r and $H_r(a_0)$ are zero for circular electric waves. Thus (68) and (73) reduce to:

$$\frac{dV_m}{dz} + j \frac{h_m^2}{\omega\epsilon} I_m = 0,$$
$$\frac{dI_m}{dz} + j\omega\epsilon V_m = -j \sum_n V_n d_n d_m \frac{k_n^2 k_m^2}{\omega\mu a_0^2} \int_0^{2\pi} T_n' T_m' \delta \, d\varphi. \quad (74)$$

The generalized telegraphist's equations represent an infinite set of coupled transmission lines. It is convenient to write transmission line equations not in terms of currents and voltages but in terms of the amplitudes of forward and backward traveling waves. Thus, let A and B be the amplitudes of the forward and backward waves of a typical mode at a certain cross section. The mode current and voltage are related to the mode amplitudes by

$$V = \sqrt{K}(A + B),$$
$$I = \frac{1}{\sqrt{K}}(A - B), \tag{75}$$

where K is the wave impedance

$$K_m = \frac{h_m}{\omega\epsilon}. \tag{76}$$

If the currents and voltages in the generalized telegraphist's equations (74) are represented in terms of the traveling-wave amplitudes, after some obvious additions and subtractions the following equations for coupled traveling waves are obtained:

$$\frac{dA_m}{dz} + jh_m A_m = -j \sum_n c_{nm}(A_n + B_n),$$
$$\frac{dB_m}{dz} - jh_m B_m = +j \sum_n c_{nm}(A_n + B_n). \tag{77}$$

The c's are coupling coefficients defined by:

$$c_{nm} = \tfrac{1}{2}\sqrt{h_n h_m}\, d_n d_m \frac{k_n^2 k_m^2}{k^2 a^2} \int_0^{2\pi} T'_n T'_m \delta \, d\varphi. \tag{78}$$

To replace the d's and T's in (78), the customary double-subscript notation for the various modes in round helix waveguide is used. Then from (66), (70) and (73) the interaction between circular electric waves and other waves in deformed helix waveguide is described by the coupling coefficients:

$p \neq 0$: $\quad c_{[0m][pn]} =$

$$N_n \sqrt{\frac{h_{pn}}{h_{0m}}} \frac{k_{0m}k_{pn}}{ka_0^2} \frac{p}{2\sqrt{\pi}} \frac{J_p^2(k_{pn})}{J'_p(k_{pn})} \int_0^{2\pi} \delta \cos p\varphi \, d\varphi, \tag{79}$$

$p = 0$: $\quad c_{[0m][0n]} = \dfrac{k_{0m}k_{0n}}{a_0^2\sqrt{h_{0m}h_{0n}}} \dfrac{1}{2\pi} \displaystyle\int_0^{2\pi} \delta \, d\varphi.$

APPENDIX B

Nonuniform Helix Waveguide with Infinite Wall Impedance

For $Z \rightarrow \infty$ the characteristic equation (61) reduces to

$$Y_n = \pm \frac{k}{p h_n} \tag{80}$$

or the two equations:

$$\begin{aligned} \frac{k_n}{p} \frac{J_{p+1}(k_n)}{J_p(k_n)} &= 1 - \sqrt{1 - \frac{k_n^2}{k^2 a_0^2}}, \\ \frac{k_n}{p} \frac{J_{p-1}(k_n)}{J_p(k_n)} &= 1 - \sqrt{1 - \frac{k_n^2}{k^2 a_0^2}}. \end{aligned} \tag{81}$$

For $k_n < ka$, an approximation for the roots of (81) is furnished by

$$\begin{aligned} J_{p+1}(k_n) &= 0, \\ J_{p-1}(k_n) &= 0. \end{aligned} \tag{82}$$

Equation (81) can be expanded about the roots of (82) to improve the approximations for k_n .

Substituting (80) for Y_n in (63) reduces the normalization factor to:

$$N_n = \frac{1}{\sqrt{\pi}\, k_n J_p(k_n)} \left(1 - \frac{p^2}{k^2 a_0^2} \mp \frac{p}{k h_n a_0^2}\right)^{-\frac{1}{2}}. \tag{83}$$

Hence the coupling coefficient is, from (23),

$$c_{[0m][pn]} = \pm \frac{1}{2} \frac{k_{0m} k_{pn}}{\sqrt{h_{0m} h_{pn}}\, a_0} \left(1 - \frac{p^2}{k^2 a_0^2} \mp \frac{p}{k h_n a_0^2}\right)^{-\frac{1}{2}} \frac{\delta_p}{a_0}. \tag{84}$$

In (84) all the subscripts have been included to identify the coupling coefficient properly.

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