Normal Modes and Mode Conversion in Helix Waveguide

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Helix waveguide, composed of closely wound insulated copper wire covered with an absorptive or reactive jacket, transmits circular electric waves with low loss. Mechanical imperfections, such as curvature and deformation, cause coupling between the circular electric waves and unwanted modes and degrade the transmission. In designing a helix waveguide for a particular application, a jacket must be found that minimizes the transmission degradation. Unwanted mode characteristics and their coupling coefficients must be known; these quantities are given by the roots of a transcendental equation involving complex Bessel functions.

A program has been set up for automatically finding the complex roots by iterative approximation. Starting from the known roots at infinite jacket conductivity, the characteristic equation is solved for all practical values of wall impedance of the jacket and all modes of interest. The representation of the mode characteristics as a function of wall impedance leads to a definite designation of modes in heterogeneous waveguide. The TE_{pn} modes of helix waveguide with $n \neq 1$ can have only a limited attenuation. These limits determine the design of mode filters. Manufacturing imperfections increase the average TE_{01} loss independently of the wall impedance. Random curvature with large correlation distance is produced by laying tolerances, but its contribution to the average loss is minimized in a helix waveguide with very large wall impedance.

I. INTRODUCTION

Helix waveguide, closely wound from insulated copper wire and covered with an absorptive or reactive jacket, is a good transmission medium for circular electric waves.[1] In long distance communication, waveguide can be designed to act as a mode filter, to negotiate bends or, particularly, to serve as the transmission line proper.[2]

As in metallic waveguide, the loss of circular electric waves decreases steadily with frequency only in a perfect helix waveguide. Any curvature of the guide axis, deformation of the cross section or deviation of the winding from a low and uniform pitch adds to the loss and degrades the transmission characteristics.[3,4]

In a perfect helix waveguide circular electric waves propagate undisturbed. Imperfections cause coupling between circular electric waves and other modes. Power is lost by conversion to unwanted modes and reconversion distorts any smooth transmission characteristics.

In order to control mode conversion and reconversion in helix waveguide with practical imperfections, and also to design helix waveguides for mode filters and intentional bends, the unwanted mode characteristics and unwanted mode coupling must be investigated. Earlier calculations have resulted in a characteristic equation which implicitly determines the properties of helix waveguide-modes,[1] and also in explicit expressions for various coupling coefficients.[2,3,4] Numerical evaluations of these equations have been very informative. They were, however, not complete enough to reveal all the unwanted mode properties and could not serve as a basis for helix waveguide design in every application.

The results of a more exhaustive numerical evaluation of helix waveguide equations will be presented here. In a few typical examples these results will be applied to helix waveguide design problems. First the equations which describe wave propagation in perfect and imperfect helix waveguide will be listed.

II. PERFECT HELIX WAVEGUIDE

A helix waveguide (Fig. 1) will be called perfect when the helix forms a straight circular cylinder and is wound with a low and uniform pitch. The mathematical model which then replaces it is an anisotropic impedance sheet at radius a conducting perfectly in circumferential direction but with a wall impedance Z in axial direction. The Z replaces the jacket surrounding the helix and takes into account the finite size of helix wires. The electromagnetic field components in a cylindrical coordinate system (r,φ,z) are then subject at $r = a$ to the boundary conditions

$$E_\varphi = 0, \tag{1}$$

$$\frac{E_z}{H_\varphi} = -Z. \tag{2}$$

Solutions of Maxwell's equations in cylindrical coordinates are Bessel

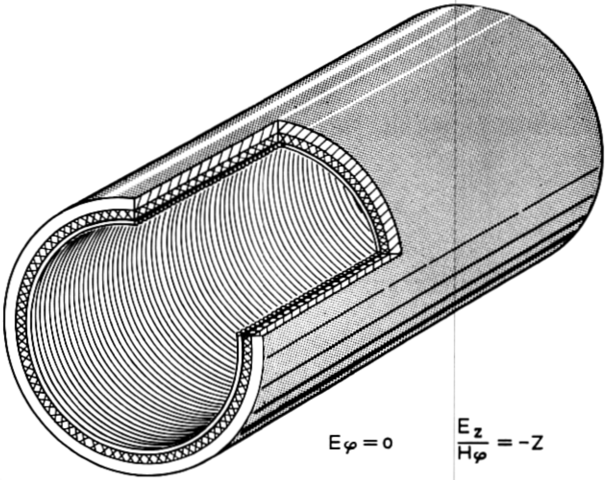

Fig. 1 — Helix waveguide and boundary conditions.

functions J_p of the radius, trigonometric functions of the azimuth and exponential functions of the axial distance:

$$\begin{Bmatrix} J_p\left(\frac{k}{a}r\right) \\ \\ J_p'\left(\frac{k}{a}r\right) \end{Bmatrix} \begin{Bmatrix} \sin p\varphi \\ \\ \cos p\varphi \end{Bmatrix} e^{-\gamma z}, \tag{3}$$

where p is the azimuthal order of the wave. The axial propagation constant γ and radial propagation constant k/a are related with the intrinsic propagation constant $\omega\sqrt{\mu\epsilon}$ of the material filling the waveguide:

$$\left(\frac{k}{a}\right)^2 = \omega^2\mu\epsilon + \gamma^2. \tag{4}$$

When the boundary conditions (1) and (2) are imposed on the solutions (3) of Maxwell's equations the following characteristic equation results:

$$j\omega\epsilon a Z - \frac{kJ_p(k)J_p'(k)}{\frac{p^2}{k^2}\frac{\gamma^2}{\omega^2\mu\epsilon}J_p^2(k) + J_p'^2(k)} = 0. \tag{5}$$

Values of γ that satisfy the characteristic equation (5) are the propagation constants of normal modes of the perfect helix waveguide. They describe wave propagation in a perfect helix waveguide completely.

III. IMPERFECT HELIX WAVEGUIDE

Wave propagation in imperfect helix waveguide has been described by generalized telegraphist's equations.[2,3,4] In a perfect helix waveguide a normal mode n of amplitude $|E_n|$ propagates independently from all other modes m:

$$\frac{dE_n}{dz} = -\gamma_n E_n . \tag{6}$$

Imperfections cause interaction between modes so that the wave amplitudes are mutually coupled:

$$\frac{dE_n}{dz} = -\gamma_n E_n - j \sum_m c_{nm} E_m . \tag{7}$$

The coupling coefficients are determined by the kind and size of the imperfection, but they are also strongly dependent on the wall impedance.

For circular electric wave applications, only coupling between these and other waves is of interest. Coupling coefficients of typical imperfections in helix waveguide will now be listed. The subscript m will refer to the TE_{0m} wave; n will refer to any of the coupled modes. A normalization factor

$$N_n = \frac{\sqrt{2}}{\sqrt{\pi}\, J_p(k_n)} \cdot \left[\frac{p^2 \gamma_n^2}{\omega^2 \mu \epsilon} (p^2 - k_n^2) Y_n^2 + \frac{1}{Y_n^2} + k_n^2 \left(1 - \frac{p^2}{\omega^2 \mu \epsilon a^2} \right) + 2 \left(\frac{1}{Y_n} - p^2 Y_n \right) \right]^{-\frac{1}{2}} \tag{8}$$

with

$$Y_n = \frac{J_p(k_n)}{k_n J_p'(k_n)} \tag{9}$$

is used to render the coupling coefficients symmetric, i.e.,

$$c_{nm} = c_{mn} . \tag{10}$$

3.1 *Curvature*[2]

There is only coupling between circular electric modes and modes of first azimuthal order in a curved helix waveguide:

$$c_{nm} = N_n \frac{\sqrt{\pi}\, J_1(k_n)}{2\omega \sqrt{\mu \epsilon}\, a} \sqrt{\frac{\gamma_n}{\gamma_m}} \frac{k_m k_n^2}{k_m^2 - k_n^2} \left[1 + \frac{\gamma_m}{\gamma_n} + \frac{\gamma_m + \gamma_n}{\gamma_m - \gamma_n} Y_n \right] \frac{1}{R}, \tag{11}$$

where R is the radius of curvature.

3.2 *Deformation of the Cross Section*[3]

The radius a_1 of a deformed guide of nominal radius a can be written

$$a_1 = a(1 + \sum_p \delta_p \cos p\varphi). \tag{12}$$

Each component δ_p will cause coupling between circular electric modes and modes of azimuthal order p:

$$c_{mn} = \frac{\sqrt{\pi}}{2} N_n \sqrt{\frac{\gamma_n}{\gamma_m}} \frac{k_m k_n^2}{\omega \sqrt{\mu\epsilon}\, a^2} p J_p(k_n) Y_n \delta_p . \tag{13}$$

3.3 *Irregular Helix Winding*[4]

In a perfect helix waveguide the angle between a helix wire and the cross section is small enough to be regarded as zero. In an irregular winding this angle can be written

$$\psi = \sum_p \theta_p \sin p\varphi. \tag{14}$$

Each component θ_p causes coupling to modes of azimuthal order p:

$$c_{nm} = \frac{\sqrt{\pi}\, k_m k_n^2 N_n J_p(k_n)}{2\omega\sqrt{\mu\epsilon}\sqrt{\gamma_m\gamma_n}\, a^3} \theta_p . \tag{15}$$

IV. NUMERICAL EVALUATION

The propagation constant of normal modes in helix waveguide is, by (4) and (5), only implicitly given as a function of frequency and waveguide parameters. The problem is to find the complex roots of a transcendental and complex equation.

With (4), γ can be eliminated from (5). Then, for a given frequency and guide radius, the characteristic equation determines k as a function of Z:

$$F(k,Z) = 0. \tag{16}$$

For $Z = 0$ the characteristic equation degenerates into

$$J_p(k) = 0, \qquad J_p'(k) = 0, \tag{17}$$

the roots of which correspond to TM and TE waves respectively of metallic waveguide.

Starting from the known roots of (17) for $Z = 0$, the solutions of (16) for helix waveguide can be traced by gradually increasing the wall impedance. If k_0 and Z_0 are a known solution of (5), then an approximate value for the solution at $Z_1 = Z_0 + \Delta Z$ is given by:

$$k_1 = k_0 - \frac{\frac{\partial}{\partial Z}[F(k_0, Z_0)]}{\frac{\partial}{\partial k}[F(k_0, Z_0)]} \Delta Z. \tag{18}$$

A better approximation is found by Newton's formula:

$$k_2 = k_1 - \frac{F(k_1, Z_1)}{\frac{\partial}{\partial k}[F(k_1, Z_1)]}. \tag{19}$$

For further improvement, the process (19) can be repeated to any desired accuracy.

The final result is the starting point for the next root at the neighboring value of wall impedance. For the numerical evaluation, the wall impedance was related to the impedance of free space $Z_0 = \sqrt{\mu/\epsilon}$:

$$\frac{Z}{Z_0} = \rho e^{j\Phi}. \tag{20}$$

The solutions were traced along lines of constant phase Φ of Z. The increment $\Delta\rho$ was varied and kept sufficiently small to insure continuity of the process.

The evaluation was programmed by Mrs. C. L. Beattie for automatic execution on an IBM 704 Data Processing System.

The characteristic equation was evaluated for all wall impedances with passive phases and amplitudes up to 5000 ohms. All those solutions were traced which for zero wall impedance start as the following metallic waveguide modes:

$$TE_{11}, TM_{11}, TE_{12}, TM_{12}, TE_{13}, TM_{13}.$$

$$TE_{21}, TM_{21}, TE_{22}, TM_{22}, TE_{23}.$$

$$TE_{31}, TM_{31}, TE_{32}, TM_{32}.$$

For some special wall impedance phases the evaluations were extended over many more modes. A value of $a/\lambda = 4.7$ was assumed corresponding to a center frequency of the proposed 35 to 75 kmc frequency band for the 2-inch inside diameter waveguide system.

The numerical results were also used to calculate from the separation constant k, the propagation constant γ in its real and imaginary parts. Figs. 2 through 6 are plotted from these results. These diagrams show

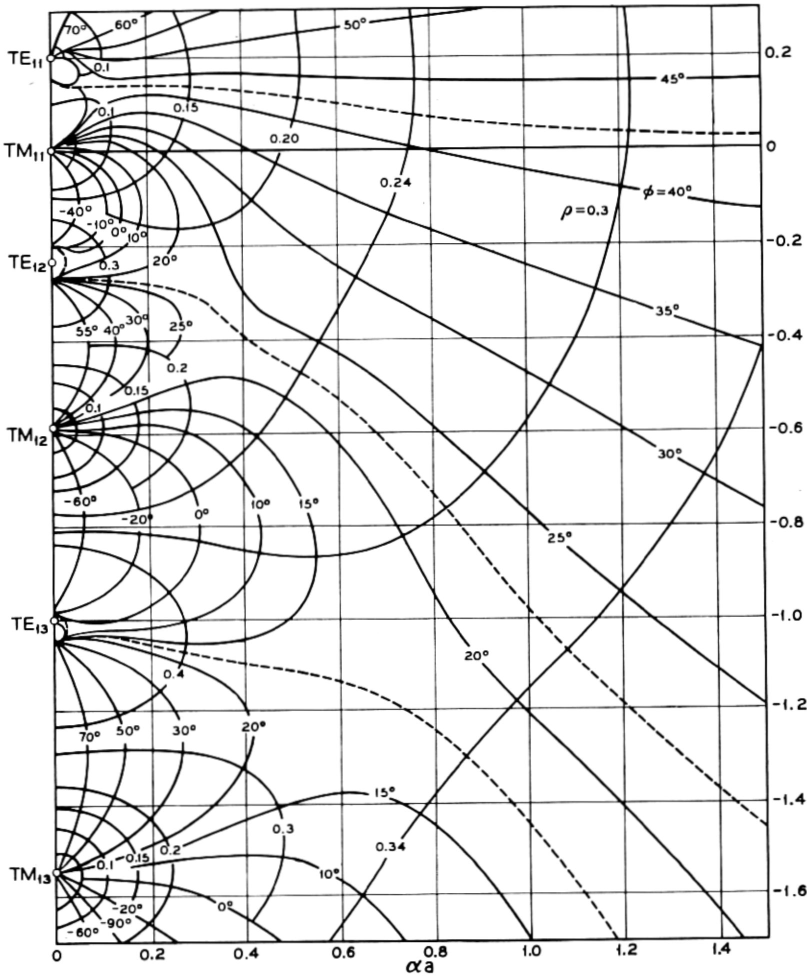

Fig. 2 — Propagation constant $\gamma = \alpha a + j(\beta_{01} + \Delta\beta)a$ in helix waveguide of wall impedance Z; contours in γ-plane of constant magnitude ρ and phase angle Φ of Z/Z_0 ; $a/\lambda = 4.7$, $p = 1$.

contour lines of constant phase Φ and constant amplitude ρ of the wall impedance drawn in the complex plane of propagation constant γ. The scale on the βa-axis has been shifted by the TE_{01} phase $\beta_{01}a = 29.305$ and represents the difference in phase constant between TE_{01} and the plotted mode.

Each diagram is for a particular value of p, specifying the respective

Curves for wall impedances with complex phase fan out into the γ-plane. Some return to the imaginary axis; others continue more and more out to ever increasing values of the attenuation constant.

The propagation constant is a multivalued function of the wall impedance. For any one wall impedance value there are as many different values of the propagation constant as there are points of zero wall im-

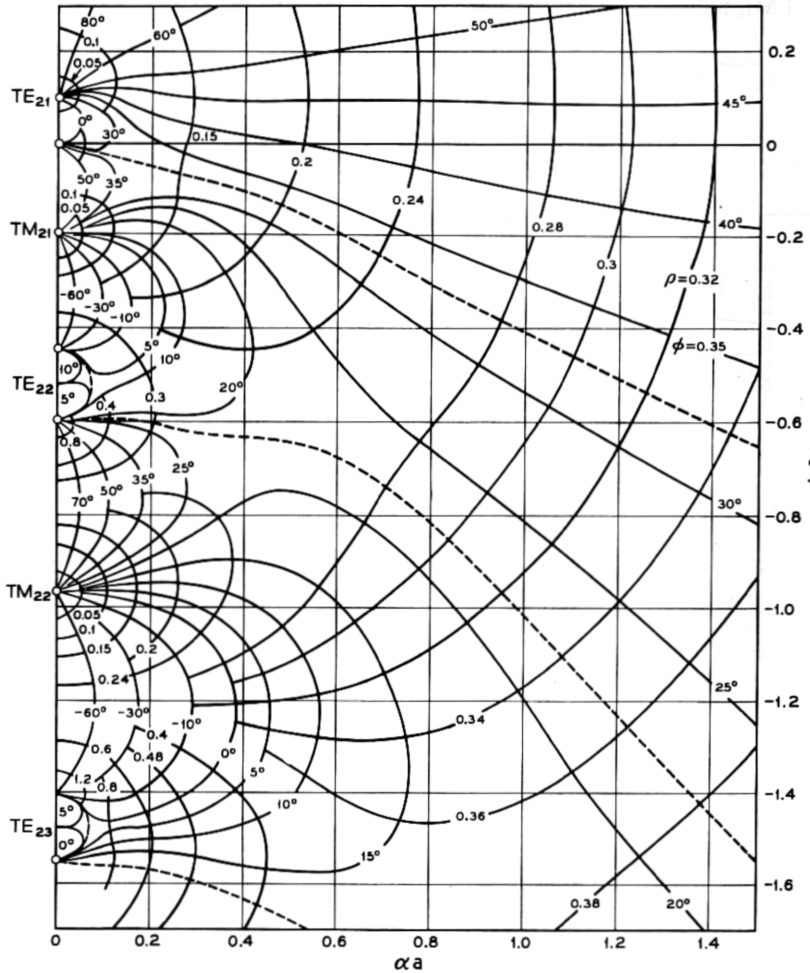

Fig. 5 — Propagation constant $\gamma = \alpha a + j(\beta_{01} + \Delta\beta)a$ in helix waveguide of wall impedance Z; contours in γ-plane of constant magnitude ρ and phase angle φ of Z/Z_0 ; $a/\lambda = 4.7$, $p = 2$.

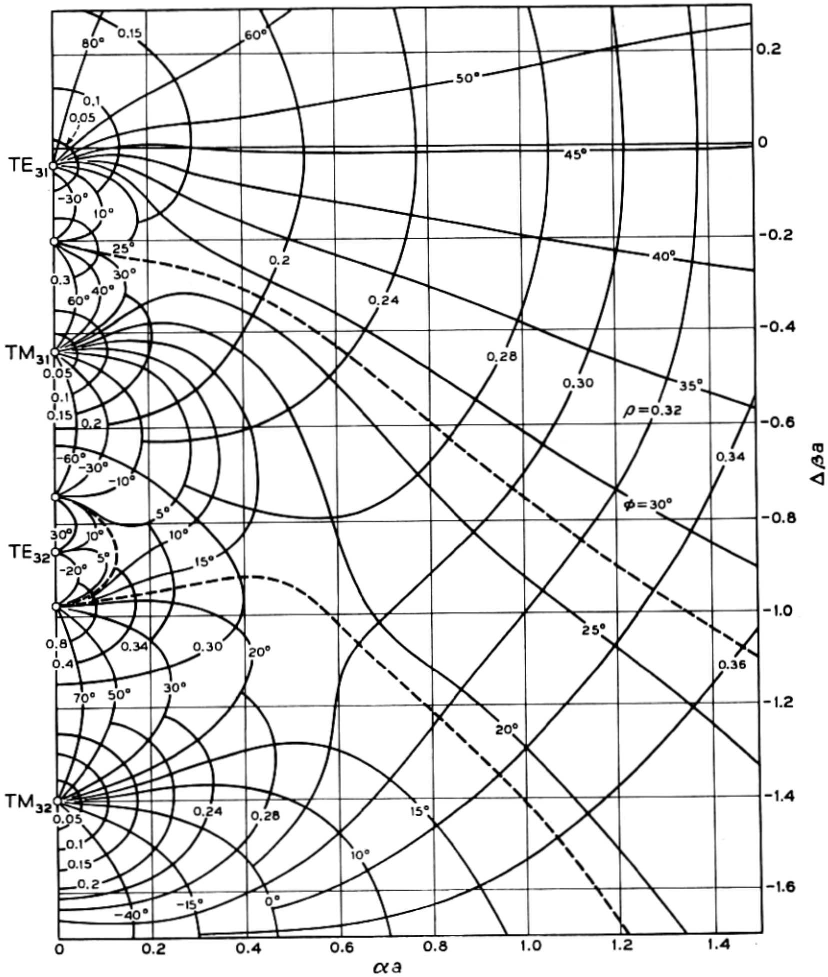

Fig. 6 — Propagation constant $\gamma = \alpha a + j(\beta_{01} + \Delta\beta)a$ in helix waveguide of wall impedance Z; contours in γ-plane of constant magnitude ρ and phase angle φ of Z/Z_0 ; $a/\lambda = 4.7$, $p = 3$.

pedance on the β-axis. Each value of propagation constant corresponds to a normal mode. The designation of these modes is not as simple as in metallic waveguide. The modes of helix waveguide are, in general, neither transverse with respect to any field component nor is their radial order well defined. Therefore, the simple designation of metallic waveguide TE_{pn} or TM_{pn} loses its significance. Nevertheless, the mode designation of metallic waveguide can be extended to helix waveguide or, for that matter, to any heterogeneous waveguide, when the γ-plane is divided up so that in each region the propagation constant is a single-valued function of the critical guide parameter. In the present case the wall impedance is the critical parameter.

The dividing lines in the γ-plane will be branch cuts of γ in the Z-plane. They separate the infinite set of branches of γ from each other, each branch corresponding to a helix waveguide mode. The branch cuts of γ should connect the branch points in the Z-plane. The branch points of γ in the Z-plane are saddle points of Z in the γ-plane. Branch cuts of γ should therefore go through the saddle points of Z in the γ-plane. As many branches will be in contact at the saddle point as is the order of the saddle point. From inspection of the diagrams all saddle points are found to be of second order; therefore, only two branches of γ are in contact at these saddle points and only one dividing line or branch cut must be made through each.

The remaining path of the branch cuts is arbitrary. They should conveniently follow a course that never cuts contour lines of constant phase of the wall impedance and ends either in infinity or on the β-axis at the points of infinite wall impedance.

For example, the branch cut between TE_{11} and TM_{11} starts in Fig. 2 at the corresponding point of infinite wall impedance and separates the contour line ($\Phi = 40°$) coming from TE_{11} from the contour line ($\Phi = 45°$) coming from TM_{11}. Somewhere in the γ-plane the dividing line hits a saddle point of $Z = f(\gamma)$. Beyond this saddle point the branch cut is continued according to the same rule, always separating contour lines of constant phase which originated at different points of zero wall impedance.

Each such region, bounded by the β-axis and the branch cuts (broken lines in the diagrams) is now designated by the metallic waveguide mode located within it. The normal modes of helix waveguide are then defined uniquely, and any further discussions can be made in terms of these modes.

This mode designation in helix waveguide can be defined in fewer words as follows: A mode in helix waveguide of finite wall impedance is identified with the metallic waveguide mode into which it degenerates when the wall impedance phase is kept constant and the wall impedance amplitude made zero.

Modes in any heterogeneous waveguide can correspondingly be identified with metallic waveguide modes when the critical parameters are subjected to the proper limiting process. All critical parameters should be kept constant except that one which in its limit changes the particular heterogeneous waveguide into a metallic waveguide. Requiring the procedure to be most direct will in general eliminate any further ambiguity.

Quite generally it is found that in the γ-plane the regions of all TE_{p1} and TM_{pn} modes in helix waveguide are unbounded while all TE_{pn} modes with $n > 1$ have a bounded region. Thus, the attenuation of all TE_{pn} modes with $n > 1$ is limited and cannot exceed a certain maximum value for any wall impedance. The attenuation constant of any of the other modes can be made arbitrarily high simply by choosing the proper wall impedance.

In most helix waveguide applications unwanted mode loss should be as high as possible. A more detailed discussion of those modes which cannot exceed a certain value of attenuation is therefore in order. A typical mode with limited attenuation is TE_{12}. Fig. 4 shows an enlarged portion of the γ-plane that contains the TE_{12} area. Besides being bounded by the β-axis this area is also bounded by an approximate semicircle as branch cut. The maximum loss of $\alpha a = 0.0363$ for TE_{12} is realized when the wall impedance is chosen

$$\left| \frac{Z}{Z_0} \right| = 0.495 \qquad \text{arc } (Z) = 4.5°, \tag{21}$$

where γ lies on the branch cut at the point of highest α. There is, however, another γ value for this wall impedance on the other side of the branch cut, a γ value that represents a TM_{11} wave. Its real part is $\alpha a = 0.0350$.

For all practical purposes it does not matter which of these points is called TE_{12} and which TM_{11}. The point with lower attenuation α is therefore the decisive one. To render the attenuation of this point as high as possible, it is moved along the branch cut into the saddle point at $\alpha a = 0.0360$. The wall impedance for this condition is

$$\left| \frac{Z}{Z_0} \right| = 0.487 \qquad \text{arc } (Z) = 4.5°. \tag{22}$$

At the same time, the other point moves also into the saddle point, and both modes degenerate into identity.

All other modes with limited attenuation behave similarly.

Using the results for the separation constant k and the propagation constant of helix waveguide modes, the coefficient of curvature coupling between TE_{01} and unwanted modes was computed for modes with first-order ($p = 1$) azimuthal dependence. For the modes of higher order in p the coupling coefficient to TE_{01} in a deformed cross section was computed. For $p = 2$ these coefficients describe coupling in an elliptical pipe. For $p = 3$ it is coupling in a trifoil deformation.

Of greater practical importance are the coefficients of curvature coupling. In Figs. 7 through 12 plots of these coupling-coefficients have been made for the modes TE_{11}, TM_{11}, TE_{12}, TM_{12}. Again, the contour lines of constant phase and the contour lines of constant amplitude of the wall impedance have been plotted as an orthogonal network in the plane of complex coupling coefficient c. Some of the lines of constant phase run out of the diagrams to very large values of c, indicating that the particular coupling coefficient has a pole in their vicinity. Comparison with the propagation constant of the respective modes shows that these poles occur at the saddle points of the $Z = f(\gamma)$ plot. Indeed inspection of (8) and

$$\frac{\partial}{\partial \gamma} [F(\gamma, Z)]$$

from (16) shows that where $\partial F/\partial \gamma$ is zero and $Z = f(\gamma)$ has a saddle point the normalization factor N_n has a pole.

Poles of the coupling coefficients might cause concern; after all, they represent very strong coupling to unwanted modes. But since the poles coincide with saddle points of $Z = f(\gamma)$ there is always strong coupling to the two degenerate modes at the saddle point. Coupling to each one of these modes is of opposite sign from the other. The total mode conversion stays in quite normal bounds.

It should be recalled on occasions like this that the normal modes of helix waveguide, like modes in any lossy structure, are not orthogonal with respect to power. Suppose, for example, that A_0 is the amplitude normalized with respect to power of a circular electric wave. Then A_0^2 is the power carried by this wave. Let the helix waveguide have a wall impedance near (22). Then the two modes TE_{12} and TM_{11} are nearly degenerate with respect to each other. Curvature will cause coupling as described by (11). Since the coupling coefficients are very large, even a short section of small curvature will generate large amplitudes A_1 of TM_{11} and A_2 of TE_{12}. One of these amplitudes alone, for example A_1, would mean seriously high mode conversion. Since TM_{11} and TE_{12} are not orthogonal with respect to power, both of the amplitudes A_1 and A_2 together compensate each other to a small total effect.

In the plots of Figs. 7 and 9 for TE_{11} and TE_{12} the wall impedance is always a single-valued function of the coupling coefficient. In Figs. 8 and 10 for TM_{11} and TM_{12}, $Z = f(c)$ is multivalued. This observation can be generalized to the following statement: Any coupling coefficient

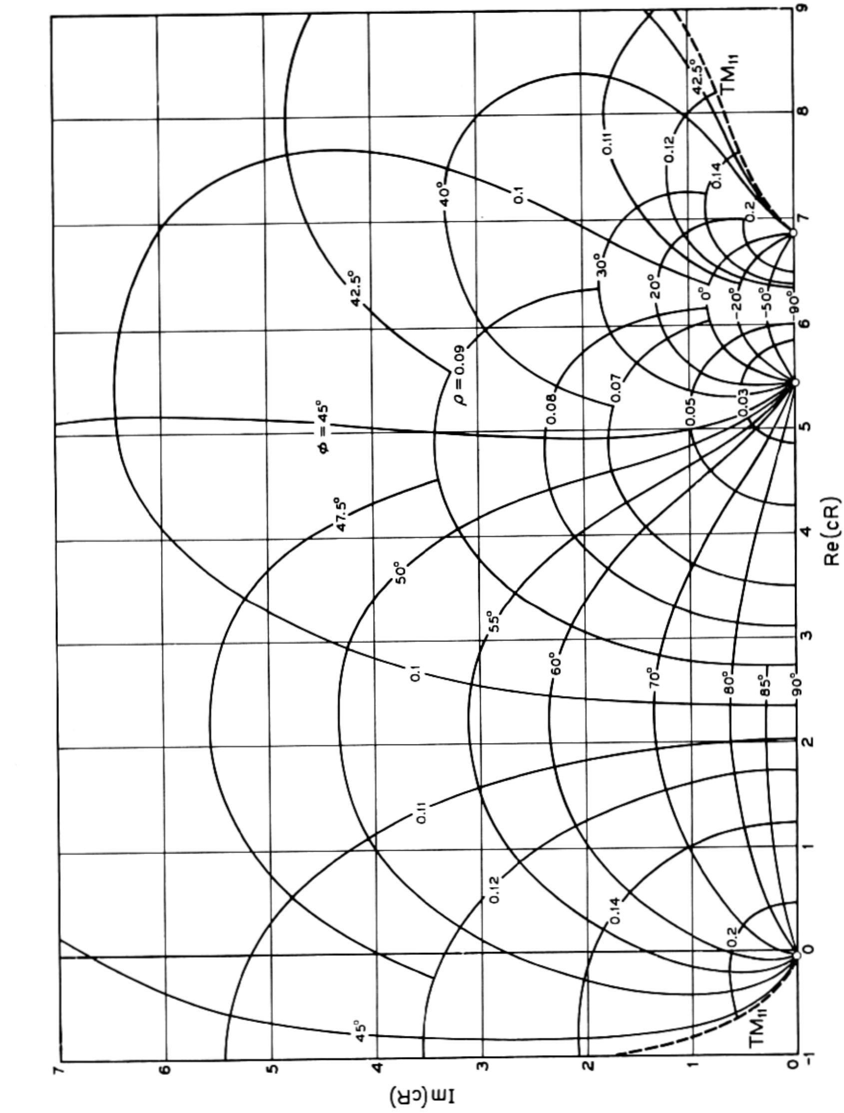

Fig. 7 — Coefficient c of curvature coupling between TE_{01} and TE_{11} in helix waveguide of wall impedance Z. Contours in (cR)-plane of constant magnitude ρ and phase angle Φ of Z/Z_0 ; $a/\lambda = 4.7$.

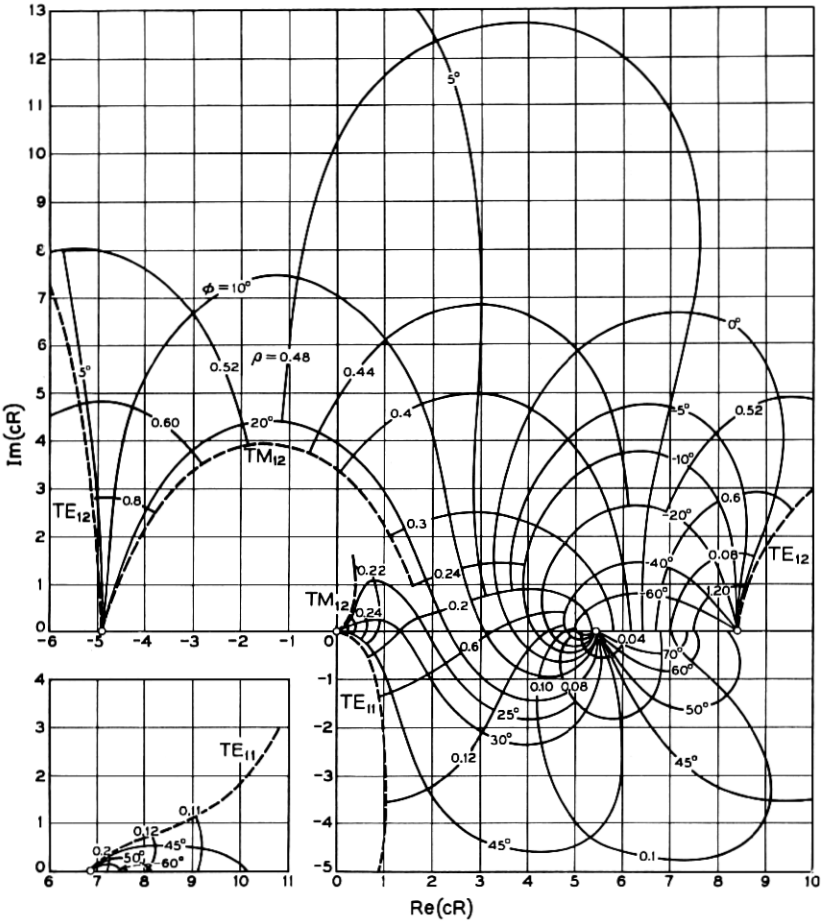

Fig. 8 — Coefficient c of curvature coupling between TE_{01} and TM_{11} in helix waveguide of wall impedance Z. Contours of constant magnitude ρ and phase angle Φ of Z/Z_0 in branches I and II of (cR)-plane; $a/\lambda = 4.7$.

c between circular electric modes and TM modes in helix waveguide of wall impedance Z is a function of Z such that its inversion $Z = f(c)$ is a multivalued function. A sufficient condition for this statement is that $c = g(Z)$ should have more than one pole, for then each of these poles gives a different value $Z = f(c)$ for the same argument $c = \infty$. Inspection of Figs. 2 through 6 shows that the area of every TM mode is adjacent to more than one saddle point of $Z = f(\gamma)$. As stated earlier, a saddle point of $Z = f(\gamma)$ corresponds to a pole of $c = g(Z)$. All TM modes

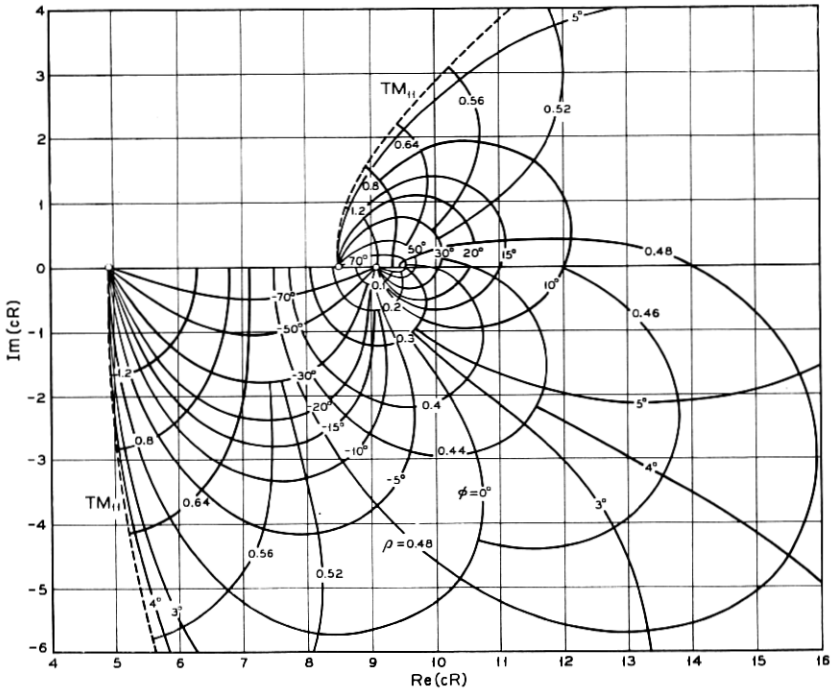

Fig. 9 — Coefficient c of curvature coupling between TE_{01} and TE_{12} in helix waveguide of wall impedance Z. Contours in (cR)-plane of constant magnitude ρ and phase angle Φ of Z/Z_0 ; $a/\lambda = 4.7$.

therefore have more than one pole of $c = g(Z)$ and $Z = f(c)$ is multivalued.

Actually the plots of Figs. 7 and 9 for TE_{11} and TE_{12} might be multivalued too. But when limiting the representation to wall impedance values with positive real part, the plots are single-valued.

To facilitate the representation of the multivalued function $Z = f(c)$ for TM_{11} and TM_{12}, branch cuts have been made in the c-plane and the different branches of c have been plotted in separate planes.

The broken lines indicate the border of a particular mode in the c-plane. They correspond to the branch cuts of γ in Figs. 2 through 6. The adjoining modes are always listed in the corresponding area.

V. APPLICATION

The results of the numerical evaluations have been applied to several problems of helix waveguide design:

5.1 *Mode Filter*

Sections of helix waveguide are inserted at intervals into plain metallic waveguide to absorb unwanted modes. For best absorption of a metallic waveguide mode the attenuation of the corresponding helix waveguide mode should be as high as possible. The most unwanted mode in metallic waveguide is TE_{12}; it most strongly degrades TE_{01} characteristics through mode-conversion effects. A good helix waveguide mode filter should therefore have a wall impedance that makes the attenuation constant of the corresponding TE_{12} mode a maximum. For the present case ($a/\lambda = 4.70$) this wall impedance value is given by (22). As high as the attenuation is for TE_{12} mode for this design, TE_{11} has quite low an attenuation constant

$$TE_{12}: \quad \alpha a = 0.0360,$$

$$TE_{11}: \quad \alpha a = 0.00686.$$

In metallic waveguide, TE_{11}, although not as objectionable as TE_{12}, is still a serious offender. A mode filter should at least represent moder-

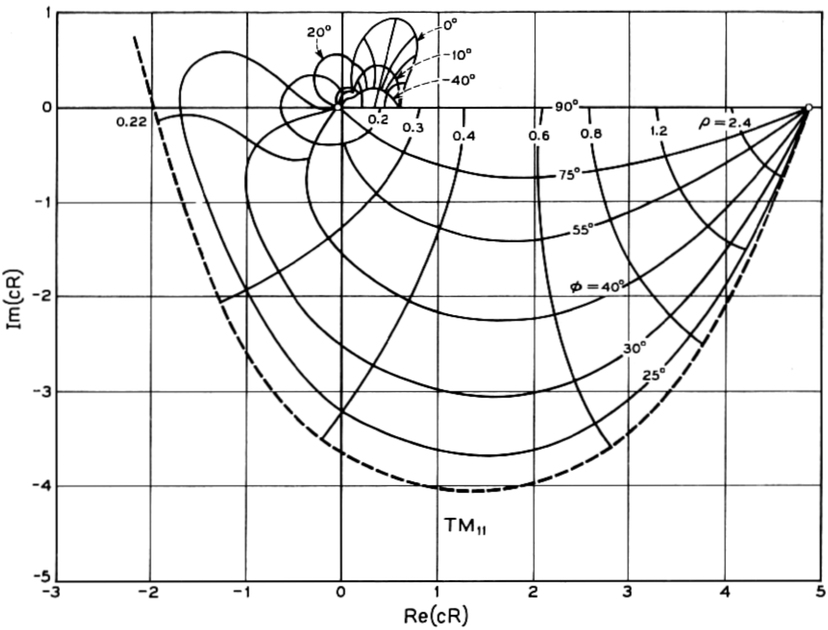

Fig. 10 — Coefficient c of curvature coupling between TE_{01} and TM_{12} in helix waveguide of wall impedance Z, contours of constant magnitude ρ and phase angle Φ of Z/Z_0 in branch I of (cR)-plane; $a/\lambda = 4.7$.

ate absorption to TE_{11}. The wall impedance for which the TE_{11} and TE_{12} attenuation are equal and a maximum is

$$\left| \frac{Z}{Z_0} \right| = 0.2975 \qquad \text{arc } (Z) = 12.0°, \tag{23}$$

and the corresponding attenuation is:

$$\alpha a = 0.01158.$$

These two wall impedance values are the limits for mode filters. Any practical design will be in between.

5.2 *Random Curvature*

Wave propagation in curved helix waveguide is described by generalized telegraphist's equations as coupling between the modes of the straight guide. For arbitrary but small coupling these equations can be solved approximately. An expression for the added TE_{01} loss can be written in terms of the coupling coefficients and the coupled mode characteristics.

Let the curvature distribution $\kappa(z)$ along the waveguide be a stationary random process with covariance

$$\sigma(u) = \langle \kappa(z)\kappa(z + u) \rangle. \tag{24}$$

According to Rowe[5] (see also Ref. 3), the average added TE_{01} loss can then be expressed in terms of the covariance of the coupling coefficient:

$$\langle \alpha \rangle = \frac{1}{L} \sum_n \int_0^L e^{-\Delta \alpha_n z} \sigma(z)(L - z)(P_n \cos \Delta\beta_n z + Q_n \sin \Delta\beta_n z)\, dz, \tag{25}$$

where L is the length of the line; $(c_n R)^2 = P_n + jQ_n$, the square of the coupling coefficient with c_n from (11); and $\Delta\alpha_n + j\Delta\beta_n = \gamma_n - \gamma_0$, the difference in propagation constant of a coupled mode n to the TE_{01} mode. The summation has to be extended over all coupled modes n.

For a mere estimate of the effects of random curvature the covariance is assumed to be exponential:

$$\sigma(z) = \langle \kappa^2 \rangle e^{-2\pi(|z|/L_0)}, \tag{26}$$

where L_0 may be regarded as a correlation distance.

When the correlation distance L_0 is small compared to the total length L of the waveguide, the average added loss is determined by the rms curvature $\sqrt{\langle \kappa^2 \rangle}$ and L_0:

$$\langle \alpha \rangle = \langle \kappa^2 \rangle L_0 \sum_n \frac{P_n(2\pi + \Delta\alpha_n L_0) + Q_n \Delta\beta_n L_0}{\Delta\beta_n^2 L_0^2 + (2\pi + \Delta\alpha_n L_0)^2}. \tag{27}$$

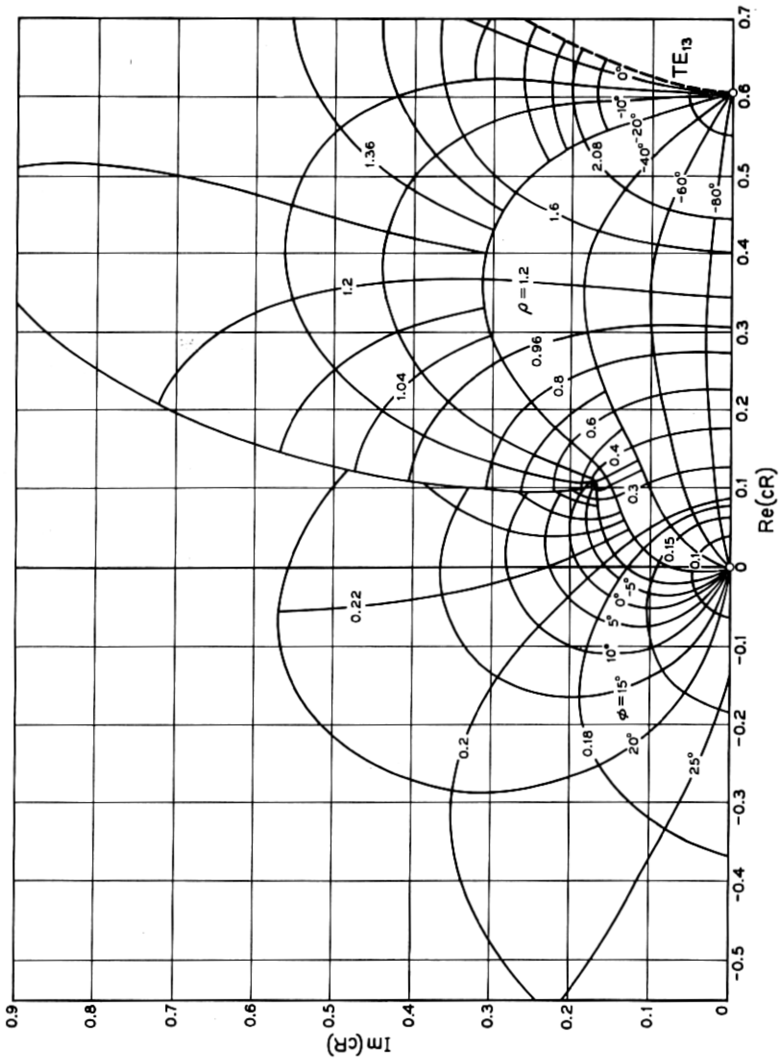

Fig. 11 — Enlarged portion of Fig. 10.

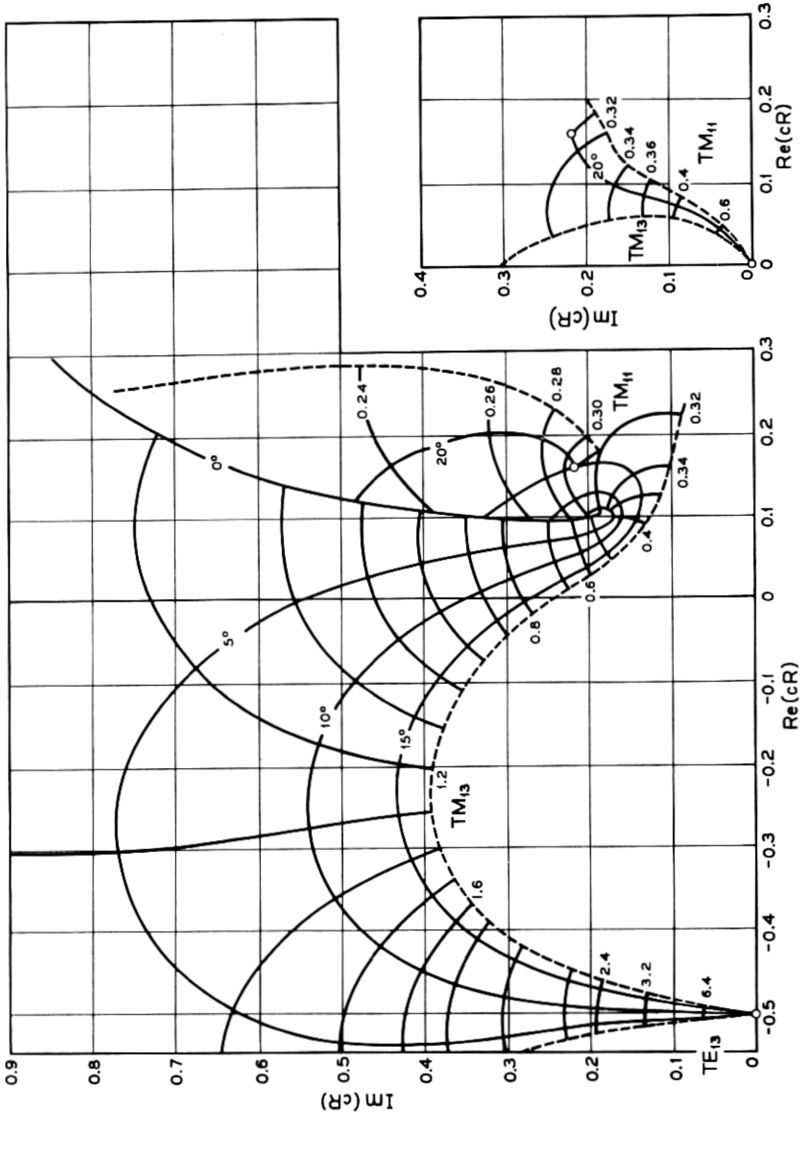

Fig. 12 — Branches II and III of Fig. 10.

Equation (27) has been evaluated for a helix waveguide, the TE_{12} attenuation of which is an absolute maximum and for a helix waveguide with equal and maximum attenuation for TE_{11} and TE_{12}. The results are plotted in Fig. 13. Also plotted in this figure are the corresponding curves for plain metallic waveguide and for helix waveguide with infinite wall impedance. The latter design of helix waveguide minimizes TE_{01} losses in intentional bends.

Shown in Fig. 13 are curves of the rms radius of curvature as a function of correlation distance L_0. This rms value would add 10 per cent of the TE_{01} loss in a perfect copper pipe to the average TE_{01} loss in the respective waveguide.

In calculating the curves of Fig. 13, coupling to the following modes of helix waveguide and metallic waveguide has been taken into account:

$$TE_{11}, \quad TM_{11}, \quad TE_{12}, \quad TM_{12}, \quad TE_{13}, \quad TM_{13}.$$

Contributions from higher-order modes are small enough to be neglected.

One important conclusion can be drawn from Fig. 13. When the correlation distance of random curvature is small enough — smaller than 10 feet in the present case — the added average loss is nearly independent of the wall impedance and nearly the same as in plain metallic waveguide. This independence is not only true for random curvature with exponential covariance but for any random curvature

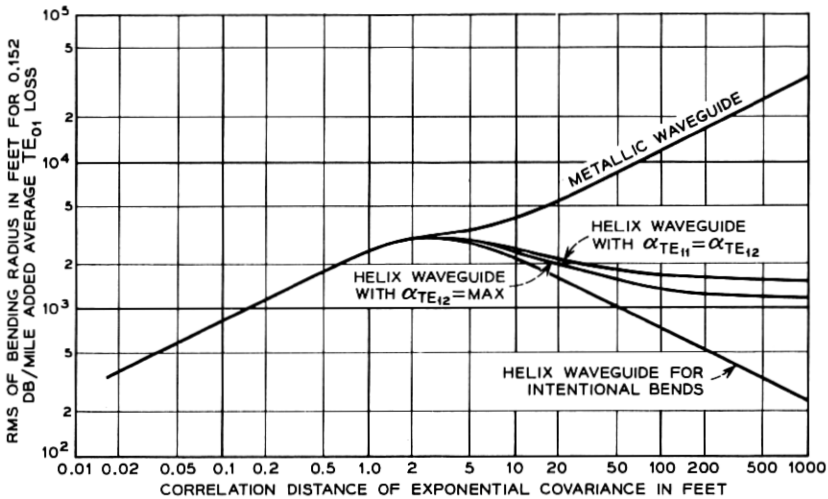

Fig. 13 — TE_{01} loss in round waveguide with random curvature; $a/\lambda = 4.7$.

with sufficiently flat spectral distribution.[6] The curves of Fig. 13 for exponential covariance demonstrate, as a typical example, over what range of correlation distance the average added loss is independent of the particular jacket structure.

Random curvature with a correlation distance smaller than 10 feet can be classified as a manufacturing imperfection. After all, the individual pipe sections which make up the line are usually only 15 feet long. Any particular choice of wall impedance therefore does not relieve the straightness tolerances which should be met in the manufacturing process.

For correlation distances larger than 10 feet the average added loss becomes more and more dependent on the wall impedance. For a specified average loss helix waveguide with infinite wall impedance — for intentional bends — may be bent most strongly. But even a helix waveguide designed optimally as a mode filter — $\alpha_{TE_{11}} = \alpha_{TE_{12}}$ or $\alpha_{TE_{12}} =$ maximum — may be bent much more than plain metallic waveguide.

Random curvature with a correlation distance larger than 10 feet may be classified as a laying imperfection. Its spectral distribution contains mainly mechanical frequencies which correspond to sine waves of 10 feet and more. Such curvature distribution arises from following right of ways or the contour of the landscape or just from not installing the pipe very carefully.

The curves in Fig. 13 have been drawn for a specified average loss. For very large correlation distance they approach asymptotically a constant value. This value corresponds to the normal circular electric mode in the particular helix waveguide with constant curvature. Helix waveguide for intentional bends, since with $Z = \infty$ it is assumed to be lossless, within the limits of the present calculation, may have an arbitrarily small radius of curvature. Uniform curvature causes no loss in this lossless structure. The curve for metallic waveguide goes to infinity. The circular electric mode is not a normal mode of the curved metallic guide.

5.3 *Random Ellipticity*

Wave propagation in elliptical helix waveguide is analyzed in a similar manner to propagation in curved helix waveguide.

Instead of (21) the covariance of the cross sectional deformation

$$\sigma(u) = \langle \delta(z)\delta(z + u) \rangle$$

is introduced and, for $(c_n a/\delta_p)^2 = P_n + jQ_n$, the coupling coefficients

$c_n = c_{n1}$ from (13) of a deformed helix waveguide are substituted. Then the average added TE_{01} loss is given by (25), and for an exponential covariance by (27).

Equation (27) has been evaluated for elliptical deformations of the same waveguides which were analyzed for random curvature before. The result is shown in Fig. 14. The rms of elliptical diameter differences $4(\sqrt{\langle\delta_1^2\rangle})a$, which would add 10 per cent of the TE_{01} loss in a perfect copper pipe to the average TE_{01} loss in the respective waveguide is plotted over the correlation distance L_0. Coupling to all modes which are propagating in the metallic waveguide has been taken into account. For $a/\lambda = 4.70$ there are 17 modes of azimuthal order $p = 2$ propagating. Contributions from higher-order modes are small enough to be neglected.

When the correlation distance is smaller than one foot the average loss is independent of the wall impedance. For larger values of correlation distance the average loss will depend on the wall impedance, but this is hardly of any practical significance. Ellipticity is a typical manufacturing imperfection, and will always have a small correlation distance. For all practical purposes, cross-sectional tolerances in helix

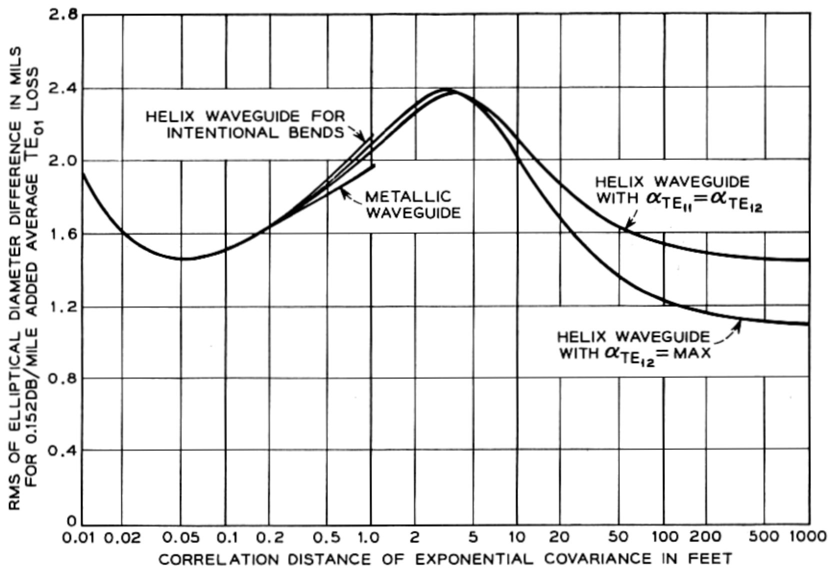

Fig. 14 — TE_{01} loss in round waveguide with random ellipticity; $a/\lambda = 4.7$.

waveguide are independent of the wall impedance and the same as in metallic waveguide.

The curves in Fig. 14 have been drawn for a specified average loss. For very large correlation distance they approach asymptotically a constant value, which corresponds to the normal circular electric mode in the particular helix waveguide with uniform ellipticity.

Metallic waveguide ($Z = 0$) and helix waveguide ($Z = \infty$) — since they are assumed to be lossless — have curves which have a never-leveling slope. Uniform ellipticity causes no loss in these lossless structures.

VI. CONCLUSION

The characteristics of normal modes in helix waveguide can be represented as a function of the wall impedance Z. The propagation constant γ is a multivalued function of the wall impedance, with each value corresponding to a normal mode. But for a specified order of azimuthal dependence the wall impedance is a single-valued function of the propagation constant. The most suitable representation of propagation characteristics of modes in helix waveguide is therefore of contour lines of Z in the γ-plane.

Appropriate branch cuts make γ a single-valued function of Z and lead to a unique mode definition: Any mode of helix waveguide is identified by the mode of metallic waveguide into which it degenerates when the wall impedance phase is kept constant and its amplitude made zero.

The attenuation constant of all TE_{pn} modes with $n \neq 1$ is limited. The attenuation constant of any other mode in helix waveguide can be made arbitrarily high with a proper choice of wall impedance.

Helix waveguide for mode filters should be designed between two extreme rules. One makes the TE_{12} attenuation an absolute maximum and leads to low TE_{11} loss; the other makes TE_{12} and TE_{11} attenuation equal and as high as possible.

Mode conversion between circular electric and other modes in curved or deformed helix waveguide can be calculated from the propagation constants and coupling coefficients of the coupled modes. For random imperfections the added average TE_{01} loss is independent of the wall impedance as long as the correlation distance is small. Manufacturing tolerances for helix waveguide are therefore independent of the particular design.

Laying tolerances produce random curvature of large correlation distance. They depend strongly on the wall impedance. An infinite wall impedance minimizes the average TE_{01} loss in helix waveguide curved randomly in this manner.

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