Relation Between Surface Concentration and Average Conductivity in Diffused Layers in Germanium

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In this paper an expression is derived for calculating the average conductivity of a diffused layer in semiconductor material as a function of the surface concentration of the diffused impurity and the background impurity concentration. Curves are presented depicting the relationship among these parameters for the case of germanium. Included are curves for both diffused impurity types for the complementary error function, gaussian, exponential and linear impurity distributions.

I. INTRODUCTION

In the design of semiconductor devices in which junctions are produced by solid state diffusion of impurities, it is of great value to know the relationships which exist between the surface concentration of the diffused impurity, C_0, the background impurity concentration, C_B, and the average conductivity of the diffused layer, $\bar{\sigma}$. These relationships can be calculated from a knowledge of the resistivity as a function of impurity concentration for material uniformly doped with a single impurity. Such calculations are presented in this paper.

II. DERIVATION OF THE AVERAGE CONDUCTIVITY EXPRESSION

For convenience, assume initially that impurity atoms are 100 per cent ionized. Therefore, the conductivity at a point in a diffused layer in semiconductor material can be given by

$$\sigma = q\mu\,(C - C_B), \tag{1}$$

where

q = electronic charge,
C = diffused impurity concentration,

C_B = background impurity concentration, and
μ = majority carrier mobility.

This expression is valid for $(C - C_B) \gg n_i$ so that minority carrier concentration is negligible. Also, for values of $C_B > 10^{14}$ mobility is primarily a function of the total number of ionized impurities present. If it is assumed that both ionized impurity types scatter a majority carrier identically, then the mobility in (1) may be considered to be a function of $(C + C_B)$.

The conductivity of material doped with a single impurity can be expressed, again assuming 100 per cent ionization of impurities, as

$$\sigma^* = q\mu N, \tag{2}$$

where N is the impurity concentration. Rewriting (1) as

$$\sigma = q\mu(C + C_B)\left(\frac{C - C_B}{C + C_B}\right) \tag{3}$$

and substituting (2) with $N = (C + C_B)$, results in

$$\sigma = \sigma^*\left(\frac{C - C_B}{C + C_B}\right). \tag{4}$$

A log-log plot of the resistivity of single impurity doped material as a function of the impurity concentration can be approximated by a set of intersecting straight lines each having an equation of the form

$$\rho = \frac{1}{\sigma^*} = \frac{1}{B} N^{-\alpha} \tag{5}$$

each of which is valid over a certain range of N. Substituting (5) in (4), again with $N = (C + C_B)$, one has

$$\sigma = B(C + C_B)^{\alpha}\left(\frac{C - C_B}{C + C_B}\right) \tag{6a}$$

$$= B(C + C_B)^{\alpha-1}(C - C_B). \tag{6b}$$

The average conductivity of a diffused layer may be obtained by integrating (6b) over values of x from the surface $(x = 0)$ to the junction $(x = x_j)$ and dividing by the junction depth, x_j. Thus,

$$\bar{\sigma} = \frac{1}{x_j}\int_0^{x_j} B[C(C_0, x) + C_B]^{\alpha-1}[C(C_0, x) - C_B]\,dx. \tag{7}$$

The values assigned to B and α at any point on the interval are determined by the value of $[C(C_0, x) + C_B]$ at that point. Equation (7) generally requires numerical integration, using experimental values for B and α.

III. APPROXIMATIONS TO RESISTIVITY CURVE

Fig. 1 shows the variation of resistivity, ρ, of single-impurity doped germanium as a function of the impurity concentration, N. Points in the range of $10^{14} \leqq N \leqq 2 \times 10^{16}$ for n-type material and in the range of $10^{14} \leqq N \leqq 6 \times 10^{16}$ for p-type material were taken from Prince.[1] Points in the range of $2 \times 10^{16} \leqq N \leqq 10^{20}$ for n-type material and $6 \times 10^{16} \leqq N \leqq 10^{20}$ for p-type material were taken from Hall effect measurements of Tyler and Soltys.[2] Hall effect measurements give the resistivity as a function of carrier concentration. However, direct measurements of resistivity as a function of impurity concentration by Trumbore and Tartaglia[3] for p-type material agree with the results of Tyler and Soltys, thus justifying the assumptions made, at least for the case of p-type material. Five straight-line approximations were made to

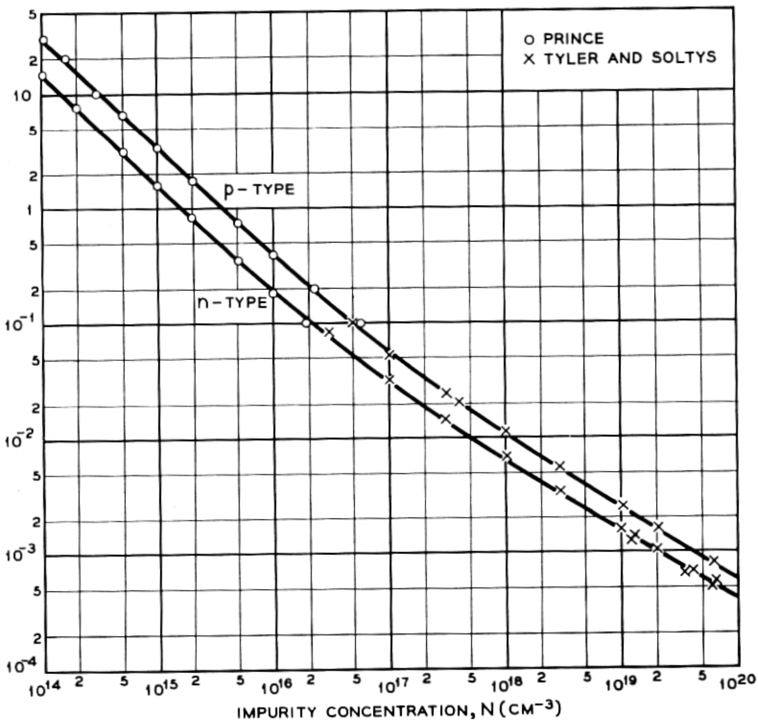

Fig. 1 — ρ vs. N for germanium at 300°K.

Table I — Constants for Approximations to ρ vs. N Curve

$$\rho = \frac{1}{B} N^{-\alpha}$$

Range	B	α
n-type		
$10^{14} \leqq N \leqq 10^{15}$	2.74×10^{-15}	0.957
$10^{15} \leqq N \leqq 10^{16}$	1.22×10^{-14}	0.914
$10^{16} \leqq N \leqq 10^{17}$	5.05×10^{-13}	0.813
$10^{17} \leqq N \leqq 10^{18}$	1.70×10^{-10}	0.664
$10^{18} \leqq N \leqq 10^{20}$	1.74×10^{-9}	0.608
p-type		
$10^{14} \leqq N \leqq 10^{16}$	1.61×10^{-15}	0.950
$10^{16} \leqq N \leqq 10^{17}$	6.28×10^{-14}	0.851
$10^{17} \leqq N \leqq 10^{18}$	1.74×10^{-11}	0.707
$10^{18} \leqq N \leqq 10^{19}$	1.61×10^{-10}	0.653
$10^{19} \leqq N \leqq 10^{20}$	1.11×10^{-9}	0.609

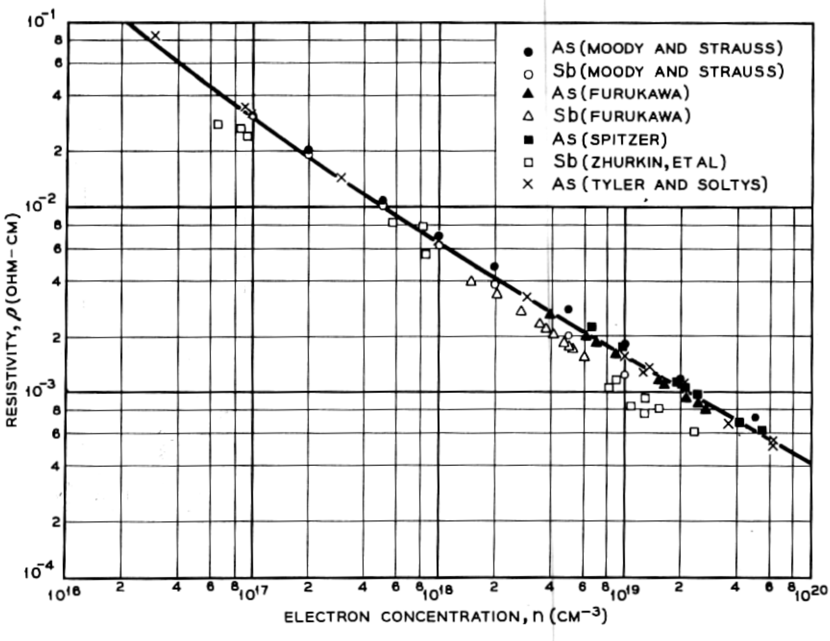

Fig. 2 — ρ vs. n for germanium at 300°K, from Hall effect measurements.

each curve giving equations of the form of (5). Values of B and α and the range of validity of each set of values are shown in Table I.

Fig. 2 shows data of resistivity as a function of electron concentration for n-type germanium as reported by Tyler and Soltys,[2] Moody and Strauss,[4] Furukawa,[5] Zhurkin et al.[6] and Spitzer.[7] Also shown in Fig. 2 is a portion of the n-type curve from Fig. 1. As can be seen, this curve represents a reasonable average of the arsenic data, for which case the present calculations are intended.

IV. RESULTS

Equation (7) was evaluated on the IBM 704 computer for various impurity distributions, and the results were checked by hand calculation of several points. Seven values of background concentration were used, and four points per decade of surface concentration were evaluated. The results are shown graphically in Figs. 3 through 10 on pages 514 through 521 for the various distributions as follows:

1. Complementary error function, Figs. 3 and 7.
2. Gaussian, Figs. 4 and 8.
3. Exponential, Figs. 5 and 9.
4. Linear, Figs. 6 and 10.

Since the conductivity of perfectly compensated material is not zero, (7) will be in error by some small amount. However, for values of C_0 and C_B such that $(C_0 - C_B) \geqq 10\ n_i$ this error will be negligible. All values of C_0 and C_B used in these calculations fulfill this requirement.

V. ACKNOWLEDGMENTS

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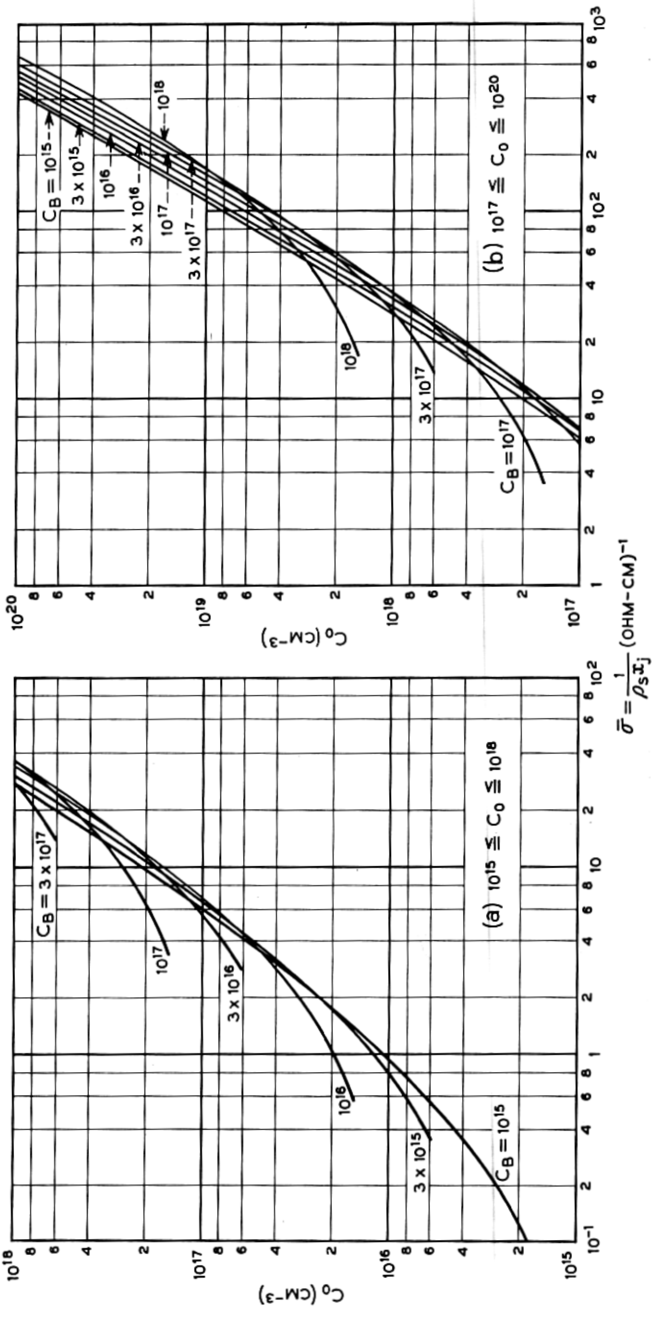

Fig. 3 — C_0 vs. $\bar{\sigma}$, p-type diffusion, ERFC distribution.

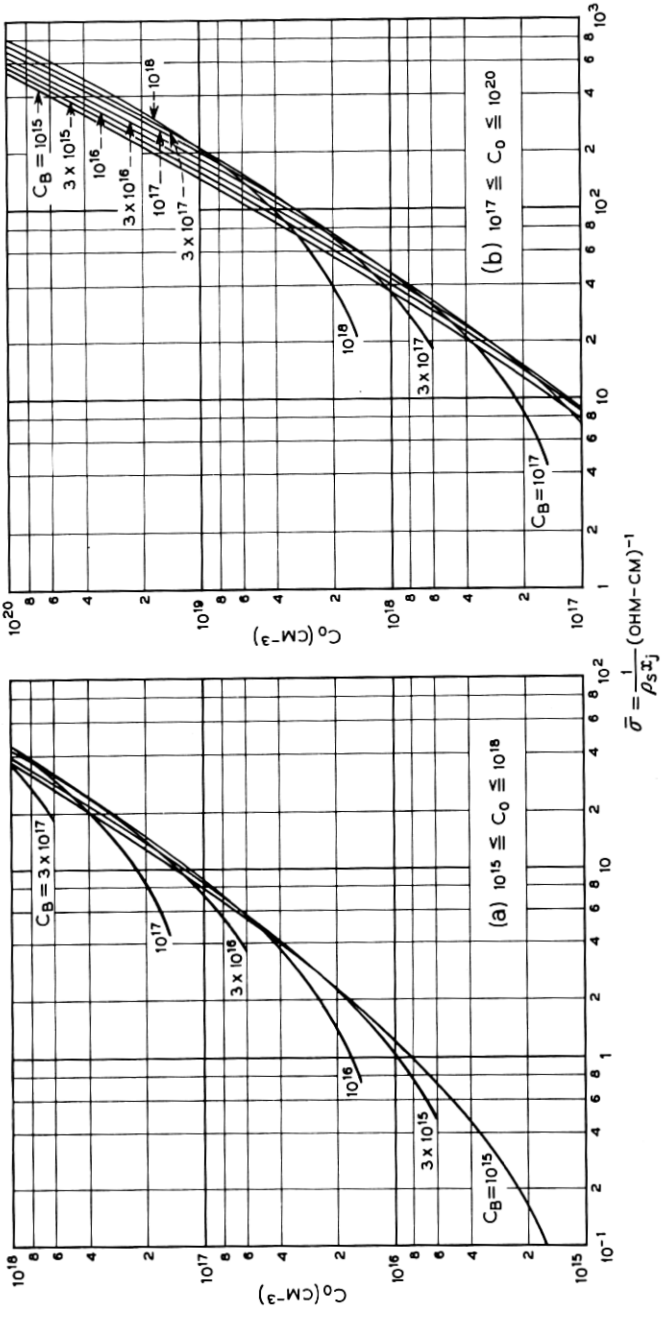

Fig. 4 — C_0 vs. $\bar{\sigma}$, p-type diffusion, gaussian distribution.

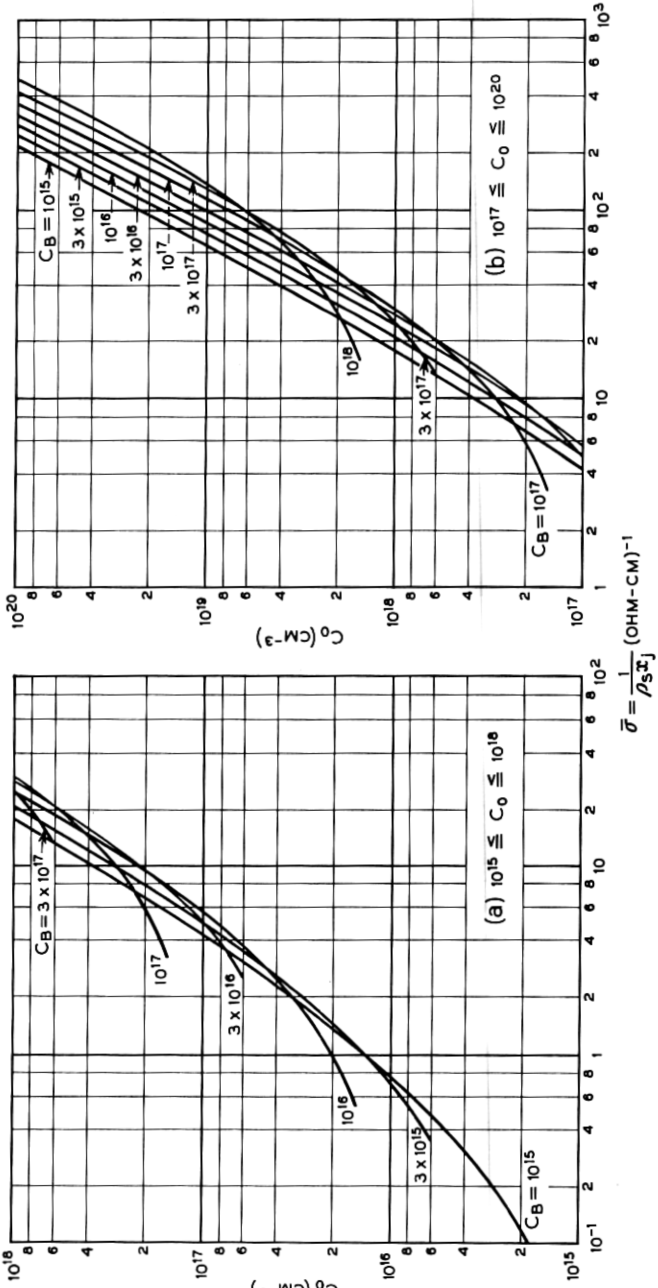

Fig. 5 — C_0 vs. $\bar{\sigma}$, p-type diffusion, exponential distribution.

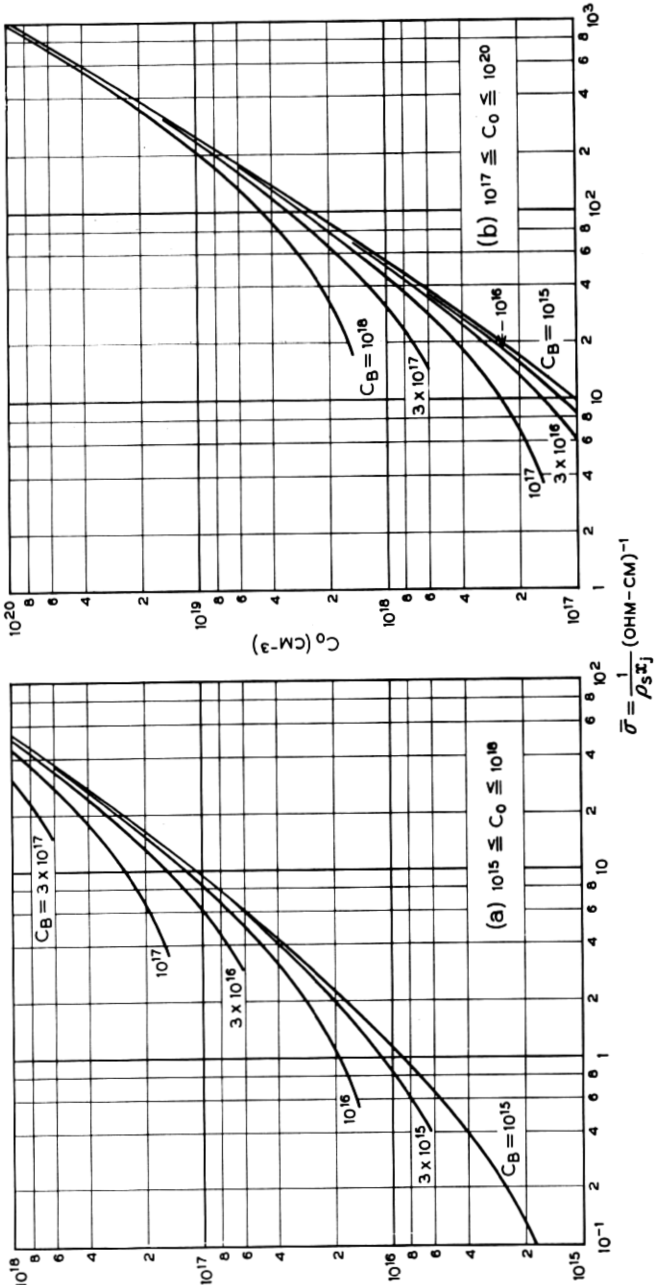

Fig. 6 — C_0 vs. $\bar{\sigma}$, p-type diffusion, linear distribution.

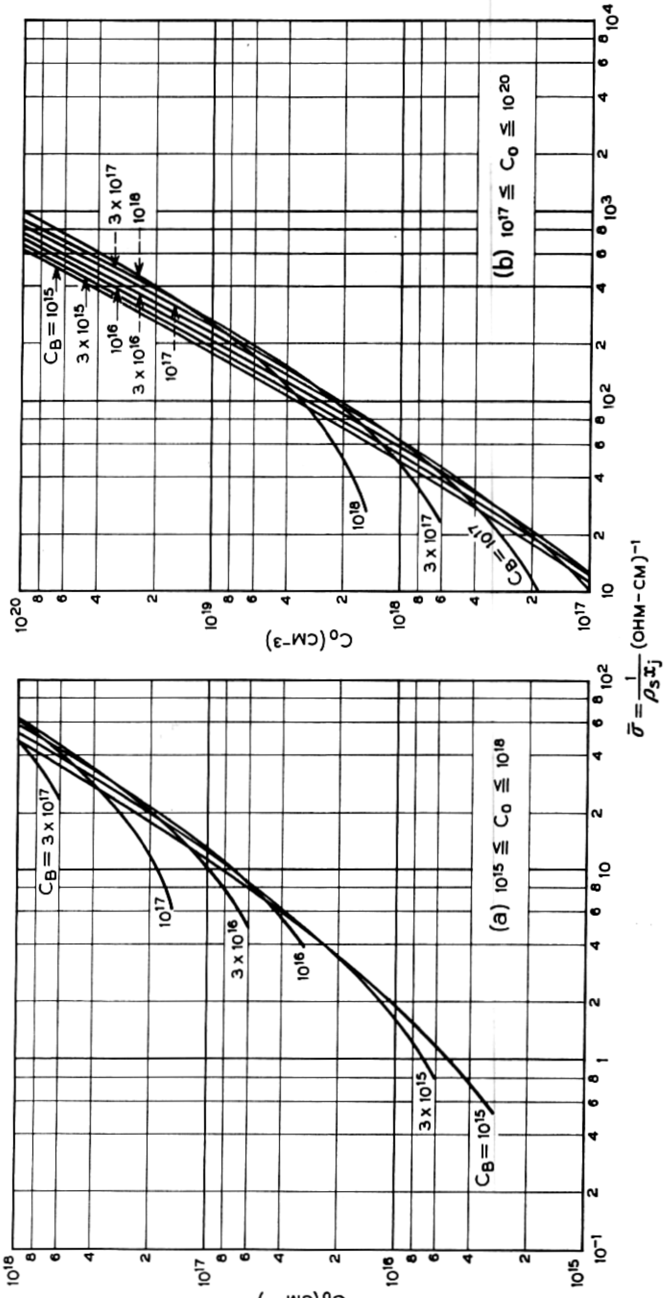

Fig. 7 — C_0 vs. $\bar{\sigma}$, n-type diffusion, ERFC distribution.

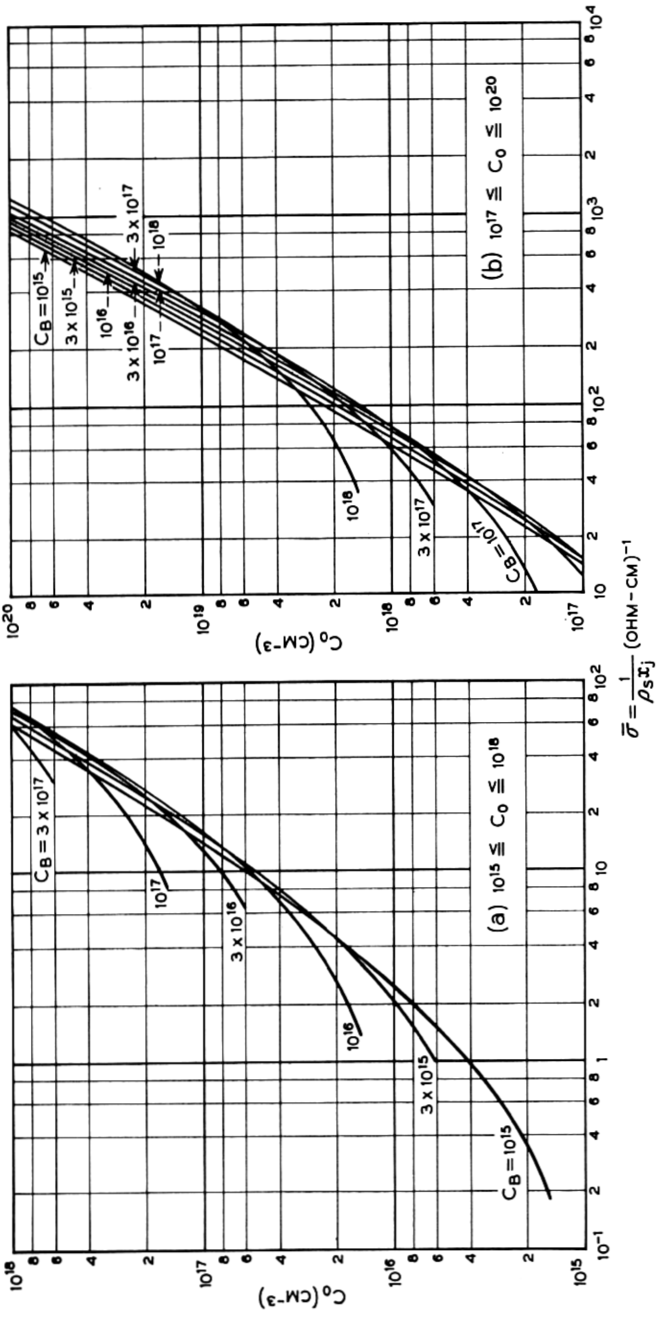

Fig. 8 — C_0 vs. $\bar{\sigma}$, n-type diffusion, gaussian distribution.

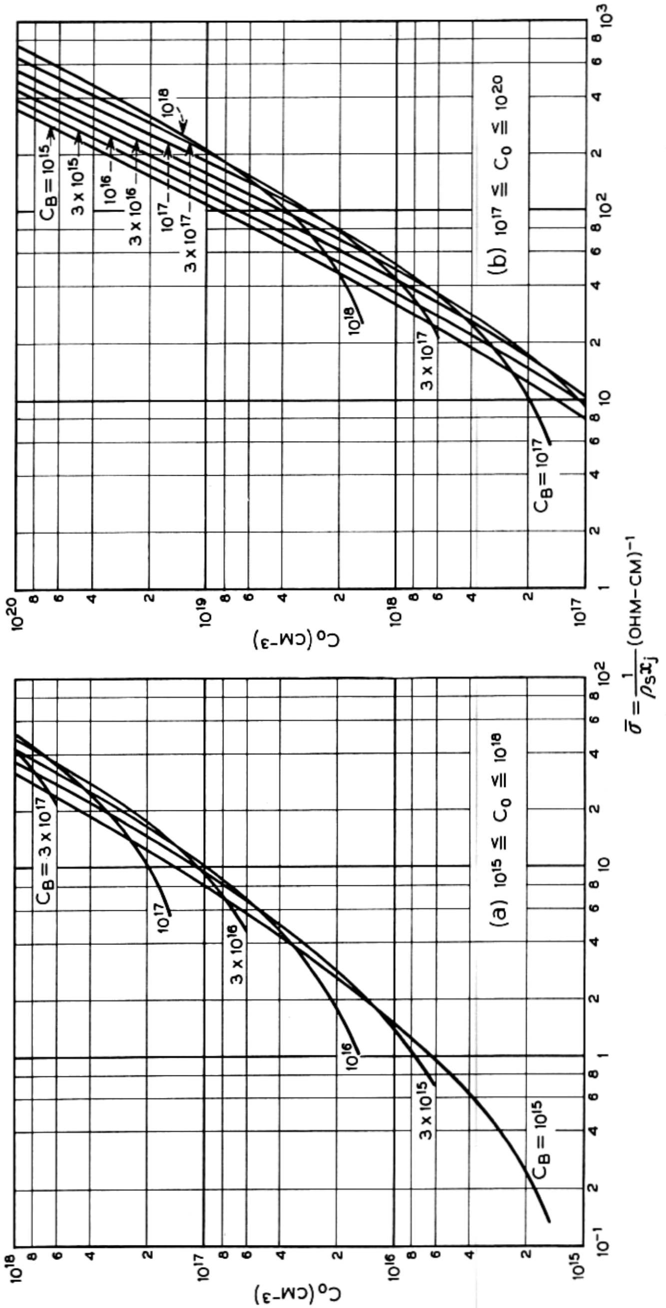

Fig. 9 — C_0 vs. $\bar{\sigma}$, n-type diffusion, exponential distribution.

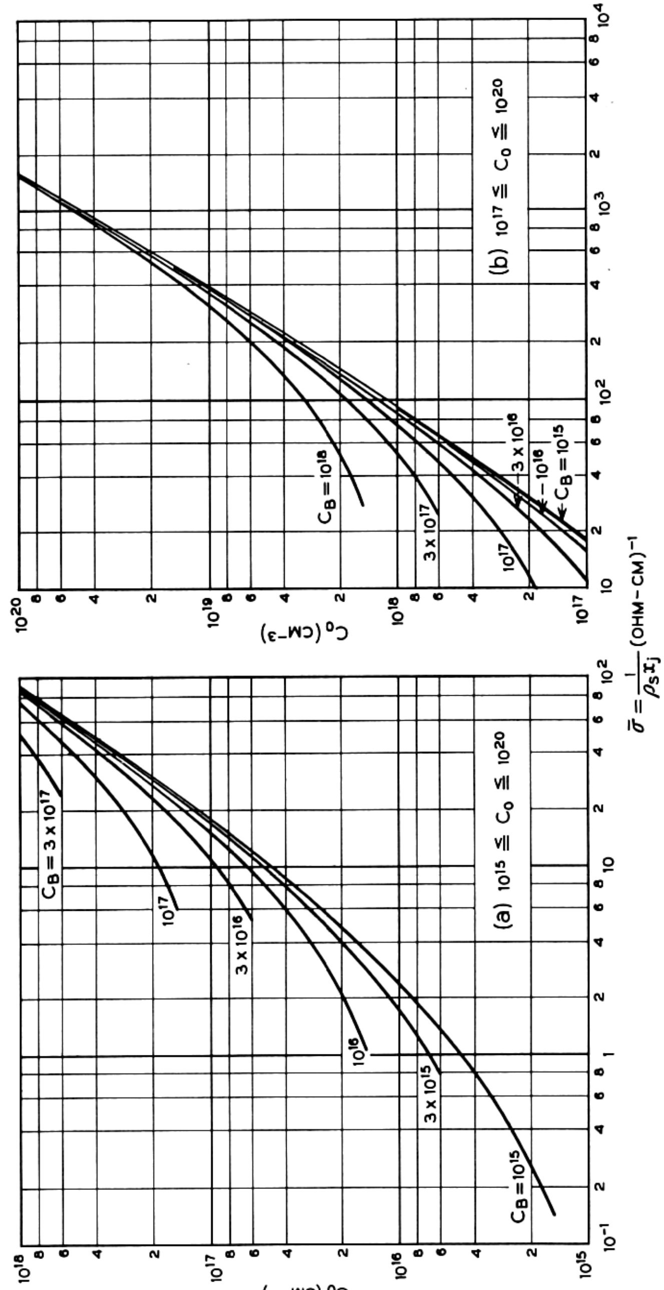

Fig. 10 — C_0 vs. $\bar{\sigma}$, n-type diffusion, linear distribution.